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Semigroup laws and free semigroups in the group of units of group rings

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SEMIGROUP LAWS AND FREE SUBSEMIGROUPS IN THE GROUP OF UNITS OF GROUP RINGS

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1. Introduction

Let KG be the group ring of the group G over the field K, and let $U(KG)$ be its group of units. We say that $U(KG)$ satisfies a semigroup law if $w_1(a,b) = w_2(a,b)$ holds in $U(KG)$, where w_1 and w_2 are distinct words in the free subsemigroup freely generated by a and b.

It would be interesting to establish when $U(KG)$ satisfies a semigroup law, but this seems very difficult. Some special cases are known. For example, a nilpotent group satisfies a semigroup law, and conditions under which $U(KG)$ is nilpotent are given in Sehgal [4].

In this note we study when $U(KG)$ satisfies a semigroup law, for G torsion and K infinite. Without much additional effort we give sufficient conditions for $U(KG)$ to contain a subsemigroup on two free generators.

Our main inspiration comes from Makar-Limanov [1].

2. Some Lemmas

Lemma 2.1: *Let G be a group and let a and b be two elements of G. For each nonnegative integer k let $a_k = b^k a b^{-k}$. Then*

(i) if G does not contain subsemigroups on two free generators the elements a_k satisfy a relation of the form

$$a_1^{i_1} \dots a_m^{i_m} = a_1^{j_1} \dots a_m^{j_m},$$

where $m > 1$, i_k and j_k are equal to 0 or 1, and $i_k - j_k$ is not always equal to 0.

(ii) if G satisfies a semigroup law then this relation can be brought to the form (*).

Proof: See [3], Lemma 4.8.

Lemma 2.2: *Let a and b be two elements of the unital ring R, with b invertible. Let be given a positive integer n, a set of rational integers $\mathbf{k} = \{k_j\} \neq \{0\}$, and a polynomial*

$$p_n(a, b, \mathbf{k}) = \sum_{j=0}^{n} k_j T^j(a),$$

where $T^j(a) = b^j a b^{-j}$.

Let $a' = ba - ab$, $s_j = \sum_{i=j}^{n} k_i$, and $p_i(a, b) = b^{-1} \sum_{j=i+1}^{n} s_j T^j(a') + (k_i + s_{i+1}) T^i(a) + \sum_{j=0}^{i-1} k_j T^j(a)$, $0 \leq i \leq n$. Then $p_0(a, b) = p_1(a, b) = \ldots = p_n(a, b)$.

Remark: We observe that

$$p_0(a, b) = \left[\sum_{j=0}^{n-1} s_{j+1} T^j(a') + s_0 ab\right] b^{-1}.$$

Proof: Let us show that $p_i = p_{i-1}$. We have

$$p_i - p_{i-1} = -b^{-1} s_i T^i(a') + (k_i + s_{i+1}) T^i(a) - (k_{i-1} + s_i) T^{i-1}(a) + k_{i-1} T^{i-1}(a)$$

$$= -b^{-1} s_i T^i(a') + s_i T^i(a) - s_i T^{i-1}(a)$$

$$= -b^{-1} s_i T^i(a') + s_i (T^i(a) - T^{i-1}(a)) = 0, \quad \text{since by definition}$$

$$T^i(a) - T^{i-1}(a) = b^{-1} T^i(a').$$ ∎

Given an element x in a ring R we define inductively, for every y in R $[y, x]^{(0)} = y$, $[y, x] = y' = yx - xy$, and for every $i \geq 0$ $[y, x]^{(i+1)} = [[y, x]^{(i)}, x]$.

Lemma 2.3: *Let R be a unital ring containing an infinite field K in its center. Let x and y be two elements of R such that, for infinitely many μ in K $\mu + x$ is invertible, and let us suppose that there are a positive integer n, a set $\mathbf{k} = \{k_j \in \mathbf{Z} \mid 1 \leq j \leq n\} \neq \{0\}$ as in Lemma 2.2 such that*

$$p_n(y, x + \mu) = 0, \quad \textit{for infinitely many } \mu \textit{ in } K. \textit{ Then } [y, x]^{(n)} = 0.$$

Proof: We will prove by induction on n that $[y,x]^{(n)} = 0$.

