

## Review Article

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# Review: Rheology concepts applied to geotechnical engineering

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**Abstract:** The effect of time on soil properties, noticeable in many earthworks, is recognized by geotechnicians. For example, secondary compression and aging pre-consolidation are considered in geotechnical design, and strain rate is standardized in geotechnical laboratory and field tests. Elastic-plastic models, from rigid-perfect plastic to Modified Cam Clay, which do not consider the effects of time, solve most geotechnical problems. However, solutions for prolonged settlements, landslides, debris flow and mudflow could profit from a deeper understanding of rheological models. In fact, rheological concepts, despite not always clearly stated, have been used to address some of these problems, and may also be important for using new materials in geotechnical practice (tailings, sludge, soil-polymer mixtures and other materials with water content higher than the liquid limit). This paper introduces basic concepts of rheology for geotechnicians, specially highlighting viscoelasticity under simple shear stress, which explains with reasonable accuracy well known phenomena dependent on time in soils. The objective is to bring geotechnicians to rheology and show another important tool to access geotechnical problems. On the other hand, a brief explanation of geotechnical tests is presented for rheologists not acquainted with geotechnical engineering. Geotechnical tests procedures are discussed in the light of rheology concepts, terminology is clarified, examples of application of rheology in geotechnics are presented, and determination of soil rheological parameters by traditional geotechnical tests as well as by tests on concrete is commented.

**Keywords:** soil rheology, viscoelasticity, geotechnical engineering, time-dependent phenomena, creep

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## 1 Introduction

The term Rheology, from the Greek “rheos”- flow and “logos” - study, was presented in 1929 by Professor Eugene Cook Bingham and defined as “*the study of deformation and flux of matter*” [1]. Rheology, as a Science, originated from the observation of real materials, which considerably differ from ideal solids (elastic) and ideal liquids (viscous) as proposed, respectively, by the Hooke and Newton models [2].

Rheology is an important topic of scientific studies, industrial applications and engineering materials, including metals and alloys, plastics, ceramics, composites, concrete, paints, inks, paper, cosmetics, food, pharmaceuticals, agrochemicals, liquid detergents, glass, oils, lubricants, greases, emulsions (liquid/liquid), suspensions (solid/liquid) [1–3], biochemical materials such as wastewater sludge, sewage sludge and water treatment sludge [4–7], soils [8–14], among others.

Rheology concepts are important to develop technological processes (e.g., cold punching and shaping), to understand the flow of different materials, such as mud stream, coal, minerals and pulps [2], to characterize materials properties, and to produce new models describing the behavior of real materials [15].

Soil rheology can be used to evaluate the time-dependent changes in the stress-strain state of soils, mainly when soil behavior fails to fit the traditional elasticity and plasticity models [15]. On the Third International Conference on Soil Mechanics and Foundation Engineering in 1953, Zurich, soil rheology was recognized as an in-

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dependent branch of soil mechanics [16], and henceforth research applied to soft soils, landslides, debris flows and mudflow has been developed [17–25]. Moreover, geotechnical engineers are nowadays concerned with waste reuse, which introduces new materials to the scope of soil mechanics, such as sewage, wastewater and water treatment sludge, mining tailings (pulps), oil drilling fluid, and polymeric suspensions for excavation stabilization, and brings forward a new interest in soil rheology.

This paper contributes with an overview of the concepts of rheology to make literature more easily understandable and available to geotechnicians. Viscoelasticity is especially highlighted, which explains with reasonable accuracy well known phenomena dependent on time in soils: secondary compression, aging pre-consolidation pressure, flow, creep and thixotropy. Terminology is clarified and some examples of the application of rheological models in geotechnics are presented: prediction of instability propagation, demonstration of geotechnical tests procedures in the light of rheology concepts, and determination of rheological parameters by traditional geotechnical tests and by tests on the rheology of concrete.

## 2 Fundamental concepts

The sum of solicitations applied to a real body may be divided into normal stresses that generate volumetric strain, and shear stresses that generate distortion (shape changes). In geotechnical engineering, symbols for normal stress, volumetric strain, shear stress and shear strain are, respectively,  $\sigma$ ,  $\epsilon$ ,  $\tau$  and  $\gamma$ . Despite the symbol for shear stress in rheology generally being  $\sigma$  (Appendix A), in this paper, we will use  $\tau$  for shear stress.

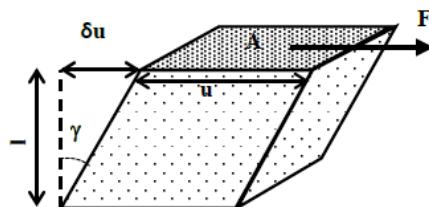


Figure 1: Body deformation by simple shear. Modified from [3].

Shear stress ( $\tau$ ) is defined as tangential force ( $F$ ) per unit of area ( $A$ ) as shown in Figure 1 and Equation 1. Shear strain ( $\gamma$ ) is defined as the angle of the displacement related to a reference layer *i.e.*, the tangential displacement ( $\delta u$ ) per distance ( $l$ ) between two parallel plates (Equa-

tion 2). When  $\gamma$  is very small,  $\tan \gamma$  is approximately equal to  $\gamma$ .

$$\tau = \frac{F}{A} \quad (1)$$

$$\gamma = \frac{\delta u}{l} \quad (2)$$

### 2.1 Hooke elasticity law

In 1678, Robert Hooke proposed the True Theory of Elasticity which established that “*the power of any spring is in the same proportion with the tension thereof*” [1]. In other words, the force is proportional to the displacement. This idea, applied to an elastic solid body, is expressed by the statement that stress is proportional to strain.

The curve shear stress versus shear strain (Figure 2a) is a straight line and the slope is equal to the shear modulus,  $G$  ( $G'$  in rheology, Appendix A), as follows in Equation 3. For Hookean materials,  $G$  is independent of the applied stress and of the strain, *i.e.* is constant and an intrinsic property of the material.

$$\tau = G\gamma \quad [\tau \text{ in Pa, } G \text{ in Pa, } \gamma \text{ dimensionless}] \quad (3)$$

For ideal elastic solids, deformation is immediate and independent of time.

### 2.2 Newton viscosity law

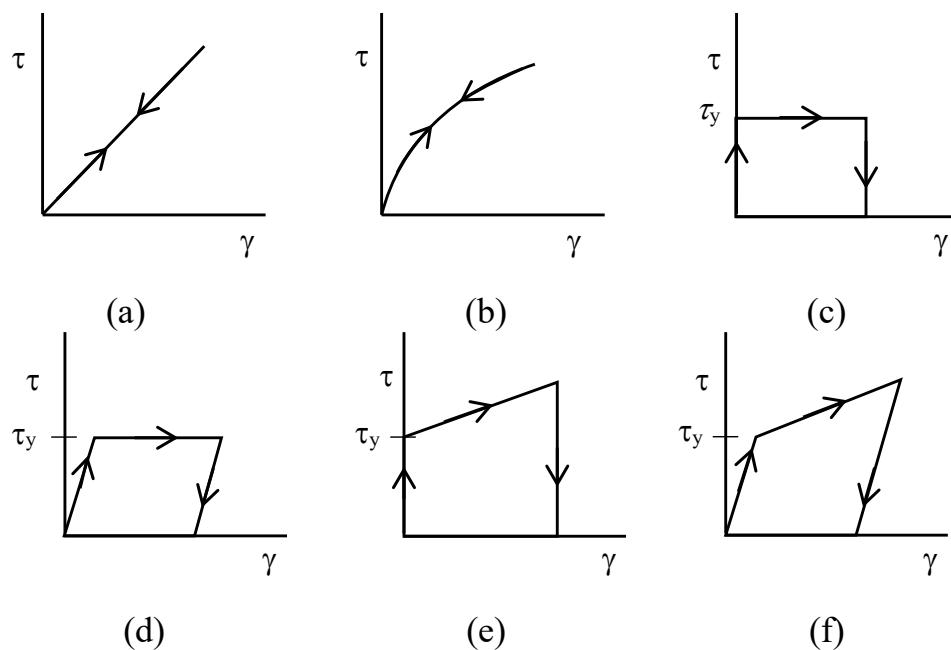
In Principia published in 1687, Isaac Newton defined the resistance of a fluid (viscosity) as “*the resistance which arises from the lack of slipperiness originating in a fluid, other things being equal, is proportional to the velocity by which the parts of the fluid are being separated from each other*” [1]. This lack of slipperiness is called viscosity  $\mu$ , ( $\eta$  in rheology, Appendix A), the result of “internal friction” which measures the “resistance to flow” [1].

