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A Remark on an Eigenvalue Condition for the Global Injectivity of Differentiable Maps of R^2

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Sobre uma condição nos autovalores para a injetividade global de aplicações diferenciáveis do $\mathbb{R}^2$

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Resumo

Usando a técnica da semi-componente de Reeb, como introduzida em [?], procuramos clarificar a relação intrínsica existente entre a injetividade de homeomorfismos locais diferenciáveis X do $\mathbb{R}^2$ e o comportamento assintótico dos autovalores reais das derivadas $DX(x)$. O resultado principal mostra que um homeomorfismo local diferenciável X do $\mathbb{R}^2$ é injetivo e tem imagem $X(\mathbb{R}^2)$ convexa, se satisfaz a seguinte condição: (*) *Não existe nenhuma seqüência* $\mathbb{R}^2 \ni x_i \to \infty$ *tal que* $X(x_i) \to a \in \mathbb{R}^2$ *e* $DX(x_i)$ *tem um autovalor real* $\lambda_i \to 0$. Quando o gráfico de X é um conjunto algébrico, a condição (*) fica sendo necessária e suficiente para que X seja um difeomorfismo global.

A remark on an eigenvalue condition for the global injectivity of differentiable maps of $\mathbb{R}^2$

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Abstract

Using the half-Reeb component technique as introduced in [10], we try to clarify the intrinsic relation between the injectivity of differentiable local homeomorphisms X of $\mathbb{R}^2$ and the asymptotic behavior of real eigenvalues of derivations $DX(x)$. The main result shows that a differentiable local homeomorphism X of $\mathbb{R}^2$ is injective and that its image $X(\mathbb{R}^2)$ is a convex set if X satisfies the following condition: (*)*There does not exist a sequence* $\mathbb{R}^2 \ni x_i \to \infty$ *such that* $X(x_i) \to a \in \mathbb{R}^2$ *and* $DX(x_i)$ *has a real eigenvalue* $\lambda_i \to 0$. When the graph of X is an algebraic set, this condition becomes a necessary and sufficient condition for X to be a global diffeomorphism. *2000 Mathematics Subject Classification:* Primary: $14R15$. Secondary: $14E07$, $14E09$, $14E40$. *Key words and phrases:* Injective differentiable maps, eigenvalue condition, polynomial diffeomorphism.

1 Introduction

Let $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a differentiable map (not necessarily C^1) and denote by $Spec(X)$ the union of all eigenvalues of $DX(x)$, for all $x \in \mathbb{R}^2$. The following result, which was recently obtained in [7], led to a positive solution of the Markus-Yamabe conjecture in the two-dimensional differentiable case.

Theorem 1. *A differentiable map X of $\mathbb{R}^2$, not necessarily C^1, is injective if for some $\epsilon > 0$*

$$Spec(X) \cap [0, \epsilon) = \emptyset. \qquad (S1)$$

This is the deepest theorem added to a long sequence of results on both the two-dimensional Markus-Yamabe conjecture and the eigenvalue condition for injectivity of map of the real plane.

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This was initiated in Olech's work [13] (in 1962) who proved that the two dimensional case of the Markus-Yamabe conjecture [11] can be reduced to the statement: *If X is of class C^1, $DetDX(x) > 0$, and the set $Spec(X)$ is contained in the open left half-complex line $\{\lambda \in \mathbb{C}^2 : Re\lambda < 0\}$, then X is injective.* In 1988, Olech and Meister ([12]) gave a positive answer for the polynomial case. In 1994, Gutierrez [10] and Fessler [8] obtained this fact from the result that X is injective if X is locally diffeomorphic and the derivations $DX(x)$ do not have positive real eigenvalues for $|x|$ large enough. A weaker C^1−version of Theorem 1 had already been proved in [5]. It is worth emphasizing that the Markus-Yamabe conjecture fails in $\mathbb{R}^3$ even for polynomial vector fields [4]. The proof of Theorem 1 presented in [7] (see also [5]) can be divided into two steps. The first step is the following theorem

Theorem 2. *A differentiable map X of $\mathbb{R}^2$ is injective if for some $\epsilon > 0$*

$$Spec(X) \cap (-\epsilon, \epsilon) = \emptyset. \qquad (S2)$$

The second it to obtain Theorem 1 by regarding a map X, which satisfies Condition $(S1)$, as the limit of a sequence of injective maps each of which is of the form $X_t(x) = X(x) - tx$, where $t \in \mathbb{R} - \{0\}$ is small, and thus X_t satisfies condition (S2). As noted in [5], there are polynomial diffeomorphisms that do not satisfy the spectral condition (S1); for instance, when $X(x, y) = (-y, x + y^2)$, $Spec(X) = S^1 \cup \mathbb{R}^*$. In this article we will show that spectral condition (S2) ensures not only the injectivity of X but also the convexity of the image $X(\mathbb{R}^2)$. This improvement was obtained by steading the geometrical behavior of differentiable maps of $\mathbb{R}^2$ under the following condition on the real spectrum of $DX(x)$.