(i) $n = 1$. Then

$Abab^{-1} + Ba = 0$ for $a = y$, and for infinitely many $b = x + \mu$. This means that $Aba + Bab = A(x+\mu)y + By(x+\mu) = 0$ holds for infinite μ's. Then $(Axy + Byx) + (Ay + By)\mu = 0$ implies that

$$\begin{cases} (A+B)y = 0 \\ Axy + Byx = 0. \end{cases}$$

Hence

$$\begin{cases} B = -A \\ A(xy - yx) = 0. \end{cases}$$

Since the relations is nontrivial $A \neq 0$, and so

$$[y,x] = 0$$

(ii) **Induction step.** Let us assume that the hypothesis is true for polynomials with $i \leq n-1$.

We have that

$$p_0(a,b) = \Big[\sum_{j=0}^{n-1} s_{j+1}T^j(a') + s_0ab\Big]b^{-1},$$

and $p_0(y, x+\mu) = 0$ for infinitely many μ in K, if and only if

$$\widetilde{p_0}(a,b) = \Big[\sum_{j=0}^{n-1} s_{j+1}b^ja'b^{(n-1)-j} + s_0ab^n\Big]b^{-1}$$

is zero for $a = y$, $b = x + \mu$ and infinitely many μ in K. But the only contribution to the coefficient of μ^n is s_0, and therefore $s_0 = 0$. Hence

$$p_0(a,b) = \Big[\sum_{j=0}^{n-1} s_{j+1}T^j(a')\Big]b^{-1}$$

is a relation that holds for the pairs $a' = [y,x]$ and $b = x + \mu$, for infinitely many μ in K. By the induction hypothesis

$$\Big[[y,x]^{(1)}, x\Big]^{(n-1)} = [y,x]^{(n)} = 0. \qquad \blacksquare$$

3. Semigroup laws

Let G be a group and let us denote by ζG the center of G.

Theorem 3.1: *Let K be an infinite field of characteristic $p \geq 0$, and let G be a group generated by torsion elements. If $U(KG)$ satisfies a semigroup law, then the subgroup of G generated by p'–elements (torsion elements if $p = 0$) is central.*

Proof: Let $g^m = h^\ell = 1$ be two torsion elements such that $p \nmid \ell$. Let μ and λ be two elements of K such that $\mu^m \neq 1 \neq \lambda^{-\ell}$. Then

$$(1-\mu g)^{-1} = (1+\mu g+\ldots+\mu^{m-1}g^{m-1}) \,/\, (1-\mu^m)$$

$$(\lambda+h)^{-1} = \lambda^{-1}(1-\lambda^{-1}h+\ldots+(-1)^{\ell-1}\lambda^{-(\ell-1)}h^{\ell-1}) \,/\, (1-\lambda^{-\ell})$$

are elements of $U(KG)$.

Let us plug

$$a_s = 1-\mu(\lambda+h)^s g(\lambda+h)^{-s}$$

in (*), Lemma 2.1. Since we have a polynomial identity in μ their first degree coefficients are equal. This implies that the nontrivial relation

$$\sum_{s=1}^{n}(i_s-j_s)b^s ab^{-s} = 0$$

is verified for infinitely many pairs $a = g$, $b = h+\lambda$, λ in K. By Lemma 2.3

$$[g,h]^{(n)} = 0.$$

Now we consider two cases:

(i) p > 0

It is easy to verify that

$$[g,h]^{(n)} = \sum_{i=0}^{n}(-1)^i C_n^i h^i g h^{n-i}.$$

Choose an integer r such that $s = p^r \geq n$. Then

$$[g,h]^{(s)} = 0 = gh^s - h^s g,$$

and since p and ℓ are relatively prime it follows that $gh = hg$, that is $h \in \zeta G$.