By definition, fluids deform (flow) continuously when submitted to shear stress. For fluids, however, the applied stress cannot be correlated with a unique value of deformation, since shear strain continues to occur along time. Therefore, the stress is correlated with the shear rate,  $\dot{\gamma}$ , which is the change of shear strain with time (Equation 4).

$$\dot{\gamma} = \frac{d\gamma}{dt} = \frac{d}{dt} \left( \frac{\delta u}{l} \right) = \frac{v}{l} \quad (4)$$

where,  $\gamma$  is the shear strain,  $t$  is the time,  $\delta u$  is the tangential displacement,  $v$  is the velocity and  $l$  is the distance between two parallel plates (Figure 1).

For ideal liquids, as proposed by Newton, the applied stress ( $\tau$ ) is proportional to the shear rate ( $\dot{\gamma}$ ), as shown in



**Figure 2:** Typical stress-strain relation for solid bodies: (a) linear elastic (Hookean solid); (b) non-linear elastic; (c) rigid-perfectly plastic; (d) elastic-perfectly plastic (elastoplastic); (e) rigid linear hardening; (f) linear elastoplastic hardening. Modified from [15, 16]

Figure 3a, and the proportionality constant is the viscosity ( $\mu$ ), as follows in Equation (5). For Newtonian fluids,  $\mu$  is independent of the shear stress and of the shear rate, *i.e.* is constant and an intrinsic property of the material.

$$\tau = \mu \dot{\gamma} \quad [\tau \text{ in Pa } \mu \text{ in Pa s, } \dot{\gamma} \text{ in s}^{-1}] \quad (5)$$

For ideal Newtonian fluids, deformation is continuous along time.

### 2.3 Historical notes of elasticity, plasticity and viscosity

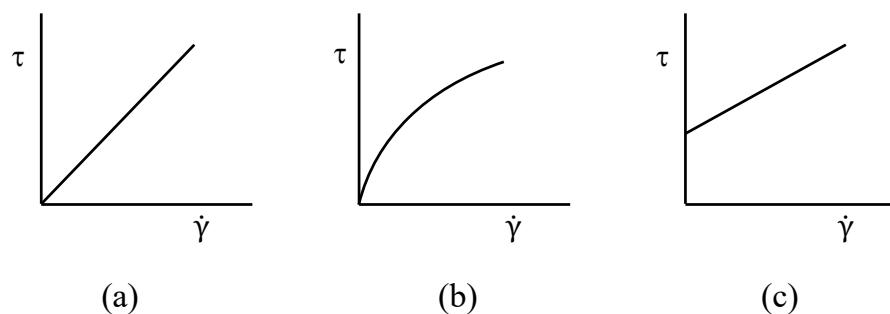
Since Hooke (Elasticity Law, 1678) and Newton (Viscosity Law, 1687), researchers from different knowledge areas have studied real materials, such as metals, silk threads and fluids, based on concepts of elasticity, plasticity and viscosity.

Elasticity is a property of a body to restore its shape and volume (solids) or its volume (liquids) when the external force (stress) is removed. In a perfect elastic body, the deformation is reversible, occurs immediately after applying a force and disappears as soon as the stress is removed. Elastic deformation can be linear (Hooke Law) or non-linear (Figure 2a, b). For an imperfect elastic solid subjected to a stress, part of the deformation will remain after the stress is removed [26] and is called plastic deformation.

Studying metal extrusion, in 1864 Tresca issued “*On the flow of a solid body subjected to high pressure*”, stating that metal flows under a certain shear stress (the “*shearing flow stress*”), and determined values of this parameter for many metals. Continuing Tresca’s studies, Saint-Venant published a pioneer paper on elastic-plastic analysis in 1871 [27].

Plasticity is a property of a body to change its shape irreversibly without failure when subjected to an external force; a minimum stress, called yield stress,  $\tau_y$  ( $\sigma_y$  in rheology, Appendix A), must be reached for the plastic deformation to occur, and when the stress is removed, at least part of the deformation is not restored. A Saint-Venant rigid-plastic body does not deform when subjected to a shearing stress lower than the yield limit (Figure 2c). The Tresca-Saint Venant condition of a perfectly plastic body assumes that a material in plastic state is incompressible and stress necessary to cause deformation of the solid body is constant (Figure 2c, d). If the stress is not constant and depends on the plastic deformation, the body presents hardening (Figure 2e, f). Plastic deformations are immediate, occur simultaneously with the application of stress and are independent of shear rate.

Other plastic models have henceforth been proposed, such as Van Mises, Guest, Mohr, Prandtl, among others [28]. Elastoplastic models have been developed in the context of soil mechanics, from the traditional Mohr-



**Figure 3:** Typical curves of stress as a function of shear rate for viscous bodies: (a) linear viscous (Newtonian fluid); (b) non-linear viscous; (c) “plastic Bingham” (viscoplastic). Modified from [15, 16]

Coulomb theory till Clam Clay and other critical state models [29] evolving lately to hypoplasticity [30, 31].

Viscosity is a parameter of energy dissipation, defined as the property of fluids (liquids and gases) to resist the motion of elemental particles relative to one another, also thought of as internal friction. The resistance is related to the shear rate at which the particles are displaced. Viscous flow is induced by any shear stress greater than zero and the deformation of viscous flow (Figure 3a, b) is totally irrecoverable [15].

In “*Fluidity and Plasticity*”, Bingham discussed the distinction between viscous and plastic deformation [26]. After that, the “plastic Bingham” behavior was introduced in rheology (Figure 3c), also known as viscoplastic behavior or plastic flow in geotechnics.

## 2.4 Liquid and solid models

Every science creates models to simplify the reality and to reflect the most important characteristics of a material, an object or a condition. The Newton and Hooke linear models are adequate to describe the behavior of many real materials. Water, alcohol, acetone and glycerin are Newtonian viscous liquids; metals, steel, rocks and minerals may be regarded as Hookean elastic solids for small deformations. Yet, the ideas of “liquid” and “solid” are also models, insufficient to describe many real materials that are in any state between solid and liquid.

Solids and liquids are usually recognized by their response to low stresses caused by gravitational forces over a short period of time (seconds or minutes). However, under a wide range of stresses applied during different time periods, liquid-like properties would be observed in solids and solid-like properties in liquids [1]. For example, paints are liquid but they do not flow down the walls like other liquids; concrete seems to be a rigid solid, but its shape changes as a liquid after long-term observation and un-

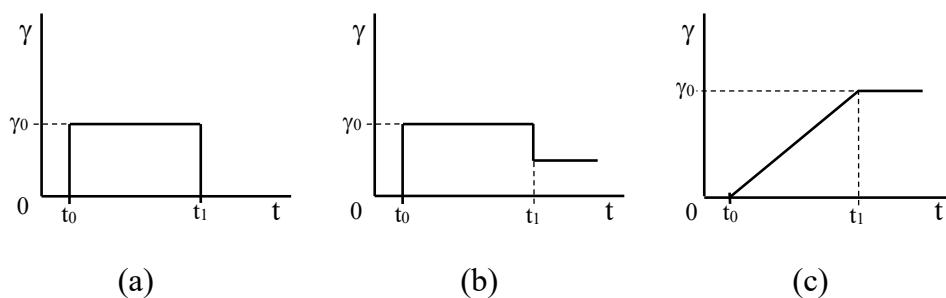
der constant solicitation; clays are considered solids, but they can be shaped and take the form of a bowl as a liquid given sufficient time [2]; soils are solids, but they can behave as liquids during a debris flow [32]; silicone assumes the recipient shape after sufficient time, but a silicone ball bounces when dropped on the floor [1].

In other words, viscous and elastic behavior may simultaneously occur for real materials at usual time scale, and the predominant behavior depends on the magnitude and duration of the applied stress or shear rate, and on time of rest. This phenomenon is called viscoelasticity.

## 3 Viscoelasticity

Since the nineteenth century, scientists have noted that the elastic response of some real materials depended on time [33]. For example, in 1847, Kohlraush observed that the stress may change with time even if under constant deformation [15]. In 1875, William Thomson (Lord Kelvin) introduced the idea of viscoelasticity of solids in the *Encyclopedia Britannica* [2]. In “*On the Dynamic Theory of Gases*”, issued in 1868, Maxwell examined the theory of stress relaxation (reduction of stress over time at constant strain) and noted there is no fundamental difference between liquids and solids. Considering that real bodies combine the properties of Hooke’s ideal elastic medium and Newton’s ideal viscous medium, Maxwell derived the rheological equation of state for a viscoelastic body [16]. In 1935, Weber noted that the elastic deformation of quartz glass and silk threads, which occurs instantaneously after the application of a tensile load, was followed by elongation, which occurs along time [15]. Other viscoelastic models besides the Maxwell model were developed to represent the many different behaviors of real materials.

Viscoelastic materials are characterized by the participation of viscous, elastic and plastic properties in different



**Figure 4:** Curves of deformation ( $\gamma$ ) versus time ( $t$ ) under constant stress: (a) elastic response; (b) plastic response; (c) viscous response. The stress is applied at  $t=t_0$  and removed at  $t=t_1$ . Modified from [3, 15]

proportions. For example, when silk, rubber or pitch (typical viscoelastic materials) are subjected to a load, an instantaneous deformation (elastic) is followed by a continuous deformation (viscous); when the load is removed, part of the deformation is recovered instantly, another part is recovered with time and, in some materials, a permanent deformation remains [33]. Elastic (reversible) and viscous (irreversible) deformations occur for any stress level different from zero, whereas the yield stress must be reached for the occurrence of plastic (irreversible) deformation.

