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YET ANOTHER REFUTATION OF ALLAIS' PARADOX

by

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Yet Another Refutation of Allais' Paradox

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We start by presenting the famous Allais' (1953) gambles:

In Situation I , you have to choose one of the following gambles:

Gamble a_1 : you win 1 million dollars with certainty.

Gamble a_2 : you win 5 million dollars with probability 0.10

1 million dollars with probability 0.89

0 dollars with probability 0.01

In Situation II , you have to choose one of the following gambles :

Gamble a_3 : you win 5 million dollars with probability 0.10

0 dollars with probability 0.90

Gamble a_4 : you win 1 million dollars with probability 0.11

0 dollars with probability 0.89

Many people prefer a_1 to a_2 in Situation I and a_3 to a_4 in Situation II. However , for any non-negative values $U(0) < U(1) < U(5)$ for the utility of , respectively, 0, 1 and 5 million dollars , we have (assuming for simplicity $U(0) = 0$) :

$$\text{Risk}(a_1) > \text{Risk}(a_2) \text{ iff } U(1)/U(5) > 10/11 \text{ , while}$$

$$\text{Risk}(a_3) > \text{Risk}(a_4) \text{ iff } U(1)/U(5) < 10/11 \text{ .}$$

Therefore, the choices a_1 and a_3 are incoherent for a follower of the neoBayesian neo-Bernoullian paradigm for decision-making which calls for maximization of unconditional utilities. The paradigm is also called the Ramsey-de Finetti-Savage point of view , as it was advanced by these authors.(Ramsey(1926), de Finetti(1952,1974,1978), Savage(1954)). The most modern formulation is due to Rubin(1987) and it is more general since it is developed from a weaker set of "rationality" axioms and does not demand (nor obtains) the separation of personal probability from the utility function.

But the strong "intuitive" appeal of choices a_1 and a_3 (together) made the Allais' gambles generate a great deal of discussion , particularly among philosophers and economists. See for example the volume edited by Allais and Hagen(1979) , where several authors present their cases for and against the incompatibility of the choices in the Allais' gambles - by now called Allais' "paradox". The authors who see no incompatibility go further to make the paradox a counterpart (for the Ramsey-de Finetti-Savage paradigm of decision-making) of the Saint Petersburg paradox (for linear utility). In other words, they show the Allais' paradox as a reason for rejecting maximization of expected utilities, which would be replaced by methods involving "psychological" or "regret" utilities.

On the other hand , adherents of the Ramsey-de Finetti-Savage school keep their point of view that the choices are incompatible and that their theory , being normative , should have no concern with what people do. Sound defenses of the incompatibility of a_1 and a_3 are found for instance in Savage(1954), part (g) of Section 5.6 (1972 Dover edition) or Raiffa (1968) , Section 4.9 . The latter suggests a connection between compatibility of a_1 and a_3 (or a_2 together with a_4) and violation of the Likelihood Principle (Basu (1975) , Berger and Wolpert (1984)).

Most justifications in favor or against the incompatibility of a_1 and a_3 exist in a framework where the two Situations are in some way or another put together. In this note we model this simultaneity in the subjectivistic way : you have a personal probability α for the event that you will be asked to play Situation I (and $1-\alpha$ for Situation II). The conclusion is , once again , that choices a_1 and a_3 are incompatible (as well as a_2 together with a_4). A more comprehensive model would have the random (for you) quantity "number of Situations to be played" with a personal probability distribution on $\{0,1,2\}$. Consideration to assumptions of additivity of utility would then have to be made, but we suspect the conclusion would remain the same. We avoid the difficulty by considering the "exactly one Situation played" only:

The space of decisions is now the set with the four choices $\{(a_1,a_3)\ ,\ (a_1,a_4)\ ,\ (a_2,a_3)\ ,\ (a_2,a_4)\}$. Let α be your probability that you will be asked to play Situation I. We will assume, without loss of generality, that your utilities are such that $U(0)=0$ and $U(1)/U(5) > 10/11$. We have:

$$\text{Risk}((a_1,a_3)) = \alpha U(1) + (1-\alpha)(0.10)U(5)$$

$$\text{Risk}((a_1,a_4)) = \alpha U(1) + (1-\alpha)(0.11)U(1)$$

$$\text{Risk}((a_2,a_3)) = \alpha(0.10U(5) + (0.89)U(1)) + (1-\alpha)(0.10U(5))$$

$$\text{Risk}((a_2,a_4)) = \alpha(0.10U(5) + (0.89)U(1)) + (1-\alpha)(0.11U(1))$$

It is easily seen that (a_1,a_4) dominates the other three pairs , i. e. , its risk (expected utility here) is larger uniformly on α. See Figure 1. If $U(1)/U(5) < 10/11$, the pair (a_2,a_3) dominates the other three options.

CONCLUSION

In this note we present a subjectivistic approach to the Allais' gambles considered *simultaneously* - as it is done, at least implicitly, in almost every confirmation or refutation of the "paradox". There is no surprise in the conclusion : the neobayesian neobernoullian paradigm is once again verified to have no paradoxical feature. While the criterion used to define the inadmissibility of pairs of choices is expected utility (non-Bayesians might claim that it is a circular argument) , it is verified the coherence of Bayesian decision-making.

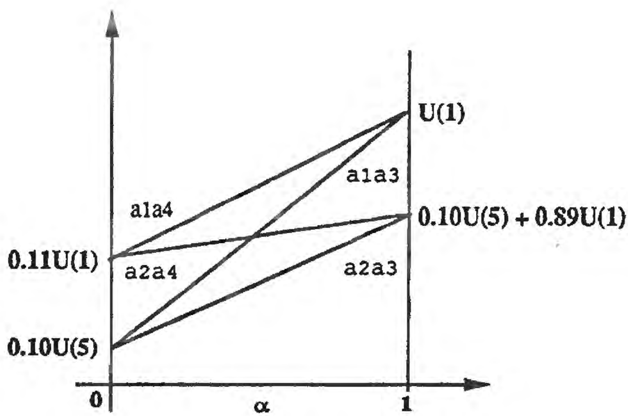

FIGURE 1

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