

Relating $D^*\bar{D}^*$ currents with $J^\pi = 0^+, 1^+$ and 2^+ to Z_c statesK. P. Khemchandani,^{*} A. Martínez Torres,[†] M. Nielsen,[‡] and F. S. Navarra[§]¹*Instituto de Física, Universidade de São Paulo, C.P 66318, 05314-970 São Paulo, São Paulo, Brazil*

(Received 22 October 2013; published 27 January 2014)

In this work we study the $D^*\bar{D}^*$ current with QCD sum rules. We write the correlation function using the general current corresponding to the $D^*\bar{D}^*$ system and then use spin projectors to obtain the correlation function in the 0^+ , 1^+ , and 2^+ spin-parity configurations. The purpose of the present work is to investigate the possibility of explaining the recently reported $Z_c(4025)$ as a $D^*\bar{D}^*$ moleculelike state. As a result, we find a state for each spin case with a very similar mass: $M^{S=0} = (3943 \pm 104)$ MeV, $M^{S=1} = (3950 \pm 105)$ MeV, $M^{S=2} = (3946 \pm 104)$ MeV. We discuss that our mass results, within error bars, for 1^+ or 2^+ are both compatible with $Z_c(4025)$. However, our results are also compatible with a possible $D^*\bar{D}^*$ bound state, in agreement with predictions of some previous works. We have also calculated the current-state coupling which turns out to be larger in the 2^+ case.

DOI: 10.1103/PhysRevD.89.014029

PACS numbers: 14.40.Rt, 12.38.Lg, 12.39.-x

I. INTRODUCTION

The present work has been motivated by the finding of a charged charmoniumlike state, named $Z_c^\pm(4025)$, in the $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm \pi^\pm$ process by the BES Collaboration [1]. The state is found in the pion recoil mass spectrum (which corresponds to the $D^*\bar{D}^*$ invariant mass spectrum) at $4026.3 \pm 2.6 \pm 3.7$ MeV, very close to the $D^*\bar{D}^*$ threshold. The spin-parity of $Z_c^\pm(4025)$ is not known although it has been assumed to be 1^+ in Ref. [1]. Its isospin is obviously 1. Interestingly, another state with very similar mass, $Z_c(4020)$, has also been found in the $\pi^\pm h_c$ mass spectrum in the $e^+e^- \rightarrow \pi^+\pi^-h_c$ process [2] although it does not seem to be clear if the states found in $(D^*\bar{D}^*)^\pm$ and $\pi^\pm h_c$ are the same or not.

The finding of these new Z_c states adds to the discovery of a series of bottomoniumlike/charmoniumlike charged states which are being reported from recent experimental studies. The special interest in these states arises from the fact that they necessarily require more than two constituent quarks to get their quantum numbers right. The existence of mesons (baryons) possessing more than two (three) constituent quarks has always been viable within QCD. However, a rigorous experimental search for such states has begun only recently, since the states made of heavy quarks are now produced with high statistics in the new B factories. This makes it easier to look for exotic hadrons by searching for charged heavy quarkoniumlike states.

In the charm sector, several charged states have been reported by now, like $Z^+(4430)$, found in the $\pi^+\psi'$ system [3,4] (also reconfirmed in Ref. [5] more recently), $Z_1^+(4050)$, $Z_2^+(4250)$ found in the $\pi^+\chi_{c1}$ invariant mass

spectrum [6], $Z_c(3900)$ in the $\pi^\pm J/\psi$ system [7,8] and now $Z_c^\pm(4025)$, $Z_c(4020)$ found in the $D^*\bar{D}^*$ [1] and πh_{c1} [2] systems, respectively. It should be mentioned, however, that the first three of these Z_c states have not been found by the BABAR Collaboration [9,10]. Hence, some of these states still need more confirmation for their existence. But if they do exist then it is interesting to notice that the masses of all these states are relatively close to each other, while the widths of these states vary between 40–200 MeV. Curiously, all of them lie very close to the threshold of some open charm meson system. Also, all the decay channels where these charged states have been found (like $\pi^+\psi'$, $\pi^+\chi_{c1}$, π^+J/ψ , $D^*\bar{D}^*$) can account for similar isospin-spin quantum numbers. Thus, these channels, in principle, can couple to each other and it is possible that a same Z_c state is seen in different $c\bar{c} - \pi$ or open charm final states. In such a situation, where states with closely spaced masses and overlapping widths are being found, it is very important to make a careful analysis to judge if all of them are different or are sometimes replicas of each other.

To add to the efforts in understanding these newly found states, we make a study of the $D^*\bar{D}^*$ moleculelike current in the isospin 1 configuration using QCD sum rules (QCDSR) and study its different spin configurations.

A system of two vector mesons can possess a total spin-parity 0^+ , 1^+ , or 2^+ when interacting in the s wave. Such configurations of two vector mesons are ideally suited to the formation of moleculelike resonances as the constituent hadrons possess little energy. Since the masses of some Z_c 's are close to the threshold of open charm meson systems, some of them could be explained within such a picture. To unambiguously separate the different spin-parity configurations of the $D^*\bar{D}^*$ systems, we apply the spin projectors discussed in our previous work [11] on the most general current written for the system. Some works have already been done on the $D^*\bar{D}^*$ system using QCD sum rules with the motivation of finding a state which can be associated to

^{*}kanchan@if.usp.br[†]amartine@if.usp.br[‡]mnielsen@if.usp.br[§]navarra@if.usp.br

$Z_c(4025)$. In Refs. [12,13] a current corresponding to 1^+ spin-parity has been studied and a state compatible with $Z_c(4025)$ has been found. The authors of Ref. [14] investigate a tetraquark current with spin-parity 1^- and 2^+ to conclude that $Z_c(4025)$ is a 2^+ tetraquark state. Some work has also been done with other formalisms to understand the nature of $Z_c(4025)$ [15–17]. The $D^*\bar{D}^*$ system has also been studied in Ref. [18] where a bound state in isospin 1 and spin-parity 2^+ was predicted with mass between 3900 – 3965 MeV and width of 160 – 200 MeV. We will later compare our results with those found in these previous works and make some conclusions.

