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by

TAVARES, H. R.
AND
ANDRADE, D. F.

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Growth Curve Models for Item Response Theory

Heliton R. Tavares and Dalton F. Andrade *

Abstract

The main goal of this work is to model the possible growth of the mean parameters of the ability distributions of groups of examinees. For instance, students from 5th, 6th, 7th and 8th grades or students from 8th grade observed in 1997, 1999, 2001 and 2003. Several regression models can be proposed to represent the true relationship between the population ability parameters and some explanatory variable, such as time or grade, for instance. The estimating equations and some simulation results are presented for polynomial models.

Key words: Item response theory, growth curve, logistic model, polynomial function, estimating equations.

1 Introduction

In many practical situations different groups of examinees are evaluated and their ability distribution parameters estimated on the same metric scale. One example would be the Basic Education National Evaluation Examination (SAEB) administered by the Brazilian Ministry of Education. Samples of students belonging to 4th, 8th and 11th grades are evaluated at every two years. One of the main goals would be to evaluate the possible growth of the mean proficiency parameters across grades and years. In most of these situations, mainly in the educational field, the interest can be in building proficiency scales (*ability*). For instance, one can build a proficiency scale to cover students from 1st to 11th grades. For each one of these 11 groups of students the mean of the population abilities needs to be estimated. If we can represent the possible growth by a function with few parameters, for instance a polynomial of degree 2, the number of parameters used to represent the mean abilities would fall from 11 to 2, decreasing substantially the computational demand and producing better results.

We will concentrate this work on the estimation of the parameters of the growth curve, regarding to the location parameters. So, we consider that the dispersion and item parameters involved in the

*Heliton R. Tavares is Associate Professor, Department of Statistics, Federal University of Pará, P.O. Box 479, 66075-900, Belém - Pará -Brazil (e-mail: heliton@ufpa.br). Dalton F. Andrade is Professor, Department of Informatics and Statistics, Federal University of Santa Catarina, P.O. Box 476, CEP: 88040-110, Florianópolis, SC - Brazil (e-mail: dandrade@inf.ufsc.br)

study were known. Thus, any variation obtained in the estimating process will be due just to the equations of the growth curve parameters, and not on the other types of parameters.

The estimation of the population parameters has been the objective of many works. Anderson and Madsen (1977) showed that the estimation of the population parameters can be done directly, without the intermediate estimation of the individual abilities. In this work they consider known the item parameters. Some following work [see Sanathanan & Blumenthal (1978), de Leeuw & Verhelst (1986) and Lindsay at al. (1991)] extended that results for the joint estimation of the item and latent distribution parameters, but just for the one parameter logistic model in the one population case. Mislevy (1984) worked on the estimation of the latent distribution based on the method of the *Marginal Maximum Likelihood* [see Bock & Lieberman (1970) and Bock & Aitkin (1981)]. Bock & Zimowski (1997) extended the previous case for situations like ours where there are several independent populations.

2 Definitions and notations

In what follows, we consider that K different groups of individuals are appraised on a certain area of knowledge, taking tests denominated *Teste 1, Test 2,..., Test K*, respectively. A sample of N_k subjects from population k takes test k composed of n_k items. The total number of items will be denoted by n, satisfying $n \leq \sum_{k=1}^{K} n_k$.

We will consider that the groups involved in the analysis correspond to the series $\mathbf{t} = (t_1, t_2, \cdots, t_K)'$. If this series is sequential, we can adopt $t_1 = 1,\ t_2 = 2, \cdots,\ t_K = K$.

Let us represent by μ_k the mean ability of the population taking the test k, $k = 1, \cdots, K$. In general, let

$$\mu_k = f(t_k|\alpha), \tag{1}$$

be a twice differentiable continuous function with p parameters and $\alpha = (\alpha_1, \alpha_2, \cdots, \alpha_p)'$ the vector of parameters of the function. The mean vector for the K levels can now be rewritten as

$$\mu = (f(t_1|\alpha), f(t_2|\alpha), \cdots, f(t_K|\alpha))'.$$

Many functions have been proposed in the theory of growth curves for longitudinal data (see Lindsey, 1993, and Singer and Andrade, 2000, for example). Frequently, simple functions, such as the linear or quadratic functions are appropriate to model the longitudinal structures, but polynomials of larger order than two, and some nonlinear functions, have been applied in several instances. Table 1 presents examples of functions thoroughly used in several situations.

