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Coherent control of nonlinear mode transitions in Bose–Einstein condensates

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Abstract

In the present work, we consider the generation of excited nonlinear topological modes of Bose–Einstein condensates. These nonlinear modes present a number of interesting properties and can be used in many applications, such as in matter-wave optics experiments. The resonant generation of nonlinear modes by modulating either the trapping potential or the scattering length have been previously reported. Here, we propose the simultaneous use of both modulations with an adjustable relative phase between them in order to coherently control the transition from the ground state to an excited mode. Within the framework of the Gross–Pitaevskii equation (GPE), we show that the transition probability and the transition duration can be controlled by means of the manipulation of the relative phase. In particular, the acceleration of the transition can be useful to avoid dissipative effects of the condensate with its surroundings. In our analysis, we employ approximate analytical techniques, including a perturbative treatment, and numerical calculations for the GPE.

Keywords: coherent control, nonlinear topological modes, non-ground-state condensates, Bose–Einstein condensates, phase control

(Some figures may appear in colour only in the online journal)

1. Introduction

The control of quantum many-body systems' dynamics is of fundamental importance for physics [1, 2]. In particular, controlling Bose–Einstein condensates (BEC) of ultracold atoms offers the possibility for investigating large-scale quantum phenomena and for implementing quantum technologies, such as quantum computers and quantum simulators [3, 4]. A relevant goal in this many-body system is the generation of condensates in excited topological modes, which exhibit several interesting features such as mode locking, critical dynamics, interference patterns and atomic squeezing [5]. Additionally, condensates prepared in such excited modes have been used in breakthrough matter-wave optics experiments [6, 7].

In the mean-field picture, nonlinear modes of BEC are represented by stationary solutions of the Gross–Pitaevskii equation (GPE), and are also termed topological modes [8]. It has been shown that a given nonlinear mode can be generated by a time-dependent modulation of the trapping potential tuned to the resonance between the ground state and the desired excited nonlinear mode [9]. Alternatively, it is also possible to generate a nonlinear mode by means of a resonant time-dependent modulation of the scattering length [10, 11], which can be produced by an alternating magnetic field close to a Feshbach resonance [11–14].

In the present work, we explore the use of both modulations to resonantly generate nonlinear topological modes. Different quantum transition amplitudes can be attributed to the modulation of the trap and to the modulation of the scattering length [9, 10]. Since these distinct excitation pathways connect an initial state to the same final target state, the total transition probability depends on the modulus squared of the sum of the corresponding complex transition amplitudes. Thus, upon changing the relative phase of these amplitudes, it is possible to modify the final yield. This interference between quantum transition amplitudes is the underlying principle of coherent control [15–22]. Here, we consider the role of the relative

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phase of the modulations on the transitions between nonlinear modes.

Coherent control concepts have contributed towards the development of quantum optimal control theory (QOCT), which seeks to find external controls to drive a given transition, maximizing some performance criteria [23–25]. There have been several experimental and theoretical successful implementations of quantum optimal control algorithms for BEC [26–30]. It is worth noting that QOCT has been applied to maximize the transition between nonlinear modes utilizing temporal and spatial modulation of both the trapping and the scattering length [31]. However, the control fields obtained from QOCT are complex and difficult to be implemented in laboratory. Additionally, the detailed role of the interference between both modulations has not been addressed.

The paper is organized as follows: in section 2, we introduce the theoretical framework for the production of the nonlinear coherent modes. In section 3, a time average technique is applied to the GPE allowing the description of the dynamics by only two modes. A perturbative treatment is used to obtain an analytic expression for the transition probability in section 4. Numerical results confirming the predictions are presented in section 5. Finally, conclusions are drawn in section 6.

2. Excitation of nonlinear modes

We consider the dynamics of a Bose gas wavefunction $\Psi(\mathbf{r},t)$ described by the Gross–Pitaevskii equation [32, 33],

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t)=\left(-\frac{\hbar^2}{2m}\nabla^2+V(\mathbf{r},t)+g(\mathbf{r},t)N|\Psi(\mathbf{r},t)|^2\right)\Psi(\mathbf{r},t),\qquad(1)$$

where m is the atomic mass and N is the number of atoms in the condensate. The trapping potential $V(\mathbf{r},t)$ is composed by two parts,

$$V(\mathbf{r},t)=V_{\mathrm{trap}}(\mathbf{r})+V_{\mathrm{mod}}(\mathbf{r},t),\qquad(2)$$

where $V_{\mathrm{trap}}(\mathbf{r})$ is a fixed trapping potential and $V_{\mathrm{mod}}(\mathbf{r},t)$ a time-dependent modulating potential. The nonlinear interaction amplitude $g(\mathbf{r},t)$ is also composed by two parts,

$$g(\mathbf{r},t)=g_0+g_{\mathrm{mod}}(\mathbf{r},t),\qquad(3)$$

where $g_0=4\pi\hbar^2a_0/m$ is a fixed nonlinearity and $g_{\mathrm{mod}}(\mathbf{r},t)=4\pi\hbar^2a(\mathbf{r},t)/m$ is a modulating nonlinearity, with the s-wave scattering length a_s near a Feshbach resonance being written as $a_s=a_0+a(\mathbf{r},t)$. The normalization condition of the wavefunction is $\int d\mathbf{r}|\Psi(\mathbf{r},t)|^2=1$.

