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LATENT TRAIT ESTIMATION THROUGH NOMINAL RESPONSE AND DICHOTOMOUS MODELS: A CASE STUDY IN A MULTIPLE CHOICE TEST.

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Palavras-Chave: Latent trait, nominal response model, dichotomous models, bayesian estimation, asymmetry distribution.

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Latent Trait Estimation through Nominal Response and Dichotomous Models : A case study in a multiple choice test

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The Bock's Nominal Response Model (Bock, 1972) was proposed to improve the **latent trait** (ability) estimation. However, a large number of real situations are modelled by the three-parameter logistic model (Baker and Kim, 2004). In this article we discuss the ability estimation under these two models and conducted a simulation study to verify the behavior of their estimatives. Basically, we use suitable estimation methods considering both item and latent trait estimation. For every model we conducted the estimation procedures using modifications of the Maximum Marginal a Posteriori (MMAP) (for the item parameters) and Expectation (EAP) and Maximum a posteriori (MAP) (for latent traits). We compare the models considering different sample sizes (number of individuals) and different levels of asymmetry for the latent density through the skew-normal distribution (Genton, 2004).

Key words : Latent trait, nominal response model, dichotomous models, bayesian estimation, asymmetry distribution.

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1. Introduction

Birnbaum (1968) proposed the well known three-parameter logistic model to analyze multiple choice items and to estimate the individual latent trait. This model simply consider whether the subject answers or not correctly the item. So, the information contained in the wrong alternatives are not considered. On the other hand, Bock (1972) proposed the **Nominal Response Model** to improve the latent trait estimation. This model, differently from the Birnbaum's model, considers the probability of choosing any alternative and in this way it uses all the information contained in the item.

Our main goal is to compare the laten trait estimation under these two models through suitable bayesian estimation methods. In next Section we present the models. In Section 3 we present and discuss briefly about the estimation methods used. Section 4 brings a simulation study and finally, in Section 5, we discuss the results obtained.

2. The Models

The three-parameter logistic model (henceforth, DRM - 3P) has the following form

$$P(Y_{ij} = 1|\theta_j, \zeta_i) = c_i + (1 - c_i)\frac{1}{1 + e^{-Da_i(\theta_j - b_i)}}, \tag{1}$$

where $Y_{ij} = 1$, if subject j answers correctly the item i, and 0 otherwise; a_i is the item parameter that represents the item's discrimination (slope); b_i is the item parameter that represents the item's difficult; c_i is the item parameter that represents the probability of a correct answer given by a low abilities' level individual; θ_j is the individual latent trait (or ability); D a scaling factor that is 1, if we want the results in the logistic metric or 1.7, if we want the results in the normal metric, see Baker and Kim (2004).

This model has two basically assumptions that are essential to the estimation processes, which are :

1. The responses of different subjects are independent;
2. Given the ability's (latent trait) subject its response to different items are independent (commonly known as **conditional independency**).

Notice that, taking $c_i = 0$ in (1) we have the two parameter logistic model (henceforth, DRM-2P), see Baker and Kim (2004).

The Nominal Response Model (henceforth NRM), see Baker and Kim (2004) for example, has the following form

$$P_{ijs} = P\left(Y_{ijs} = 1|\theta_j, \zeta_i\right) = \frac{e^{a_{is}(\theta_j - b_{is})}}{\sum_{h=1}^{m_i} e^{a_{ih}(\theta_j - b_{ih})}}, \tag{2}$$

where Y_{ijs}, is the random variable that assumes value 1, if subject j, $j = 1, 2, \ldots, n$ chooses the alternative s, $s = 1, 2, \ldots, m_i$ of item i, $i = 1, 2, \ldots, I$ and 0, otherwise,

a_{is} e b_{is} represent the parameters related to the **discrimination** and the **difficult** of the category, respectively. In this model we may have negative values for both type of parameters. For the parameter discrimination we expect negative, or small values, for the wrong alternatives and positive value for the right alternative. This means that a higher value for individual ability is associated with higher probability of this subject chooses the right alternative. The difficult parameter represents, in some way, the ability that a subject must have to choose the referred alternative.

Notice that, the two parameter logistic model is not a particular case of the NRM with only two categories.

This model has three basically assumptions that are essential to the estimation processes. Two of them are that ones considered in DRM-3P and the third one is related to the probability that a subject chooses an alternative of a specific item, which can be modelled by the **multivariate Bernoulli model**, that is,

$$P(\boldsymbol{Y}_{ij.} = \boldsymbol{y}_{ij.}|\theta, \zeta_i) \equiv P(\boldsymbol{Y}_{ij.} = \boldsymbol{y}_{ij.}) = \prod_{s=1}^{m_i} P_{ijs}^{y_{ijs}},$$

where $\boldsymbol{y}_{ij.} = (y_{ij1}, \ldots, y_{ijm_i})^t$ represents a specific set of responses of a subject to a specific item.