II. FORMALISM

We write the interpolating current corresponding to the $\bar{D}^{*0}D^{*+}$ molecule as

$$j_{\mu\nu}(x) = [\bar{c}_a(x)\gamma_\mu u_a(x)][\bar{d}_b(x)\gamma_\nu c_b(x)], \quad (1)$$

where a, b denote the color indices. With this current we construct the two-point correlation function

$$\Pi_{\mu\nu\alpha\beta}(q^2) = i \int d^4x e^{iqx} \langle 0 | T [j_{\mu\nu}(x) j_{\alpha\beta}^\dagger(0)] | 0 \rangle \quad (2)$$

and then apply the spin projectors discussed in Ref. [11] to it. The 0^+ , 1^+ , and 2^+ components of the correlation function written in Eq. (2) can be obtained using the following projectors:

$$\begin{aligned} \mathcal{P}^{(0)} &= \frac{1}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta}, \\ \mathcal{P}^{(1)} &= \frac{1}{2} (\Delta^{\mu\alpha} \Delta^{\nu\beta} - \Delta^{\mu\beta} \Delta^{\nu\alpha}), \\ \mathcal{P}^{(2)} &= \frac{1}{2} (\Delta^{\mu\alpha} \Delta^{\nu\beta} + \Delta^{\mu\beta} \Delta^{\nu\alpha}) - \frac{1}{3} \Delta^{\mu\nu} \Delta^{\alpha\beta}, \end{aligned} \quad (3)$$

where $\Delta_{\mu\nu}$ is defined in terms of the metric tensor, $g^{\mu\nu}$, and the four-momentum q of the correlation functions as

$$\Delta_{\mu\nu} \equiv -g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}. \quad (4)$$

These projectors were obtained in Ref. [11] by building an analogy with the work done in Ref. [19] where the s -wave $D^*\rho$ interaction was studied using effective field theory. Some of these projectors coincide with those determined in Ref. [20], where projectors for more spin-parity cases are given. As mentioned earlier, we are interested in studying 0^+ , 1^+ , and 2^+ configurations of $D^*\bar{D}^*$ keeping in mind that the low energy interaction of these two mesons is dominated by s -wave scattering which is a favorable situation for the formation of moleculelike states.

The motivation behind separating only the positive parity components is to look for moleculelike states with mass

close to the threshold of the constituent hadrons, in which case there is little energy available for the hadrons, which as a consequence interact in the s wave. A moleculelike picture for $Z_c(4025)$ seems to be quite plausible since its mass is merely 8 MeV away from the $\bar{D}^{*0}D^{*+}$ threshold. In other words, here we want to see if $Z_c(4025)$ can be interpreted as a 1^+ or 2^+ resonance of the $\bar{D}^{*0}D^{*+}$ system. The 0^+ assignment is ruled out for $Z_c(4025)$ by spin-parity conservation for the $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm \pi^\pm$ process. However, some other Z_c resonance with 0^+ might exist.

As is well known, the QCDSR method is based on the dual nature of the correlation function: it can be interpreted as quark-antiquark fluctuations at short distances, which is usually referred to as the QCD side, while it can be related to hadrons at large distances, which is referred to as the phenomenological side. In this method, one thus calculates the correlation function within both interpretations and equates the two results with the conviction that the two sides must be equivalent in some range of q^2 [21–24]. The calculation from the QCD side leads to a quark propagator form of the correlation function which is written in terms of the operator product expansion (OPE) and the coefficients of the series are calculated perturbatively [21–25].

In practice, one calculates the spectral density which is related to the correlation function through the dispersion relation

$$\Pi_{\text{OPE}}(q^2) = \int_{s_{\min}}^{\infty} ds \frac{\rho_{\text{OPE}}(s)}{s - q^2} + \text{subtraction terms.} \quad (5)$$

Proceeding with the standard scheme, then, we obtain the spectral density, corresponding to the spin-projected correlation function, by going in the OPE series up to dimension six in the present case

$$\rho_{\text{OPE}}^S = \rho_{\text{pert}}^S + \rho_{\langle\bar{q}q\rangle}^S + \rho_{\langle g^2 G^2 \rangle}^S + \rho_{\langle\bar{q}g\sigma Gq\rangle}^S + \rho_{\langle\bar{q}q\rangle^2}^S + \rho_{\langle\bar{q}^3 G^3\rangle}^S. \quad (6)$$

The different spin-projected OPE results are given in the Appendix of the paper.

Next, to make the calculations from the phenomenological side we assume, as usually done, that the spectral density can be written as a sum of a narrow, sharp state, which precisely corresponds to the one we are looking for, and a smooth continuum

$$\rho_{\text{phenom}}^S(s) = \lambda_S^2 \delta(s - m_S^2) + \rho_{\text{cont}}^S(s). \quad (7)$$

In Eq. (7), S denotes the spin, $s = q^2$ is the squared four-momentum flowing in the correlation function, λ_S is the coupling of the current to the state we are interested in, and m_S denotes its mass. The density related to the continuum of states is assumed to vanish below a certain value of s called the continuum threshold; let us call it s_0 . Above s_0 one usually considers the ansatz [21–24]

$$\rho_{\text{cont}}(s) = \rho_{\text{OPE}}^S(s) \Theta(s - s_0). \quad (8)$$

Using this parametrization of the spectral density, the correlation function from the phenomenological side can be written as

$$\Pi_{\text{phenom}}^S(q^2) = \frac{\lambda_S^2}{m_S^2 - q^2} + \int_{s_0}^{\infty} ds \frac{\rho_{\text{OPE}}^S(s)}{s - q^2}. \quad (9)$$

To get closer to the idea of the dual nature of the correlation function, a Borel transform of Eqs. (5) and (9) is taken. This suppresses the contribution of the continuum on the phenomenological side and divergent contributions arising due to the long range interactions on the OPE side. Equating the Borel transformed results, we get the following expression for the mass:

$$m_S^2 = \frac{\int_{4m_c^2}^{s_0} ds s \rho_{\text{OPE}}^S(s) e^{-s/M^2}}{\int_{m_c^2}^{s_0} ds \rho_{\text{OPE}}^S(s) e^{-s/M^2}}, \quad (10)$$

and that for the coupling λ_S

$$\lambda_S^2 = \frac{\int_{4m_c^2}^{s_0} ds \rho_{\text{OPE}}^S(s) e^{-s/M^2}}{e^{-m_S^2/M^2}}. \quad (11)$$

