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Letter

System-environment quantum information flow

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The return of the information from the environment to the system is a phenomenon that can be related to the existence of non-Markovian mechanisms in the environment, and such a transformation of resources can be useful for quantum information applications. Thus, understanding the details of the system-environment information dynamics, i.e., the transference of quantum resources, is of key importance to design noise-resilient quantum technologies. In this Letter, we show how a quantum resource propagates from the main system to an environment, using as a model a single qubit coupled to two linear chains of qubits, and also the information dynamics among the environment qubits. In this way, we characterize the propagation of information leaving the main qubit and going through the environment. Finally, we connect the conditions for the emergence of this dynamics to the existence of quantum Darwinism.

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Introduction. In an open quantum system, a backflow of information from the environment can take place, thus giving the origin to non-Markovian dynamics. Such a feature can be exploited in a number of quantum tasks, including in the emerging domain of quantum thermodynamics [1–4]. Characterizing the information flow entailed by non-Markovian dynamics is thus important both fundamentally and practically.

Various quantities have been proposed to quantitatively address the non-Markovian behavior of a given quantum evolution [5,6]. Among them, the one proposed by Breuer, Laine, and Piilo (BLP) [7,8] associates non-Markovianity to the lack of dynamical state homogenization: The distinguishability of two initial states of a quantum system—as measured by the trace norm—is a contractive function of time under a Markovian quantum channel, signaling asymptotic state homogenization. Thus, a phenomenology resulting in the violation of such monotonicity can be interpreted as a signature of non-Markovianity, and used to quantify the *degree* of departure of a given evolution from the Markovian framework. A significant body of literature (both theoretical and experimental) on this subject has been produced to date, and much has been established of the mathematical foundations and applications of non-Markovianity [1,3,5,6,9–16]. However, there are fewer studies about the information flow between the system and environment and details of the mechanisms of creation or destruction of quantum resources [11,17–20].

Establishing the formalism able to describe such flow and identifying instances of physical situations that allow for its quantitative assessment would be very important. Besides shedding further light on the phenomenology of non-Markovian quantum dynamics, it will enable the identification of the fundamental mechanisms leading to the quantum-to-classical transition, as established, for instance, by the framework of quantum Darwinism [21,22]. There, the quantum information shared by different observers is central to the assessment of the process leading the quantum state of a system to classicality through the emergence of redundant information encoding: When the same information is spread across the elements of a multipartite environment, and can be collected by associated observers, classicality is present [21–23]. The emergence of quantum Darwinism can thus be signaled by quantifying mutual information between the main system and growing-in-size subparts of the environment [23]. Several dynamical models have been studied theoretically through the quantum Darwinism framework [24–29] and on experimental platforms [30–32]. The key nature of quantum Darwinism as a diagnostic tool for the understanding of how quantum information flows from the system to the environment makes it well suited to study quantum non-Markovianity.

In this Letter we use the BLP measure [7,8] to study the link between non-Markovianity of the dynamics of a quantum system and the flow of information to a finite-size environment. Remarkably, we are able to characterize such flow fully through the degree of initial quantum coherence in the state of the system. We study the conditions for information trapping

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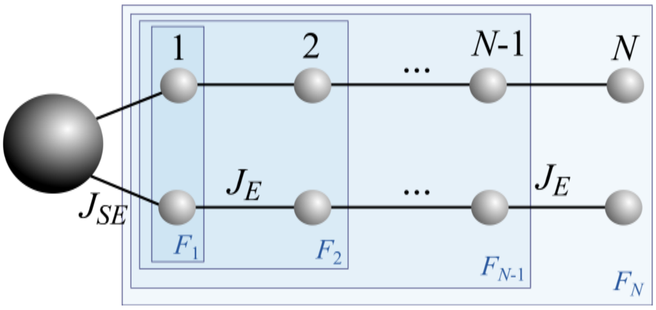

FIG. 1. Sketch of the physical situation considered. The model consists of a system qubit (dark gray dot) coupled to an environment comprising two distinct chains of qubits (small bright dots). Each blue-colored rectangle refers to the mth fragment F_m of the environment. Here, J_{SE} (J_E) is the coupling rate between the system and the first element of each chain (the intrachain coupling rate).

within the environment, and return to the system. The assessment of dynamically created coherence, and the propagation of information from the system, paves the way to the study of the occurrence of quantum Darwinism. We show that the instants of time when information is sent to the environment or returns from it, correspond to when equal amounts of information are present across different fragments of the environment, thus identifying unquestionably the manifestation of quantum Darwinism.