The response of elastic, plastic and viscous behaviors when a body is subjected to a constant stress at  $t=t_0$  and removed at  $t=t_1$  is shown in Figure 4. According to Figure 4a, for a perfect elastic body (Hookean), the deformation occurs instantaneously ( $\gamma_0$ ) after the application of the stress and does not change if the stress is constant; when the stress is removed, the dimension of the body is completely restored (reversible deformation). For a plastic body, part of the deformation is not restored after the stress is removed (Figure 4b). When a perfect viscous liquid (Newtonian) undergoes stress (even if very small), flow occurs (deformation increases linearly with time); when the stress is removed, the deformation remains the same (irreversible deformation), as shown in Figure 4c.

Polymers are the most important example of viscoelasticity, because viscoelastic response is observed in ordinary time-scale (seconds, minutes, hours, decades), while for other materials this observation may require a very short time (fraction of seconds), such as water, or a very long time (years), such as metals, concrete and soils. For instance, the “pitch drop experiment” held at Trinity College in Dublin since the 1940s [34], aimed to demonstrate the high viscosity of pitch, and in 2014, more than 7 decades after the first drop, the 9th drop fell from the funnel. Based on these results, the viscosity of this pitch is estimated to be about two million times that of honey, and 20 billion times the viscosity of water.

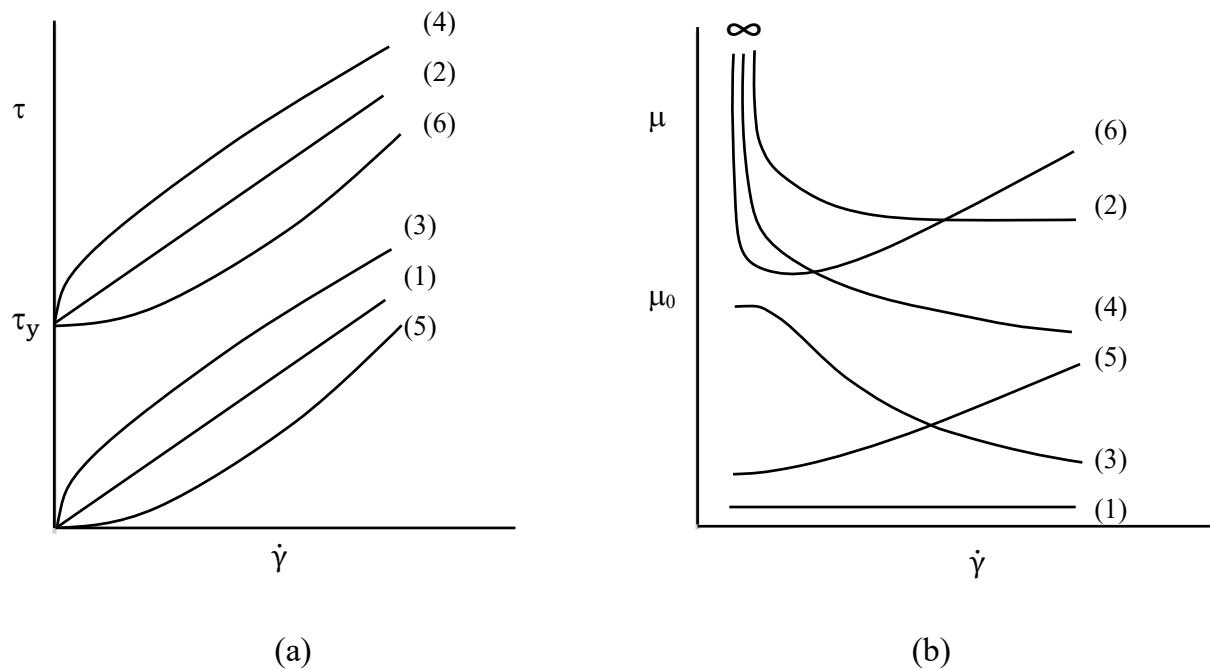
Time-scale effects are due to rearrangements of the inherent structure of the materials, which are manifested as changes in properties, such as viscosity and shear modulus. In the Newtonian systems, viscosity is independent of shear rate; while in the non-Newtonian systems, there is no linearity between shear rate and shear stress, and viscosity is not an intrinsic property of the fluid. For real materials, viscosity may change with stress and shear rate, and these changes may occur instantaneously or throughout a long period of time. Hence, viscosity varies with shear rate, but it may vary or not with time. The rheological behavior of real materials may be divided into two classes: (a) dependent on shear rate but without alteration along time (time-independent behavior); or (b) dependent on shear rate and time (time-dependent behavior).

Therefore, the viscoelastic response depends on the stress, the shear rate, the time of observation (or experiment) and the partial contributions of the elastic, plastic and viscous properties of real materials.

## 4 Time-independent behavior

Figure 5 shows different curves of stress ( $\tau$ ) and viscosity ( $\mu$ ) as a function of shear rate ( $\dot{\gamma}$ ), called **flow curves**, for time-independent behavior.

For **Newtonian systems**, the shear rate is proportional to the shear stress and the viscosity is the constant of proportionality, independent of the applied shear rate. **Bingham (plastic) systems** present similar behavior to the Newtonian fluids; however, a minimum stress, called yield stress ( $\tau_y$ ), must be reached before flow occurs. When a fluid is subjected to a shear stress lower than the yield stress, viscosity tends to infinite and the fluid behaves as a solid. For **shear thinning systems (pseudoplastic)**, the shear rate is not linearly proportional to the shear stress. There is no yield stress, yet there is a limit viscosity ( $\mu_0$ ) for small shear rates, called “residual” viscosity. Viscos-



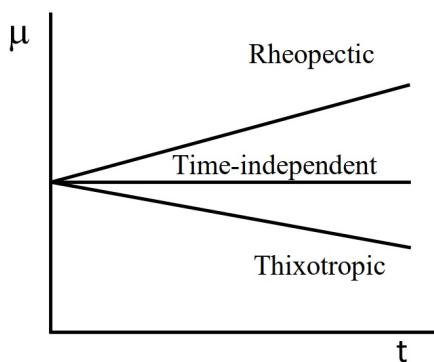
**Figure 5:** Rheological behavior of different fluids: (a) flowcurve; (b) viscosity as a function of shear rate. (1) Newtonian, (2) Bingham (Plastic), (3) Shear thinning (pseudoplastic), (4) Shear thinning with yield stress ( $\tau_y$ ), (5) Shear thickening (dilatant), (6) Shear thickening with yield stress ( $\tau_y$ ). Modified from [3]

ity decreases with the increase of shear rate. **Shear thinning with yield stress** systems present similar behavior of shear thinning fluids, but they present yield stress. For **shear thickening systems (dilatant)**, the resistance to flow (viscosity) increases with the increase of shear rate. **Shear thickening with yield stress** presents similar behavior to shear thickening fluids, yet they present yield stress.

Numerous rheological models with different degrees of complexity can be found in the literature to represent the behavior of real materials, most of them empirical (Table 1). The Herschel-Bulkley equation is reduced to the Power Model when  $\tau_\beta = 0$ , to Bingham model for  $\mu = 1$  and becomes the Newton model when  $\tau_\beta = 0$  and  $\mu = 1$ . The Power Law fits the experimental results for many non-Newtonian systems and is more versatile than the Bingham model; Casson is a semi-empirical model that fits the flow curves of many paints and printing inks; in turn, the Cross model is used to shear thinning systems, reducing to Power model when  $\mu \ll \mu_0$  and  $\mu \gg \mu_\infty$ , and to Sisko model if  $\mu \ll \mu_0$  [3].