III. RESULTS AND DISCUSSIONS

In this article we are investigating the possibility of interpreting some of the recently found Z_c states, which essentially need more than two valence quarks, with a $\bar{D}^{*0} D^{*+}$ molecular current. For this, as explained in the previous section, we use the QCDSR method for which we write the general current corresponding to $\bar{D}^{*0} D^{*+}$ system [Eq. (1)] and then use spin projectors [Eq. (3)] to obtain the correlation function with spin-parity 0^+ , 1^+ , and 2^+ . In general, a state can couple to different currents and one could consider admixture of several of them. In fact, such a work was done in Ref. [26] and it was found that an admixture of different currents, with the same quantum numbers, leads to almost the same mass in the QCDSR approach. As a consequence, the results found in our work are not expected to change much by considering admixtures of different currents. Besides, the $\bar{D}^{*0} D^{*+}$ current is unique in the sense that it couples to different spin configurations. A possible reorganization of the $\bar{D}^* D^*$ current could be $J/\Psi \rho$ but the mixing of these two currents is expected to be suppressed since $\bar{D}^* D^* \rightarrow J/\Psi \rho$ requires an exchange of an open (heavy) charm meson. We do not consider admixtures of yet other types of currents in our study since we must admix different currents to different spin-projected correlation functions, which would necessarily imply the introduction of new parameters in the calculation, on which we do not have much information to rely. This would be too big a price to pay, given that the

changes in the results are expected to be small (as shown in Ref. [26]).

We calculate the spectral density within a valid ‘‘Borel window’’ in which the results can be relied upon. A valid Borel window is that range of Borel mass where, on the QCD side, the OPE series converges and where the contribution from the pole term dominates over the one of the continuum on the phenomenological side. In order to carry out these calculations we need to fix the value of the continuum threshold, $\sqrt{s_0}$, which should be a reasonable value above the mass of the state we are interested in. Usually it is taken to be 0.5 GeV above the mass of the state since, phenomenologically, the average difference between the masses of a hadron and its first excited state is found to be around 0.5 GeV. Since the principal motivation of our work is to find a description for the recently found $Z_c(4025)$ [1], we take $\sqrt{s_0} \sim 4.5$ GeV. We will, actually, vary the value of $\sqrt{s_0}$ around 4.5 GeV and test the stability of our results against the variation of this value. We find that a valid Borel window exists for the calculations done with the three spin-projected correlation functions. It remains to give the values of the other inputs required for the numerical calculations, like the quark condensate, the gluon condensate, the constituent charm quark mass, etc. We use the same values for these inputs as those used in our previous work [27]. For the readers convenience we also list them here in Table I.

In the left panel of Fig. 1 we show the contributions of the pole and continuum terms obtained by calculating the correlation function from the phenomenological side. This figure shows the results for the correlation function for spin 0 and for $\sqrt{s_0} \sim 4.45$ GeV, as an example. The results for other configurations and other values of $\sqrt{s_0}$ around 4.5 GeV are similar. The right panel shows the results obtained for the different terms in the OPE series for the same spin and $\sqrt{s_0}$. The OPE results shown in Fig. 1 are the relative contributions of the different terms of the series. This means that the result for dimension zero (labeled by ‘‘dim 0’’ in Fig. 1 is divided by the sum of all the terms in the series of Eq. (6). Then the dimension three results are added to dimension zero and the result is, once again, divided by the sum of all the terms in Eq. (6) (labeled by ‘‘dim 3’’ in Fig. 1. Similarly, one keeps going to the next higher dimension. Thus the legend labels in Fig. 1 indicate

TABLE I. Values of the different inputs required for numerical calculations.

Parameters	Values
m_c	1.23 ± 0.05 GeV
$\langle \bar{q}q \rangle$	$-(0.23 \pm 0.03)^3$ GeV ³
$\langle g^2 G^2 \rangle$	(0.88 ± 0.25) GeV ⁴
$\langle g^3 G^3 \rangle$	(0.58 ± 0.18) GeV ⁶
$\langle \bar{q}\sigma \cdot Gq \rangle$	$0.8 \langle \bar{q}q \rangle$ GeV ²

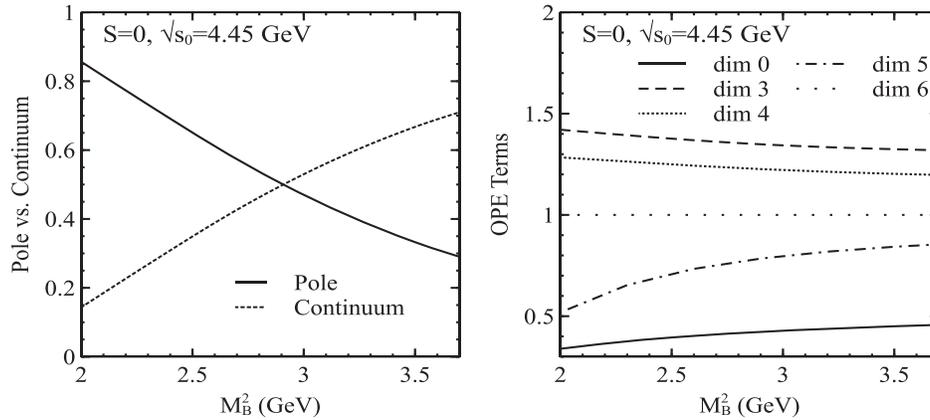


FIG. 1. Left panel: The pole and continuum contributions for the correlation function corresponding to the $D^*\bar{D}^*$ system with $J^\pi = 0^+$. Right panel: Contribution of the different terms of the OPE series for the same. The label S in the figure denotes the spin ($= 0$) of the system, while $\sqrt{s_0}$ denotes the continuum threshold which is taken as 4.45 GeV to obtain the results shown here. M_B^2 is the squared Borel mass.

the dimension up to which the OPE terms, weighted by the sum of all terms in Eq. (6), have been considered.