3 Presentation of the Model

Let

$$P_{kji} = P(U_{kji} = 1|\theta_{kj}, \zeta_i), \tag{2}$$

Table 1: Models for growth curves

Logistic Triple	$\mu_t = \frac{\gamma_1}{1+e^{-\alpha_1(t-\beta_1)}} + \frac{\gamma_2}{1+e^{-\alpha_2(t-\beta_2)}} + \frac{\gamma_3}{1+e^{-\alpha_3(t-\beta_3)}}$
Gompertz	$\mu_t = \gamma_1 + \gamma_2 e^{-e^{-\alpha(t-\beta)}}$
Jenns	$\mu_t = \alpha + \beta t - e^{\gamma+\delta t}$
Count	$\mu_t = \alpha + \beta t + \gamma \log t$
Mitscherlish	$\mu_t = \alpha - \beta e^{-\gamma t}$
Polinomial	$\mu_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \cdots + \alpha_d t^d$

be a twice-differentiable item response function that describes the conditional probability of a correct response to item i, $i = 1, 2, \cdots, n_k$, of individual j, $j = 1, 2, \cdots, N_k$, in group k, $k = 1, 2, \cdots, K$, where U_{kji} represents the (binary) response, θ_{kj} the ability (latent trait) and ζ_i the vector of the item parameters. Examples of such function would be the logistic 1, 2 and 3 parameters models (see Andrade et al. (2000) and Hambleton et al. (1991) for details). Assuming the conditional independence of the responses to the items in test k, given the ability θ, we have that

$$P(\boldsymbol{U}_{kj.}|\theta, \boldsymbol{\zeta}) = \prod_{i \in I_k} P(U_{kji}|\theta, \zeta_i), \tag{3}$$

where $\boldsymbol{I}_k$ is the set of the indexes of those items presented in test k, $\boldsymbol{U}_{kj.} = (U_{kj1}, U_{kj2}, \cdots, U_{kjn_k})'$ is the $(n_k \times 1)$ vector of responses of individual j in test k and $\boldsymbol{\zeta} = (\zeta_1', \zeta_2', \cdots, \zeta_n')'$. Note that, for convenience and without loss of generality we dropped the indexes from the ability parameter because we are interested in the distribution of the abilities and not in any individual ability.

Assuming that the ability for population k follows a continuous distribution, that we will denote its probability density function by $g(\theta|\boldsymbol{\eta}_k)$, with parameters $\boldsymbol{\eta}_k$ of finite elements, the unconditional, or marginal, probability of pattern $\boldsymbol{U}_{kj.}$ can be written as

$$P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k) = \int_{\boldsymbol{R}} P(\boldsymbol{U}_{kj.}|\theta, \boldsymbol{\zeta}) g(\theta|\boldsymbol{\eta}_k) d\theta. \tag{4}$$

By independence of the responses of the individuals in group k, $k = 1, \cdots, K$, we have that

$$P(\boldsymbol{U}_{k..}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k) = \prod_{j=1}^{N_k} P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k) \tag{5}$$

and by the independence between groups we have

$$P(\boldsymbol{U}_{...}|\boldsymbol{\zeta}, \boldsymbol{\eta}) = \prod_{k=1}^{K} P(\boldsymbol{U}_{k..}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k) \tag{6}$$

with $\boldsymbol{U}_{k..} = (\boldsymbol{U}_{k1.}', \boldsymbol{U}_{k2.}', \cdots, \boldsymbol{U}_{kN_k.}')'$ being the $(N_k n_k \times 1)$ vector of responses of group k, $\boldsymbol{U}_{...} = (\boldsymbol{U}_{1..}', \boldsymbol{U}_{2..}', \cdots, \boldsymbol{U}_{N_K..}')'$ the $(\sum_{k=1}^{K} N_k n_k \times 1)$ vector of responses in all tests and $\boldsymbol{\eta} = (\boldsymbol{\eta}_1', \cdots, \boldsymbol{\eta}_K')'$.

