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QUANTUM TURBULENCE IN TRAPPED BEC: NEW PERSPECTIVES FOR A LONG LASTING PROBLEM

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Abstract. Quantum turbulence, and turbulence in general, is an open topic of research with more questions to be answered than answers given in the last decades. The use of atomic Bose-Einstein condensates to study turbulence is fairly recent and has potential to grow in the next few years. In this paper we review some of our contributions to the field of quantum turbulence in Bose-Einstein condensates as well as point some directions towards which we believe the field should move in the next future.

1. INTRODUCTION

Turbulence is since long recognized as one of the most difficult and long-lasting unsolved [1] problems tackled by science. Although numerous scientists, starting as far as Leonardo da Vinci, studied turbulence in its various forms, our understanding of this phenomena remains limited.

In modern times, major advances in the understanding of turbulence have been made by A. Kolmogorov *et al.* [2, 3]. By describing a stationary turbulent flow, where energy is pumped at large energy scales and dissipated at the same rate at small scales, he was able to predict an universal scaling behavior for the whole energy spam involved in the turbulent decay process. This is known as the Kolmogorov's scaling law and it predicts the energy decay in turbulence to be a power law with a exponent of $-5/3$. Indeed, Kolmogorov's scaling law turned out to be fairly accurate in turbulent flows within a wide variety of classical fluids.

With the advent of superfluid liquid Helium, the study of turbulence got a whole new playground. Named Quantum Turbulence (QT) after the nature of the quantum (super)fluid where it was created, this new environment for turbulence promised a whole new set of frontiers to be explored. From the nature of the fluid itself to the nature of its vortices, now carrying only quantized amounts of angular momentum, many aspects in QT are *a priori* different from classical turbulence. However, despite the differences, Kolmogorov's law still holds for QT in liquid Helium.

When the first evidences of Bose–Einstein condensation in dilute atomic gases [4, 5] have been observed 20 years ago, once more a new frontier has been opened. It is no surprise that, soon after the study of vortices in this new system started [6], the first mentions of turbulence in atomic gases have been made in the scientific literature [7, 8], followed by our experimental discovery [9].

Indeed, dilute atomic BECs have several desirable features for the study of turbulence: vortices can be directly observed by optical means, as well as reconnections, tangles and related aspects. Also, the ultracold clouds are extremely tunable: density, number of particle, interatomic interactions can be manipulated almost at will. Finally, the models used to describe and simulate atomic BECs are far simpler and straightforward than for superfluid liquid Helium and even classical fluids, allowing one to perform deep theoretical investigations in such samples.

This report focus exactly on this topic: the investigation of quantum turbulence in atomic BECs. The paper is organized as follows: in the first section we review some theoretical and experimental achievements made in the study of QT in Bose-Einstein condensates, those we have considered contain the key points to understanding QT in atomic superfluids. Although there are numerous groups working on the subject, here we stick to the detailed description of our recent results [10–13]. Then we look to the future, summarizing the theoretical perspectives and experimental challenges to be faced and tackled. A short section with the conclusions closes this contribution.

2. THEORETICAL AND EXPERIMENTAL ACHIEVEMENTS

We start reviewing some of our contributions, both theoretically and experimentally, to the field of quantum turbulence. Since Bose-Einstein condensates are small systems and, unlike liquid Helium, they are far from the homogeneous limit, one has to consider finite-size effects in any study to be made. Here we describe how that was applied to estimate the number of vortices held by our finite BEC sample, and how it is connect with the threshold of a turbulent regime. Next we discuss our experimental observation and theoretical model for the self-similar expansion of a turbulent BEC: after released from an anisotropic harmonic trap, while thermal clouds evolve to a unitary aspect ratio, the regular BECs present a well-known aspect ratio inversion during expansion [14]. A turbulent BEC, otherwise, maintains its in-trap aspect ratio during the expansion. That unusual expansion we so-called self-similar fashion. Finally we investigate the possibility to extract the momentum distribution to test Kolmogorov's scaling law from the images of an expanding turbulent cloud.

2.1. FINITE SYSTEMS

The traditional way to investigate BECs theoretically is through the Gross-Pitaevskii Equation (GPE), which is a model in the limit of zero temperature, where essentially all the atoms exist in the Bose-Einstein condensate phase. In many experiments the condensate exists at well below the BEC transition temperature such that this approximation is justified. Even at $T = 0$, however, the BEC system which can be currently created in laboratory contain a small number of atoms (few hundred thousand); hence do not sustain the number of quantum vortices present in helium experiments. This brings to light the issue of length scales and the question if Kolmogorov's scaling still holds.

In spite of the limited number of vortices present in trapped BECs, numerical results of these system [7, 15–17] suggest that kinetic energy is distributed over the length scales in agreement with the Kolmogorov scaling $k^{-5/3}$ (where k is the wavenumber), which is observed in ordinary turbulence. The presence of the this scaling, even over the small range of lengths available, would prove that the finite BEC system does really become turbulent.