In the next section we present the estimation methods.

3. Estimation Methods

There is an interesting and wide literature concerning estimation methods in IRT. We may cite Baker and Kim (2004), Patz and Junker (1999 a and 1999 b), Albert (1992) among others. In these references, the authors discuss different approaches, both classical and bayesian ones, to fit IRT models. Based on the results of these works, we will use in this article the MMAP procedure, see Baker and Kim (2004), to the item parameter estimation and two bayesian methods with suitable modification, to estimate the latent trait. Such modification is to consider a skew-normal distribution to the EAP and to the MAP methods. For both MMAP and EAP procedures, we use different quadrature weights based on this priori distribution. These quadrature points will be discussed ahead. Then, we conduct the estimation processes in two steps using the marginal bayesian approach, see Mislevy (1986), instead of the full bayesian one (Patz and Junker, 1999 a). We briefly describe the two methods, EAP and MAP, suitable modified and the maximum likelihood to the NRM latent trait estimation. The correspondent estimation methods to the DRM are obtained using a different IRF (item response function), see Baker and Kim (2004), for example.

3.1. Maximum Likelihood Estimation

As described in Baker and Kim (2004) and Azevedo (2003) we have the following log-likelihood to an individual latent trait

$$l(\theta_j, \widehat{\Gamma}) \equiv l(\theta_j) = \sum_{i=1}^{I} \sum_{s=1}^{m_i} y_{ijs} \ln P_{ijs},$$

where $\widehat{\Gamma}$ represents an estimate of the item free parameters (Azevedo 2003). After some algebra, we obtain the score function,

$$S(\theta_j) = \sum_{i=1}^{I} \alpha_i^t T_i \left[y_{ij.} - P_{ij.} \right], \tag{3}$$

and Hessian matrix

$$H(\theta_j) = -\sum_{i=1}^{I} \left\{ \alpha_i^t T_i W_{ij} T_i^t \alpha_i \right\}, \tag{4}$$

where α_i, T_i and W_i are suitable matrices which do not depend on the responses. We may notice, by (4), that the Hessian Matrix is non-stochastic, then the Information Matrix is only $I(\theta_j) = -H(\theta_j)$. Thereafter, considering $\widehat{\theta}_j$ an estimative of θ_j in iteration t, we can define the Newton-Raphson / Fisher Scoring method (Baker and Kim, 2004) as

Newton-Raphson

$$\widehat{\theta}_j^{(t+1)} = \widehat{\theta}_j^{(t)} - H\left(\widehat{\theta}_j^{(t)}\right)^{-1} S\left(\widehat{\theta}_j^{(t)}\right), \tag{5}$$

Fisher Scoring

$$\widehat{\theta}_j^{(t+1)} = \widehat{\theta}_j^{(t)} + I\left(\widehat{\theta}_j^{(t)}\right)^{-1} S\left(\widehat{\theta}_j^{(t)}\right), \tag{6}$$

$t = 1, 2, \ldots$, up to reach a convergence criteria.

In the next subsection we present the modal and expectation bayesian estimation.

3.2. Bayes Modal Estimation

In a general way, in the bayesian estimation we need to use the posterior distribution which follows directly from the **Bayes Theorem** (Bernardo and Smith, 1998),

$$g_j^*(\theta_j) \equiv Cg(\theta_j|y_{.j.},\widehat{\Gamma},\eta) = C\,P(Y_{.j.}|\theta_j,\widehat{\Gamma})g(\theta_j|\eta) \propto P(Y_{.j.}|\theta_j,\widehat{\Gamma})g(\theta_j|\eta), \tag{7}$$

where $Y_{.j.} = (Y_{1j1},\ldots,Y_{1jm_i},\ldots,Y_{Ij1},\ldots,Y_{Ijm_i})^t$, $P(Y_{.j}|\theta_j,\widehat{\Gamma})$ is the profile likelihood, $g(\theta_j|\eta)$ is a convenient priori, η are the **hyperparameters** (or the populational parameters) and C is a normalization constant. To evaluate the **bayes modal** we need to maximize equation (7). Thereafter, taking its natural logarithm we have

$$\ln g_j^*(\theta_j) \equiv l_j^*(\theta_j) = l(\theta_j) + \ln g(\theta_j|\eta) + const. \tag{8}$$

Differentiating (8) we have the **bayesian estimating equation**

$$S(\theta_j)_B = \frac{\partial l_j^*(\theta_j)}{\partial\theta_j} = \frac{\partial l(\theta_j)}{\partial\theta_j} + \frac{\partial \ln g(\theta_j|\eta)}{\partial\theta_j}. \tag{9}$$