(ii) $p = 0$

Let p_1 and p_2 be two distinct rational prime integers. It is clear that $[g,h]^{(n)} = 0$ in $(\mathbb{Z}/p_i\mathbb{Z})G$, $i = 1,2$, and so, by what has been proved before, there are integers t_1 and t_2, such that the elements $h^{p_1^{t_1}}$ and $h^{p_2^{t_2}}$ are in the centralizer of g in G.

But $rp_1^{t_1} + sp_2^{t_2} = 1$, for convenient integers r and s, and so $h = (h^{p_1^{t_1}})^r\,(h^{p_2^{t_2}})^s$ commutes with g, that is $h \in \zeta G$. ∎

Proposition 3.2: *Let K be an infinite field of characteristic $p > 0$, let G be a group such that KG is algebraic over K, and let us assume that $U(KG)$ satisfies a semigroup law. Then there exists a positive integer n such that*

$$U(KG)^{p^n} \subseteq \zeta U(KG).$$

Proof: First we observe that G is torsion. Otherwise, let x be a nontorsion element of G. Then $\{x^i \mid i \in \mathbb{Z}\}$ is linearly independent over K, in contradiction with the hypothesis that KG is algebraic over K.

Next we note that if

$$f(X) = X^r + a_1X^{r-1} + \ldots + a_{r-1}X + a_r$$

is the minimal polynomial of $u \in KG$ over K, then u is invertible if and only if $a_r \neq 0$.

Now, let $u \in KG$ be a unit.

We claim that $u + \lambda$ is a unit for infinitely many λ in K.

Indeed, let

$$h(X) = f(X - \lambda) = X^r + b_1X^{r-1} + \ldots + b_{r-1}X + b_r.$$

Then $h(X)$ is the minimal polynomial of $u + \lambda$ over K, and the constant term of $h(X)$ is

$$h(0) = f(-\lambda) = b_r.$$

Since $f(X)$ has at most r roots in K and K is infinite, we have infinitely many choices for λ in K such that $b_r \neq 0$. This proves the claim.

Let $u \in KG$, and let $g \in G$ be such that $g^m = 1$. Let λ and μ be elements of K such that $u + \lambda$ and $1 - \mu g$ are units. Then, arguing as in Theorem 3.1, we conclude that there exists a positive integer n, depending only of the identity, such that

$$u^{p^n} g = g u^{p^n}.$$

This implies that $U(KG)^{p^n} \subseteq \zeta U(KG)$. ■

Before stating the next theorem, let us remember that a group G is called *p-abelian* if the commutator subgroup G' of G is a finite p-group. A ring R is called *m-Engel*, for a positive integer m, if for any x and y in R we have that $[y,x]^{(m)} = 0$.

For a group G and a subgroup H of G we define inductively, for $i \geq 1$, $(H,G) = (H, {}_1G)$ and $(H, {}_{(i+1)}G) = ((H, {}_{(i)}G), G)$, the subgroup generated by $(x,y) = xyx^{-1}y^{-1}$, with x in $(H, {}_{(i)}G)$ and y in G. .

Theorem 3.3: *Let K be an infinite field of characteristic $p > 0$, and let G be a locally finite p-group. Then are equivalent:*

(i) $U(KG)$ satisfies a semigroup law;

(ii) G is nilpotent, the FC-subgroup ϕ of G is p-abelian, and $[G:\phi] < \infty$;

(iii) there exists a positive integer m such that KG is Lie m-Engel;

(iv) there exists a positive integer n, such that $U(KG)$ satisfies the identity

$$x_1^{p^n} x_2 = x_2 x_1^{p^n}.$$

Proof: (i) $\Rightarrow$ (ii). Since G is locally finite KG is algebraic over K. By Proposition 3.2 there exists a positive integer n such that

$$U(KG)^{p^n} \subseteq \zeta U(KG).$$

Let x, y and z be elements of KG and let us denote by $\Delta(G,G')$ the kernel of the canonical map $KG \to K(G/G')$. Considering, instead of G, the subgroup generated by the supports of x, y and z, we can assume that G is a finite p-group. Then $\Delta(G,G')$ is nilpotent and $xy - yx$ belongs to $\Delta(G,G')$. Hence $1 + xy - yx \in U(KG)$, and by the first part

$$(1 + xy - yx)^{p^n} z = z(1 + xy - yx)^{p^n}.$$

By [2], Theorem V.2.14 we have $[G : \phi] < \infty$ and $|\phi'| < \infty$.