4 The Likelihood Equation

Let now index j represent one of the s_k different response patterns for group k, $s_k \leq \min(N_k, 2^{n_k})$, instead of just individual j, $r_{kj} > 0$ be the number of occurrences of the response pattern j for group k, $j = 1, 2, \cdots, s_k$, and $\boldsymbol{R}_k = (r_{k1}, r_{k2}, \cdots, r_{ks_k})$ be the $(s_k \times 1)$ vector of frequencies of the observed pattern $\boldsymbol{U}_{kj.}$. From the independence among the responses of the different individuals for each group, $\boldsymbol{R} = (\boldsymbol{R}_1, \cdots, \boldsymbol{R}_K)$ follows a Product-Multinomial distribution given by

$$P(\boldsymbol{R}|\boldsymbol{\zeta}, \boldsymbol{\eta}) = \prod_{k=1}^{K} \frac{N_k!}{\prod_{j=1}^{s_k} r_{kj}!} \prod_{j=1}^{s_k} (P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k))^{r_{kj}}.$$

Therefore, the log-likelihood function is given by

$$\log L(\boldsymbol{\zeta}, \boldsymbol{\eta}) = constant + \sum_{k=1}^{K} \sum_{j=1}^{s_k} r_{kj} \log P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k).$$

The set of estimating equations for the population parameters $\boldsymbol{\eta}$ is given by

$$\frac{\partial \log L(\boldsymbol{\zeta}, \boldsymbol{\eta})}{\partial \boldsymbol{\eta}} = 0. \tag{7}$$

The evaluation of (7) involves some intermediate results that follow. First of all, note that

$$\frac{\partial \log L(\boldsymbol{\zeta}, \boldsymbol{\eta})}{\partial \boldsymbol{\eta}} = \sum_{k=1}^{K} \sum_{j=1}^{s_k} r_{kj} \frac{1}{P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k)} \frac{\partial P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k)}{\partial \boldsymbol{\eta}}, \tag{8}$$

where

$$\begin{aligned} \frac{\partial P(\boldsymbol{U}_{kj.}|\boldsymbol{\zeta}, \boldsymbol{\eta}_k)}{\partial \boldsymbol{\eta}} &= \int_{\boldsymbol{R}} P(\boldsymbol{U}_{kj.}|\theta, \boldsymbol{\zeta}) \left(\frac{\partial g(\theta|\boldsymbol{\eta}_k)}{\partial \boldsymbol{\eta}}\right) d\theta \\ &= \int_{\boldsymbol{R}} P(\boldsymbol{U}_{kj.}|\theta, \boldsymbol{\zeta}) g(\theta|\boldsymbol{\eta}) \left(\frac{\partial \log g(\theta|\boldsymbol{\eta}_k)}{\partial \boldsymbol{\eta}}\right) d\theta. \end{aligned} \tag{9}$$

The order of the derivative and of the integral was able to be exchanged based on the Dominated Convergence Theorem of Lebesgue (Chow & Teicher, 1978). It follows from (8) that the estimating equation for $\boldsymbol{\eta}$ is

$$\begin{aligned}\frac{\partial \log L(\zeta,\eta)}{\partial \eta} &= \sum_{k=1}^{K}\sum_{j=1}^{s_k} r_j \frac{1}{P(U_{kj.}|\zeta,\eta)} \int_{\boldsymbol{R}} P(U_{kj.}|\theta,\zeta)g(\theta|\eta_k)\left(\frac{\partial}{\partial \eta}\log g(\theta|\eta_k)\right)d\theta \\ &= \sum_{k=1}^{K}\sum_{j=1}^{s_k} r_j \int_{\boldsymbol{R}}\left(\frac{\partial}{\partial \eta}\log g(\theta|\eta_k)\right) g^*_{kj.}(\theta)d\theta = 0, \qquad (10)\end{aligned}$$

where

$$g^*_{kj.}(\theta) = \mathbb{P}(\theta|U_{kj.},\zeta,\eta) = \frac{P(U_{kj.}|\theta,\zeta)g(\theta|\eta)}{P(U_{kj.}|\zeta,\eta)}. \qquad (11)$$

Note that the probability in the numerator is given by (3).