With both modulations turned off, i.e., $V_{\mathrm{mod}}(\mathbf{r},t)=0$ and $g_{\mathrm{mod}}(\mathbf{r},t)=0$, the system can be described by the nonlinear Hamiltonian H_0,

$$H_0[\phi(\mathbf{r})]=-\frac{\hbar^2}{2m}\nabla^2+V_{\mathrm{trap}}(\mathbf{r})+g_0N|\phi(\mathbf{r})|^2.\qquad(4)$$

We consider the nonlinear topological modes of H_0, which are solutions of the eigenvalue problem [9],

$$H_0[\phi_n(\mathbf{r})]\phi_n(\mathbf{r})=\mu_n\phi_n(\mathbf{r}),\qquad(5)$$

with n generally being a multi-index label for the quantum states and μ_n the corresponding chemical potential. Here, we are concerned with inducing transitions between stationary solutions $\phi_n(\mathbf{r})$. This task can be accomplished by means of modulating the trapping potential with an oscillatory field with frequency ω_t [9, 34]. Alternatively, one may also modulate the atomic scattering length with frequency ω_g [10]. We assume that both modulations are present and that they possess a phase difference given by θ,

$$V_{\mathrm{mod}}(\mathbf{r},t)=V(\mathbf{r})\cos(\omega_t t+\theta),\qquad(6)$$

and

$$g_{\mathrm{mod}}(\mathbf{r},t)=g(\mathbf{r})\cos(\omega_g t).\qquad(7)$$

For definiteness, we consider a transition between an initial state $\phi_1(\mathbf{r})$ to a final state $\phi_2(\mathbf{r})$, with $\mu_1<\mu_2$, and we associate the resonance frequency $\omega_{21}=(\mu_2-\mu_1)/\hbar$ with this transition. As we have already pointed out, the transition can be induced by resonant modulations and in this case the system can be approximately described solely by the topological modes involved in the transition [9, 34]. Thus, we assume that the modulating frequencies ω_t and ω_g are close to ω_{21}. More specifically, we assume that $|\Delta\omega_t/\omega_t|\ll 1$ and $|\Delta\omega_g/\omega_g|\ll 1$, with the detunings defined by $\Delta\omega_t=\omega_t-\omega_{21}$ and $\Delta\omega_g=\omega_g-\omega_{21}$.

3. Two-level approximation

In order to simplify the dynamical equations, we consider that the wave function $\Psi(\mathbf{r},t)$ can be written as an expansion in terms of nonlinear modes [9],

$$\Psi(\mathbf{r},t)=\sum_m c_m(t)\phi_m(\mathbf{r})\exp\left(-i\mu_m t/\hbar\right),\qquad(8)$$

and that the following condition is valid,

$$\frac{\hbar}{\mu_m}\left|\frac{dc_m}{\mathrm{d}t}\right|\ll 1,\qquad(9)$$

meaning that the $c_m(t)$ are slow functions of time in comparison with $\exp(-i\mu_m t/\hbar)$.

Substituting expansion (8) in the GPE and performing a time-averaging procedure, with the coefficients $c_m(t)$ treated as quasi-invariants, will result in a set of coupled nonlinear differential equations for the coefficients $c_m(t)$ (see reference [8] for details). As a consequence, if at the initial time only the levels $n=1,2$ are populated and the frequencies of the modulations are close to ω_{21}, the only relevant coefficients are $c_1(t)$ and $c_2(t)$ and we obtain the set of equations,

$$i\frac{dc_1}{\mathrm{d}t}=\alpha_{12}|c_2|^2c_1+\frac{1}{2}\beta_{12}c_2\mathrm{e}^{i(\Delta\omega_t t+\theta)}+\frac{1}{2}\mathrm{e}^{-i\Delta\omega_g t}c_2^*{c_1}^2\gamma_{21}+\frac{1}{2}\mathrm{e}^{i\Delta\omega_g t}\left(|c_2|^2c_2\gamma_{12}+2|c_1|^2c_2\gamma_{21}^*\right),\qquad(10a)$$

$$i\frac{dc_2}{\mathrm{d}t}=\alpha_{21}|c_1|^2c_2+\frac{1}{2}\beta_{12}^*c_1\mathrm{e}^{-i(\Delta\omega_t t+\theta)}+\frac{1}{2}\mathrm{e}^{i\Delta\omega_g t}c_1^*{c_2}^2\gamma_{12}+\frac{1}{2}\mathrm{e}^{-i\Delta\omega_g t}\left(|c_1|^2c_1\gamma_{21}+2|c_2|^2c_1\gamma_{12}^*\right),\qquad(10b)$$