(*) *There does not exist a sequence $\mathbb{R}^2 \ni x_i \to \infty$ such that $X(x_i) \to a \in \mathbb{R}^2$ and $DX(x_i)$ has a real eigenvalue $\lambda_i \to 0$.*

This condition is sufficient for the injectivity and the convexity of the image of differentiable local homeomorphisms.

Theorem 3. *Suppose $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a differentiable local homeomorphism. Then:*

(i) If X satisfies Condition (*), then X is injective and its image $X(\mathbb{R}^2)$ is a convex set.

(ii) X is a global homeomorphism of $\mathbb{R}^2$ if and only if X satisfies Condition (*) and its image $X(\mathbb{R}^2)$ is dense in $\mathbb{R}^2$

Condition (*) is somewhat weaker than condition $(S1)$ and is not a necessary condition for injectivity. For example, each of the maps $X(x, y) = (\exp(x), \exp(\pm y))$ is injective and has convex image though does not satisfy Condition (*). However, from the principle " Injectivity ⇒ Bijectivity" (which asserts that every continuous injection from $\mathbb{R}^n$ into itself, with algebraic graph, must be bijective, [9], [14]) and Theorem 3 one may easily obtain the following.

Theorem 4. *A differentiable local homeomorphism $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ with algebraic graph is a global homeomorphism of $\mathbb{R}^2$ if and only if X satisfies Condition (*).*

By a canonical procedure, Theorem 3 (i) allows us to re-obtain theorem 2. We have already explained how to obtain Theorem 1 from Theorem 2. We will also show the following:

Theorem 5. *Suppose that $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ is a local homeomorphism. Then,*

(i) X is an injective map having convex image if and only if X has no half-Reeb components;

(ii) X is a global homeomorphism of $\mathbb{R}^2$ if and only if X has no half-Reeb components and its image $X(\mathbb{R}^2)$ is dense in $\mathbb{R}^2$.

Our approach is based on the so-called half-Reeb component technique, as introduced in [10], and will be described in &2.

The problem of characterizing the injectivity of differentiable maps of an arbitrary dimensional space $\mathbb{R}^n$ in terms of spectral conditions has been studied, for instance, in [3], [15], [6]. These works are related to the Jacobian conjecture, which can be reduced to the question of *whether every polynomial map $X : \mathbb{C}^n \longrightarrow \mathbb{C}^n$ of the form $X(x) = x + H(x)$ with 3-degree homogeneous $H(x)$ must be injective if $Spec(X) = \{1\}$* (see [2]).

2 Half-Reeb component

2.1 Definition of half-Reeb component

Let $\epsilon > 0$ and let $\beta, \gamma : (-\epsilon, \epsilon) \longrightarrow \mathbb{R}^2$ be injective C^0-curves such that $\beta(0) = \gamma(0)$. We say that β is transversal (resp. tangent) to γ at $\beta(0) = \gamma(0)$ if there exist local C^0- coordinates in a neighborhood of $\beta(0) \in \mathbb{R}^2$ such that in these coordinates $\beta(t) = (t, 0)$ and $\gamma(t) = (0, t)$ (resp. $\beta(t) = (0, \beta_2(t))$; with $\beta_2(t) \geq 0$).

Let $h : \mathbb{R}^2 \longrightarrow \mathbb{R}$ be a given continuous function such that the levels of h form a (non-singular) C^0-foliation $F(h)$. Let $h_0(x, y) = xy$ and consider the set

$$B = \{(x, y) \in [0, 2] \times [0, 2] : 0 < x + y \leq 2\} \ .$$

We will say that $A \subset \mathbb{R}^2$ is a *half-Reeb component* for $F(h)$ (or simply a hRc for $F(h)$) if there exists a homeomorphism $H : B \longrightarrow A$ which is a topological equivalence between $F(h)_{|A}$ and $F(h_0)_{|B}$ and is such that

(1) The segment $\{(x, y) \in B : x + y = 2\}$ is sent by H onto a transversal section for the foliation $F(h)$ in the complement of $H(1, 1)$; this section is called the compact edge of A.