Notice that the first term in the right-hand side of (9) is exactly the **score function** defined in (3) and, considering a skew-normal distribution as the prior, that is, $\theta_j|\eta_j \sim SN(\mu_{\theta_j},\sigma^2_{\theta_j},\lambda_{\theta_j})$ (Genton, 2004) it follows that

$$S(\theta_j)_B = S(\theta_j) - \frac{\theta_j-\mu_\theta}{\sigma^2_\theta} + \left\{\Phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\right\}^{-1}\phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\frac{\lambda_{\theta_j}}{\sigma_{\theta_j}},$$

where $\eta_j = (\mu_{\theta_j},\sigma^2_{\theta_j},\lambda_{\theta_j})^\top$, μ_{θ_j} is the mean, $\sigma^2_{\theta_j}$ is the variance, λ_{θ_j} is the asymmetry parameter, $\phi(.)$ and $\Phi(.)$ are the density and cumulative function of a standard normal distribution. It follows that, the hessian matrix and the Fisher information are given by

Hessian Matrix

$$H(\theta_j)_B = H(\theta_j) - \left\{\frac{1}{\sigma^2_\theta} + h(\lambda_{\theta_j},\theta_j)\right\},$$

where $H(\theta_j)$ is given by (4) and,

$$
\begin{aligned}
h(\lambda_{\theta_j},\theta_j) &= \left\{\Phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\right\}^{-2}\phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\left(\frac{\lambda_{\theta_j}}{\sigma_{\theta_j}}\right)^2 \\
&\times \left\{\Phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\frac{\lambda_{\theta_j}}{\sigma_{\theta_j}}(\theta_j-\mu_{\theta_j})+\phi\left[\lambda_{\theta_j}\left(\frac{\theta_j-\mu_{\theta_j}}{\sigma_{\theta_j}}\right)\right]\right\}, \qquad (10)
\end{aligned}
$$

Fisher Information

$$
I(\theta_j)_B = I(\theta_j) + \frac{1}{\sigma_\theta^2} + h(\lambda_{\theta_j},\theta_j).
$$

The Fisher information is only the symmetric value of hessian matrix in the NRM. The iterative process may be applied as described in (5) and (6). In the next subsection we present the bayes expectation estimator.

3.3. Bayes Expectation

From (7) we have that the bayes expectation is given by

$$
\mathbb{E}\left[\theta_j|\boldsymbol{y}_{.j.},\widehat{\boldsymbol{\Gamma}},\boldsymbol{\eta}\right] = \frac{\int_{\Re}\theta P(\boldsymbol{Y}_{.j.}|\theta_j,\widehat{\boldsymbol{\Gamma}})g(\theta_j|\boldsymbol{\eta})d\theta}{\int_{\Re}P(\boldsymbol{Y}_{.j.}|\theta_j,\widehat{\boldsymbol{\Gamma}})g(\theta_j|\boldsymbol{\eta})d\theta}. \qquad (11)
$$

Generally, the integrals in (11) do not have an explicit form and then they must be solved by some numerical method of integration (Robert and Casella, 1999). Again, considering a $SN(\mu_\theta,\sigma_\theta^2,\lambda_\theta)$ with $\lambda_{\theta_j}=0$, i.e., a normal distribution, the integrals may be solved by **Gauss-Hermite** method (Stroud and Secrest, 1966) or by generating quadrature points considering any value to λ_θ. Then, in terms of the quadrature points, (11) becomes

$$
\begin{aligned}
\mathbb{E}\left[\theta_j|\boldsymbol{y}_{.j.},\widehat{\boldsymbol{\Gamma}},\boldsymbol{\eta}\right] &\approx \mathbb{E}\left[\overline{\theta}_j|\boldsymbol{y}_{.j.},\widehat{\boldsymbol{\Gamma}},\boldsymbol{\eta}\right] = \frac{\sum_{l=1}^{q}\overline{\theta}_l P(\boldsymbol{Y}_{.j.}|\overline{\theta}_l,\widehat{\boldsymbol{\Gamma}})g(\overline{\theta}_l|\boldsymbol{\eta})}{\sum_{l=1}^{q}P(\boldsymbol{Y}_{.j.}|\overline{\theta}_l,\widehat{\boldsymbol{\Gamma}})g(\overline{\theta}_l|\boldsymbol{\eta})} \\
&= \frac{\sum_{l=1}^{q}\overline{\theta}_l P(\boldsymbol{Y}_{.j.}|\overline{\theta}_l,\widehat{\boldsymbol{\Gamma}})A_l}{\sum_{l=1}^{q}P(\boldsymbol{Y}_{.j.}|\overline{\theta}_l,\widehat{\boldsymbol{\Gamma}})A_l},
\end{aligned}
$$

where $\overline{\theta}_l$ and A_l, are the quadrature points and the quadrature weights, respectively. A measure of precision of the EAP estimation is given by the Variance a Posteriori (VAP) that has the following form

$$Var\left[\overline{\theta}_j|y_{.j.},\widehat{\Gamma},\eta\right] = \frac{\sum_{l=1}^{q}\left\{\overline{\theta}_l - E\left[\overline{\theta}_j|y_{.j.},\widehat{\Gamma},\eta\right]\right\}^2 P(Y_{.j.}|\overline{\theta}_l,\widehat{\Gamma})A_l}{\sum_{l=1}^{q} P(Y_{.j.}|\overline{\theta}_l,\widehat{\Gamma})A_l}.$$

We need to point out that for both MAP and EAP methods we used aa asymmetry sample estimative for λ_θ using the observed scores. To generate quadrature points considering any value for λ_θ, it was written a R-function that generates quite similar values to those used for Bilog (Mislevy and Bock, 1990) when $\lambda_\theta = 0$.