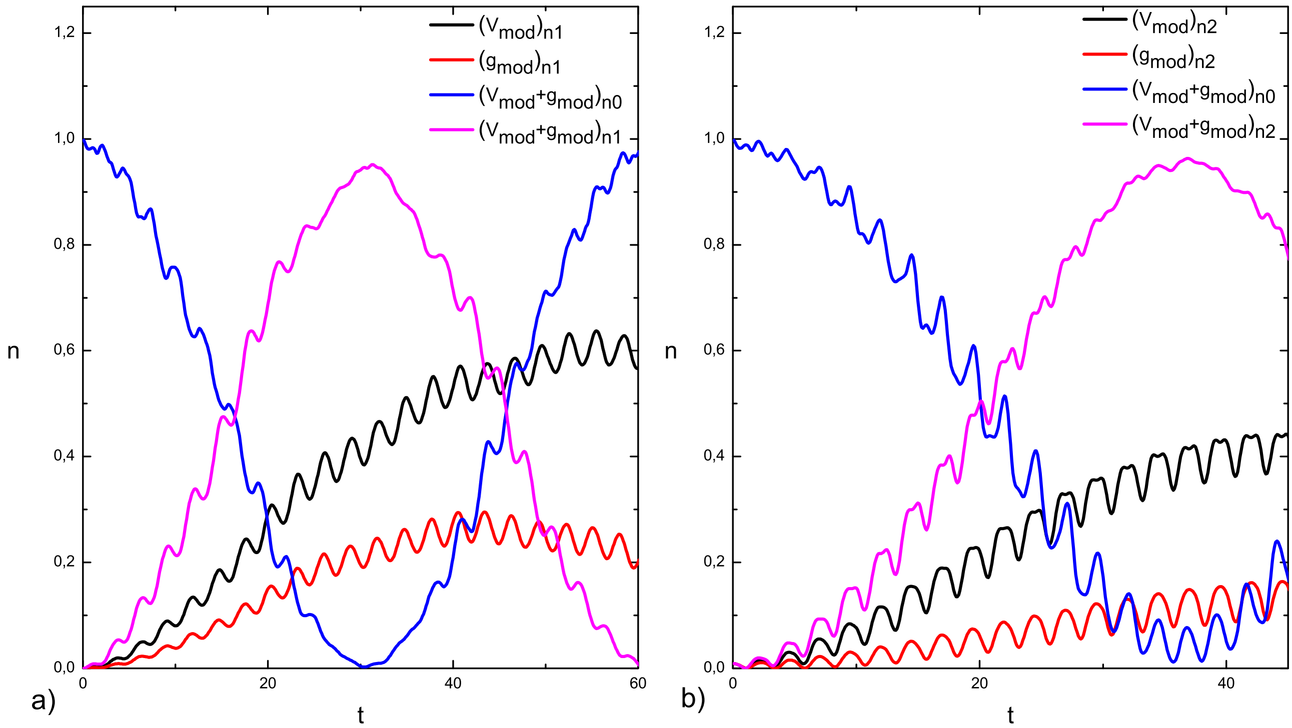

Figure 1. (a) Populations $n_j \equiv |\langle \phi_j|\ \Psi(x,t)\rangle|^2$ of the ground and the first excited modes versus time for double modulation and relative phase $\theta = 0$ ($V_{\text{mod}} + g_{\text{mod}}$), along with the population of the first mode for trap-only modulation (V_{mod}) and for nonlinearity-only modulation (g_{mod}), with $A_t = 0.1$ and $A_g = 0.3$ amplitudes. (b) The same as (a) but for the population of the second excited mode and for $A_t = 0.08$ and $A_g = 0.4$.

with the coupling constants α_{mn}, β_{mn} and γ_{mn} given by

$$\alpha_{mn} \equiv g_0 \frac{N}{\hbar} \int d\mathbf{r} |\phi_m(\mathbf{r})|^2 \left[2|\phi_n(\mathbf{r})|^2 - |\phi_m(\mathbf{r})|^2\right], \quad (11)$$

$$\beta_{mn} \equiv \frac{1}{\hbar} \int d\mathbf{r} \phi_m^*(\mathbf{r}) V(\mathbf{r}) \phi_n(\mathbf{r}), \quad (12)$$

and

$$\gamma_{mn} \equiv \frac{N}{\hbar} \int d\mathbf{r} \phi_m^*(\mathbf{r}) g(\mathbf{r}) |\phi_n(\mathbf{r})|^2 \phi_n(\mathbf{r}). \quad (13)$$

In order to fulfill condition (9), the couplings are assumed to be much smaller than the transition frequency, i.e., $|\alpha_{mn}/\omega_{21}| \ll 1$, $|\beta_{mn}/\omega_{21}| \ll 1$ and $|\gamma_{mn}/\omega_{21}| \ll 1$ [8].

From the dynamical equation (10), we note that the modulation of the trap couples the modes by means of the linear term containing β_{mn} and the nonlinear term with α_{mn}, while the modulation of the scattering length couples the modes by means of distinct nonlinear terms containing γ_{mn}. Therefore, the linear and the nonlinear terms can interfere with each other and by varying the phase θ, this interference can be controlled. We also note that when the modulation of the scattering length is absent, $g_{\text{mod}}(\mathbf{r}, t) = 0$, an approximate analytic solution to (10) has been derived, which shows that the population oscillates between the two states with a Rabi-like chirped frequency [9]. This chirped frequency depends on the populations $|c_m(t)|^2$. Unfortunately, such approximate solutions for $g_{\text{mod}}(\mathbf{r}, t) \neq 0$ is not possible due to the presence of terms with $c_m(t)^2$. Thus, we resort to perturbation theory to gain more insight into the role of θ in the transition.