Experimentally, a small vortex tangle has been created in a harmonically trapped BEC through the combination of rotation and an external oscillating perturbation [9, 18]. The experimental evolution of the number of vortices as a function of both, the amplitude and duration of the oscillatory excitation, has been reported in [11], with the turbulent regime appearing for higher values of these excitation parameters. Figure 1 shows the various structures and associated regimes seen by means of the oscillatory excitation, mapped in a diagram of the amplitude and duration of the excitation. The transition from the vortices to the turbulence regions in this diagram was analyzed in [10] (see Figure 2), based on an analytically model that establishes a critical number of vortices according with the size of the atomic sample.

In a recent publication [19], the authors applied numeric calculations to simulate the experimental condition in [9, 18]. Varying the excitation parameter, they characterized the regime of the atomic cloud analyzing the spatial variation nor only in the density, but also in the phase. They explored the particular aspects of the granular regime, a heterophase turbulence characterized by drops of Bose-condensate atoms surrounding by a rarified atomic sample.

2.2. SELF-SIMILAR EXPANSION

The most common diagnostic of trapped atomic clouds is done by imaging it, not in the trap, but after some time of free expansion. That turns easy to identify the differences between a thermal and a Bose-condensed cloud. The thermal cloud shows a gaussian profile that evolves to a isotropic density distribution at long times of expansion. Otherwise, in the Thomas-Fermi regime, the quantum cloud shows a profile that reflects the shape of the confining trap. Then, for a cigar-shaped trap, for example, the BEC cloud expands dramatically faster in the radial than in the axial direction. That causes the signature inversion of the BEC cloud aspect ratio during

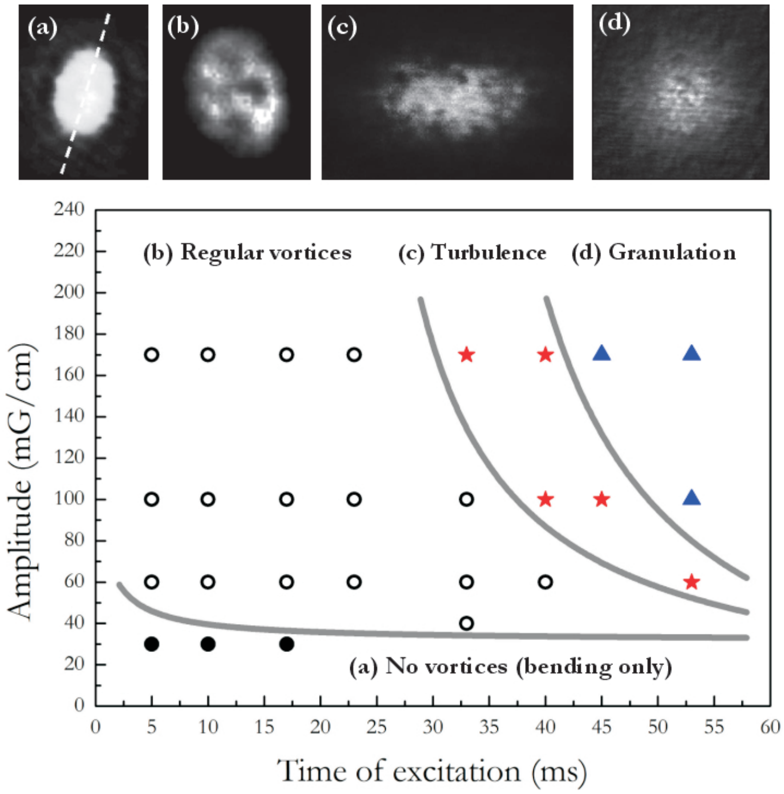

Fig. 1 – Diagram of the excitation amplitude A_{exc} versus its duration t_{exc} presenting regions where four different effects were observed. (After Ref.[11].)

the free expansion. Those evolutions can be seen respectively in columns (a) and (b) of Fig. 3. Considering now a turbulent BEC cloud, besides the evidences of the tangle vortices configuration with the vacancies in the density profile absorption image, the cloud free expansion dynamics also differ due to the presence of vorticity. In fact, for the cigar shape trap used in [9], the turbulent condensate expands with a nearly constant aspect ratio once released from its confinement. In Fig. 3 (c) we show this evolution, clearly self-similar, of the turbulent cloud.