Model and resulting dynamics. We take inspiration from the quantum model proposed in Ref. [33], consisting of a single-qubit system coupled to a finite-size environment, to construct a mechanism for open system dynamics where information travels through the environment in discrete time steps. We thus consider the adamantane molecule ($C_{10}H_{16}$) in the presence of a strong static magnetic field as the platform for our investigation. The compound consists of six CH_2 and four CH groups. The carbon spin from the CH_2 group plays the role of a system qubit, which is coupled to an environment comprising two linear chains of N qubits each. Such a configuration has been realized experimentally in a nuclear magnetic resonance (NMR) platform [33–36]. The total number of qubits in the full system is then $2N+1$, as sketched in Fig. 1.

Reference [33] showed that the nuclear spins of the hydrogen in the molecule can be considered as a thermal bath for the nuclear spin of a single ^{13}C atom, which embodies the main system (we have approximately one ^{13}C nuclear spin for 160 ^{1}H spins, which thus represent a spin bath for the carbon nuclear spin). The natural Hamiltonian of the compound reads [37] $H^{(0)} = H_z^S + H_z^E + H_{SE}^{(0)} + H_E^{(0)}$, where H_z^S and H_z^E represent the Zeeman energies induced by the interaction of system and environment with the external magnetic field, respectively. They can be discarded in a suitable rotating frame at the Larmor frequencies associated with such Zeeman terms [34–36]. The corresponding Hamiltonian reduces to a term describing the system-environment coupling, and one accounting for the interaction among the elements of the environment, and can be written as $H = H_{SE} + H_E$ with (we assume units such that $\hbar = 1$ throughout the text) [33]

$$H_{SE} = J_{SE} \sum_{\alpha=a,b} \left(2\sigma_z \varepsilon_z^{\alpha,1} + \sigma_x \varepsilon_x^{\alpha,1} + \sigma_y \varepsilon_y^{\alpha,1}\right), \tag{1}$$

and

$$H_E = J_E \sum_{\alpha=a,b} \sum_{k=1}^{N-1} \left[2\varepsilon_z^{\alpha,k}\varepsilon_z^{\alpha,k+1} - \left(\varepsilon_x^{\alpha,k}\varepsilon_x^{\alpha,k+1} + \varepsilon_y^{\alpha,k}\varepsilon_y^{\alpha,k+1}\right)\right]. \tag{2}$$

Here, σ_μ and $\varepsilon_\mu^{\alpha,k}$ ($\mu = x, y, z$) are the system and environment spin operators, respectively. They satisfy (anti)commutation relations of spin-1/2 particles and are thus akin to Pauli matrices. The label α identifies the environmental chains, whose elements are labeled as $k = 1, \ldots, N-1$. The coupling rate between the main system and the first qubit of each chain is denoted by J_{SE}, while J_E stands for the interaction rate between neighboring qubits in each chain.

In this context, we are interested in characterizing the dynamics of the main system. It is know that a system coupled to an environment with finite degrees of freedom can present a non-Markovian behavior [6] and one particular form to quantify such a characteristic is based on the indistinguishability of quantum states, as measured by the trace distance [7,8]. As such a quantity is contractive under Markovian evolution, its nonmonotonic behavior signals non-Markovianity stemming from a flow of information from the environment back to the system. In this light, we will follow the dynamics of the trace distance between two distinct states of the system and identify the cases in which this quantity is nonmonotonic over time.