We claim that G is nilpotent.

This argument is due to Sehgal [4], Theorem V.6.1, and we reproduce it here for sake of completeness.

To begin with, we note that G acts as a finite p-group of automorphisms of ϕ'. Applying [4], Lemma V.4.1 to G/ϕ'' and $A = \phi'/\phi''$, we obtain that

$$((\phi', G), \ldots, G) = (\phi', {}_sG) \subseteq \phi''.$$

Repeating this procedure with ϕ'', ϕ''', etc., we eventually obtain that $(\phi', {}_rG) = 1$. This means that $\phi' \subseteq \zeta_r G$, the r-th term of the upper central series of G. So, we can assume that $\phi' = 1$.

Therefore, with this simplification, we have an abelian normal subgroup $A = \phi$ of G such that, G/A is a finite p-group, and $G^{p^n} \subseteq \zeta G$. We are allowed to simplify further factoring by the central subgroup A^{p^n}. Hence $A^{p^n} = 1$, and by [4], Lemma V.4.1, there exists a positive integer t such that

$$(A, {}_tG) = 1,$$

that is, $A \subseteq \zeta_t G$. Therefore G is nilpotent.

(ii) $\Rightarrow$ (iii). This follows by [4], Theorem V.6.1.

(iii) $\Rightarrow$ (iv). Since KG is Lie n-Engel, for every x and y in K

$$[y, x]^{(p^n)} = yx^{p^n} - x^{p^n} y = 0.$$

In particular

$$U(KG)^{p^n} \subseteq \zeta U(KG).$$

Therefore, for every x_1, x_2 in $U(KG)$, we have

$$x_1^{p^n} x_2 = x_2 x_1^{p^n}.$$

(iv) $\Rightarrow$ (i). Obvious. ∎

Corollary 3.4: *Let K be an infinite field of characteristic p. Let G be a torsion group such that, if $p > 0$ then the set S_p of p-elements of G forms a locally finite group. Then $U(KG)$ satisfies a semigroup law if and only if*

(i) $p = 0$ and G is abelian;

(ii) $p > 0$ and G is nilpotent, $S_{p'} \subseteq \zeta G$, the FC-subgroup ϕ of S_p is p-abelian and S_p/ϕ is finite.

At this point a very natural question arises: *If G is torsion and K is infinite can we recover the characterization of nilpotence of $U(KG)$, given in [4]?*

The answer is yes, and we will present it here. First we need

Definition: *Let G be a group. We say that G is T-nilpotent if, given any sequence $(x_i)_{i\in \mathbf{N}}$ of elements of G, there exists a positive integer n, depending on the sequence, such that*

$$((x_0, x_1), \ldots, x_n) = 1.$$

Proposition 3.5: *Let K be a field of characteristic $p > 0$, and let G be a group possessing a central element h of order p. Then $U(KG)$ T-nilpotent implies that, for every $g \in G$, $[\phi : C_\phi(g)] < \infty$. Here $C_\phi(g)$ denotes the centralizer of g in ϕ – the FC-subgroup of G.*

Proof: Let us assume that there exists an element g in G such that $[\phi : C_\phi(g)] = \infty$, and let $\eta = \sum_{i=0}^{p-1} h^i$. We construct inductively sequences $\{g_i \mid g_i \in \phi\}$, and $\{b_i \mid b_i = (g_i, g) \in \phi\}$, such that

(i) $g_{n+1} \in C_\phi\{g_1, \ldots, g_n, b_1, \ldots, b_n\}$ and

(ii) $\eta(1-b_1)\ldots(1-b_n) \neq 0$ for all n.