To characterize the anomalous expansion of the turbulent sample, a generalized Lagrangian approach [20] can be applied to describe the dynamics of the expanding condensate cloud (*i.e.*, collective modes and free expansion). To produce a model expansion, the kinetic term contribution of a tangle vortex configuration is added to the Lagrangian system and the resulting Euler-Lagrange dynamic equations are derived. The theoretical aspect-ratio calculations agreed with the experimental curves showing that the physics behind the anomalous expansion of the turbulent system was well explained by the hydrodynamic model [12]. This model will be briefly re-

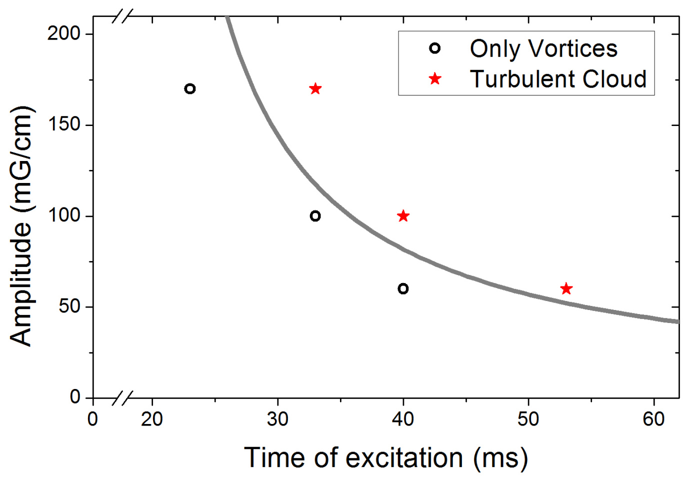

Fig. 2 – Diagram of the excitation amplitude A_{exc} versus its duration t_{exc} presenting the transition from a non–turbulent regime with regular vortices to a turbulent one. (After Ref.[10].)

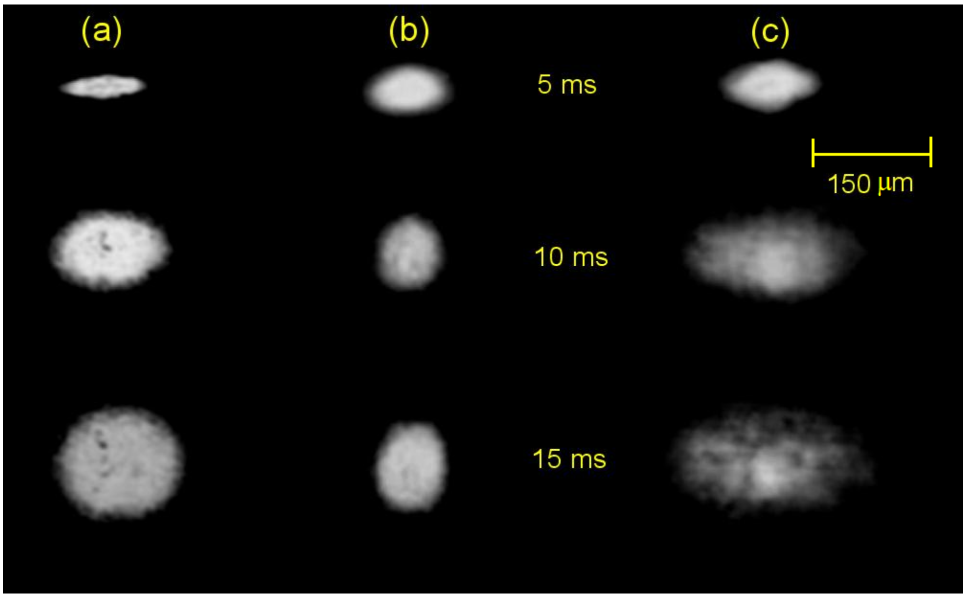

Fig. 3 – Free expanding (a) thermal cloud , (b) normal BEC and (c) turbulent cloud.

viewed below.

The basic Hamiltonian density is

$$\varepsilon[\psi] = \frac{\hbar^2 |\nabla\psi|^2}{2m} + V_{tr}|\psi|^2 + \frac{1}{2}g|\psi|^4 \tag{1}$$

where $g = 4\pi\hbar^2 a_s/m$, with a_s the s-wave scattering length, and $V_{tr}(\mathrm{r})$ is the harmonic trap potential. The Lagrangian approach relies on the functional

$$\mathcal{L} = \int d^3r \left[i\frac{\hbar}{2}\left(\psi^* \frac{\partial\psi}{\partial t} - \psi\frac{\partial\psi^*}{\partial t}\right) - \varepsilon[\psi]\right] \tag{2}$$

for a trial function ψ that depends on variational parameters (here, the time-dependent condensate TF radii R_j and phases β_j).