4. Perturbative approximation

We assume that modulating fields (6) and (7) can be considered as small perturbations in order to apply canonical perturbation theory [35]. To this end, we introduce a perturbation parameter $\lambda \ll 1$ such that the Hamiltonian can be written as

$$H[\Psi] = H_0[\Psi] + \lambda \left[V_{\text{mod}}(\mathbf{r}, t) + g_{\text{mod}}(\mathbf{r}, t) N |\Psi|^2\right]. \quad (14)$$

The choice of a single parameter for both modulations is justified in order to consider the modulations on equal footing and consequently to simplify the subsequent treatment. We are interested in the transition probability from state ϕ_1 to state ϕ_2, often defined as $P_{1\to2}(t) = |\langle \phi_2 | \Psi(\mathbf{r}, t)\rangle|^2$ and we assume the initial conditions $c_1(0) = 1$ and $c_2(0) = 0$. However, from the approximations of the last section, one can deduce the normalization condition for the coefficients, $\sum_m |c_m(t)|^2 = 1$. Thus, despite of the fact that the set of nonlinear modes is not orthogonal, we can define the transition probability simply as $P_{1\to2}(t) = |c_2(t)|^2$.

As usual, we write the coefficients $c_j(t)$ as a power series in λ,

$$c_j(t) = c_j^{(0)}(t) + \lambda c_j^{(1)}(t) + \lambda^2 c_j^{(2)}(t) + \cdots, \quad (15)$$

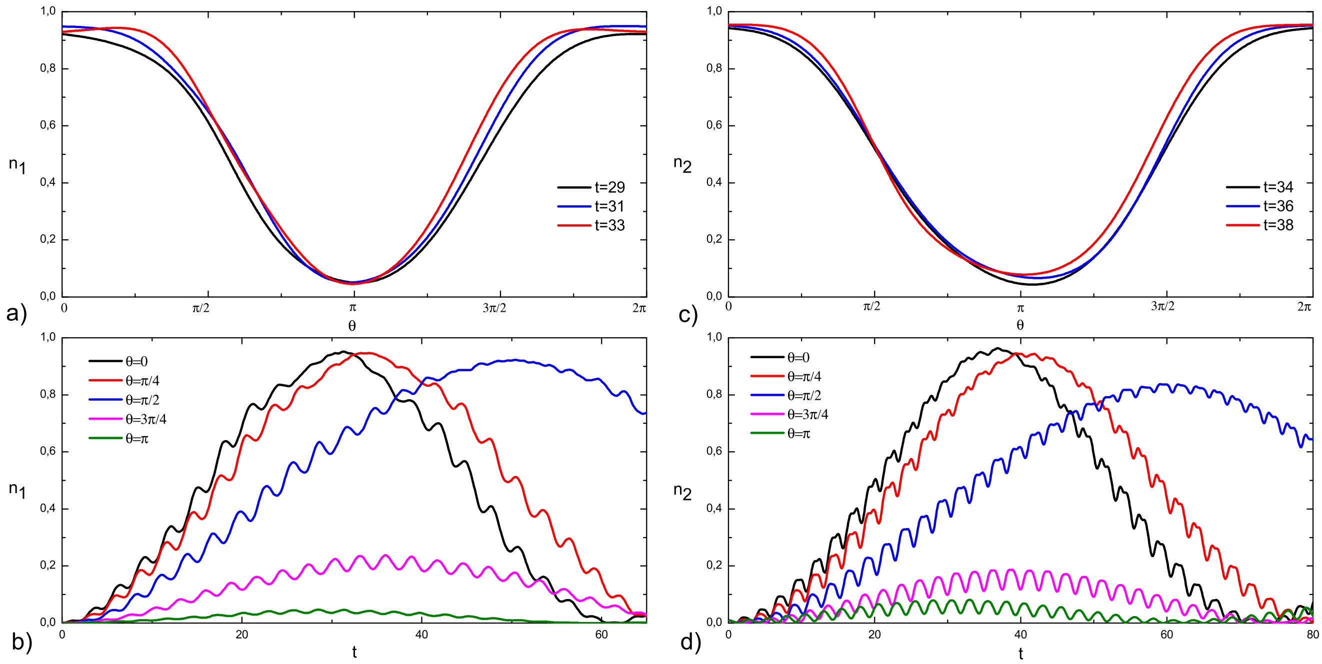

Figure 2. (a) Population of the first excited mode versus relative phase of the modulations at some fixed times and parameters of panel (a) of figure 1. (b) Population of the first excited state versus time for some fixed phases [same parameters of (a)]. (c) Population of the second excited state at some fixed times and parameters of panel (b) of figure 1. (d) Population of the second excited mode versus time for some fixed phases [same parameters of (c)].

and substitute this series into dynamic equation (10) equating the like powers of λ. To zeroth order, this yields,

$$i\frac{dc_1^{(0)}}{\mathrm{d}t} = \alpha_{12}|c_2^{(0)}|^2 c_1^{(0)}, \tag{16a}$$

$$i\frac{dc_2^{(0)}}{\mathrm{d}t} = \alpha_{21}|c_1^{(0)}|^2 c_2^{(0)}. \tag{16b}$$

And we obtain the zeroth order solution as being $c_1^{(0)}(t) = 1$, $c_2^{(0)}(t) = 0$.