## 5 Time-dependent behavior

Time-dependent behavior means that, under a constant shear rate or shear stress, the viscosity decreases (thixotropy) or increases (rheopecty) along time. This reversible time effect occurs because structural rearrangements in these materials require time to develop. Figure 6 presents curves of viscosity as a function of time for thixotropic, rheopectic and time-independent materials obtained by tests performed at constant shear rate. For



**Figure 6:** Typical curves of viscosity as a function of time at constant shear rate for thixotropic (positive thixotropy), rheopectic (negative thixotropy) and time-independent materials

**Table 1:** Rheological models and state equations

Model	Behavior	Equation
Newton Law	Newtonian	$\tau = \mu \dot{\gamma}$ (5)
Bingham plastic	Viscoplastic	$\tau = \tau_y + \mu \dot{\gamma}$ (6)
Power Law	Shear thinning (Pseudoplastic) ( $n < 1$ ) Shear thickening (Dilatant) ( $n > 1$ )	$\tau = \kappa \dot{\gamma}^n$ (7)
Herschel-Bulkley	Shear thinning + yield stress ( $n < 1$ ) Shear thickening + yield stress ( $n > 1$ )	$\tau = \tau_y + \kappa \dot{\gamma}^n$ (8) $\tau = \tau_y + \kappa \dot{\gamma}^n$ (9)
Casson	Viscoplastic	$\tau^{1/2} = \tau_c^{1/2} + \mu c^{1/2} \dot{\gamma}^{1/2}$ (10)
Cross	Shear thinning	$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \frac{1}{1 + K \dot{\gamma}^m}$ (11)
Carreau		$\frac{\mu - \mu_\infty}{\mu_0 - \mu_\infty} = \frac{1}{(1 + (K \dot{\gamma})^2)^{m/2}}$ (12)
Sisko		$\mu = \mu_\infty + K \dot{\gamma}^{n-1}$ (13)

$\mu$  is the viscosity,  $\dot{\gamma}$  is the shear rate,  $\tau_y$  is the yield stress,  $\kappa$  is the consistency index,  $n$  is the shear thinning index,  $\tau_c$  is the Casson yield stress calculated from the intercept of the curve  $\tau c^{1/2}$  versus  $\dot{\gamma}^{1/2}$ ;  $\mu c$  is the Casson viscosity calculated from the slope of the curve,  $\mu_0$  is the limit viscosity that occurs at low shear rates and  $\mu_\infty$  is the limit viscosity corresponding to high shear rates,  $K$  is the Cross constant parameter of time,  $m$  is the Cross constant.

time-independent materials at constant shear rate, structural changes occur instantaneously.

Thixotropy refers to a reversible isothermal decrease of the viscosity along time during shearing at constant or variable shear rate, followed by an increase of the viscosity when the shearing is removed, *i.e.* when a system is sheared during some time, the viscosity decreases (destruction of structure); however, when the system is left to rest, the viscosity is recovered. Examples of thixotropic materials are solutions of corn starch, jellies, yogurts, some clayey soils [35–37] and water treatment sludge [7, 38].

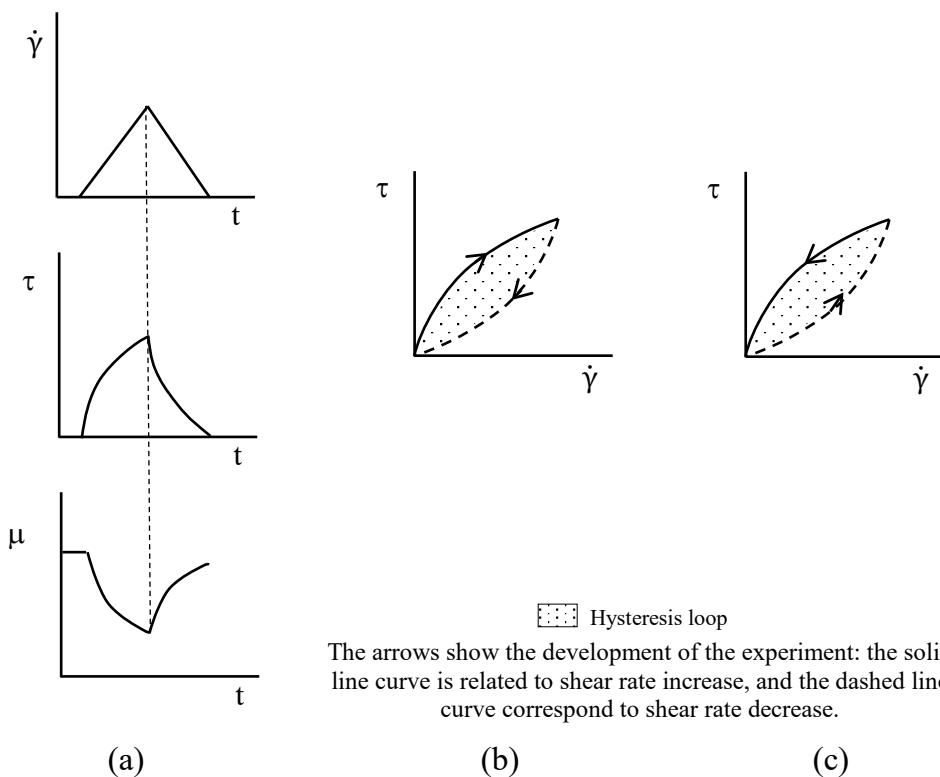
Rheopexy is the reversible increase of the viscosity along time, and can also be called negative-thixotropy or anti-thixotropy. When the system is sheared, the viscosity increases (building of structure); however, when the system is left to rest, the viscosity decreases (the initial viscosity is recovered). This concept must be applied for inert materials; viscosity of reactive materials, such as cement, lime, and plaster, increases over time independently of applied stress or shear rate, but this does not mean that they are rheoplectic materials.

Rheological tests may be performed by varying the shear rate to obtain shear stress and viscosity. Figure 7a presents one of the most common test methods: the *loop test* (the shear rate is increased continuously and linearly with time, from zero to a maximum value, and then decreased to zero in the same way). Shear stress versus shear rate response of the loop tests are presented for thixotropic (Figure 7b) and rheoplectic (Figure 7c) behaviors. Since the processes of structure rupture and rebuilding do not occur

at the same pace, curves for shear rate increase (acceleration) and decrease (deceleration) are not coincident and form a hysteresis loop.

The hysteresis area between the curves of shear stress increase and decrease represents the energy consumed in structure rupture or building. Hence, the hysteresis loop area may be a quantitative measure of thixotropic and rheoplectic properties [2, 39]. Yet, the curves of Figure 7 could also be obtained for shear thinning and shear thickening materials that are not thixotropic or rheoplectic: when shear rate is changed, stress also changes, even if viscosity does not alter with time. Therefore, only using the hysteresis loop to characterize thixotropy may lead to erroneous conclusions. Also, reactive materials, such as concrete or soil-cement, may present response similar to rheoplectic materials as a consequence of hardening by chemical reactions; however, the hardening is not reversible, thus discarding rheopexy.

Although thixotropy is a well-known phenomenon [1, 2, 33, 40, 41] which occurs in many real materials [7, 35, 42, 43], the terms thixotropy and rheopexy are not generally well defined and are frequently mixed with other viscoelastic effects [2]. In soil mechanics, many authors have suggested definitions to the term thixotropy, such as [36, 44, 45]. However, thixotropy has been sometimes erroneously used to describe all kinds of soil aging effects [46]. Actually, thixotropy, rheopexy, dilatancy (shear thickening), pseudoplasticity (shear thinning), creep and relaxation responses may occur associated to each other [47–49].

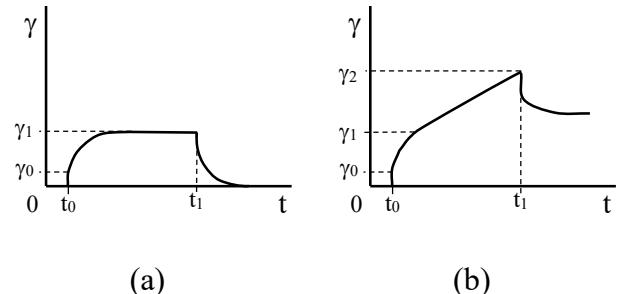


**Figure 7:** Loop test: (a) Shear rate, stress and viscosity as a function of time for an ideal (inelastic) thixotropic fluid; (b) Stress as a function of shear rate for thixotropic material; (c) Stress as a function of shear rate for rheopectic material. Modified from [1] and [3]

## 6 Creep and stress relaxation

Two important solicitation conditions for materials in general are creep and relaxation, *i.e.* constant stress and constant deformation, respectively. There are laboratorial tests applied to characterize viscoelastic behavior based on these conditions: the creep test, in which the stress is maintained constant along time and deformations are measured, and the relaxation test, in which an imposed deformation is maintained constant along time and stresses are measured.