Let S be prepared in the initial states $\rho_S^{(\pm)}(0) = |\pm\rangle\langle\pm|_S$ with $|\pm\rangle_S = (|0\rangle \pm |1\rangle)_S/\sqrt{2}$ and $\{|0\rangle, |1\rangle\}$ embodying the computational basis of our problem (we have shifted the energy of such states in a way that $\sigma_z|0\rangle = |0\rangle$ and $\sigma_z|1\rangle = -|1\rangle$). The initial state of the environment is $\rho_E(0) = \bigotimes_{\alpha=a,b}\bigotimes_{k=1}^{N}|0\rangle\langle 0|_{\alpha,k}$ and we consider uncorrelated system-environment initial states in the form $\rho_{SE}^{(\pm)}(0) = \rho_S^{(\pm)}(0) \otimes \rho_E(0)$. The evolution is governed by the Liouville–von Neumann equation

$$\frac{d}{dt}\rho_{SE}^{(\pm)}(t) = -i\left(H, \rho_{SE}^{(\pm)}(t)\right), \tag{3}$$

which can be solved to find $\rho_{SE}^{(\pm)}(t)$ and, in turn, the reduced states of the system $\rho_S^{(\pm)}(t)$. The distinguishability between such states is then defined as

$$D_S\left(\rho_S^{(+)}(t), \rho_S^{(-)}(t)\right) = \left\|\rho_S^{(+)}(t) - \rho_S^{(-)}(t)\right\|_1, \tag{4}$$

with $\|A\|_1 = \frac{1}{2}\mathrm{Tr}(\sqrt{A^\dagger A})$. Such a quantity decreases monotonically under Markovian dynamics, which implies

$$\sigma_S(t) = \frac{d}{dt}D_S\left(\rho_S^{(+)}(t), \rho_S^{(-)}(t)\right) < 0. \tag{5}$$

Any revival of this quantity that leads to its change of sign (from negative to positive) can be related to a backflow of information from the environment [7,8]. In the case at hand here, $D_S(\rho_S^{(+)}(t), \rho_S^{(-)}(t))$ coincides with the difference between the coherence of the two states of the system since the initial states are orthogonal between them [7,8,38,39] [cf. the Supplemental Material (SM) [40] for a detailed derivation of this]. Thus, in the remainder of this Letter, we will exploit the behavior of the degree of distinguishability between the chosen initial states of S to relate the information exchanged between the system and environment to the variation of the quantum resource embodied by coherence [41].

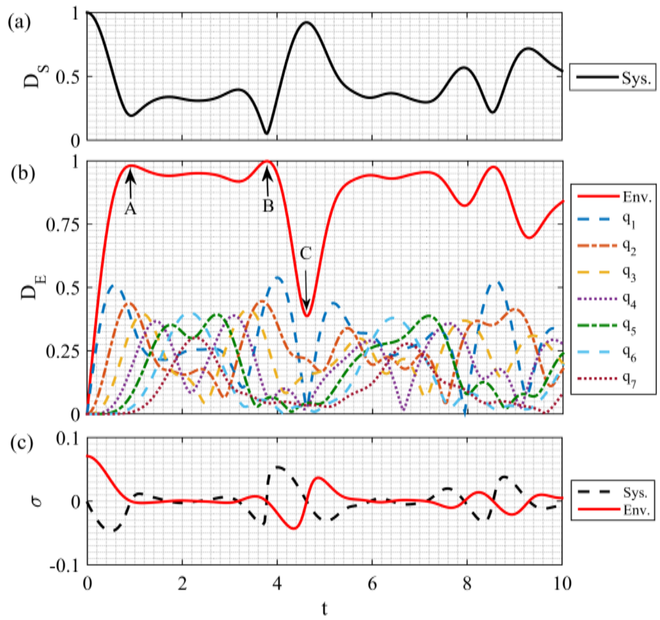

FIG. 2. (a) Evolution of the trace distance between states $\rho_S^{(+)}(t)$ and $\rho_S^{(-)}(t)$. (b) Evolution of the trace distance between the states of the complete environment, $\rho_E^{(+)}(t)$ and $\rho_E^{(-)}(t)$, and between the states of each qubit of the chain. (c) Derivative in time of the trace distances between the states of the system and the states of the full environment. Parameters used: $2N = 14$ and $J_{SE} = 0.71 J_E$.