To first order in λ, the equations are

$$\begin{aligned} i\frac{dc_1^{(1)}}{\mathrm{d}t} &= \alpha_{12}\left[\left(c_2^{*(0)}c_2^{(1)} + c_2^{*(1)}c_2^{(0)}\right)c_1^{(0)} + |c_2^{(0)}|^2 c_1^{(1)}\right] \\ &\quad + \frac{1}{2}\beta_{12}c_2^{(0)}\mathrm{e}^{i(\Delta\omega_l t+\theta)} + \frac{1}{2}\mathrm{e}^{i\Delta\omega_g t} \\ &\quad \times \left[|c_2^{(0)}|^2 c_2^{(0)}\gamma_{12} + 2|c_1^{(0)}|^2 c_2^{(0)}\gamma_{21}^*\right] \\ &\quad + \frac{1}{2}\mathrm{e}^{-i\Delta\omega_g t}c_2^{*(0)}c_1^{(0)2}\gamma_{21}, \end{aligned} \tag{17a}$$

$$\begin{aligned} i\frac{dc_2^{(1)}}{\mathrm{d}t} &= \alpha_{21}\left[\left(c_1^{*(0)}c_1^{(1)} + c_1^{*(1)}c_1^{(0)}\right)c_2^{(0)} + |c_1^{(0)}|^2 c_2^{(1)}\right] \\ &\quad + \frac{1}{2}\beta_{12}^* c_1^{(0)}\mathrm{e}^{-i(\Delta\omega_l t+\theta)} + \frac{1}{2}\mathrm{e}^{-i\Delta\omega_g t} \\ &\quad \times \left[|c_1^{(0)}|^2 c_1^{(0)}\gamma_{21} + 2|c_2^{(0)}|^2 c_1^{(0)}\gamma_{12}^*\right] \\ &\quad + \frac{1}{2}\mathrm{e}^{i\Delta\omega_g t}c_1^{*(0)}c_2^{(0)2}\gamma_{12}. \end{aligned} \tag{17b}$$

Substituting the zeroth order solutions into (17a) and (17b), these equations simplify to

$$i\frac{dc_1^{(1)}}{\mathrm{d}t} = 0, \tag{18a}$$

$$i\frac{dc_2^{(1)}}{\mathrm{d}t} = \alpha_{21}|c_1^{(0)}|^2 c_2^{(1)} + \frac{1}{2}\beta_{12}^*\mathrm{e}^{-i(\Delta\omega_l t+\theta)} + \frac{1}{2}\gamma_{21}\mathrm{e}^{-i\Delta\omega_g t}. \tag{18b}$$

Thus, within first order, $c_1^{(1)}(t) = 0$ and

$$\begin{aligned} c_2^{(1)}(t) &= -\frac{1}{2}\frac{\beta_{12}^*}{(\alpha_{21} - \Delta\omega_l)}\mathrm{e}^{-i\theta}\left(\mathrm{e}^{-i\Delta\omega_l t} - 1\right) \\ &\quad - \frac{1}{2}\frac{\gamma_{21}}{(\alpha_{21} - \Delta\omega_g)}\left(\mathrm{e}^{-i\Delta\omega_g t} - 1\right). \end{aligned} \tag{19}$$

Thus, we can write the transition probability within first order as

$$\begin{aligned} P_{1\to 2}(t) &\approx \frac{|\beta_{12}|^2}{2|\alpha_{21} - \Delta\omega_l|^2}[1 - \cos(\Delta\omega_l t)] + \frac{|\gamma_{21}|^2}{2|\alpha_{21} - \Delta\omega_g|^2} \\ &\quad \times \left[1 - \cos(\Delta\omega_g t)\right] + \frac{\beta_{12}\gamma_{21}}{4(\alpha_{21} - \Delta\omega_l)^*(\alpha_{21}\Delta\omega_g)}\mathrm{e}^{i\theta} \\ &\quad \times \left[1 + \mathrm{e}^{i(\Delta\omega_l - \Delta\omega_g)t} - \mathrm{e}^{i\Delta\omega_l t} - \mathrm{e}^{-i\Delta\omega_g t}\right] \\ &\quad + \frac{\gamma_{21}^*\beta_{12}^*}{4(\alpha_{21} - \Delta\omega_g)^*(\alpha_{21} - \Delta\omega_l)}\mathrm{e}^{-i\theta} \\ &\quad \times \left[1 + \mathrm{e}^{-i(\Delta\omega_l - \Delta\omega_g)t} - \mathrm{e}^{-i\Delta\omega_l t} - \mathrm{e}^{i\Delta\omega_g t}\right]. \end{aligned} \tag{20}$$