In next section we present the simulation study.

4. Simulation Study

First of all, we want to clarify that the our main goal is to compare the models not the estimation methods. In this sense, it is very likely that, depending on the situation, other estimation methods may work better.

To conduct the study of simulation we consider three levels of asymmetry for the latent distribution $\lambda_\theta = (-2, 0, 2)^t$, under the parameterization considered in Genton (2004) and three sample sizes, $n = (500, 1000, 3000)^t$.

Concerning the test we considered the following situation. We generated individual's answers using two item parameter sets related to NRM, one of them related to $\lambda_\theta = 0$ and $\lambda_\theta = 2$ and the another one concerning to $\lambda_\theta = -2$, in order to ensure the covering of the latent trait range. After this, the answers were corrected as right/wrong, considering the most difficult alternatives as the right ones. This was made in order to use the DRM (both the 2P and 3P models).

For each one of 9 response sets we used MMAP method to the item parameter estimation and, given these estimatives, we used the EAP and MAP to obtain the latent trait estimatives. All the simulated data and the calculations were done using specific programs written by the authors in R language, see R Development Core Team (2006), and may be asked for directly from the authors.

We have also to keep in mind that the real model that generates the responses in a real multiple-choice test is not necessarily neither the NRM or the DRM, but we are considering that the former and/or the latter ones are reasonable mechanisms to the represent this process.

The statistics used in the simulation study were,

- Correlation : the mean of all correlations between the true latent traits and their estimatives among all replicas.

- Mvar : the mean of all variances associated to all replicas and all estimatives.

- MSR : the mean of squared residuals among all replicas and latent traits, that is,, $\frac{1}{nR}\sum_{r=1}^{n}\sum_{j=1}^{R}(\theta_j - \widehat{\theta}_{jr})^2$.

- Mbias: the sum of two former statistics.

Table 1 shows statistics for all of three models. We may see that highest correlations are associated with NRM and also that, for negative asymmetry, the NRM seems more appropriate than the other models. The estimatives from NRM with EAP present the smallest variances associated to the latent trait estimation as well as the smallest residuals, considering null and negative asymmetry. The NRM with MAP presents the smallest statistics, variance and residuals, for positive asymmetry. These behaviors are more clear in the highest samples sizes. In general, one may see that the bias related to the estimatives are smallest for the NRM approach with both EAP and MAP methods.

Figures 1 to 9 present the scatter plots of true latent traits and their estimatives. It is clear that to the negative asymmetry, the estimatives are worse than the other situations because the distance between them and true values. Probably, this happens because the estimation procedures. For the null asymmetry the results are closer to those related to the presence of positive asymmetry. In these analysis is also evident the superiority of NRM approach.

Figures 10 to 18 show the variance associated to the estimatives along the latent traits values. For the negative asymmetry, the variances are smaller in the DRM approaches, for small values of latent traits and the variances of NRM approach are smaller for other values of latent trait. The same happens to the null asymmetry but is more evident the superiority of NRM approach. However, for the positive asymmetry, it seems that the NRM with EAP presents the best results for all latent trait values.

Figures 19 to 27 present the mean of squared residuals. We may see that, for the negative asymmetry, the NRM presents the smallest values, for negative latent trait values and the highest for the positive latent trait values, even though this last difference is very slight. For symmetry situation, the NRM shows the smallest values independently of the latent trait value, specially for smaller values of them. Finally, for the positive asymmetry, we may notice that the NRM produces the smallest values for the positive latent traits and the highest for the negative values of latent trait. Again, the difference in favor to the NRM is higher than that against it.

Figures 28 to 36 present the bias of the estimatives obtained from the three models. Basically, the results go toward to that from mean of squared residuals.