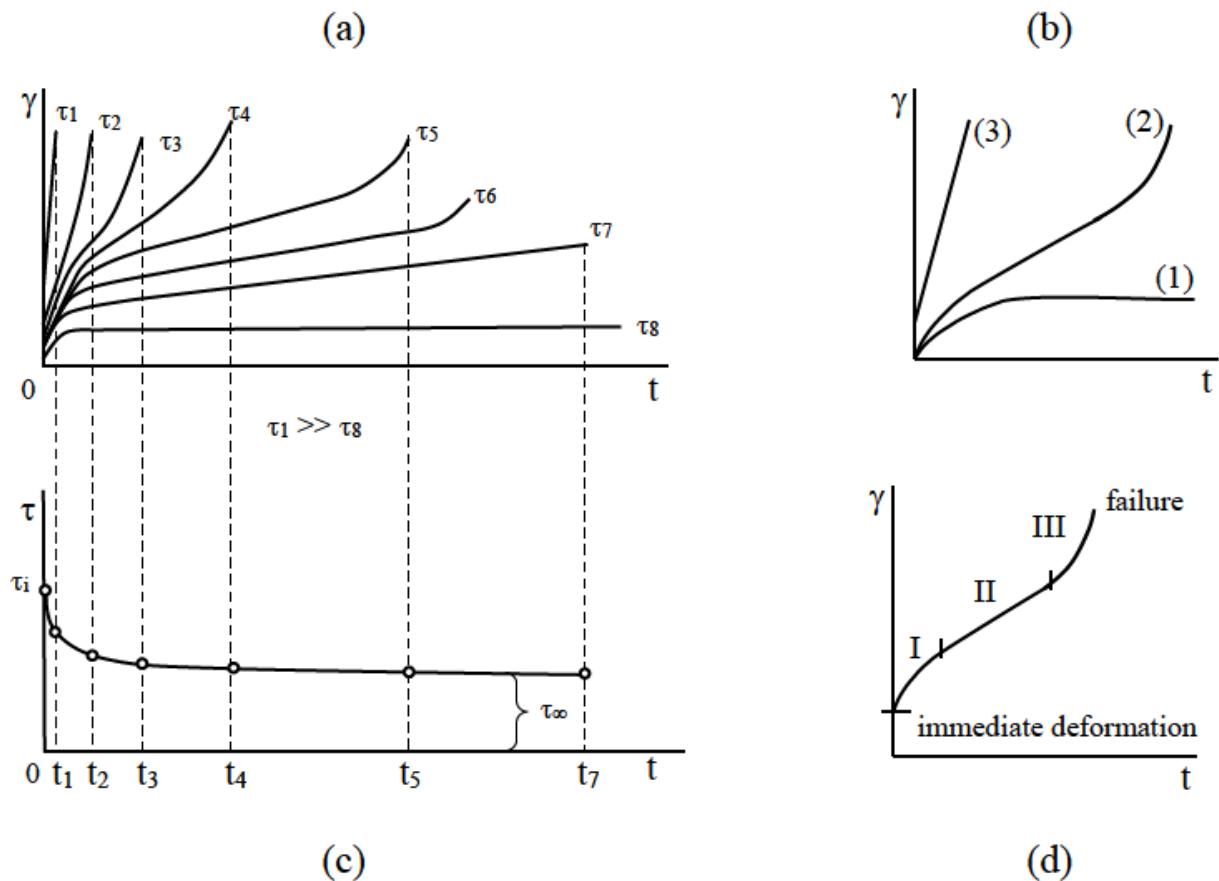
**Creep** is defined as the ***continuous, prolonged and slow deformation of a material at constant stress***. Figure 8 shows two examples among the different types of creep response for viscoelastic materials, in which a constant stress is applied at  $t=t_0$  and removed at  $t=t_1$ . In the first example, for a viscoelastic solid, after the application of the constant stress (Figure 8a), part of the deformation occurs instantaneously ( $0$  to  $\gamma_0$ ), then the deformation increases with time ( $\gamma_0$  to  $\gamma_1$ ) tending to the maximum value ( $\gamma_1$ ); when the stress is removed ( $t_1$ ), part of the deformation is restored immediately (elastic response), and part is recovered along time (creep recovery). For viscoelastic liquids (Figure 8b), part of deformation also occurs instant-



**Figure 8:** Typical creep curves of strain versus time under constant stress: (a) viscoelastic solid response; (b) viscoelastic liquid response. The stress is applied at  $t=t_0$  and removed at  $t=t_1$ . Modified from [15, 16]

aneously ( $0$  to  $\gamma_0$ ), then the deformation continuously increases along time ( $\gamma_0$  to  $\gamma_2$ ); when the stress is removed ( $t_1$ ), the deformation is partially restored; only the elastic deformation is recovered.

Studying silk threads between 1835 and 1841, Weber observed the complete recovery of creep after stress relief [16], as the behavior depicted in Figure 8a. However, creep deformation is not always recovered. [50] was the first to investigate the creep of clayey soils and called attention to the importance of their prolonged deformation.



**Figure 9:** (a) Family of creep curves for a given soil at different constant loads: the higher the load, the faster the failure; (b) Creep curves for different soils at a given load: (1) attenuating or fading creep, (2) accelerated or non-attenuating creep, (3) rapid failure; (c) Long-term strength curve related to creep: where  $\tau_\infty$  is the ultimate long-term strength and  $\tau_i$  is the instantaneous strength; (d) Stages of creep of accelerated or non-attenuating creep: I – attenuating creep, II – steady-state creep flow, and III – accelerated creep. Modified from [15] and [16]

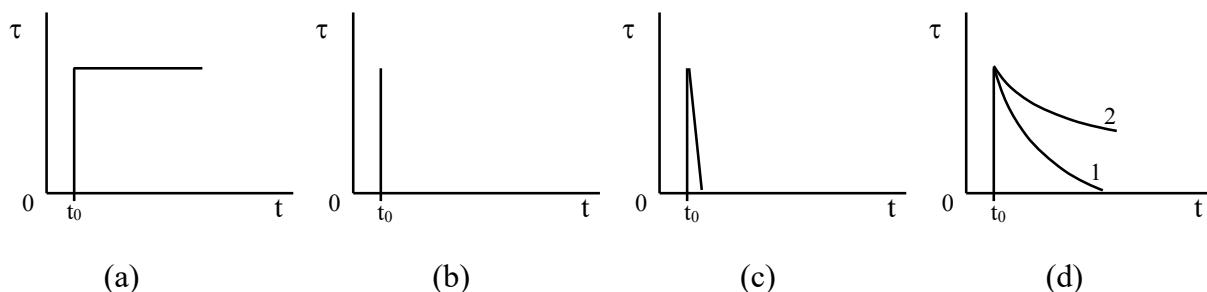
Figure 9a shows a family of creep curves for a given soil, *i.e.* curves of deformation along time for different constant loads. A high constant load leads to high deformations developing at a high rate, which may lead to failure. Hence, rapid failure may occur under large stresses, called **accelerated** [15] or **non-attenuating creep** [16], which culminates in the total **loss of strength** and soil failure. Low loads may result in a lower deformation rate, named **attenuating** [15] or **fading creep** [16].

When comparing different soils under a given load, phenomena varying from attenuating creep to rapid failure may occur, depending on the mechanical properties of the soils (Figure 9b). Different soils may show different creep response under natural conditions, depending on their mechanical properties, state conditions, magnitude of the load in the field, and time of observation.

Figure 9a also shows that for each constant applied stress ( $\tau_1, \tau_2, \tau_3$ , etc.), there is a time associated to the

failure of the soil, and the higher the load, the faster the failure. If the soil (that displays creep) is loaded fast to the point of failure, the stress at failure is called instantaneous strength [15] or initial short-term instantaneous strength [16], usually associated to the concept of “*ultimate strength*” from material science. Similarly, for any given time interval, there is a stress that causes failure, called **long-term strength** (Figure 9c). Long-term strength is a consequence of creep and culminates in a total loss of strength of the soil. There is a limit stress value ( $\tau_\infty$ ), called **ultimate long-term strength** [15] or **long-term strength limit** [16], above which there is failure, and below which the creep curve will be attenuating, and failure is not reached. Hence, the ultimate long-term strength may be defined as the resistance to failure due to the application of a constant load along time.

**Relaxation** is defined as the **slow decrease of stress over time at constant deformation**. Hookean solids show



**Figure 10:** Typical relaxation curves of stress versus time under constant strain: (a) Hookean solid; (b) Newtonian liquid; (c) Water; (d) Viscoelastic liquids (1) and solids (2)

no stress relaxation, *i.e.* the stress is constant under a constant deformation (Figure 10a). For Newtonian liquids, stress relaxation almost occurs immediately after deformation is applied (Figure 10b). In real materials, stress relaxation is not instantaneous and occurs over time (Figure 10c, d). For viscoelastic liquids the stress is dissipated and may decay to zero, and for viscoelastic solids, stress decreases along time but does not decay to zero.

The concept of relaxation is much wider, though. Maxwell used relaxation to designate delayed restoration of a molecular structure of matter distorted by any external factor. Thus, time effect is a consequence of the transformation of material structure and the kinetics of restoration of an equilibrium state [2]. Relaxation is the result of a redistribution of elastic (instantaneous) deformation ( $\gamma_e$ ) and viscous deformation ( $\gamma_t$ ).

Considering that the total (elastic and creep) deformation ( $\gamma_0$ ) is constant and that the shear modulus is constant, stress decreases with the increase of creep deformation, as follows.

$$\gamma_0 = \gamma_e + \gamma_t = \text{constant} \quad (14)$$

$$\frac{\tau_0}{G} = \frac{\tau_e}{G} + \gamma_t = \text{constant} \quad (15)$$

Where  $G$  is the shear modulus or rigidity modulus of the soil, and at  $t_0 = 0$ ,  $\gamma_t = 0$ .