Several distributions can be proposed for the ability distribution. In this work we will consider the univariate normal distribution.

4.1 Normal ability distribution case

In this section we will explore the case in that the ability distribution for population k is normal with parameters $\eta_k = (\mu_k, \sigma_k^2)'$, where $\mu_k = f(t_k|\alpha)$ is the mean and σ_k^2 is the known variance for population k. Specifically, we have that $\eta = \alpha$ and

$$\frac{\partial \log g(\theta|\eta_k)}{\partial \alpha} = \left(\frac{\partial \log g(\theta|\eta_k)}{\partial \mu_k}\right)\left(\frac{\partial \mu_k}{\partial \alpha}\right) = \frac{(\theta-\mu_k)}{\sigma_k^2}\left(\frac{\partial \mu_k}{\partial \alpha}\right). \qquad (12)$$

So, the estimating equations for α is

$$\alpha: \quad \sum_{k=1}^{K}\sum_{j=1}^{s_k} r_{kj}\int_{\boldsymbol{R}} \frac{(\theta-\mu_k)}{\sigma_k^2}\left(\frac{\partial \mu_k}{\partial \alpha}\right) g^*_{kj}(\theta)d\theta = 0. \qquad (13)$$

4.2 Polynomial growth curve

Consider that the growth curve for the mean components is given by the expression

$$\mu_k = f(t_k|\alpha) = \alpha_0 + \alpha_1 t_k + \alpha_2 t_k^2 + \cdots + \alpha_d t_k^d = \sum_{s=1}^{d} \alpha_s t_k^s, \qquad (14)$$

from which we get

$$\frac{\partial \mu_k}{\partial \alpha_s} = t_k^s$$

and so

$$\frac{\partial \mu_k}{\partial \alpha} = (1, t_k, t_k^2, \cdots, t_k^d)' \equiv l_k.$$

Substituting this expression in (13) we get

$$\alpha : \quad \sum_{k=1}^{K} \sum_{j=1}^{s_k} r_{kj} \int_{\mathbb{R}} \frac{(\theta - \mu_k)}{\sigma_k^2} l_k g_{kj}^*(\theta) d\theta = 0. \tag{15}$$

(16)

There is no closed form solution for these equations and some iterative method, such as Newton-Raphson or Fisher Scoring algorithms, needs to be considered in order to solve them.

5 Simulation results

In this section we present one application of the proposed methodology in simulated data. The data were generated based on $N_k = 1000$ individuals for group k, with $K = 5$ groups. The total simulation consisted of 1000 replications. The ability distribution and the composition of each one of the 5 tests are discussed below.

5.1 The item response function

It was considered the 3 parameter logistic model (LM3) with $D = 1.7$, given by

$$P(U_{ijk} = 1|\theta, \zeta_i) = c_i + (1 - c_i) \frac{1}{1 + e^{-Da_i(\theta - b_i)}}, \qquad \zeta_i = (a_i, b_i, c_i)', \tag{17}$$

where a_i is the discrimination parameter, b_i is the difficulty parameter and c_i is the guessing parameter. D is a scale factor, constant and equal to 1 or 1.7. The 1.7 value is used when it is wished that the logistic function yields results similar to that of the normal function.

5.2 The K tests

In order to generate the data it was assumed that at each one of the five tests, 1000 examinees were submitted to a test composed of 24 items with 6 common items between two consecutive grades. The items were of multiple choice with five categories of response. The total number of different items considered in the five tests was 96. Their parameters values and distribution along the tests are presented in Appendix A. For both, linear and quadratic, growth curves, the values for parameter a (discrimination) varied from 0.6 (low discrimination) to 1.4 (high discrimination) and the values for parameter b (difficulty) varied from -0.7 to 4.7. The items with higher (lower) values of b were allocated to those groups where the examinees have higher (lower) mean abilities. This was done in order to avoid estimation problems. For the guessing parameter c it was considered only one value (0.20).