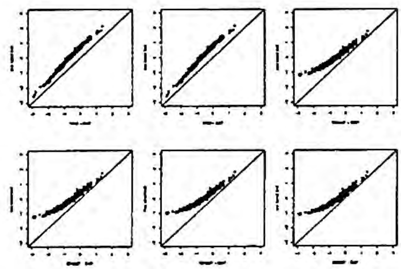

Figure 1. Correlation of estimatives and true values: $\lambda_\theta = -2$ and n= 500

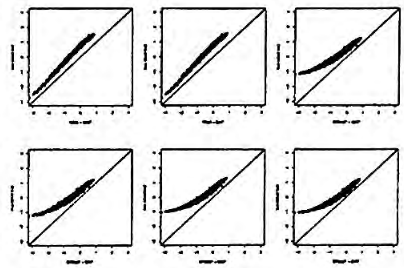

Figure 2. Correlation of estimatives and true values: $\lambda_\theta = -2$ and n= 1000

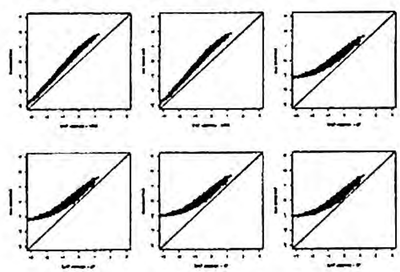

Figure 3. Correlation of estimatives and true values: $\lambda_\theta = -2$ and n= 3000

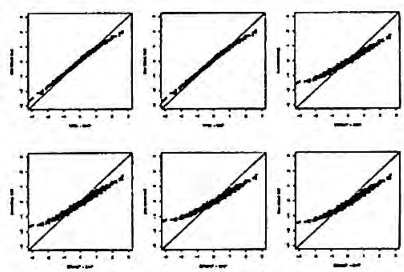

Figure 4. Correlation of estimatives and true values: $\lambda_\theta = 0$ and n= 500

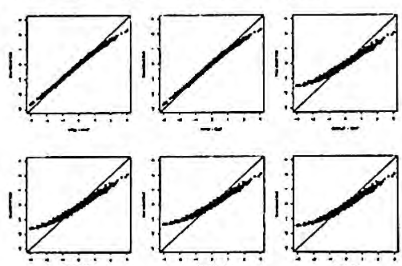

Figure 5. Correlation of estimatives and true values: $\lambda_\theta = 0$ and n= 1000

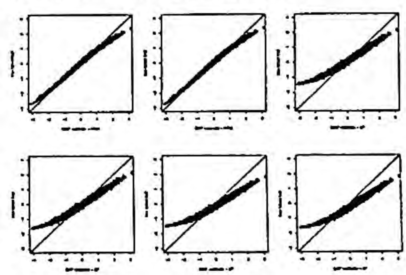

Figure 6. Correlation of estimatives and true values: $\lambda_\theta = 0$ and n= 3000

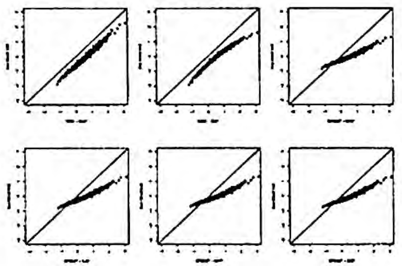

Figure 7. Correlation of estimatives and true values: $\lambda_\theta = 2$ and n= 500

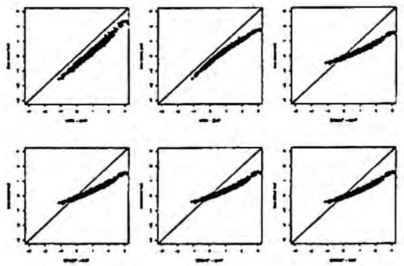

Figure 8. Correlation of estimatives and true values: $\lambda_\theta = 2$ and n= 1000

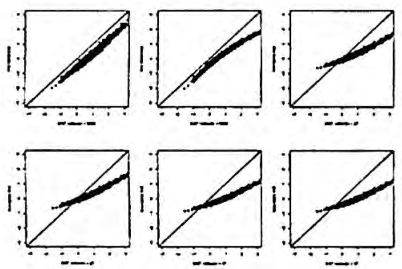

Figure 9. Correlation of estimatives and true values: $\lambda_\theta = 2$ and n= 3000