The Thomas-Fermi (TF) trial function becomes

$$\begin{aligned}\Psi_{TF}(\vec{r},t) &= e^{iS(\vec{r},t)} \prod_{j=x,y,z} e^{i\beta_j m x_j^2/2\hbar} \sqrt{n_0} \left(1 - \sum_{j=x,y,z} \frac{x_j^2}{R_j^2}\right)^{1/2} \\ &\quad \times\Theta\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right),\end{aligned} \tag{3}$$

and the normalization integral yields

$$n_0 = \frac{15N}{8\pi}\frac{1}{R_x R_y R_z}. \tag{4}$$

This trial function includes the irrotational velocity potential $\phi_j = \beta_j\, M\, x_j^2/2\hbar$ that leads to a linear velocity field $u_j = \beta_j\, x_j$, with $j = x,y,z$. With this trial function, Eq. (2) provides

$$\begin{aligned}\mathcal{L} &= -\frac{Nm}{14}\left(R_x^2\dot{\beta}_x + R_y^2\dot{\beta}_y + R_z^2\dot{\beta}_z\right) - \frac{Nm}{14}\left(\beta_x^2 R_x^2 + \beta_y^2 R_y^2 + \beta_z^2 R_z^2\right) \\ &\quad -\frac{Nm}{2}\langle \upsilon_0^2\rangle - \frac{Nm}{2}\langle\delta\upsilon^2\rangle - \frac{15}{7}\frac{\hbar^2\, a_s\, N^2}{m}\frac{1}{R_x R_y R_z}.\end{aligned} \tag{5}$$

In the calculations above we consider

$$\frac{\hbar}{m}\nabla S = \upsilon_0 + \delta\upsilon, \tag{6}$$

where υ_0 is the local center of mass motion induced by the vorticity, and $\delta\upsilon$ is the additional local velocity associated with the individual vortices.

For a given randomly oriented vortex, there is an energy per unit length

$$\frac{\pi n\hbar^2}{m}\ln\left(\frac{\ell}{\xi}\right), \tag{7}$$

where ℓ is the effective intervortex separation, $\xi = \frac{\hbar}{\sqrt{2\pi mng}}$ is the healing length (vortex core size) and n is the local number density. Assume that there is a mean vortex line length L per unit volume. Thus the total energy from the random vortices is

$$\frac{\pi N\hbar^2}{m} L\ln\left(\frac{\ell}{\xi}\right). \tag{8}$$

It is more appropriate to use a scaling based on an inverse squared length, so

$$L(R) \propto (R_x^2 + R_y^2 + R_z^2)^{-1}. \tag{9}$$

When the vortex energy in Eq. (8) is substituted into Eq. (5), we obtain the total Lagrangian

$$\begin{aligned}\mathcal{L} \;=\; & -\frac{Nm}{14}\left(R_x^2\dot{\beta}_x + R_y^2\dot{\beta}_y + R_z^2\dot{\beta}_z\right) - \frac{Nm}{14}\left(\beta_x^2 R_x^2 + \beta_y^2 R_y^2 + \beta_z^2 R_z^2\right) \\ & -NU(R_x, R_y, R_z)\end{aligned} \tag{10}$$

with U the effective potential given by

$$U(R_x,R_y,R_z) = \frac{m}{2}\langle v_0^2\rangle + \frac{15}{7}\frac{a_s\hbar^2 N}{m}\frac{1}{R_xR_yR_z} + \frac{\pi\hbar^2}{m}\frac{L_0(R_{x0}^2+R_{y0}^2+R_{z0}^2)}{R_x^2+R_y^2+R_z^2}\ln\left(\frac{\ell}{\xi}\right) \tag{11}$$

and L_0 the initial value of the turbulent vortex line density. The Lagrangian equations yields the dynamical equation

$$\ddot{R}_j = 14\pi\,\frac{\hbar^2}{m^2}\,\frac{L_0(R_{x0}^2+R_{y0}^2+R_{z0}^2)R_j}{(R_x^2+R_y^2+R_z^2)^2}\ln\left(\frac{\ell}{\xi}\right) + 15\,\frac{\hbar^2\,a_s}{m^2}\,\frac{N}{R_jR_xR_yR_z} \tag{12}$$

Typically, the rapid expansion arises from two sources: the vorticity (the first term on the right-hand side) and the repulsive atomic interaction (the second term on the right-hand side). For large time, when the expansion yields large values of the mean radii, the vortex term should dominate since it scales with three inverse powers, whereas the interaction term has four. At long times, when the vortex term dominates over the interaction, the equations have a simple form with $\ddot{R}_j/R_j$ independent of the index j, which should lead to asymptotic scale invariance (self-similarity).

The hydrodynamic model also allows for a comparison between the interaction and kinetic energy of the cloud for all times. For a vortex free condensate it is well known that the energy that comes from the repulsive interaction between the trapped atoms dictates completely the aspect ratio of the cloud *in situ* and during expansion [20]. However, from the solution of the dynamic equations, we can calculate the energy evolution for a turbulent system and show how the extra kinetic energy provided by the vortices has an important contribution to the release energy.