The mobilized stress decreases from the initial value  $\tau_0$  tending to a limit value, while  $\gamma_t$  increases, also tending to constancy. The limit mobilized stress is the ultimate long-term strength or long-term strength limit. Therefore, relaxation and creep are entwined phenomena.

The time necessary for the material to recover a stable structure after the removal of external forces, dependent on the level of structure organization, is called time of relaxation. An almost infinite time is required for the movement of the atoms of solids; conversely, the movement of the atoms of liquids or gases is very fast, as exemplified in Table 2.

**Table 2:** Time of relaxation of some materials

Material	Time of relaxation
Newtonian fluids	0
Water	$10^{-11}$ - $10^{-12}$ seconds
Lubricating oils between gear teeth	$10^{-6}$ seconds
Polymer melts during plastics processing	few seconds
Glass	100 years
Rocks	Billions of years
Hookean solids	$\infty$

## 7 Comments on rheology applied to soils

Elastic-plastic models, used to describe soil behavior and to solve most geotechnical problems, do not consider the time effect on the stress and strain state of the soil: every phenomenon occurs instantaneously after the application of an external force. However, soils and other potential geotechnical materials, such as water treatment sludge, mining tailings etc. may present more complex behavior when submitted to a load for a long time.

Despite not always clearly displayed, rheology concepts have been used to **create models for soil behavior**, mainly for **soft soils**, to understand the dynamic of **debris flows, mudflows and landslides**, and to elaborate **laboratorial tests**.

The interaction between water and soil particles has an important role in the strength of soils, since soil behavior is commanded by effective stresses ( $\sigma_e$ ) generated at the contact between soil particles. Terzaghi [50] postulated that the effective stress is the total applied stress ( $\sigma$ ) minus the pore pressure ( $u$ ): ( $\sigma_e = \sigma - u$ ). Solicitations under drained conditions, which allow dissipation of pore pres-

sure, and therefore higher effective stress, are related to higher soil strength, while saturated soils under rapid solicitations respond with lower strength.

Most geotechnical tests and devices were designed to semi-solids and solid materials, *i.e.* compactable and moldable materials. There are also geotechnical devices specially designed to measure mechanical strength of soft soils (vane and cone tests). The precision of laboratorial vane test corresponds to an undrained shear strength of approximately 0.2 kPa, and the precision of the cone test is even lower. New geotechnical materials with very low strength, such as sludge and tailings, may not be precisely characterized by these tests; on the other hand, rheometry tests, developed to characterize the mechanical behavior of liquids, pastes and concentrated suspensions with good precision, may not allow to measure the strength of these materials in high stages of consolidation or drying. The limit in terms of material characteristics for which one needs to move from usual rheometry tests to usual tools in geotechnics is not established and depends on the material consistency, device limitations and application purposes.

## 8 Geotechnical time-dependent phenomena

The most acknowledged time-dependent phenomena in geotechnics are:

- **aging** (alteration of geotechnical properties or characteristics as a consequence of different processes that occur along time);
- **thixotropy** (regain of strength along time of remolded soft clays); and
- **creep or long-term deformation** (continuous, prolonged and low deformation at constant stress).

Some manifestations of these phenomena are prolonged settlements, tilts of structures, and landslides. Since the work of [50] on creep of clayey soils, many authors have studied time-dependent phenomena [35–37, 51, 52]. However, as remembered by [46], everything changes along time, everything ages. Time-dependent phenomena related to soft soils are well known in soil mechanics, but also occur to sands and gravels, as shown by [46, 51, 52].

According to [46], aging comprises many events that may change the state of the soil along time: alteration of the stress state and groundwater level, swelling and desiccation, freezing and thawing, chemical and biologic attack, and earthquakes, among others. Yet the author

pointed out the importance of “pure” aging effects (involving only the passage of time) and showed many cases of normally consolidated soils from a geologic point of view, which presented overconsolidation ratio (OCR) higher than 1 resulting from time, *viz.* secondary compression (creep). This phenomenon was named “**aging pre-consolidation**” by [46] and “**hardening**” by [53]. The effective vertical stress that separates recompression and compression, obtained by oedometric consolidation tests with undisturbed samples from soil deposits, when higher than the overburden effective vertical stress, was named “**critical pressure**” by [54] and “**yielding pressure**” by [55].

The importance of considering this gain of “pre-consolidation pressure” in the design of embankments over soft soils was thoroughly discussed by [55]: starting from the historical evolution of the correlation between undrained strength/effective stress ( $\frac{S_u}{\sigma'}$ ) and plasticity index (PI) proposed by [56], the author analyzes all classical researches [29, 54, 57–68] to estimate the design undrained shear strength. [69], based on Skempton and Bjerrum, proposed a method to predict final settlements of embankments on soft soils considering primary and secondary compression by normalization of oedometric compression curves.

[35] used the term “**hardening**”, and [36] “**aging**” to describe the phase when a thixotropic soil returns to its original state (strength regain) after remolding. Some confusion is noticeable about the usage of the terms aging and thixotropy in geotechnics. Even though thixotropy has been defined in the scope of geotechnics [35, 36, 44, 45], this term is still commonly used to describe all kinds of aging effects. According to [46], both phenomena have similar effects on the soils (changes in the soil structure, stiffening and strengthening) and involve flocculation/dispersion phenomena. However, thixotropy occurs in a very high void ratio system (or suspensions), very low effective stress, and under constant volume (undrained condition), while aging happens when there is drainage, the effective stress increases and the volume decreases. Aging may also be related to creep. This rheological concept, applied to continuous materials, is explained by structural rearrangement and does not regard effective stresses or pore pressures.

The terms secondary compression and creep are also commonly used as synonymous in soil mechanics. However, the former is generally associated with long-term deformation after the dissipation of excess pore pressures (primary consolidation) caused by vertical surcharges, while the latter is commonly related to long-term deforma-

tion of slopes by self-weight. Actually, both are manifestations of long-term deformation under constant stress, *i.e.* creep. That is also why there is discussion about the beginning of secondary compression: creep starts immediately as the vertical stress is applied, but due to the dissipation of excess pore pressures, primary settlements are more remarkable in the beginning; settlements due to creep gain importance along time and are usually noticed after the end of the primary consolidation.

An interesting example of a geotechnical solution considering creep is the design of foundations of the viaducts of the Imigrantes Highway in the Brazilian coastal mountain chain Serra do Mar. The air compressed caissons, embedded in the bedrock of biotite gneiss, crossed a superficial layer of colluvial silty clay with sand up to 4-m thick (*talus*) and underlying layers of altered residual sandy soil and altered rock (*saprolite*). Unprotected piles presented cracks in the shaft because of creep of the *talus* superficial layer. Concrete rings (protection jackets) were placed around the piles across the total depth of the *talus* layer to prevent the movement of the piles, providing an annular void to reduce or to eliminate horizontal stresses in the shaft of the piles. The diameter of the concrete rings were calculated for an estimated displacement of up to 0.5 m, so that the horizontal movement of the talus may cause the displacement of the concrete ring without touching the pile in the long run [70].

Among many other examples, creep deformation in soils are also regarded in the design of roads traversing marine and alluvial soft soils, and in buildings built in coastal areas, as “secondary compression” vertical settlements and as “negative skin friction” on piles in soft soils. [71] commented on the existence of about hundred buildings inclined due to differential settlements in the coastal area of Santos, Brazil, and used a technique to jack upright, without evacuation of the inhabitants, two buildings tilted because of creep of a soft clay layer located below a compact sand layer which supported the shallow foundations.

Constitutive functions for soils may also take into account time-dependent phenomena. For example, [72] employed the viscous-hypoplastic model proposed by [73] and the constitutive functions proposed by [30] to simulate results from undrained isotropic compression triaxial tests. The equations were applied to investigate relaxation and creep, and to analyze the influence of viscous parameters on the model response.

## 9 Flow in geotechnics

In geotechnical engineering, the term flow has been used as a generic word to describe time-dependent deformation. Furthermore, an unconfined plastic deformation under a steady load is also referred to as flow, apparently by analogy with the flow of a liquid, despite not being identical to the notion of viscous flow, as pointed out by [15]. For a **viscous fluid**, **flow** or **viscous flow** represents the continuous and unconfined change of shape, *i.e.* the shear **deformation along time at constant rate** (Newtonian liquids). For **solids**, **flow** also describes a **slowly progressing continuous deformation**. Considering that “*everything flows, if you wait long enough*” as stated by Professor Marcus Reiner [1], soils also flow under long-term observation and flow is dependent of time.