As can be seen from Fig. 1, the pole term, on the phenomenological side, dominates up to a squared Borel mass ~ 2.9 GeV² while the convergence of the OPE series is good beyond the squared Borel mass ~ 2.65 GeV² (where the contribution of the second last term in the series is $\leq 25\%$ of the last term).

Further, in Fig. 2 we show the results obtained for the mass in the three spin configurations for three different values of $\sqrt{s_0}$. The valid Borel window for the different cases is marked by a filled rectangle in the figure. These results show that there is a reasonable stability in the value of the mass of the states. However, it is further important to check the uncertainty present in our results due to the lack of precision with which the different condensates and the constituent mass of the charm quark are known. We thus vary all these inputs, one by one, and make an average over all the results obtained. The range within which the mass varies in the whole process gives us an idea of the total uncertainty present in our calculations. The masses obtained, considering all these sources of uncertainties, in the three cases are

$$\begin{aligned}
 M^{S=0} &= (3943 \pm 104) \text{ MeV} \\
 M^{S=1} &= (3950 \pm 105) \text{ MeV} \\
 M^{S=2} &= (3946 \pm 104) \text{ MeV}
 \end{aligned} \tag{12}$$

These results indicate the existence of three nearly spin-degenerate states, although the masses given in Eq. (12) contain an error bar of about 100 MeV and, thus, may differ by the size allowed by the error bars. The question which now arises is if any spin degeneracy is expected in systems like $D^*\bar{D}^*$. The answer might be searched by recalling the presence of heavy quarks in such systems. Hadrons

containing a heavy quark Q ($m_Q \gg \Lambda_{\text{QCD}}$) together with light (q) ones are known to possess a spin symmetry (in the heavy quark limit, $m_Q \rightarrow \infty$) which leads to the existence of a pair of nearly degenerate states with total angular momentum $(j - 1/2)$ and $(j + 1/2)$, where j is the total angular momentum of the cloud of the light degrees of freedom. Indeed, one observes such a symmetry in nature; for example, the mass difference between the D and D^* mesons (ground states) is of the order of the pion mass. Similar is the case of η_c and J/Ψ . The mass difference is further reduced in corresponding systems with a bottom quark. As has been discussed earlier by several authors (see, for example, Refs. [28–32], the $Q\bar{q}$ meson is similar to the hydrogen atom, with the heavy quark playing the role of the nucleus and the light quark playing the role of the electron. Consequently, the heavy quark spin symmetry is analogous to the one involved with atoms, where the hyperfine levels are nearly degenerate. This analogy can further be extended to the system of two heavy hadrons (like D^* mesons), and expect that the masses of the resulting states with different spin may not be very different. In fact, some studies dealing with $D^* - \bar{D}^*$ (or $D_s^* - \bar{D}_s^*$) mesons within formalisms based on heavy quark symmetry find nearly spin-degenerate results [17,33].

Further, in terms of the quark model, the two heavy quarks can be considered to form a diquark (as discussed in Refs. [34–36]) while the other two light quarks form another diquark. In this way, we again have a mesonlike system made of heavy and light subsystems, which, once again, possesses the spin symmetry discussed above in case of a hadron made of a heavy and light quarks. In fact, our currents can be reorganized using the Fierz transformation as a tetraquark (QQqq) system while the results will remain the same (as shown in Ref. [26]), i.e., giving nearly spin-degenerate states as can be expected for a QQqq-like system from the above discussions. Thus, the finding of

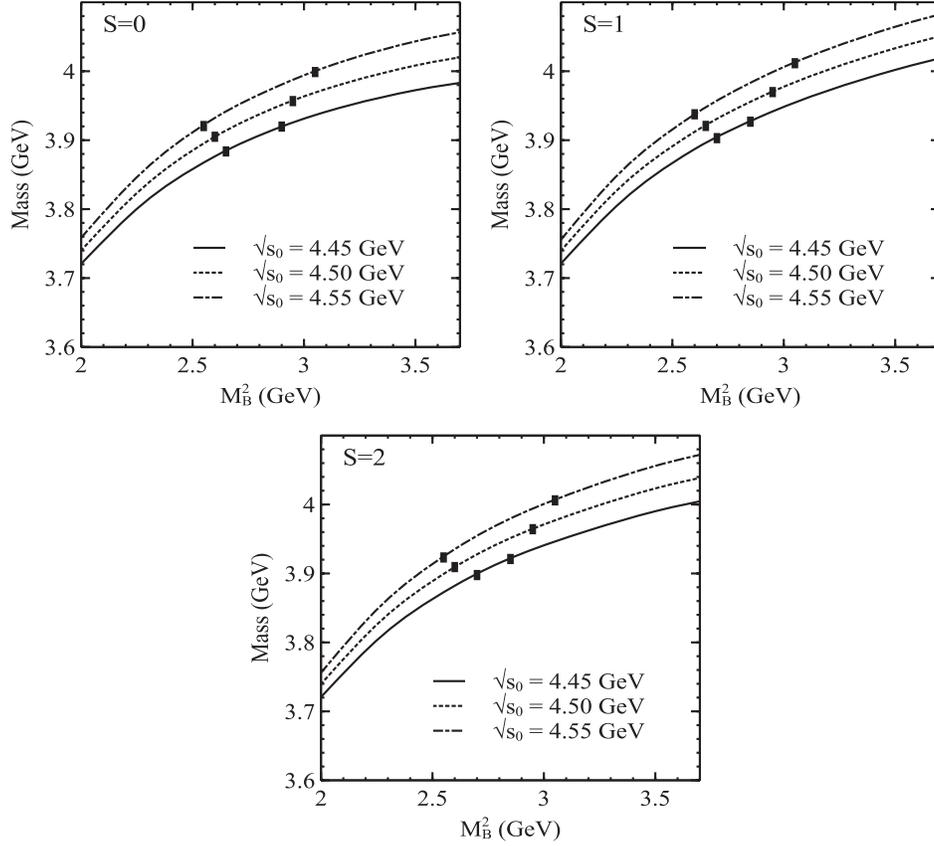


FIG. 2. Masses of the states obtained with spin-parity 0^+ , 1^+ , and 2^+ . The meaning of $\sqrt{s_0}$, S , and M_B^2 here is same as in Fig. 1. The valid Borel windows are indicated in the figures by filled rectangles.

similar masses in different spin configurations of the $D^*\bar{D}^*$ system should not come as a surprise.