(2) Both segments $\{(x, y) \in B : x = 0\}$ and $\{(x, y) \in B : y = 0\}$ are sent by H onto full half-trajectories of $F(h)$. These two semi-trajectories of $F(h)$ are called the non–compact edges of A. Let $X = (f, g) : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ be a local homeomorphism of $\mathbb{R}^2$. For each $\theta \in \mathbb{R}$ let us denote by R_θ the linear rotation $(x, y) \mapsto (x\, cos\theta - y\, sin\theta, x\, sin\theta + y\, cos\theta)$ and $X_\theta := (f_\theta, g_\theta) = R_\theta \circ X \circ R_{-\theta}$. In other words, $X_\theta = (f_\theta, g_\theta)$ is the representation of X in the linear coordinates of $\mathbb{R}^2$ associated with the rotation R_θ.

Definition 1. *(Half-Reeb component for X).* By a *half-Reeb component of X* we mean a half-Reeb component for the foliation $F(h)$, where $h \in \{f_\theta, g_\theta\}$ for some $\theta \in \mathbb{R}$.

Let A be a half-Reeb component of $F(f_\theta)$. While the geometry of A is very simple, it is very useful to examine the behavior of $X_\theta|_A$ around infinity. We have that,

1. Two non-compact edges of A are subsets of one level of f_θ, say $f_\theta = c$. The map X_θ sends diffeomorphically these two edges onto a pair of disjoint half-open intervals of the line $x = c$, say $I_1 = \{c\} \times [\alpha, \beta)$ and $I_2 = \{c\} \times (\gamma, \delta]$, $\beta < \gamma$.

2. X_θ sends diffeomorphically the compact edge of A to a compact path L lying on one-side of the line $x = c$ and intersecting it only at the points (c, α) and (c, δ).

3. The image $X_\theta(A)$ is the simple connected domain bounded by $L \cup I_1 \cup I_2 \cup I_3$, where $I_3 := \{c\} \times [\beta, \gamma]$.

4. The foliation in $\mathbb{R}^2$, by vertical lines, induces in $intX_\theta(A)$ (the interior of $X_\theta(A)$) a trivial fibration by open-interval-fibers; moreover, $X_\theta : intA \longrightarrow intX_\theta(A)$ is a homeomorphism giving a topological equivalence between this fibration and the foliation $F(f_\theta)$ restricted to $intA$.

5. The essential point is that while every level of f_θ, restricted to $intA$, is connected, *the intersection of the level $f_\theta = c$ with the closure $\overline{A}$ of A must have at least two components contained in the frontier ∂A of A* .

2.2 Half-Reeb component and injective map of convex image

In this subsection we shall prove Theorem 5. A result related to the following proposition can be found in [10].

Proposition 1. *Suppose $h : I\!R^2 \longrightarrow I\!R$ is a continuous function such that the levels of h form a non-singular C^0-foliation $F(h)$. If h has a disconnected level , then h has a hRc.*

Proof. Assume that h has a disconnected level, say $h = 0$. Let C_1 and C_2 be two distinct connected components of the level $h = 0$. Take two points $p_i \in C_i$, $i = 1, 2$. Let $\Omega(p_1, p_2)$ be the set of compact arcs of $\mathbb{R}^2$ whose endpoints are p_1 and p_2 and which meet transversally C_1 and C_2 at $\{p_1, p_2\}$.

(a) Among all elements of $\Omega(p_1, p_2)$, take $\Gamma \in \Omega(p_1, p_2)$ which minimizes the number of tangencies of Γ with (leaves of) $F(h)$.

We claim that:

(b) $C_i \cap \Gamma = \{p_i\}$, for $i = 1, 2$.

If we assume, by contradiction, that $C_1 \cap \Gamma$ contains properly $\{p_1\}$, we may find $q \in \Gamma \setminus \{p_1, p_2\}$ and a closed subinterval C of C_1, with endpoints p_1, q, such that $C \cap \Gamma = \{p_1, q\}$. We may assume that Γ is transversal to C at q. Let γ denote the connected component of $\Gamma \setminus \{q\}$ containing $\{p_2\}$. We can see that $C \cup \gamma$ is an arc connecting p_1 and p_2 and also that Γ is tangent to $F(h)$ at some point of $\Gamma \setminus (\gamma \cup \{p_1\} \cup \{q\})$. Under these conditions, we may approximate $C \cup \gamma$ by an arc of $\Omega(p_1, p_2)$ which has less number of tangencies, with $F(h)$, than Γ. This contradiction with (a) proves (b).