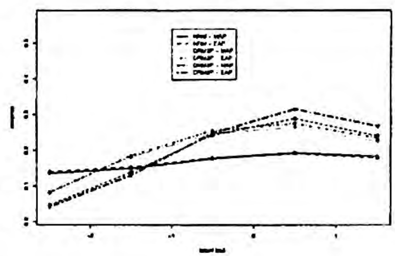

Figure 10. Variance of estimatives : $\lambda_\theta = -2$ and n= 500

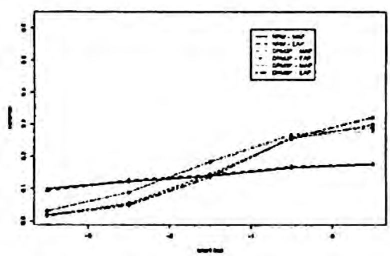

Figure 11. Variance of estimatives : $\lambda_\theta = -2$ and n= 1000

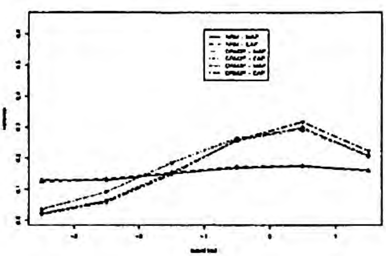

Figure 12. Variance of estimatives : $\lambda_\theta = -2$ and n= 3000

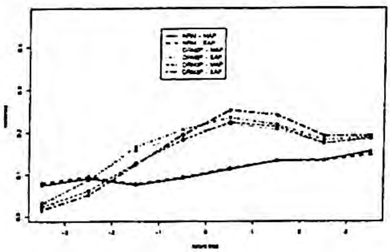

Figure 13. Variance of estimatives : $\lambda_\theta = 0$ and n= 500

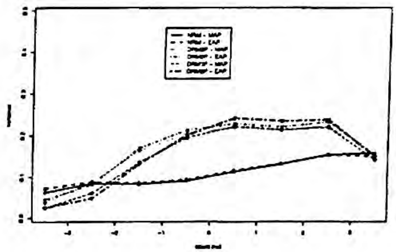

Figure 14. Variance of estimatives: $\lambda_\theta = 0$ and n= 1000

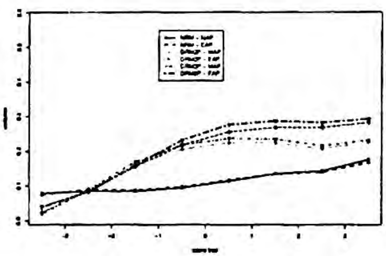

Figure 15. Variance of estimatives: $\lambda_\theta = 0$ and n= 3000

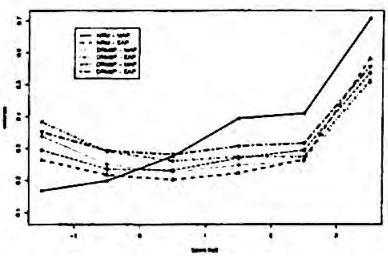

Figure 16. Variance of estimatives : $\lambda_\theta = 2$ and n= 500

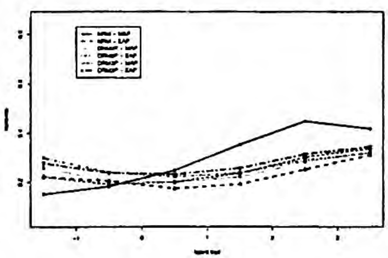

Figure 17. Variance of estimatives : $\lambda_\theta = 2$ and n= 1000

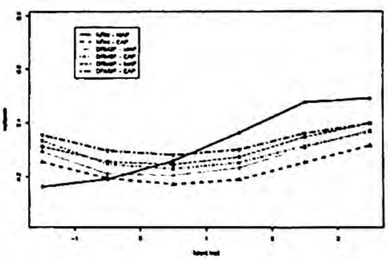

Figure 18. Variance of estimatives : $\lambda_\theta = 2$ and n= 3000

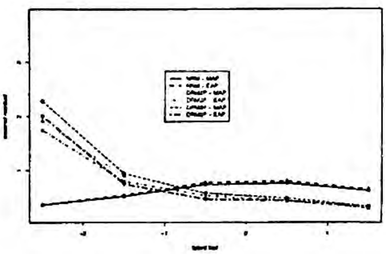

Figure 19. Mean of Squared Residual : $\lambda_\theta = -2$ and n= 500

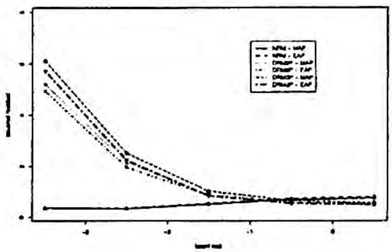

Figure 20. Mean of Squared Residual : $\lambda_\theta = -2$ and n= 1000

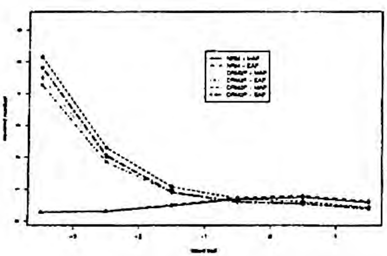

Figure 21. Mean of Squared Residual : $\lambda_\theta = -2$ and n= 3000

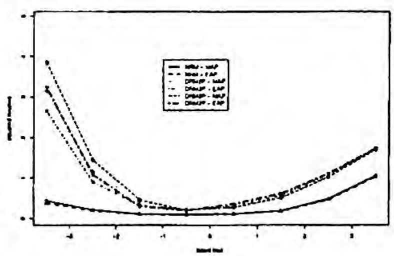

Figure 22. Mean of Squared Residual : $\lambda_\theta = 0$ and n= 500

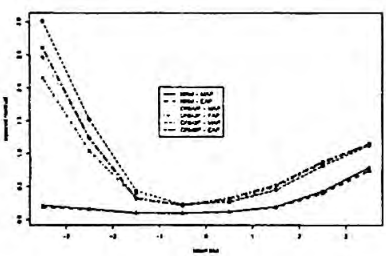

Figure 23. Mean of Squared Residual : $\lambda_\theta = 0$ and n= 1000

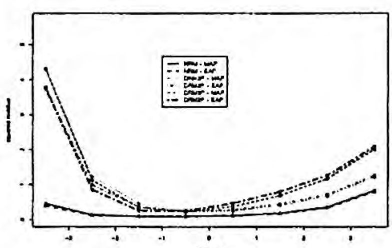

Figure 24. Mean of Squared Residual : $\lambda_\theta = 0$ and n= 3000

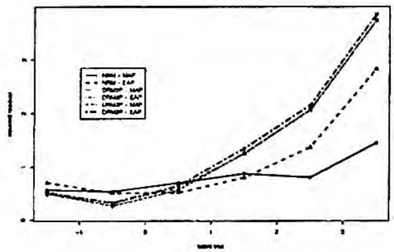

Figure 25. Mean of Squared Residual : $\lambda_\theta = 2$ and n= 500

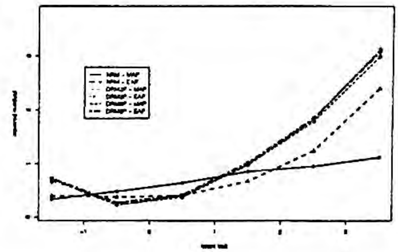

Figure 26. Mean of Squared Residual : $\lambda_\theta = 2$ and n= 1000