In geotechnical engineering, the term **flow** is usually used as a synonym of a special stage of **creep** (Figure 9d), when the deformation occurs at constant rate [15], or as synonym of the movement of a liquefied mass of soil and fragmented rock in landslides with high pore-water pressure such as in **debris flows**. The term plastic flow is commonly used in geotechnics as well. As mentioned in the item *Historical notes of elasticity, plasticity and viscosity*, in the theory of plasticity, the term **plastic flow** designates an instantaneous plastic deformation that occurs when a minimum load is reached (yield point or yield stress). In continuum mechanics, plastic flow or **viscoplastic flow** is related to the viscous flow which occurs when the load reaches a minimum value, *e.g.* Figure 5a curve (2) for a Bingham or other “plastic” solids according to [74]. The term **viscous flow** is also used to describe creep at variable rates and the usage of terms such as ideal viscous, viscoelastic and elasto-viscoplastic are common [16].

This analysis suggests that the term flow was introduced into soil mechanics related to mass movements observed in macroscale. Adjectives were added to describe the type of movement, for instance, a moving liquefied mass during a debris flow would be a “viscous” or “viscoplastic” flow; a slope failure would be a “plastic” flow; and so on. On the other hand, progressive long-term settlements of apparently static soil masses are referred to as “aging”, “creep” and “secondary compression”. However, according to rheological definitions, we have shown in this article that a creep curve can be related to a mudflow or to an overconsolidation ratio of a soft clay deposit incompatible with the geological history.

## 9.1 Landslide, debris flow and mudflow

Natural disasters, such as landslide and debris flow, often occur around the World [75]. According to [23], in USA, losses from landslides and other ground failures surpass the sum of all other natural hazards. Also in Brazil, damages caused by debris flow reached millions of dollars and affected one of the most important oil refineries located in Cubatao, São Paulo [76]. Hence, the importance of understanding debris flow and landslide mechanisms is evident to geotechnical engineering.

In 1950, Terzaghi compared creep and landslide, noting that both phenomena are similar, and the difference is the velocity of the movement; while landslide refers to a quick movement of the soil mass, creep is related to a movement that occurs at an unnoticeable rate. In 1964, Skempton investigated landslides in railway embankments which illustrated the loss of strength of clayey soils over time as a consequence of creep [16]. Figure 9 explains and corroborates the findings of Terzaghi and Skempton: depending on stress magnitude and shear rate, a soil slope may reach rapid failure or present attenuating creep along time. Likewise, for given values of stress and shear rate, due to the slope geometry, rapid failure or attenuating creep may take place depending on the mechanical characteristics of the soil.

The utilization of rheology and rheometry has increased in researches on landslides, debris flow and mudflows. Rheological behavior models, such as Herschel and Bulkley, Cross, Power law and Bingham, are often used to simulate natural flow, such as snow avalanches [77], landslides, debris flows and mudflows [17–21, 23–25, 78].

Determining rheological parameters of soils is important to study, to understand and to predict the dynamics of soil mass flow, and also to calibrate numerical rheological models used to predict the propagation of instabilities [22, 25]. Different methods and apparatus have been used for determining rheological parameters of soils, such as laboratory rheometer, inclined plane test, large scale rheometer, and field tests [20, 23].

[25] recommend an integrated laboratory-field approach to evaluate rheological parameters for debris flow simulations and to investigate the dynamics of flows. Satisfactory calibration of numerical models of the propagation of debris flow was obtained by associating the viscosity measured by rotational rheometer and inclined plane tests in laboratory [24] to yield stress estimated by field observations.

The laboratorial and numerical rheological approach is also useful to understand triggering factors for mudflows and debris flows. [23] measured the rheological prop-

erties of a natural mudflow deposit in Colorado by rotational viscometer and showed that viscosity and yield stress increased exponentially, in three orders of magnitude, as the solid concentration of the silt-clay mixtures increased from 0.10 to 0.40; additionally, a sand content higher than 20% was necessary to increase the viscosity of the mixtures and to alter the properties of the mudflow. [21], studying the influence of flow characteristics of fine-grained soils (two natural soils from Canada - St-Alban clays and Baie des Ha!Ha!, a soil collected from debris from La Valette landslide, and iron ore tailings) on the mobility of landslides and debris flow, observed that the flow behavior was strongly influenced by the volumetric concentration of solids; besides, the larger the particle size, the lower was viscosity. [18], studying debris flow in Rio de Janeiro, Brazil, found out that, for the tested samples, the higher the sand content, the lower were the viscosities, and cited other researches with concordant and conflicting results. Therefore, the particle size distribution does not seem to be a good indicator of soil viscosity, possibly because also important is the packing of grains [79], as observed in other materials, such as cement pastes [80] and concretes [81, 82].

[19] studied the effect of soil organic carbon content on the outbreak and evolution of mudflow in a catastrophic landslide in Cervinara, South Italy. Samples of slurry from the mudflow deposit were submitted to rheological tests to obtain the yield stress and flow curves. Viscosity and yield stress decreased with the reduction of soil organic carbon (SOC). Removal of the dissolved organic carbon (DOC), which was approximately 6% of SOC for the studied samples, reduced yield stress from 26.4 to 5.4 kPa. The authors concluded that SOC has a stabilizing role in the slurry microstructure and hypothesized that rainfall, concurrent to the trigger of the landslide, may have “washed” the soil and removed part of the natural DOC.

## 9.2 Geotechnical tests

Rheology concepts were important to design laboratorial and field devices, besides standard tests procedures in soil mechanics. A classic example is the sedimentation method to determine the particle size of soils in laboratory, designed from Stoke's Law, who studied the movement of spherical balls falling into a viscous fluid in 1849.

The creation of the liquid limit test [83], defined by Atterberg and standardized by Casagrande, was also based on rheology concepts. During the test, a cup drops at a constant height (10 mm) and at a constant rate (two drops per second), allowing both halves of the soil (divided by

a grooving tool) to **flow**, at constant shear stress (height) and shear rate (rate of drop), respectively. If the water content is high, the soil will behave like a liquid because the particles interaction will be weak [84], causing flow with a low number of drops. The plot of water content versus number of drops (in log scale) is also called **flow curve**. The liquid limit is the water content relative to 25 drops in the flow curve, and it is equivalent to 2.5 kPa of undrained shear strength [85] or 2.66 kPa by the cone fall test [86], based on the measurement of the cone penetration into the soil contained in a cup. At the liquid limit, the soil is at the boundary between the liquid and plastic states of consistency, but is still not a liquid and needs a very low magnitude of shear stress to flow. The term **flow curve** in this test is not a coincidence: the **geotechnical flow curve**, obtained from the liquid limit test, describes the flow behavior of the soil as a function of the water content at constant shear stress and shear rate, whereas the **rheological flow curve** (shear stress versus shear rate – Figure 4), may be obtained by varying the shear rate at a constant water content.

Both field and laboratorial **Vane tests** [87, 88] are also good examples of how rheology concepts were used to design devices and procedures. A vane blade screwed into the base of a rod is pushed vertically into soft soil and then rotated at a constant slow rate, while the torque is measured at regular time intervals. Vane tests are performed in a single condition of solicitation, determined by the velocity of rotation applied at constant rate (6-90°/min) during the whole test. In other words, each Vane test result is a single point on the **rheological flow curve**. Indeed, the Vane test apparatus may be roughly compared to a rheometer that works at a single velocity (just one constant shear rate), while rheometers may be programmed to vary the shear rate. This simplification of the design of the Vane apparatus has advantages and disadvantages. Maintaining a constant rotation rate, the results from different researchers may be compared. However, the Vane test is performed very fast (the soil fails in 2-3 minutes) when compared to solicitations imposed in the field by earth works. Therefore, Vane tests tend to overestimate the undrained strength of the soil and the Bjerrum's correction factor [57] must be applied: the faster the test, the higher the stress of failure ("ultimate stress"), as discussed before (Figure 9).

Classical laboratory geotechnical tests, such as oedometric compression [9], direct-shear [10], cone penetration [89], Vane [90, 91], and triaxial tests [92] have been used to determine rheological parameters of soils.

**Oedometric tests** are performed by applying increasing (and, in sequence, decreasing) predetermined vertical loads to a soil sample (a circular disc of soil placed

inside a rigid metal confining ring that restrains horizontal displacements and allows only vertical displacements) and measuring the deformation response for each step of applied load during 24 hours. The results, interpreted by Terzaghi's consolidation theory, are used to predict one-dimensional settlements under applied vertical loads and the time necessary to develop these settlements. In this test the pore pressure is expected to increase immediately after each step of load is applied (excess pore pressure due to surcharge), and to decrease to null during 24 hours; however, vertical settlement is measured instead of pore pressure, considering that the total dissipation of excess pore pressure corresponds to the end of vertical displacements.