Since $h(p_1) = h(p_2) = 0$, Γ is tangent to $F(h)$ at some point q different from p_1 and p_2. By considering the leaves of $F(h)$ around q, we may see that there exist subintervals $(p, q]$, $[q, Tp)$ of Γ with $(p, q] \cap [q, Tp) = \{q\}$, and a homeomorphism $T : (p, q] \to [q, Tp)$ such that,

(c1) $Tq = q$ and, for every $x \in (p, q]$, there is an arc $[x, Tx]_h$ of a leaf of $F(h)$, starting at x, ending at Tx and meeting Γ exactly and transversally at $\{x, Tx\}$, (c2) the family $\{[x, Tx]_h : x \in (p, q]\}$ depends continuously on x and tends to $\{q\}$ as $x \to q$.

From now on, suppose that

(d) $(p, q]$ is maximal with respect to properties (c1)-(c2) above.

We claim that

(e) there is no leaf $[p, Tp]_h$ of $F(h)$ connecting p and Tp such that the family $\{[x, Tx]_h : x \in (p, q]\}$ approaches continuously $[p, Tp]_h$ as x tends to p.

In fact, suppose that (e) is false. Then, by using (d) we conclude $[p, Tp]_h$ is tangent to Γ at least at one of the points of $\{p, Tp\}$. Under these circumstances, it is not difficult to approximate the curve, which is the union of $[p, Tp]_h$ with $\Gamma \setminus ((p, q] \cup [q, Tp))$, by a curve $\Gamma_1 \in \Omega(p_1, p_2)$ which has less tangencies with $F(h)$ than Γ. This contradiction with (a) proves (e). Therefore, the subinterval $[p, q] \cup [q, Tp]$ is the compact edge of a half-Reeb component of $F(h)$ made up of two half-leaves of $F(h)$ starting at p and Tp, respectively, together with the union of the arcs $[x, Tx]_h$, with $x \in (p, q]$. This finishes the proof. □

We next return to the case of a local homeomorphism $X = (f, g) : I\!R^2 \to I\!R^2$. It is easy to see that if X is not injective, then both f and g must have disconnected levels. So we get

Proposition 2. *(Lemma 1 of [10]). A non-injective local homeomorphism $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ must have half-Reeb components.*

Proof of Theorem 5.

Let us prove (i), that is: a local homeomorphism $X : I\!R^2 \to I\!R^2$ is injective and has convex image if and only if X has no half-Reeb components.

Suppose that X has no half-Reeb components. By Proposition above, X is injective. Let us prove the convexity of $X(I\!R^2)$. Let $p, q \in X(I\!R^2)$ and $[p, q] = \{tp + (1-t)q : 0 \le t \le 1\}$. Take $\theta \in I\!R$ so that $R_\theta([p, q])$ is contained in a straight line of the form $x = c$. Therefore, as (by Proposition 1) the level set $f_\theta = c$ is connected, $X_\theta(f_\theta = c)$ is a connected subset of $x = c$ containing $\{R_\theta(p), R_\theta(q)\}$; that is $R_\theta([p, q]) \subset X_\theta(I\!R^2)$, which implies $[p, q] \subset X(I\!R^2)$ and so $X(I\!R^2)$ is convex.

Conversely, suppose now that X is injective and has convex image. As X is injective, by the Invariance of Domain Theorem, $X(I\!R^2)$ is an open subset of $I\!R^2$ and X takes homeomorphically $I\!R^2$ onto $X(I\!R^2)$. If we assume, by contradiction, that for some $\theta \in I\!R$, $F(f_\theta)$, has a Reeb component $\mathcal{A}$, then there exists $c \in I\!R$ such that the non–compact edges of $\mathcal{A}$ are contained in the level set $f_\theta = c$ and so they are mapped homeomorphically, by X_θ, onto the union of two intervals of the form $\{c\} \times [a, b)$ and $\{c\} \times (d, e]$, with $b < d$. As $X_\theta(I\!R^2)$ is convex, the compact interval $\{c\} \times [a, e]$ is contained in the open set $X_\theta(I\!R^2)$ and therefore $X_\theta^{-1}([a, e])$ is a compact arc containing the non-compact edges of $\mathcal{A}$. This contradiction finishes the proof of (i) of Theorem 5.