In this same context, it is also possible to demonstrate how the asymmetry of the trap changes the energy balance between kinetic and interaction energy. The information about the internal structure comes from the assumption that the kinetic energy has now a crucial role in the expansion, attenuating the effects of the interaction energy that, for our cigar shape trap configuration, affects mainly the more confined direction. Still with the turbulent cloud in a cigar shape trap, the inhibition of the aspect ratio inversion, that stays stationary in a constant value bellow one [9], is related to a higher asymptotic velocity field along the less confined direction (axial direction of the cigar shape trap). It is easy to explain the influence of the kinetic vortex term in the axial direction, based on the preferential alignment of the vortices lines along the short axis. Since the latter presents diminished probability for vortex line bending, the centrifugal contribution from the vortex velocity field will be concentrated perpendicular to that direction.

2.3. MOMENTUM DISTRIBUTION OF A TURBULENT TRAPPED BEC

To investigate turbulence experimentally, atoms are held for 20 ms in the trap, after the drive has been terminated, before they are released for imaging. The atoms are measured by a time of flight (TOF) absorption technique. In [13], TOF was used as a probe for the momentum distribution of a turbulent condensate, assuming that the turbulent state dictates the particle distribution between the momentum states. To reinforce this point, we used the hydrodynamic model described before to calculate the energy of a turbulent and normal BEC cloud during the free expansion. In Fig. 4 we can see how the energy of the trapped cloud transfer to its release energy. While in a normal BEC cloud we clearly see an important contribution given by the interatomic interaction energy, in the turbulent cloud the kinetic energy associated to the presence of the vortex circulation appears more relevant than the interactions. The first is the dominant effect for long time expansion of a turbulent cloud. The contact interaction, dominant term in a normal Thomas-Fermi BEC, is strongly attenuated due to the presence of the tangle vortices configuration, as reflected by the self-similar expansion of the turbulent cloud observed in [9] and discussed above.

The momentum distribution is extracted from the measurements of the absorption image of the expanded cloud. The projected image is a distorted two-dimensional shadow of the real atomic distribution. As a result, the spatial density has contributions from many wavenumbers along the path of the imaging light. As the interaction is assumed to be negligible, the ballistically expanding atoms allow for an experimental Fourier conversion of the real space density distribution after a TOF to an $in\,situ$ momentum distribution. The radii of the expanded cloud are con-

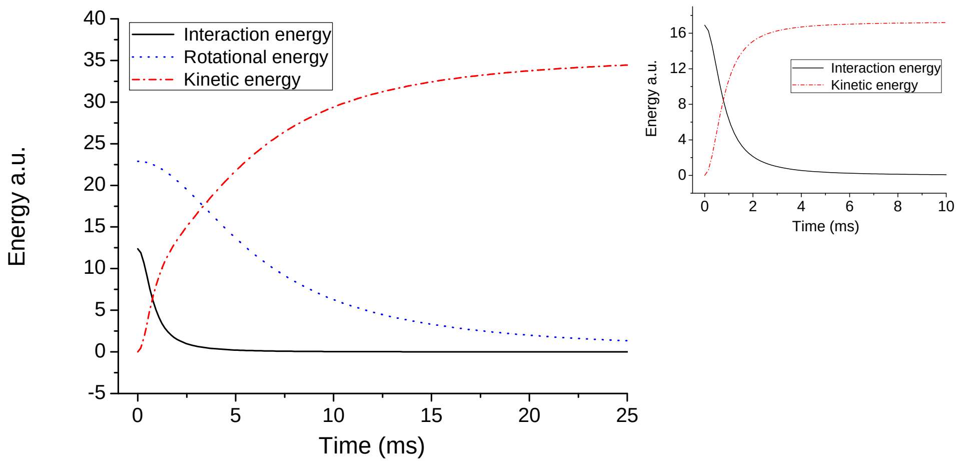

Fig. 4 – Energy of the turbulent and normal BEC cloud (inset) as function of the free expansion time. The kinetic energy associated to the vortex configuration is labeled as the "rotational energy". (After Ref. [13].)

verted in momentum shells. Then, we could count the number of atoms in each one of the shells to construct the momentum distribution of the sample.

It is recognized that the initial interaction energy of a Bose condensed system alters the momentum distribution. During the early stages of the condensate expansion, the interaction energy is converted into kinetic energy prior to the ballistic regime, and then, it can distort the conversion described before. In [13], however, it is argued that the interaction is small relative to the turbulent kinetics and does not qualitatively change the momentum distribution measured with this technique. Another important remark that support their assumption is reflected in the resulted turbulent spectra distribution, that presented a population of atoms in a higher momentum window not present in a normal BEC. This particular region of the turbulent spectra shows a linear decreasing distribution, and from its slope they could associated a power law.