Figure 2(a) shows the dynamics of the trace distance in Eq. (4), while Fig. 2(c) (black dashed line) shows the trend followed by $\sigma_S(t)$ (time has been rescaled as $t = J_E\tau$, where τ is the actual evolution time and $J_E = 700$ rad s^{-1} is the coupling rate between environment qubits [33]). The oscillations that are visible in $D_S(\rho_S^{(+)}(t), \rho_S^{(-)}(t))$, and the sign switches showcased by $\sigma_S(t)$, suggest a significant backflow of information from the environment to the system. As we mentioned above, since $\sigma_S(t)$ is related to a coherence measure, such a quantum resource can be the quantity transmitted to and recovered from the environment across the dynamics. To analyze such a conjecture, in what follows we take the standpoint of the environmental system and verify the dynamics of resources.

Information flow. In order to verify the dynamics of the information inside the environment, we use a measure proposed in Refs. [42,43] (cf. SM [40] for further details),

$$D_E\big(\rho_E^{(+)}(t), \rho_E^{(-)}(t)\big) = \big\|\rho_E^{(+)}(t) - \rho_E^{(-)}(t)\big\|_1, \qquad (6)$$

and its derivative $\sigma_E(t)$, where $\rho_E^{(\pm)}(t)$ stands for the state of the environment at a given instant of time t, conditioned to the initial state of the system to be $\rho_S^{(\pm)}(0)$, thus obtained as $\rho_E^{(\pm)}(t) = \mathrm{Tr}_S[\rho_{SE}^{\pm}(t)]$. The behavior of Eq. (6) is presented in Fig. 2(b). The trace distance between conditional environmental states, in particular, has been studied by considering the elements of E both collectively and individually (i.e., by addressing each element in each chain as $\rho_{E,k}^{(\pm)}(t) = \mathrm{Tr}_{\overline{k}}[\rho_E^{(\pm)}(t)]$, where $\mathrm{Tr}_{\overline{k}}$ denotes the partial trace over all qubits in the environment except the kth). Our simulations, which were performed by considering two seven-element chains, show a growing degree of distinguishability from an initial situation where the states of E are perfectly indistinguishable. The maximum of Eq. (6) is achieved when Eq. (4) is minimum [cf. points A and B in Fig. 2(b)], while the minimum of Eq. (6) is achieved simultaneously to the maximum of Eq. (4) (cf. point C). This is strongly suggestive of a process where information is transferred from S to E and back. As the environmental elements are arranged symmetrically in the two chains, the dynamics of the qubits of both chains are equivalent and, therefore, the results shown refer to the qubits of a single chain only. We also notice that, as time evolves, quantum coherence is transferred from a layer of environmental qubit to the next, until it reaches the end of the chain, approximately halfway through the plateau between points A and B. Then, the information goes back to the first qubit again.

The patterns identified above are quasiperiodic: When reaching point C, the process of information transfer is repeated—the quantum coherence of the state of S travels throughout the environments and returns to the main system. The time needed to transfer most of the information from S to E, i.e., to reach the plateau, is of the order of $t_A = 1/J_{SE}$ and the time at which the plateau ends, i.e., the amount of time the information travels inside the environment (point B), is of the order of $t_B = N/2J_E$. Therefore, we have a relationship between sending information from the system qubit to the environment characterized in the system-environment coupling constant J_{SE}, whereas the coupling constant between the environment qubits J_E establishes the information return. In SM [40] we show the dynamics of our system against the coupling rates.

As mentioned before, the violation of monotonic decrease in Eq. (5) gives a signature of non-Markovian behavior of the dynamics [7,8]. Figure 2(c) shows the evolution of $\sigma_S(t)$ (black dashed line) and $\sigma_E(t)$ (red solid line). It is interesting to note that both figures present plateaus of stability, suggesting that during the evolution, the information flow has ceased.