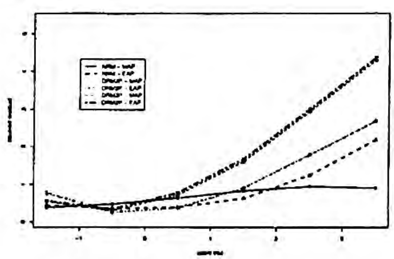

Figure 27. Mean of Squared Residual : $\lambda_\theta = 2$ and n= 3000

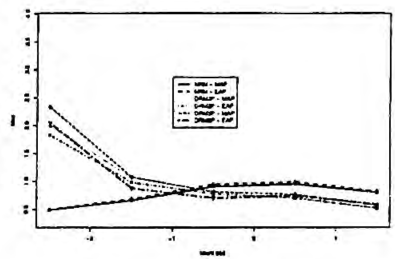

Figure 28. Bias of Estimatives : $\lambda_\theta = -2$ and n= 500

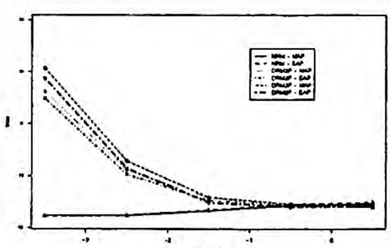

Figure 29. Bias of Estimatives : $\lambda_\theta = -2$ and n= 1000

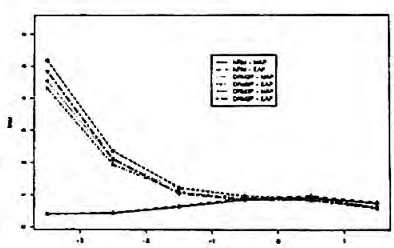

Figure 30. Bias of Estimatives : $\lambda_\theta = -2$ and n= 3000

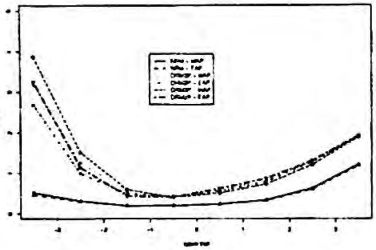

Figure 31. Bias of Estimatives : $\lambda_\theta = 0$ and n= 500

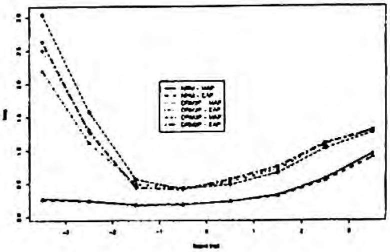

Figure 32. Bias of Estimatives : $\lambda_\theta = 0$ and n= 1000

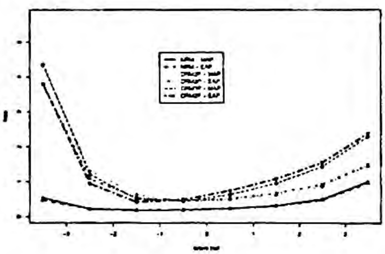

Figure 33. Bias of Estimatives : $\lambda_\theta = 0$ and n= 3000

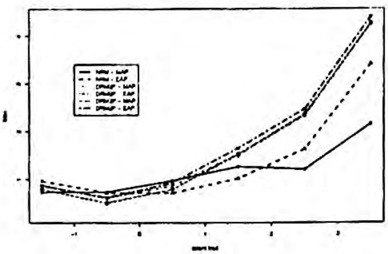

Figure 34. Bias of Estimatives : $\lambda_\theta = 2$ and n= 500

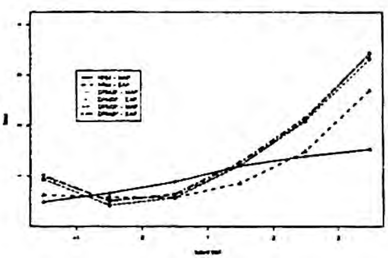

Figure 35. Bias of Estimatives : $\lambda_\theta = 2$ and n= 1000

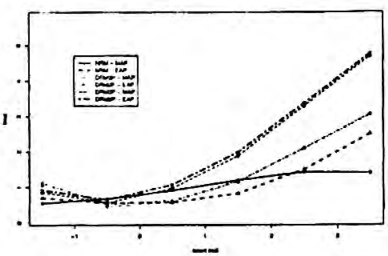

Figure 36. Bias of Estimatives : $\lambda_\theta = 2$ and n= 3000

Table 1
Statistics of the latent trait estimation

Statistic	Model	Estimation Method	λ_θ = -2, n = 500	-2, 1000	-2, 3000	0, 500	0, 1000	0, 3000	2, 500	2, 1000	2, 3000
Correlation	NRM	EAP	0.994	0.996	0.995	0.995	0.996	0.996	0.987	0.989	0.989
		MAP	0.994	0.996	0.995	0.996	0.996	0.996	0.989	0.989	0.989
	DRM-3P	EAP	0.968	0.960	0.965	0.983	0.985	0.986	0.978	0.978	0.979
		MAP	0.971	0.963	0.967	0.984	0.986	0.987	0.977	0.977	0.979
	DRM-2P	EAP	0.978	0.972	0.972	0.988	0.989	0.988	0.981	0.981	0.981
		MAP	0.979	0.972	0.972	0.988	0.989	0.988	0.980	0.980	0.980
Mvar	NRM	EAP	0.172	0.162	0.167	0.105	0.107	0.109	0.213	0.186	0.184
		MAP	0.170	0.159	0.164	0.103	0.104	0.106	0.302	0.276	0.287
	DRM-3P	EAP	0.216	0.228	0.230	0.206	0.208	0.243	0.292	0.244	0.293
		MAP	0.212	0.225	0.228	0.189	0.196	0.228	0.246	0.213	0.260
	DRM-2P	EAP	0.231	0.242	0.242	0.207	0.211	0.217	0.270	0.232	0.243