There are many reasons, however, that do not allow the direct connection of the extracted power law in this trapped system with the Kolmogorov law. As mentioned before, the range of length scales available experimentally is much smaller than those of other classical or quantum fluids. Besides the finite size of the sample, another peculiarities of their system are the uncontrolled turbulent injection mechanism, that prohibits the stationary turbulent condition to settle and the establishment of an appropriate inertial range in the spectra. Each of these experimental situations affect the measured anomalous momentum distribution and need to be better controlled to give conclusive results.

3. THEORETICAL PERSPECTIVES

Atomic Bose-Einstein condensates (BEC) have become a very important tool for the development of theoretical and experimental insights into some of the unanswered questions of quantum turbulence. The relatively low density of trapped atomic superfluids allows a clearer visualization of the vortices lines distribution, as well as the direct observation of their dynamic. Vortex reconnections [21], which changes the geometry of the vortex lines, always play an important role in quantum turbulence. Experimental verifications in BEC can bring a better understanding about the microscopic details of the process involving the vortices, which was first observed by indirect measurements in liquid helium.

The finite aspects of the atomic trapped superfluid systems can bring new effects concerning the evolution of the turbulent regime and its decay. Also, it may place limits on the maximum number of allowed vortices existing in the superfluid cloud before it evolves to turbulence. This fact may have dramatic implications on the dynamical behavior of the turbulent states, which can be easily detected and better understood in their many aspects.

The direct Kolmogorov energy cascade mechanism has been established numerically for trapped atomic condensates [22, 23], but it is yet to be confirmed experimentally. Trapped condensates may be a suitable system in which to determine the lower bound in the system size and vortex line density for which Kolmogorov scaling may be observed. Tracking the length of individual vortices is also possible numerically, and may help answer questions about the relationship of an energy cascade to a Richardson cascade. Similarly, if there is a Richardson cascade process in quantum turbulence (QT), it may be directly observable in trapped atomic condensates. Establishing the existence or absence of a Richardson cascade process coupled to a direct Kolmogorov cascade is one of the most exciting prospects in future experimental and numerical studies of turbulent vortex tangle in trapped atomic BEC [24].

The numerical simulations of few-vortex systems in $3D$ trapped condensates showed that Kelvin waves (transverse excitations taking place in the vortices lines [25]) facilitate the decay of vortices [26]. Since those excitations modes are strongly affected by the condensate dimensionality and by the effective confinement along the vortex line direction [26], the vortex decay processes are expected to be connected with the vortices line configuration and the trap inhomogeneity. However, the main process to dissipate the incompressible kinetic energy and how this scales with condensate size in zero temperature trapped atomic condensates remain unknown [24].

One last remark in the BEC system rely in the possibility of adjust its intrinsic atomic properties. The strength of atom-atom interactions can be controlled and even driven from attractive to repulsive by tuning an external magnetic field. Also, the system dimensionality can be modified through an optical trap potential. Therefore, those system create conditions for studying both $2D$ turbulence and $3D$ turbulence, opening up the possibility of investigating transitions between a $2D$ and $3D$ QT.

4. EXPERIMENTAL PERSPECTIVES

The experimental study of quantum turbulence in ultracold atomic gases, although incipient, is a natural step when one considers both the historical development of the field and the versatility provided by such systems.

Historically there have always been experiments dedicated to reproduce in ultracold atomic systems the phenomena observed long before in liquid Helium: consequences of Bose-Einstein statistics like critical temperature and condensed fraction, superfluid properties in general, like quantized vortices and vortex lattices, superfluid critical velocity and the speed of sound, the excitation spectrum, and many other phenomena.

In the other hand, while the study of turbulence in superfluid Helium has provided many fruitful results towards the understanding of such fundamental process of nature, those systems are not easily handled also not very tunable. In contrast, ultracold atomic systems can be tuned in their geometry, dimensionality, interparticle interactions and density and number of particles. Besides, since densities are much smaller, vortex cores are comparatively bigger allowing for direct, optical image of vortex tangles, although this has also been made possible in liquid Helium.

Naturally, there are not only advantages in studying turbulence in Bose-Einstein condensates: since we are dealing with small systems, with a million particles at best in most experimental setups, the number of vortices is also limited and turbulence probably differs quite a bit in these systems compared to classical fluids and superfluid liquid Helium.

With this context in mind, namely: the possibilities allowed by ultracold systems and their limitations, we discuss below the perspectives on the study of quantum turbulence, from an experimental point-of-view, for the next years.

Mechanism of vortex tangle generation - Turbulence is characterized by the presence of a vortex tangle: a set of vortices distributed in the sample with no clear ordering or preferential direction. While in liquid Helium the mechanism widely believed to be the responsible for vortex generation is the viscosity coming from the counter-flow of superfluid and normal components, in BECs the situation is not clear.