Link to quantum Darwinism. The plateaus pinpointed in the analysis above suggest very strongly a potential connection with the selection of environment eigenstates and, thus, a link to quantum Darwinism [21,22]. Mutual information is a perfectly apt tool to quantify the information shared between S and each of the *fragments* into which the environmental chains comprised in E can be partitioned (as per Fig. 1).

Unlike what we did previously, when we traced the environment out qubit by qubit, here we will trace different environment layers, thus building fragments of growing size. By doing this we will be able to retain the correlations between the main system and E, and explore the dynamics of information to different observers. Such correlations have been proven to be key in characterizing both the nature of the dynamics induced by a finite-size environment such as the one addressed here and the phenomenology of quantum Darwinism [43,44]. Thus, we consider an environment divided into independent fragments of different sizes (Fig. 1), exploring the possibility of achieving a plateau in the mutual information between the system and such a fragment, which is a qualitative witness of the emergence of Darwinism [21,27–29,45–47]. While in SM [40] we address the nature of the correlations under scrutiny, here we report on the behavior of the mutual information

$$I(\rho_S : \rho_{F_m}) = \mathcal{S}(\rho_S) + \mathcal{S}(\rho_{F_m}) - \mathcal{S}(\rho_{SF_m}), \qquad (7)$$

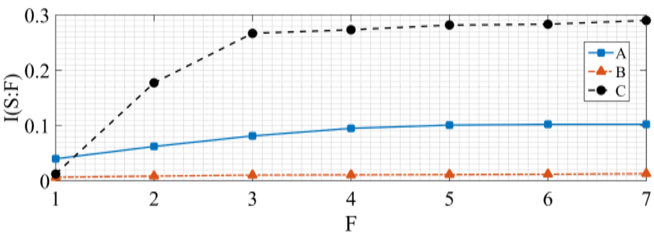

FIG. 3. Mutual information at points A, B, and C shown in Fig. 2 for a whole system of 15 qubits.

where $\mathcal{S}(\rho) = -\mathrm{tr}(\rho \log \rho)$ is the von Neumann entropy [48] and F_m stands for the mth fragment (F_m thus comprises the elements with $k = 1, \ldots, m$ of both chains with $m = 1, \ldots, N$).

In Fig. 3 we show $I(\rho_S : \rho_{F_m})$ against m for $N = 7$ and at points A (information travels to the environment), B (backflow of information to the system), and C (instant of time at which the process is complete and starts over) in Fig. 2. While for A and B only a partial Darwinistic behavior is achieved, when C is considered and $m = N$, we have $I(S : E) = 2S(\rho_S)$ [22], signaling a full establishment of objective reality stemming from a system-environment interaction, and reinforcing the qualitative link with the behavior of the information exchanged by S and E at the core of the discussion above.

Conclusions. We have studied the dynamics of information transfer between a system and its environment in a compound inspired by an adamantane molecule-based NMR setting [33–36]. We have been able to provide a very detailed description of the phenomenology of such a transfer, pinpointing the time at which information is sent to the environment and back. Remarkably, the occurrence of dynamical plateaus in the trace distance that quantifies the degree of distinguishability between different input states of the system and characterizes the non-Markovianity of the ensuing dynamics allowed us to establish a qualitative link with the phenomenon of redundant encoding of information that is at the core of quantum Darwinism. Such a link has been made robust through the study of mutual information, which clearly highlighted the uniform spreading of information on S across growing fractions of E.

The detailed characterization of such information transfer processes in an open system dynamics will be the key to understanding the redistribution of energy and quantum coherence across a quantum compound, which could in turn help characterize dynamical processes relevant, for instance, in quantum thermodynamics.

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[40] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevA.110.L040401 for the SM provides (a) calculation of relationship between trace distance and quantum coherence, (b) trace distance between the system and the environment considering correlations during the dynamics, (c) markovianity measurement by changing parameters of the system and (d) study of the strong quantum Darwinism.

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