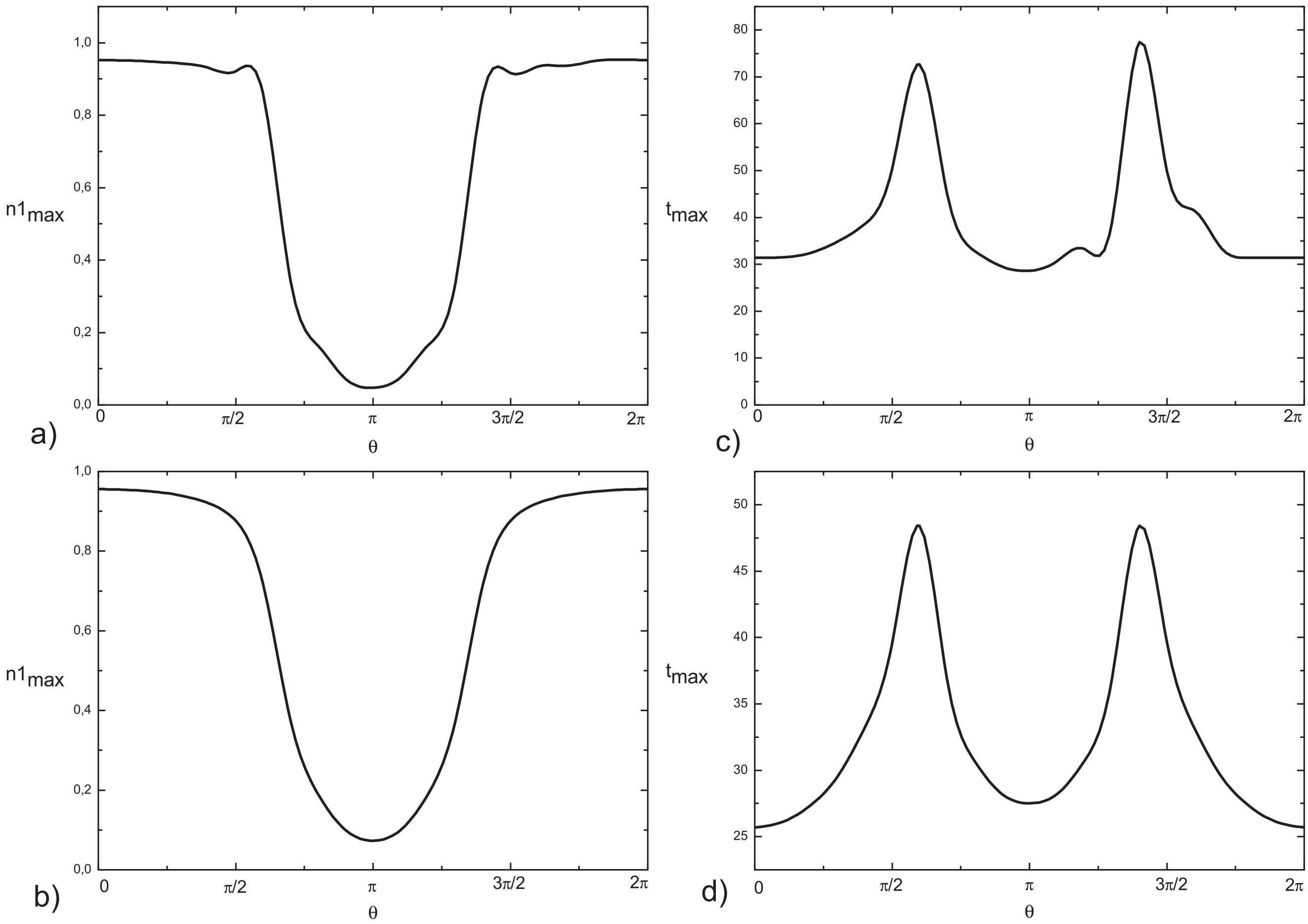

Figure 3. (a) Maximum population of the first excited mode versus relative phase of the modulations. (b) Same of (a) obtained from the two-level approximation. (c) Time to reach the maximum population versus relative phase. (d) Same of (c) obtained from the two-level approximation.

When the frequencies of the modulations are equal, $\Delta\omega_t = \Delta\omega_g = \Delta\omega$, the expression for the transition probability simplifies to

$$P_{1\to2} \approx \left[\frac{|\beta_{12}|^2 + |\gamma_{21}|^2 + 2\Re\{\beta_{12}\gamma_{21}\ \exp(i\theta)\}}{|\alpha_{21} - \Delta\omega|^2}\right] \sin^2\left(\frac{\Delta\omega t}{2}\right), \tag{21}$$

where $\Re\{\cdot\}$ stands for the real part.

Although the above expression is only valid for very short times, for which the population of the state ϕ_1 is still very close to 1, expression (21) evidences the role of the relative phase θ on the transition. For instance, if $(\beta_{12}\gamma_{21})$ is real and positive, then for $\theta = \pi$ the modulations will act destructively decreasing the transition probability, whereas for $\theta = 0$ they will act constructively. The extent of the interference will be dictated by the magnitude of the couplings parameters β_{12} and γ_{12}. Additionally, according to (21), if the modulation of the scattering length is absent, then variation of θ plays no role in the dynamics.