Theoretical simulations point in two opposite paths: some point that a certain degree of dissipation has to be introduced in order generate vortices and vortex tangles, being the dissipative part a phenomenological account of the effects of the thermal cloud, other are able to produce vortices with a pure, zero-temperature sim-

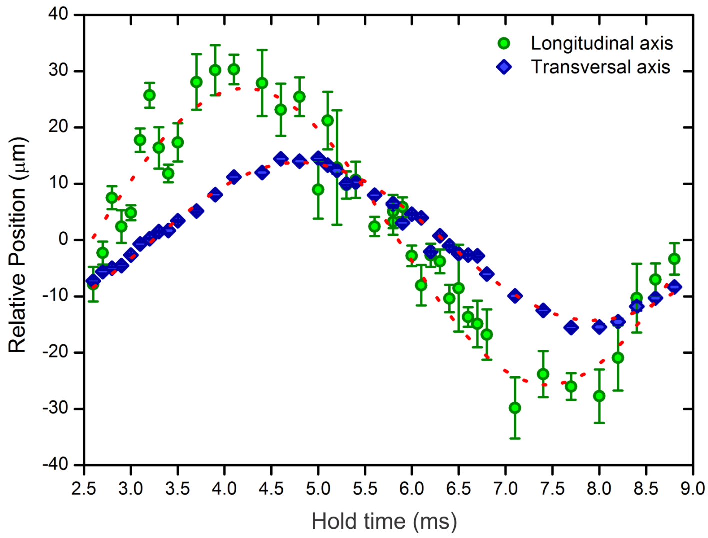

Fig. 5 – Evidence of counter-flow between thermal and condensate cloud as possible mechanism for turbulence formation in trapped atomic BECs.

ulation of the Gross-Pitaevskii equation.

In previous works we have seen evidences of counter-flow in our excited Bose-Einstein condensate, with thermal and condensate components moving out-of-phase and against each other [27]. We have also observed the presence of round structures forming in the border of a BEC and the thermal cloud, indicating that the thermal cloud has indeed some degree of contribution in the vortex generation.

In that sense, the study of vortices generation and evolution to a vortex tangle as a function of the size of the thermal component is definitely a road one has to navigate in order to investigate those mechanisms. Clearly, one cannot get rid of the thermal cloud completely, but very small thermal components can be produced as well as vanishingly small BEC components. In fact, one question that is fair to ask is if one can generate turbulence in a purely thermal cloud as a model of a classical fluid. We will come back to this topic when discussing the decay of the turbulent state.

Kolmogorov scales - As already mentioned in the previous section, the characteristic vortex tangle of turbulence in not completely random but it obeys some scaling law which, astonishingly, is, to date, valid for classical and quantum fluids wherever it has been tested. The verification of this $-5/3$ Kolmogorov law would be the by far one of the most spectacular results towards an universal description of turbulence. However, there a few *a priori* questions one has to ask in order to investigate Kolmogorov's law. Does turbulence in BECs achieve the stationary state necessary

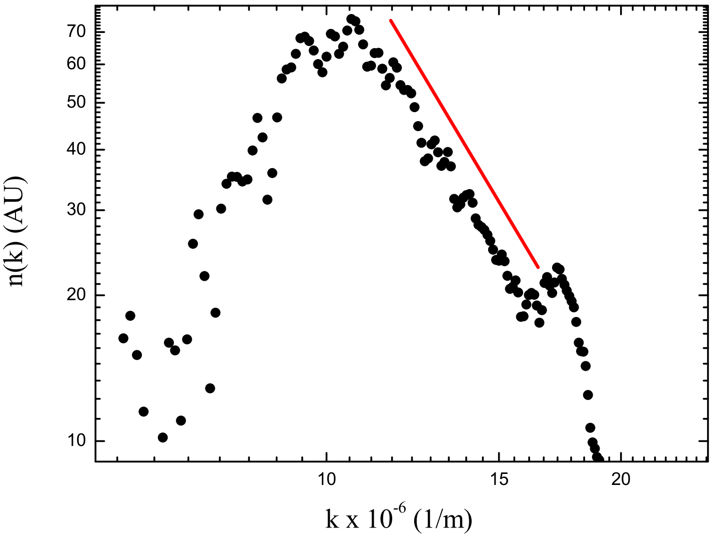

Fig. 6 – Modeling the momentum distribution showing indicatives of a scaling law.

to the Kolmogorov scale the settle? Is the number of vortices in the BEC enough to characterize a distribution as depicted by Kolmogorov or those are to few? What is the role of the inhomogeneity of the system in the evolution of the BEC to the stationary state: does it occur in one dimension but not in other? Even with those conditions valid, is the decay of turbulence, for any reason, not obeying Kolmogorov's scaling?