Item (ii) is an easy consequence of (i). □

3 Half-Reeb component and Condition (*)

In this subsection we will show the essential fact that the Condition (*) ensures the non-existence of half-Reeb components.

First, let us state a kind of stability of a half-Reeb component. Its proof can be found in [7] (see also [10]), where it was used as a technical tool.

Proposition 3. *(Lemma 1 of [10]). Let* $X = (f, g)\colon \mathbb{R}^2 \to \mathbb{R}^2$ *be a non-injective, differentiable map such that* $0 \notin \operatorname{Spec}(X)$*. Let* $\mathcal{A}$ *be a hRc of* $F(f)$ *and let* $(f_\theta, g_\theta) = R_\theta \circ X \circ R_{-\theta}$, $\theta \in \mathbb{R}$*. If* $\Pi(\mathcal{A})$ *is bounded, where* $\Pi : \mathbb{R}^2 \to \mathbb{R}$ *is given by* $\Pi(x, y) = x$*, then there is an* $\epsilon > 0$ *such that, for all* $\theta \in (-\epsilon, 0) \cup (0, \epsilon)$*,* $F(f_\theta)$ *has a hRc* $\mathcal{A}_\theta$ *such that* $\Pi(\mathcal{A}_\theta)$ *is an interval of infinite length.*

An analogous statement to the proposition below was proved in [5], [7], where Conditions (S1-S2) were used instead of Condition (*) (see Section 1). The proof of proposition below can be done in a similar way to that of [7] and so it will be presented in geometrical terms.

Proposition 4. *A differentiable local homeomorphism* $X : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$ *satisfying Condition (*) has no half-Reeb components.*

Proof. Suppose by contradiction that $X = (f, g)$ has a hRc. By Proposition 3, we may assume that f has a half-Reeb component $\mathcal{A}$ such that $\Pi(\mathcal{A})$ is an unbounded interval. To simplify matters, let us suppose that $[b, \infty) \subset \Pi(\mathcal{A})$. Then, when $a > b$ is large enough and $x \geq a$, the vertical line $\Pi^{-1}(x)$ intersects exactly one leaf $C_x \subset \mathcal{A}$ of $F(f)|_{\mathcal{A}}$ such that $\Pi(C_x) \cap (x, \infty) = \emptyset$. In other words, x is the maximum for the restriction $\Pi|_{C_x}$. The leaf C_x is a continuous curve and the set $C_x \cap \Pi^{-1}(x)$ is a compact subset of $\mathcal{A}$. So we can define functions $H : (a, +\infty) \longrightarrow \mathbb{R}$ by

$$H(x) = \sup\left\{y : (x, y) \in C_x \cap \Pi^{-1}(x)\right\}$$

and $\varphi : (a, +\infty) \longrightarrow \mathcal{A}$ by $\varphi(x) := f(x, H(x))$. As proved in [7], φ is a bounded, strictly monotone function such that, for some full measure subset $M \subset (a, +\infty)$, φ is differentiable on M and, for $x \in M$,

$$DX(x, H(x)) = \begin{pmatrix} \varphi'(x) & 0 \\ g_x(x, H(x)) & g_y(x, H(x)) \end{pmatrix}$$

In the other words, on M the derivative $\varphi'(x)$ exists and it is an eigenvalue of $DX(x, H(x))$.

Now, for convenience we may assume that φ is strictly increasing. So, we have that $\varphi'(x) \geq 0$ on M. As φ is bounded, there is a constant $K > 0$ such that for all $c \geq a$, we have $0 \leq \varphi(c) - \varphi(a) \leq K$. Then, we obtain

$$0 \leq \int_a^\infty \varphi'(x)dx = \lim_{c \to \infty} \int_a^c \varphi'(x)dx \leq \lim_{c \to \infty} (\varphi(c) - \varphi(a)) \leq K$$

Hence $\liminf_{x \to \infty} \varphi'(x) = 0$. It follows that there exists a sequence $(x_i, H(x_i)) \to \infty$ such that $DX(x_i, H(x_i))$ has an eigenvalue $\varphi'(x_i) \to 0$ and $X(x_i, H(x_i))$ tends to a finite value in the closure $\overline{X(\mathcal{A})}$ (which is compact). This contradicts Condition (*). □

Proof of Theorem 3. By Proposition 4, Condition (*) ensures that X does not have any half-Reeb component. Then, the conclusion is immediate from Theorem 5.

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