Certainly, we can choose $g_1 \in \phi$ such that $b_1 = (g_1, g)$ and $\eta(1-b_1) \neq 0$. Indeed, since $\phi = \cup_{i \in I} h_i C_\phi(g)$, $\# I = \infty$, for $i \neq j$ we have

$$h_i g h_i^{-1} \neq h_j g h_j^{-1}, \quad \text{which implies that}$$

$$h_i g h_i^{-1} g^{-1} = (h_i, g) \neq (h_j, g) = h_j g h_j^{-1} g^{-1}.$$

Since we have an infinite number of conjugates we have an infinite number of choices for $(h_i, g) \in \phi$. Moreover, the right anihilator of a nonzero element of KG has only a finite number of elements of the form $g-1$, $g \in G$.

Suppose now that the elements $g_1, g_2, \ldots, g_n$ with the required properties have already been chosen. Then $g_i, b_j \in \phi$, $1 \leq i, j \leq n$, and by Poincare's Theorem $H = C_\phi\{g_i, b_j \mid 1 \leq i, j \leq n\}$ has finite index in ϕ, and $[H : C_H(g)] = \infty$. The latter equality is true, otherwise $[\phi : C_\phi(g)] \leq [\phi : C_H(g)] = [\phi : H][H : C_H(g)] < \infty$,. in contradiction with the way in which g was chosen. Therefore the set $\{(h, g) \mid h \in H\}$ is infinite, and there exists an element $g_{n+1} \in H$ such that $1 - b_{n+1} = 1 - (g_{n+1}, g)$ does not lie in the right anihilator of $\eta(1-b_1)\ldots(1-b_n)$. Now we prove by induction that

$$(1+\eta g, g_1) = (1+\eta g) g_1 (1-\eta g) g_1^{-1} = 1 + \eta(1-b_1) g,$$

$$(1+\eta g, g_1, g_2) = (1+\eta(1-b_1)g, g_2) = 1 + \eta(1-b_1)(1-b_2) g, \quad \text{and}$$

$$(1+\eta g, g_1, \ldots, g_n) = 1 + \eta(1-b_1)\ldots(1-b_n) g.$$

But these are never 1, contradicting the T-nilpotent of $U(KG)$. ∎

Corollary 3.6. *Let K be an infinite field of characteristic p, and let G be a torsion group. Then $U(KG)$ is nilpotent if and only if:*

(i) G is abelian, if G has no p-elements;

(ii) G is nilpotent and p-abelian, if G has p-elements.

Proof: (i) Follows from Collary 3.4.

(ii) Let us assume that $U(KG)$ is nilpotent and G has a p–element. Then G is a nilpotent group with a central p–element. Moreover $G = S_p \times S_{p'}$, the direct product of a p–group by a p'–group. By Corollary 3.4 $S_{p'} \subseteq \zeta G$, and it is enough to show that S_p is p–abelian. From Corollary 3.4 $[S_p : \phi] < \infty$, and if $S_p \neq \phi$ by Proposition 3.5 we have that $[S_p : \phi] = \infty$, a contradiction.

For sufficiency see [4], Theorem VI.3.1. ■

Finally, some words about the existence of free subsemigroups in $U(KG)$. It is now easy to show:

Theorem 3.7: *Let K be an uncountable field of characteristic p, and let G be a group generated by p'–elements. Then, either G contains a subsemigroup on two free generators or G is abelian.*

Theorem 3.8: *Let G be a locally finite p–group and let K be an uncountable field of characteristic p. Then, either $U(KG)$ contains a subsemigroup on two free generators or $U(KG)/\zeta U(KG)$ is torsion.*

Sketch of Proof: Let us see, for example, how to give a proof for Theorem 3.7. Let $g^m = h^\ell = 1$ be two p'–generators of G. Let μ and λ be two elements of K such that $\mu^m \neq 1 \neq \lambda^{-\ell}$. If $U(KG)$ does not contain subsemigroups on two free generators then, by Lemma 2.1 and the Pigeon-Hole Principle, there exists a sequence of integers $(n, i_1, \ldots, i_n, j_i, \ldots, j_n)$ such that, the relation (*) holds for

$$a_s = (\lambda + h)^s \, (1 - \mu g)(\lambda + h)^{-s},$$

and uncountably many μ and λ. The proof now proceeds as in Theorem 3.1. ■

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