Let us now question if the states found in Eq. (12) can be related to any known states. Within the error bars, one of the states given in Eq. (12) could correspond to the recently observed $Z_c(4025)$ [1] for which the spin-parity is assumed to be 1^+ in Ref. [1]. The spin-parity conservation excludes the 0^+ assignment to this state since it has been found in the $D^*\bar{D}^*$ mass spectrum in the $e^+e^- \rightarrow \pi D^*\bar{D}^*$ process. The 0^+ state obtained here can probably be related with the $Z_1^+(4050)$ state found in Ref. [6] in the $\pi\chi_{c1}$ system. A possibility of understanding $Z_1^+(4050)$ as a $D^*\bar{D}^*$ molecule has earlier been investigated in some works [37,38].

The 1^+ and 2^+ states given in Eq. (12) are both compatible with the $Z_c(4025)$ found in the $D^*\bar{D}^*$ mass spectrum in Ref. [1] although they could also correspond to a state below the threshold (in agreement with Ref. [18] as we will discuss shortly). The 2^+ result is in agreement with other work done using QCDSR with a tetraquark current [14]. The similarity in our results and those found in Ref. [14] are expected since it is well known that a tetraquark current and a molecular current with same quantum numbers can be related by a Fierz

transformation [26]. Our 1^+ result is compatible with previous works where $D^*\bar{D}^*$ moleculelike currents were studied [12,13], although the currents in Refs. [12,13] were constructed directly for an axial-vector state.

Resonance generation in the $D^*\bar{D}^*$ system has been studied within effective field theories earlier [17,18]. In Ref. [17] the $D^*\bar{D}^*$ system has been investigated with a formalism based on heavy quark symmetry and as a result an isospin 1 resonance with $J^\pi = 1^+$ and mass ranging between 3950 – 4017 MeV has been found as dynamically generated. This state has been associated to $Z_c(4025)$. On the other hand, in Ref. [18] a state with only $J^\pi = 2^+$, in the case of isospin 1, and mass 3900 – 3965 MeV has been found in the $D^*\bar{D}^*$ system. As can be seen from Eq. (12), our results in 1^+ and 2^+ configurations are both in good agreement with those found in Refs. [17,18].

From our work, thus, both 1^+ and 2^+ spin-parity assignments seem to be plausible for $Z_c(4025)$, but our mass results, within the error bars, in the two cases are compatible with having a resonance or a bound state in the $D^*\bar{D}^*$ system.

We have also calculated the coupling of the states found in our work to the corresponding currents. We find

$$\begin{aligned}
\lambda^{S=0} &= (17 \pm 4) \times 10^{-3} \text{ GeV}^5, \\
\lambda^{S=1} &= (30 \pm 6) \times 10^{-3} \text{ GeV}^5, \\
\lambda^{S=2} &= (39 \pm 8) \times 10^{-3} \text{ GeV}^5,
\end{aligned} \tag{13}$$

where the error bars have been obtained following the procedure mentioned earlier to calculate the uncertainty in the mass values. Equation (13) indicates that both currents with spin 1 and 2 couple strongly to a $D^*\bar{D}^*$ state although the current-state coupling in spin 2 seems to be slightly larger. From these results too, like in the case of the masses found in Eq. (12), both the spin-parity 1^+ and 2^+ seem to be equally associable to $Z_c(4025)$. In principle, with the results for the mass given in Eq. (12), it is also not possible to exclude the possibility of associating the 1^+ state found in our work with $Z_c(3900)$. Although the $D^*\bar{D}^*$ threshold is about 120 MeV far from the mass of $Z_c(3900)$ and the $D^*\bar{D}^*$ component should have a weight far too small in the wave function of $Z_c(3900)$, the coupling between the two can exist. Then if the 1^+ state found here could be related to $Z_c(3900)$, we would be left with a 2^+ assignment for $Z_c(4025)$. Of course, within our formalism, it is not possible to clarify this issue and also if the states found here are resonances or bound states, which even questions the existence of $Z_c(4025)$. In fact very different explanations seem plausible for the signal found by the BES Collaboration [1] when the $D^*\bar{D}^*$ mass spectrum is calculated for the process $e^+e^- \rightarrow \pi^\pm(D^*\bar{D}^*)^\pm$ considering the production of resonances/bound states with spin-parities $J^\pi = 1^+$ or 2^+ in different partial waves [39].

Finally, we would like to discuss the isospin 0 $D^*\bar{D}^*$ current, which can be written as

$$\begin{aligned}
j'_{\mu\nu}(x) &= \frac{1}{\sqrt{2}} \{ [\bar{c}_a(x)\gamma_\mu d_a(x)][\bar{d}_b(x)\gamma_\nu c_b(x)] \\
&\quad + [\bar{c}_a(x)\gamma_\mu u_a(x)][\bar{u}_b(x)\gamma_\nu c_b(x)] \}.
\end{aligned} \tag{14}$$

Clearly, with this current we obtain, up to a normalization factor, the same correlation function as the one obtained in the case of isospin 1. Thus, our results for the isospin 0 case would be same as those obtained in the isospin 1. This finding is also compatible with the light flavor symmetry expected in the presence of the heavy quarks (considering that $m_q \ll M_Q$), as discussed in Refs. [28–32,40]. Indeed some states have been found in recent experimental studies to which our isospin 0 states can be related, for example, $Y(3940)$ found in the $J/\Psi\omega$ spectrum [41] and $Y(4140)$ found in the $J/\Psi\phi$ system [42]. In fact, the $D^*\bar{D}^*$ molecular interpretation of $Y(3940)$ and $Y(4140)$ has been discussed by several authors [18,37,40,43,44], where spin-parity 0^+ and 2^+ has been assigned to these states, respectively.