5. Numerical results

We have carried out direct numerical calculations of the GPE solving equation (1) in its 1D version,

$$\mathrm{i}\frac{\partial}{\partial t}\Psi(x,t) = H[\Psi]\Psi(x,t), \tag{22}$$

with the nonlinear Hamiltonian given by

$$H[\Psi] = -\frac{\partial^2}{\partial x^2} + V(x,t) + g(x,t)|\Psi(x,t)|^2, \tag{23}$$

and considering arbitrary units such that $\hbar = m = N = g_0 = 1$. The nonlinear Hamiltonian operator has been written as a matrix over a grid of points according to the Chebyshev spectral method [36, 37].

In order to solve the time-dependent equation (22), we express the corresponding time evolution operator, which connects the initial time $t = 0$ to the final time $t = t_f$, in N small time-step Δt evolution operators,

$$U(t_f, 0) = \prod_{k=1}^{N} U\,(k\Delta t, (k-1)\Delta t)\,. \tag{24}$$

Each one of the small time-step evolution operators is calculated as an expansion in Chebyshev polynomials [38–40],

$$U\,(k\Delta t, (k-1)\Delta t) \approx \sum_{n=0}^{N_p} a_n\chi_n(-iH[\Psi((k-1)\Delta t)]\Delta t), \tag{25}$$

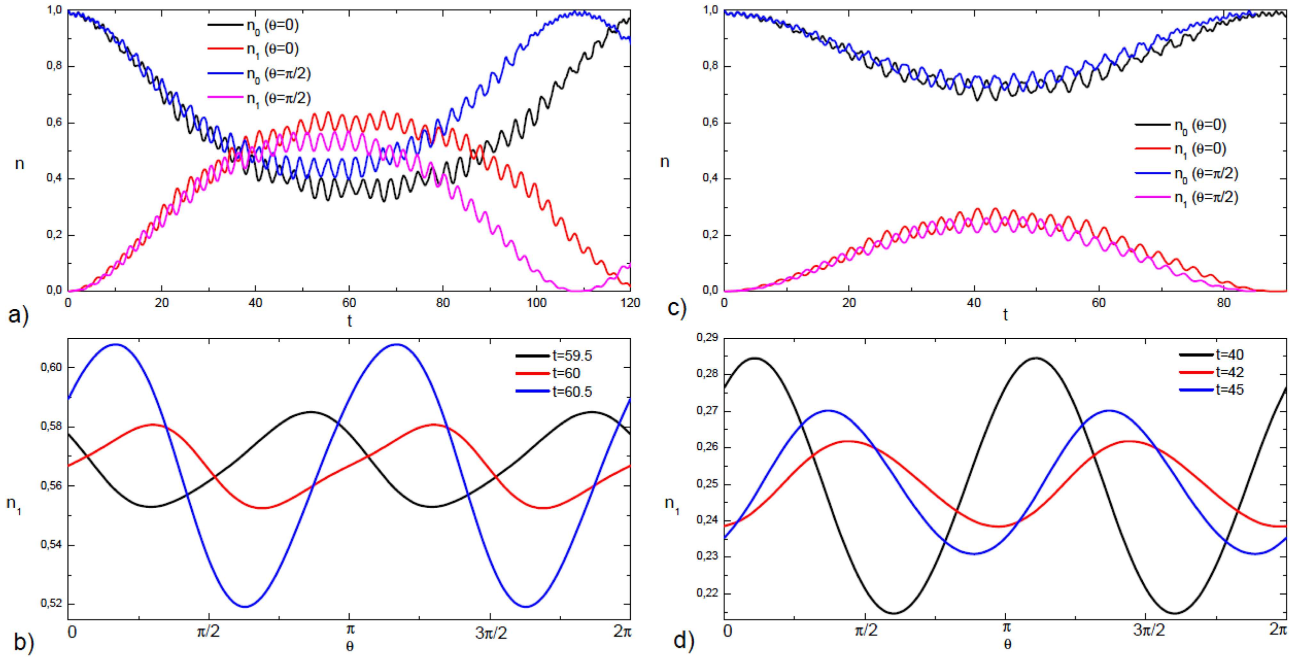

Figure 4. (a) Population of ground state and the first excited mode versus time for a system driven by the trap modulation only, with amplitude $A_t = 0.1$ and relative phase $\theta = 0$ and $\theta = \pi/2$. (b) Population versus absolute phase for some fixed times [same parameters as (a)]. (c) Population of first excited state versus time for a system driven by the scattering length modulation only with fixed $A_g = 0.3$ and relative phase $\theta = 0$ and $\theta = \pi/2$. (d) Population of the first excited mode versus absolute phase for some fixed times [same parameters as (c)].

where a_n are the expansion coefficients, χ_n are the complex Chebyshev polynomials and N_p sets the number of terms in the expansion. The propagation of the wavefunction in the kth time step is obtained by applying $U(k\Delta t, (k-1)\Delta t)$ to the wavefunction calculated in the previous step $\Psi((k-1)\Delta t)$. The relaxation method, which in essence consists in performing propagation with imaginary time $t \to \mathrm{i}t$, has been applied to obtain the ground state [41]. The excited modes of the condensate have been determined by the spectrum-adapted scheme described in reference [31]. We have also found very good agreement comparing our results for the time evolution with those from references [31, 42].