We have tried to answer those questions by analyzing absorption images of vortex tangles and, at this point, we cannot offer any definitive answer, since our measurements are not conclusive. Nevertheless, there is still a lot of room for improvements, in the conditions of the measurements, in the method of analysis of the images and in the understanding on how to extract the momentum distribution of the cloud. Specifically this last point, how to extract the momentum distribution from 2D integrated density profiles of 3D vortex tangles constitutes a huge challenge for experimentalist working on quantum turbulence.

Decay of turbulence - One important aspect related to a turbulent flow is how it decays. It is widely accepted that turbulence decays through Richardson cascades by vortex recombination and that is one of the reasons behind Kolmogorov's scaling laws. Observing such process, dynamically, the way it was done recently in liquid Helium would be a breakthrough observation.

For these processes to be observable, one would have to produce turbulence in a system that combines two features together: high-resolution imaging, like the recent developed atomic-microscopes and non-destructive imaging of the cloud. In particular, this last feature is fairly important: since turbulence is a dynamical, chaotic process, no experimental realization will be the same. In that way, one would have to be able to observe and follow a vortex reconnection live, while it happens, preferably in-trap.

Besides, observing a vortex recombination process is a definitive proof of the turbulence decay process in a BEC? How can one quantify this decay experimentally and how are the experimental observable that allow to quantify decay rates? We are most likely left with bulk quantities to observe like the volume of the cloud, its heating or simple atom-loss. While theorist play with those possible observable, experimentalists have to find ways to pin down which of those effects are related to the decay of turbulence and can be good witnesses to quantify it.

Tuning Turbulence - One interesting direction the study of turbulence can take only on ultracold atoms is the possibility to tune inter particle interactions almost at will in the systems. The interaction energy in ultracold gases is usually the dominant energetic contribution, despite being very simply described: a single parameter, the s-wave scattering length is enough to characterize completely the interatomic interaction energy. Making use of Fano-Feshbach resonances, one is able to tune the scattering length in a wide range. For some atomic species, it is possible to bring the scattering length very close to zero, allowing the realization of an ideal gas model. It can be brought to negative (attractive) values, where BECs become unstable against collapse or to the very positive (repulsive) range.

How turbulence, in all its aspects, formation, decay and scaling laws, would react to those different interaction environments? To our knowledge, no theoretical prediction has been made on this issue. Experimentally, one would have to be able to generate turbulence in a sample with easily tunable interactions, like ^{39}K, ^{6}Li or ^{85}Rb and play around with them, maybe changing interactions before the formation of the turbulent state and during its evolution/decay. Are there interaction values where turbulence formation is prevented or enhanced? Can we protect the turbulent state if we change the interaction strength while it evolves? Those and many other questions could be investigated as one study turbulence in tunable BECs.

In fact, in this context, one question arises concerning energy balance in the turbulent state. As we have seen before, the free expansion of a turbulent cloud shows a self-similar behavior, unlike regular BECs or thermal clouds. This phenomena base been explained by an energy balance consideration: when the turbulent state takes place, the rotation energy is dominant over the interaction energy and defines the free evolution of the cloud during expansion. In the other hand, regular BECs are not stable against collapse when interactions are made attractive. The question is: in a turbulent BEC where interactions are suddenly made attractive, would collapse still takes place or the rotational energy present would also dominate the dynamical behavior of the cloud? This is possibly an interesting challenge to both theory and experiments to observe.

Two quantum fluids turbulence - Another interesting direction one could take experimentally in the study of turbulence (and even single vortices) is how it can be transferred from one fluid to a second one. The idea here is to produce a mixture of two superfluids, which is already common-place in several experimental setups, with, e.g., K and Rb, or Li and Na. Then, using a magical wavelength for one of the components, one could stir the second component in order to produce vortices and eventually turbulence in one of the components only, not affecting the other. The effect of the presence of the second component in the formation of the vortices and turbulence would already be interesting to observe [28]. Besides, one could track how turbulence decays and/or is transferred between the two components. In fact, here, tunable interactions would also play an interesting role: one could generate turbulence in one of the components and quickly ramp the interspecies scattering length to an attractive side, forcing the formation of molecules. Is turbulence preserved, transformed, or simply vanishes as the individual atoms bind in diatomic molecules? Does turbulence prevent the formation of molecules? All those questions could be addressed in an experimental setup of mixtures and turbulence.

5. CONCLUSIONS

To summarize, atomic BECs are currently the most accessible system in which to study the small scale properties of turbulent vortex flow. Theoretical investigations have begun to build up a picture of vortex dynamics and the processes contributing to the forcing and decay of turbulence at small scales and there is a favorable scenario for experimental verification of those mechanisms in atomic BECs. At large scales, the features of classical turbulence is an emergent feature of quantum turbulence [29], suggesting that research with BEC systems may provide insight into some of the outstanding questions of turbulence. The study and understanding of turbulence in Bose-Einstein condensation of trapped neutral atoms is still at the very beginning, but has a great potential to become an important and exciting research field.

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