IV. SUMMARY

We have studied the $D^*\bar{D}^*$ system using QCD sum rules with the motivation to find if a moleculelike state could associated to the recently found $Z_c(4025)$ in the $(D^*\bar{D}^*)^\pm$ mass spectrum in the process: $e^+e^- \rightarrow \pi^\pm(D^*\bar{D}^*)^\pm$ [1]. The spin-parity of this state is not known. The $D^*\bar{D}^*$ system can have total spin 0, 1 or 2 and isospin 0 or 1. We argue that the two mesons interact in the s wave since a very little energy is available to the $D^*\bar{D}^*$ system and thus such a state should have a positive parity. With this argument we obtain the correlation function corresponding to spin-parity 0^+ , 1^+ , 2^+ using spin projectors of Ref. [11]. With these correlation functions we carry out the calculation up to dimension six on the OPE side. We find the pole dominance and good OPE convergence for reasonable values of continuum thresholds. As a result, we find three states with spin-parity 0^+ , 1^+ , 2^+ each, both in the isospin 0 and 1 cases. The masses of all the states turn out to be very similar. We conclude that both 1^+ and 2^+ assignments could be associated with $Z_c(4025)$. However, our results are also compatible with the formation of bound states in line with the findings of Ref. [18]. The 0^+ state found here can be related with $Z_1^+(4050)$ [6]. To conclude the article, we would like to say that it is very important to obtain further experimental confirmation of the existence of these new states: $Z_c(4025)$ and $Z_c(4020)$.

ACKNOWLEDGEMENTS

The authors would like to thank the Brazilian funding agencies FAPESP and CNPq for their financial support. The authors also acknowledge useful discussions with Professors E. Oset and C. Z. Yuan.

APPENDIX: OPE RESULTS

In this section we give the results obtained for the different OPE terms in Eq. (6). The superscript on ρ in the following expressions denotes the spin of the current. Further, to write these results in a compact form we define the following functions:

$$\begin{aligned}
F_{\alpha_1, \alpha_2} &= m_c^2(\alpha_1 + \alpha_2) - q^2\alpha_1\alpha_2, \\
g_{\alpha_1, \alpha_2} &= 1 - \alpha_1 - \alpha_2, \\
h_{\alpha_1, \alpha_2} &= q^2\alpha_1\alpha_2, \\
F_{\eta_1, \eta_2} &= \frac{mc^2(\eta_1 + \eta_2)}{\eta_1\eta_2}, \\
g_{\eta_1, \eta_2} &= 1 - \eta_1 - \eta_2,
\end{aligned} \tag{A.1}$$

where $\alpha_1, \alpha_2, \eta_1, \eta_2$ are variables of integration, m_c is the constituent mass of the charm quark, M_B is the Borel mass, and q is the running momentum in the correlation function.

$$\begin{aligned} \rho_{\text{pert}}^0 &= \frac{1}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \frac{g_{\alpha_1, \alpha_2}^3 \left(\frac{F_{\alpha_1, \alpha_2}}{4} (F_{\alpha_1, \alpha_2} - 16h_{\alpha_1, \alpha_2}) + 4h_{\alpha_1, \alpha_2}^2 \right) F_{\alpha_1, \alpha_2}^2}{2^{10} \alpha_1^3 \alpha_2^3} \right. \\ &\quad \left. + \frac{3g_{\alpha_1, \alpha_2} F_{\alpha_1, \alpha_2}^4}{2^{11} \alpha_1^3 \alpha_2^3} - \frac{3g_{\alpha_1, \alpha_2}^2 \left(\frac{F_{\alpha_1, \alpha_2}^4}{24} - \frac{1}{3} h_{\alpha_1, \alpha_2} F_{\alpha_1, \alpha_2}^3 \right)}{2^8 \alpha_1^3 \alpha_2^3} \right\}, \\ \rho_{\text{pert}}^1 &= \frac{1}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \frac{3F_{\alpha_1, \alpha_2}^4 g_{\alpha_1, \alpha_2}}{2^{11} \alpha_1^3 \alpha_2^3} - \frac{9g_{\alpha_1, \alpha_2}^2 \left(\frac{F_{\alpha_1, \alpha_2}^4}{24} - \frac{F_{\alpha_1, \alpha_2}^3 h_{\alpha_1, \alpha_2}}{3} \right)}{2^8 \alpha_1^3 \alpha_2^3} \right\}, \\ \rho_{\text{pert}}^2 &= \frac{1}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \frac{F_{\alpha_1, \alpha_2}^2 g_{\alpha_1, \alpha_2}^3 \left[\frac{F_{\alpha_1, \alpha_2}}{4} (F_{\alpha_1, \alpha_2} - 16h_{\alpha_1, \alpha_2}) + 4h_{\alpha_1, \alpha_2}^2 \right]}{2^9 \alpha_1^3 \alpha_2^3} \right. \\ &\quad \left. + \frac{3F_{\alpha_1, \alpha_2}^4 g_{\alpha_1, \alpha_2}}{2^{11} \alpha_1^3 \alpha_2^3} + \frac{3g_{\alpha_1, \alpha_2}^2 \left(\frac{F_{\alpha_1, \alpha_2}^4}{24} - \frac{F_{\alpha_1, \alpha_2}^3 h_{\alpha_1, \alpha_2}}{3} \right)}{2^8 \alpha_1^3 \alpha_2^3} \right\}, \\ \rho_{\langle \bar{q}q \rangle}^0 &= -\frac{m_c \langle \bar{q}q \rangle}{2^7 \pi^4} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left(\frac{\alpha_1 + \alpha_2}{\alpha_1^2 \alpha_2^2} \right) (g_{\alpha_1, \alpha_2} [F_{\alpha_1, \alpha_2}^2 - 4F_{\alpha_1, \alpha_2} h_{\alpha_1, \alpha_2}] + F_{\alpha_1, \alpha_2}^2), \\ \rho_{\langle \bar{q}q \rangle}^1 &= -3\rho_{\langle \bar{q}q \rangle}^0, \\ \rho_{\langle \bar{q}q \rangle}^2 &= 5\rho_{\langle \bar{q}q \rangle}^0, \end{aligned}$$