For harmonic trapping potentials and modulating fields with linear behavior with distance, no transition to excited modes is possible through modulation of the trap [5]. Thus, we have fixed the trapping potential to $V_{\text{trap}}(x,t) = x^4/4$, allowing for a simple form of the spatial dependence of $V(x)$. For this trap, we have obtained the chemical potentials $\mu_0 = 0.808$, $\mu_1 = 1.857$, and $\mu_2 = 3.279$, for the ground, first and second nonlinear modes, respectively.

We have considered transitions from the ground state to the first and to the second excited modes. In the first case, we have set $g(x) = A_g x$ and $V(x) = A_t x$, while in the second case, $g(x) = A_g x^2$ and $V(x) = A_t x^2$. The frequencies of the modulations are set to be equal $\omega_t = \omega_g = \omega$ and are chosen to satisfy the resonance condition for each target. The error between the frequency applied and the true one is estimated for a detuning of $\Delta\omega \approx 0.01$.

Figure 1 compares single modulation with double modulation for $\theta = 0$ by showing the corresponding target population dynamics, denoted by $n_j \equiv |\langle\phi_j|\Psi(x,t)\rangle|^2$. In panel (a), the target is the first excited state, while in panel (b) the target is the second excited state. In both cases, we observe the double modulation performing a faster transition than the individual modulations. Additionally, the double modulation enhances the target population beyond that of the sum of the individual modulations, which is an evidence of quantum interference.

Panels (a) and (c) of figure 2 show the population of the target modes, the first and second modes, respectively, as a function of the relative phase of the modulations for some fixed times. For $\theta = \pi$ the transition is essentially inhibited, whereas for $\theta = 0$ the target population is enhanced, in agreement with the perturbative analysis. Panels (b) and (d) show the corresponding population dynamics of the target modes for some fixed phases. We observe that as the phase varies from 0 to $\pi/2$, the transitions become slower, while transferring about the same number atoms. As the phase varies from $\pi/2$ to π, the transitions time is shortened, but the transfer of atoms is significantly decreased. This behavior has not been captured by the perturbative expression and may be attributed to the nonlinear character of the GPE.

Panel (a) of figure 3 shows the maximum value of n_1 as a function of the relative phase. We observe that the maximum population has an abrupt decrease as θ goes from $\pi/2$ to π and an abrupt increase as θ goes from π to $3\pi/2$. Panel (c) shows the time span to reach the corresponding maximum value of n_1. For θ just above $\pi/2$ and for θ just below $3\pi/2$ there is a significant increase of the time to accomplish the transition. For comparative purpose, panels (b) and (d) present the corresponding results obtained from the two-level approximation by solving equation (10) using a fourth-order Runge–Kutta method.

We have obtained for the coupling parameters: $\alpha_{21} \approx 0.124$, $\beta_{12} \approx 7.7 \times 10^{-2}$ and $\gamma_{21} \approx 4.6 \times 10^{-2}$. It is observed an overall qualitative agreement between the numerical solution of the GPE with the two-level approximation.

Figure 4 considers the impact of the phase when only a single modulating field is present. Here we are considering the role of the absolute phase of each modulation in order to compare with the role of the relative phase. In panels (a) and (b), the modulation of the nonlinearity is turned off $g_{\text{mod}}(x,t) = 0$, whereas in panels (c) and (d) the modulation of the trap is turned off $V_{\text{trap}}(x,t) = 0$ and equation (7) reads $g_{\text{mod}} = g(x)\cos(\omega_g t + \theta)$. The upper panels show the population dynamics of the ground and first modes for $\theta = 0$ and $\theta = \pi/2$, while the lower panels show the population of the first mode as a function of θ for some fixed times. We note that the variation of the absolute phase has more impact for the trapping modulation than for the modulation of the scattering length. But in both cases, there are small changes in the populations transfer and in the speed of the transition. The effects of the absolute phase on the dynamics can be mainly attributed to the nonlinear nature of the GPE. Considering the modulation of the trapping potential with fixed scattering length, modifying the absolute the trapping modulation causes changes in the dynamics of the wavefunction, which in turn changes the nonlinear term of the GPE, leading to modifications of the population dynamics. Nevertheless, the impact of the absolute phase is small compared to the relative phase when the two modulating fields are present.

6. Conclusion

We have investigated the simultaneous resonant modulation of the trapping potential and of the scattering length to generate nonlinear topological modes. In particular, we have focused on the impact of the relative phase of the modulations on the transition from the ground state to excited modes in the framework of the GPE. Numerical as well as approximated analytical methods have been applied. We have shown that the relative phase can be used to coherently control the transition to the excited modes by enhancing or suppressing the transition probability. We have also shown that the relative phase can affect the speed of the transitions. Thus, by adjusting the relative phase, the desired transition can be accelerated, which may be useful to avoid dissipative effects of the condensate with its surroundings. This behavior, which is not often found in ordinary quantum dynamics, can be attributed to the nonlinear nature of the GPE. The present work should motivate the study of different control problems in BEC using double modulation, such as in the excitation of collective modes.

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