On Connections between Forcing Relations

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Forcing was first introduced by Paul J. Cohen in his work on the independence of the Continuum Hypothesis, see [1] and [3]. Inspired by the ideas of Cohen, other formulations of forcing appeared using Model Theory [7], Boolean-valued Models [2], and Topos Theory [4]. There is a well-known claim that these three approaches are the *same*, at least at the level of their *mathematical content*[1]. In this talk, we will present two results not found in the literature toward establishing connections between these versions of forcing.

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[1] As Mac Lane and Moerdijk mention in [4] page 283.