$$\begin{aligned} \rho_{\langle g^2 G^2 \rangle}^0 &= \frac{\langle g^2 G^2 \rangle}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \left(-\frac{\alpha_1 + \alpha_2}{24\alpha_1^2 \alpha_2^2} \right) \left[\frac{g_{\alpha_1, \alpha_2}}{2^6} \left(\frac{F_{\alpha_1, \alpha_2}^2}{2} - 2F_{\alpha_1, \alpha_2} h_{\alpha_1, \alpha_2} \right) \right. \right. \\ &\quad \left. \left. - \frac{F_{\alpha_1, \alpha_2}^2}{2^9} - \frac{g_{\alpha_1, \alpha_2}^2 \left(\frac{3}{2} F_{\alpha_1, \alpha_2} (F_{\alpha_1, \alpha_2} - 8h_{\alpha_1, \alpha_2}) + 4h_{\alpha_1, \alpha_2}^2 \right)}{2^9} \right] + m_c^2 \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \right. \\ &\quad \left. \times \left[-\frac{g_{\alpha_1, \alpha_2}^3 (4h_{\alpha_1, \alpha_2} - F_{\alpha_1, \alpha_2})}{3 \cdot 2^{12}} + \frac{F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}}{2^{11}} - \frac{g_{\alpha_1, \alpha_2}^2 (m_c^2 (\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})}{3 \cdot 2^{11}} \right] \right\} \\ &\quad - \frac{m_c^6 \langle g^2 G^2 \rangle}{9 \cdot 2^{10} \pi^6} \int_{\eta_{1\text{min}}}^{\eta_{1\text{max}}} d\eta_1 \int_{\eta_{2\text{min}}}^{\eta_{2\text{max}}} d\eta_2 \frac{g_{\eta_1, \eta_2}^3 (\eta_1 + \eta_2)^2 (\eta_1^3 + \eta_2^3)}{\eta_1^4 \eta_2^4} \delta(s - F_{\eta_1, \eta_2}), \end{aligned}$$

$$\begin{aligned} \rho_{\langle g^2 G^2 \rangle}^1 &= \frac{\langle g^2 G^2 \rangle}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \left(\frac{\alpha_1 + \alpha_2}{\alpha_1^2 \alpha_2^2} \right) \left(\frac{3F_{\alpha_1, \alpha_2}^2}{2^{12}} \right) - \frac{m_c^2}{2^{11}} \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \right. \\ &\quad \left. \times (g_{\alpha_1, \alpha_2}^2 [m_c^2 (\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2}] - F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}) \right\}, \end{aligned}$$

$$\begin{aligned} \rho_{\langle g^2 G^2 \rangle}^2 &= \frac{\langle g^2 G^2 \rangle}{\pi^6} \int_{\alpha_{1\text{min}}}^{\alpha_{1\text{max}}} d\alpha_1 \int_{\alpha_{2\text{min}}}^{\alpha_{2\text{max}}} d\alpha_2 \left\{ \left(\frac{\alpha_1 + \alpha_2}{24\alpha_1^2 \alpha_2^2} \right) \left[-\frac{g_{\alpha_1, \alpha_2}}{2^5} \left(\frac{F_{\alpha_1, \alpha_2}^2}{2} - 2F_{\alpha_1, \alpha_2} h_{\alpha_1, \alpha_2} \right) \right. \right. \\ &\quad \left. \left. - \frac{7F_{\alpha_1, \alpha_2}^2}{2^9} + \frac{g_{\alpha_1, \alpha_2}^2 \left(\frac{3}{2} F_{\alpha_1, \alpha_2} (F_{\alpha_1, \alpha_2} - 8h_{\alpha_1, \alpha_2}) + 4h_{\alpha_1, \alpha_2}^2 \right)}{2^8} \right] + m_c^2 \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \right. \\ &\quad \left. \times \left[-\frac{g_{\alpha_1, \alpha_2}^3 (4h_{\alpha_1, \alpha_2} - F_{\alpha_1, \alpha_2})}{3 \cdot 2^{11}} + \frac{F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}}{2^{11}} + \frac{g_{\alpha_1, \alpha_2}^2 (m_c^2 (\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})}{3 \cdot 2^{11}} \right] \right\} \\ &\quad - \frac{m_c^6 \langle g^2 G^2 \rangle}{9 \cdot 2^9 \pi^6} \int_{\eta_{1\text{min}}}^{\eta_{1\text{max}}} d\eta_1 \int_{\eta_{2\text{min}}}^{\eta_{2\text{max}}} d\eta_2 \frac{g_{\eta_1, \eta_2}^3 (\eta_1 + \eta_2)^2 (\eta_1^3 + \eta_2^3)}{\eta_1^4 \eta_2^4} \delta(s - F_{\eta_1, \eta_2}), \end{aligned}$$

where the variable $s = q^2$.

$$\rho_{\langle \bar{q}g\sigma Gq \rangle}^0 = \frac{-m_c \langle \bar{q}g\sigma Gq \rangle}{2^8 \pi^4} \int_{\alpha_{1 \min}}^{\alpha_{1 \max}} \frac{d\alpha_1}{\alpha_1} \left\{ \frac{(m_c^2 - s\alpha_1(1 - \alpha_1))}{(1 - \alpha_1)} + \int_{\alpha_{2 \min}}^{\alpha_{2 \max}} \frac{d\alpha_2}{\alpha_2} (m_c^2(\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})(\alpha_1 + \alpha_2) \right\},$$

$$\rho_{\langle \bar{q}g\sigma Gq \rangle}^1 = -3\rho_{\langle \bar{q}g\sigma Gq \rangle}^0,$$

$$\rho_{\langle \bar{q}g\sigma Gq \rangle}^2 = 5\rho_{\langle \bar{q}g\sigma Gq \rangle}^0,$$

$$\rho_{\langle \bar{q}q \rangle^2}^0 = \int_{\alpha_{1 \min}}^{\alpha_{1 \max}} d\alpha_1 \frac{m_c^2 \langle \bar{q}q \rangle^2}{2^4 \pi^2},$$