**Direct shear tests** are conducted on specimens placed inside a shear metal box, which height is split in two parts, one fixed and one mobile. Initially, a vertical confining stress is applied to the specimen. When consolidation under the vertical stress is completed, a constant horizontal strain rate is applied to the mobile part of the box until the specimen fails (a discontinuity develops along the soil horizontal plane between the upper and lower parts of the box). The test is repeated for at least three different confining vertical stresses, and the pairs of confining vertical stress and horizontal failure stress determine the shear strength parameters of the soil, cohesion and friction angle. The rate of horizontal strain depends on the soil: slower rates are usually used for granular (permeable) soils, to allow dissipation of pore pressures (drained condition) and determination of effective strength parameters; whereas, for fine-grained (low permeability) soils the drained condition would exceedingly extend the test, therefore higher strain rates are applied so that practically no dissipation of pore pressure occurs, and total strength parameters are determined.

**Triaxial compression tests** also provide the shear strength parameters: a cylindric soil specimen, involved by an impermeable membrane, is placed inside a cylindric test cell. The cell is filled with water, and the water is pressurized to apply radial compression to the specimen. Then continuous increasing axial stress is applied, or a constant strain rate is applied vertically, until the specimen fails. The test is repeated for at least three different confining radial stresses to yield the pairs of confining pressure and correspondent vertical failure stress. The two stages, application of radial confining pressure and application of vertical stresses, may be drained (with measurement of volumetric deformation) or undrained (with measurement of pore pressure).

[9] used long-term oedometric compression tests to fit [93–95] models for consolidation of a Brazilian marine soft soil and to separate the viscous and the elastic frac-

tions of the excess pore pressure. Trying to reduce the duration of creep and long-term strength tests for clayey soils, [10] performed direct-shear tests under various loading conditions and concluded that accelerated methods can reduce tests duration from weeks or months to some hours. [89] proposed the use of the laboratory fall cone tests and Casson's model to estimate the viscosity of soils at liquidity index of 0.3-2.1; results indicated that kaolin viscosity decreases exponentially with the increase of liquidity index. [90, 91] compared the Vane test to rheometer tests for estimating the yield stress of highly concentrated suspensions (titanium dioxide suspensions and "red mud"-bauxite tailings), and recommended the Vane test as a direct, low cost and less consuming method to accurately estimate the yield stress. However, there are no standards to obtain rheological soil parameters from single-point geotechnical tests; a methodology should be standardized, or the applied shear rate should be chosen in accordance to the expected solicitations in the field on a case-by-case basis, to use the Vane test as proposed by the authors.

Thixotropy of soils was also investigated by Vane tests [7, 90, 96], fall cone tests [38] and triaxial tests [92].

New devices and methods also have been created to determine rheological parameters with a view to establish a **correlation between geotechnical and rheological properties of soils** and to predict soil behavior for many applications in soil mechanics [18, 21, 97]. [21], based on the results from rheological tests with an axial cylinder viscometer, proposed relationships between geotechnical and rheological properties (yield stress, viscosity and liquidity index) of a Canadian soil to study mobility of landslides and debris flows.

[98] applied compressive and rotational rheometry to evaluate the rheological properties of soil from a slope in Vila Albertina, in the northern area of São Paulo city, Brazil, and obtained linear relationships of apparent viscosity and yield stress as a function of moisture content for liquidity index varying from 0 to 1.6 by rotational rheometry. Compressive effort was similar for liquidity indexes of 0 and 0.6, however 95% lower for liquidity indexes of 1.4 and 1.6. These results may help to understand the causes of a local landslide, relating soil behavior to water content.

[97] created a new device, called "Flow Box", using the Bingham model and the trap door principle formulated by Terzaghi [99], which continuously determines change in the soil viscosity while the soil moves from a plastic state to a liquid state; the goal of this research was to obtain the relationship between the initial viscosity and the liquidity index that leads to the initiation of mudflow.

Furthermore, some researchers [18, 100–102] were inspired by studies performed in other fields of science, par-

ticularly the rheology of concrete, to bring innovation to soil mechanics. This promising research strategy was formerly encouraged by Terzaghi and in [16], Terzaghi's statement is cited: "*cohesive soils may be compared with concrete, wherein sand and dust serve as the skeleton and colloidal silt serves as the cement.*" Hence, due to similarities between the structure and the properties of clayey soils and concrete, some devices and methodologies applied to the concrete technology may be used to improve researches on soil behavior. [100] proposed a new laboratory test routine using a flow table, originally used for concrete investigations, to characterize the flow behavior of artificially mixed clay-sand soils and to understand the flow characteristics of any excavated material involved in Earth Pressure Balance (EPB) tunnel drive. [101] performed flow tests by means of a plate-plate rheometer, used in concrete researches, to understand the mechanical behavior of water treatment sludge from Cubatao, São Paulo, Brazil. [102] found sound explanations for the loss of shear strength and increase of compressibility of a compacted tropical lateritic soil in contact with caustic soda from alumina tailings ("red mud") by means of zeta potential and rheometer tests. [18], studying four tropical soils from Rio de Janeiro, Brazil, proposed a methodology to evaluate the viscosity of soils for debris flow analysis by means of a combined modified slump test apparatus and a plate-plate rheometer. The methodology developed by the authors uses the modified slump test to evaluate the shear rate at which the material flows under its own weight (condition supposedly similar to that in the field after soil saturation and structure breakdown); the range of shear rate determined by the modified slump tests is then used at rheometer tests to determine soil rheological parameters. A good relationship between water content, shear rate and viscosity was observed. Based on the correlations obtained from the experimental and analytical results and numerical back-analysis of debris flow occurred in the Rio de Janeiro, the authors concluded that the methodology can provide adequate input parameters for debris flow analysis.

## 10 Conclusions

The geo-mechanical behavior of soils results from the superimposition of elastic, plastic and viscous behaviors in different proportions, varying with the soil inherent characteristics and state conditions, type and duration of solicitation (shear rate and stress magnitude), and time-scale of observation. Traditional elastic-plastic models (instantaneous responses) solve most geotechnical prob-

lems. Yet other models are required to describe some soil phenomena, such as secondary compression, aging pre-consolidation, flow, creep, relaxation and thixotropy. Rheological viscoelastic models may be a powerful tool to analyze these phenomena, which depend on the time necessary to rearrange the inherent structure of soils.

The lack of an effective geotechnical terminology for phenomena that occur along time causes some confusion. In soil mechanics, mass movements are usually referred to as "flow", and long-term settlements of apparently static soil masses, as "creep" or "secondary compression". However, creep as defined by rheology (continuous, prolonged and slow deformation of a material at constant stress) may be responsible for triggering soil mass instabilities, as well as for the overconsolidation of soft soils classified as normally consolidated by geological history. Aging and thixotropy; flow and creep; viscous flow and creep; viscoplastic flow, plastic flow and flow, among others, are terms commonly and sometimes equivocally used as synonyms. Hence, terminology standardization is mandatory to clarify some of these concepts and definitions.

The review performed herein showed the importance of rheological models, such as Herschel-Bulkley, Cross, Power law and Bingham to simulate the dynamics of soil mass flow and to predict propagation of instabilities.

The use of rheology concepts can also be applied to understand soil behavior that triggers landslides events and to determine the relationships between stress and deformation during the soil movement process.

In addition, rheology concepts have also been important to design laboratorial and field devices, besides standard tests procedures (sedimentation to determine grain size, Vane test, liquid limit test).

Furthermore, the World demand for sustainability has required the reuse of waste, introducing new materials to the scope of geotechnics, such as sewage, wastewater, water treatment sludge and mining tailings. Rheology, widely used to study the industrial application of polymers, ceramics, concrete, paints, cosmetics, food, mining tailings, sludge, among others, may be useful to analyze geotechnical processes.

Considering Terzaghi's thought about the similarities between clayey soils and concrete and how the knowledge of other materials helped to understand soil behavior in the past, we believe that the concepts of rheology and rheometry applied to the development of other materials, especially concretes, may be helpful to understand geotechnical problems.

We hope this paper contributes to the necessary terminology standardization, which should be the goal for future researches on soft soils, soil-additive mixtures, active

clays (bentonites, zeolites, vermiculites etc.) and waste materials.

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## Appendix A

Terminology	Normal stress	Shear stress	Shear stress	Units
	Geotechnical Engineering		Rheology	
Stress	$\sigma$	$\tau$	$\sigma$ or $\tau$	$\text{N/m}^2$ , Pa
Strain	$\epsilon$	$\gamma$	$\gamma$	dimensionless
Elasticity modulus or Young's modulus	$E$	-	-	$\text{N/m}^2$ , Pa
Shear modulus or Rigidity modulus	-	$G$	$G'$	$\text{N/m}^2$ , Pa
Stress-strain relation	$\sigma = E\epsilon$	$\tau = G\gamma$	$\sigma = G'\gamma$ or $\tau = G'\gamma$	$\text{N/m}^2$ , Pa
Shear rate	-	-	$\dot{\gamma}$	$\text{s}^{-1}$
Yield stress	-	$\sigma_y$ or $\tau_y$	$\sigma_y$ or $\tau_y$	$\text{N/m}^2$ , Pa
Viscosity	-	$\mu$	$\eta$	$\text{Pa}\cdot\text{s}$