		MAP	0.224	0.236	0.237	0.195	0.203	0.206	0.238	0.210	0.219
MSR	NRM	EAP	0.686	0.673	0.664	0.137	0.126	0.123	0.623	0.514	0.497
		MAP	0.653	0.634	0.627	0.138	0.127	0.123	0.729	0.689	0.683
	DRM-3P	EAP	0.594	0.705	0.741	0.399	0.353	0.446	0.847	0.629	1.069
		MAP	0.730	0.848	0.876	0.387	0.347	0.405	0.765	0.584	1.003
	DRM-2P	EAP	0.633	0.725	0.750	0.372	0.339	0.329	0.795	0.610	0.578
		MAP	0.694	0.788	0.811	0.370	0.338	0.331	0.764	0.601	0.563
Mbias	NRM	EAP	0.858	0.835	0.831	0.242	0.232	0.231	0.836	0.700	0.681
		MAP	0.823	0.793	0.790	0.241	0.231	0.229	1.031	0.965	0.971
	DRM-3P	EAP	0.810	0.932	0.971	0.605	0.560	0.689	1.139	0.873	1.362
		MAP	0.942	1.073	1.104	0.576	0.544	0.633	1.010	0.797	1.263
	DRM-2P	EAP	0.865	0.967	0.992	0.580	0.550	0.546	1.065	0.842	0.821
		MAP	0.917	1.024	1.048	0.565	0.541	0.537	1.002	0.810	0.782

5. Discussion

In this work we compared the NRM and two types of DRM to estimate the latent traits in a multiple choice test. We used simulated data and developed specific programs in R language (R development Core Team, 2006) to do the simulations and also all the calculations. Furthermore, we study the effects of latent trait asymmetry in their estimation process which may be viewed as an extension of NRM proposed by Bock

(1972).

The results point out the best fit using NRM-EAP approach, even though, for the asymmetry situations, the estimatives were far from true values. We believe that the main reason for this is that the estimation methods used did not take into the asymmetry account in a suitable way. To use the observed scores to estimate the asymmetry parameter and then to use this value for the latent trait estimation procedure, may produce poor results as we observed. Better results could be obtained using MCMC methods, or another suitable approach, that would allow the estimation of all parameters concurrently.

6. Acknowledgments

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