$$\rho_{\langle \bar{q}q \rangle^2}^1 = -3\rho_{\langle \bar{q}q \rangle^2}^0,$$

$$\rho_{\langle \bar{q}q \rangle^2}^2 = 5\rho_{\langle \bar{q}q \rangle^2}^0,$$

$$\rho_{\langle g^3 G^3 \rangle}^0 = \frac{\langle g^3 G^3 \rangle}{\pi^6} \int_{\alpha_{1 \min}}^{\alpha_{1 \max}} d\alpha_1 \int_{\alpha_{2 \min}}^{\alpha_{2 \max}} d\alpha_2 \left\{ \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \left[-\frac{g_{\alpha_1, \alpha_2}^3 (4h_{\alpha_1, \alpha_2} - F_{\alpha_1, \alpha_2})}{3 \cdot 2^{14}} + \frac{F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}}{2^{13}} - \frac{g_{\alpha_1, \alpha_2}^2 (m_c^2(\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})}{3 \cdot 2^{13}} \right] \right. \\ \left. + 2m_c^2 \left(\frac{\alpha_1^4 + \alpha_2^4}{\alpha_1^3 \alpha_2^3} \right) \left[\frac{g_{\alpha_1, \alpha_2}^3}{3 \cdot 2^{14}} - \frac{g_{\alpha_1, \alpha_2}^2}{3 \cdot 2^{13}} + \frac{g_{\alpha_1, \alpha_2}}{2^{13}} \right] \right\} + \frac{m_c^4 \langle g^3 G^3 \rangle}{3 \cdot 2^{12} \pi^6} \int_{\eta_{1 \min}}^{\eta_{1 \max}} d\eta_1 \int_{\eta_{2 \min}}^{\eta_{2 \max}} d\eta_2 \left(\frac{g_{\eta_1, \eta_2}^2 (\eta_1 + \eta_2)}{\eta_1^4 \eta_2^4} \right)$$

$$\times \left[-\frac{g_{\eta_1, \eta_2} (\eta_1^3 + \eta_2^3)(\eta_1 + \eta_2)}{3} + 2 \left(\frac{g_{\eta_1, \eta_2}}{3} \left[\frac{F_{\eta_1, \eta_2}}{M_B^2} + 1 \right] - 1 \right) (\eta_1^4 + \eta_2^4) \right] \delta(s - F_{\eta_1, \eta_2}),$$

$$\rho_{\langle g^3 G^3 \rangle}^1 = \frac{\langle g^3 G^3 \rangle}{\pi^6} \int_{\alpha_{1 \min}}^{\alpha_{1 \max}} d\alpha_1 \int_{\alpha_{2 \min}}^{\alpha_{2 \max}} d\alpha_2 \left\{ \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \left[-\frac{g_{\alpha_1, \alpha_2}^2 (m_c^2(\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})}{2^{13}} + \frac{F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}}{2^{13}} \right] \right. \\ \left. + m_c^2 \left(\frac{\alpha_1^4 + \alpha_2^4}{\alpha_1^3 \alpha_2^3} \right) \left(\frac{g_{\alpha_1, \alpha_2}}{2^{12}} - \frac{g_{\alpha_1, \alpha_2}^2}{2^{12}} \right) \right\} - \frac{m_c^4 \langle g^3 G^3 \rangle}{2^{11} \pi^6} \int_{\eta_{1 \min}}^{\eta_{1 \max}} d\eta_1 \int_{\eta_{2 \min}}^{\eta_{2 \max}} d\eta_2 \left(\frac{g_{\eta_1, \eta_2}^2 (\eta_1 + \eta_2)(\eta_1^4 + \eta_2^4)}{\eta_1^4 \eta_2^4} \right) \delta(s - F_{\eta_1, \eta_2}),$$

$$\rho_{\langle g^3 G^3 \rangle}^2 = \frac{\langle g^3 G^3 \rangle}{\pi^6} \int_{\alpha_{1 \min}}^{\alpha_{1 \max}} d\alpha_1 \int_{\alpha_{2 \min}}^{\alpha_{2 \max}} d\alpha_2 \left\{ \left(\frac{\alpha_1^3 + \alpha_2^3}{\alpha_1^3 \alpha_2^3} \right) \left[-\frac{g_{\alpha_1, \alpha_2}^3 (4h_{\alpha_1, \alpha_2} - F_{\alpha_1, \alpha_2})}{3 \cdot 2^{13}} + \frac{F_{\alpha_1, \alpha_2} g_{\alpha_1, \alpha_2}}{2^{13}} \right. \right. \\ \left. \left. + \frac{g_{\alpha_1, \alpha_2}^2 (m_c^2(\alpha_1 + \alpha_2) - 3h_{\alpha_1, \alpha_2})}{3 \cdot 2^{13}} \right] + 2m_c^2 \left(\frac{\alpha_1^4 + \alpha_2^4}{\alpha_1^3 \alpha_2^3} \right) \left[\frac{g_{\alpha_1, \alpha_2}^3}{3 \cdot 2^{13}} + \frac{g_{\alpha_1, \alpha_2}^2}{3 \cdot 2^{13}} + \frac{g_{\alpha_1, \alpha_2}}{2^{13}} \right] \right\} \\ + \frac{m_c^4 \langle g^3 G^3 \rangle}{3 \cdot 2^{11} \pi^6} \int_{\eta_{1 \min}}^{\eta_{1 \max}} d\eta_1 \int_{\eta_{2 \min}}^{\eta_{2 \max}} d\eta_2 \left(\frac{g_{\eta_1, \eta_2}^2 (\eta_1 + \eta_2)}{\eta_1^4 \eta_2^4} \right) \left[-\frac{g_{\eta_1, \eta_2} (\eta_1^3 + \eta_2^3)(\eta_1 + \eta_2)}{3} \right. \\ \left. + 2 \left(\frac{g_{\eta_1, \eta_2}}{3} \left[\frac{F_{\eta_1, \eta_2}}{M_B^2} + 1 \right] + \frac{1}{2} \right) (\eta_1^4 + \eta_2^4) \right] \delta(s - F_{\eta_1, \eta_2}).$$

The limits of integration in above expressions are

$$\alpha_{1 \min} = \frac{1 - \sqrt{1 - \frac{4m_c^2}{q^2}}}{2}, \quad \alpha_{1 \max} = \frac{1 + \sqrt{1 - \frac{4m_c^2}{q^2}}}{2},$$

$$\alpha_{2 \min} = \frac{m_c^2 \alpha_1}{(\alpha_1 q^2 - m_c^2)}, \quad \alpha_{2 \max} = 1 - \alpha_1,$$

$$\eta_{1 \min} = 0, \quad \eta_{1 \max} = 1,$$

$$\eta_{2 \min} = 0, \quad \eta_{2 \max} = 1 - \eta_1.$$

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