5.3 Ability distribution and growth curve parameters

It was assumed that the ability for group k follows a normal univariate distribution with parameters $\eta_k = (\alpha, \sigma_k^2)$, where α are the parameters of the grow curve. All variances are considered to be equal to 1. Table 2 contains the values for the ability distribution parameters used in the simulation process. These values were stated taking into account the standard normal distribution. The values for the means and grow curve parameters (gcp) were selected to have populations close to each other and also apart from each other.

Table 2: Values of the ability distribution parameters used in the simulation

Grow	Location parameters					gcp		
Curve	μ_1	μ_2	μ_3	μ_4	μ_5	α_0	α_1	α_2
Linear	0	1	2	3	4	-1	1	
Quadratic	0	1.4	2.4	3.0	3.2	-1.8	2.0	-0.2

5.4 The computational program

All calculations were done via a computer program developed by the authors using the computer language *Ox* (see Doornik, 2000). In this program we use only the expression (15) in the internal maximization function, namely MaxBFGS.

5.5 Results, comments and suggestions

Observing Tables 3 e 4, we see that the average estimates are very close of the true values, proving the effectiveness of the estimating process of the growth curve parameters. Also the values of the standard deviations are very small.

Table 3: Descriptive statistics for the estimates: Linear case

	Grow curve parameters	
Curve	*Linear*	*Angular*
Mean	-1.0001	1.0000
Standard deviation	0.0220	0.0061
Minimum	-1.0768	0.9790
Maximum	-0.9272	1.0204

The mean abilities estimates were -0.0207, 0.9905 2.0016 3.0127 and 4.0239 for the linear case, and -0.0838, 1.3736, 2.4091 3.0227 and 3.2144 form the quadratic case, very close to the true values too.

Table 4: Descriptive statistics for the estimates: Quadratic case

	Grow curve parameters		
Curve	*Linear*	*Angular*	*Quadratic*
Mean	-1,7954	1,9968	-0,1995
Standard deviation	0,0784	0,0492	0,0071
Minimum	-2,0366	1,8454	-0,2221
Maximum	-1,5493	2,1533	-0,1775

Comments: In this work we assumed that all the parameters, but the mean distribution parameters were known. Also we considered two situations were we have a perfect fit using polynomial models. In future works we will consider the situations were the dispersion ability distribution parameters and the item parameters are unknown, and the case where the means are not in a perfect polinomial curve. Some nonlinear growth curve models will also be considered.

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A Appendix

Table A: Simulated item parameters - linear grow curve

Item	Test	a_i	b_i	c_i	Item	Test	a_i	b_i	c_i	Item	Test	a_i	b_i	c_i
1	1	0.6	-0.7	0.2	33	2	1.4	1.1	0.2	65	4	1.0	3.0	0.2
2	1	1.0	-0.7	0.2	34	2	0.6	1.3	0.2	66	4	1.4	3.0	0.2
3	1	1.4	-0.7	0.2	35	2	0.6	1.5	0.2	67	4	0.6	3.1	0.2
4	1	0.6	-0.5	0.2	36	2	0.6	1.7	0.2	68	4	1.0	3.1	0.2
5	1	1.0	-0.5	0.2	37	2,3	1.0	1.3	0.2	69	4	1.4	3.1	0.2
6	1	1.4	-0.5	0.2	38	2,3	1.0	1.5	0.2	70	4	0.6	3.3	0.2
7	1	0.6	-0.3	0.2	39	2,3	1.0	1.7	0.2	71	4	0.6	3.5	0.2
8	1	1.0	-0.3	0.2	40	2,3	1.4	1.3	0.2	72	4	0.6	3.7	0.2
9	1	1.4	-0.3	0.2	41	2,3	1.4	1.5	0.2	73	4,5	1.0	3.3	0.2
10	1	0.6	-0.1	0.2	42	2,3	1.4	1.7	0.2	74	4,5	1.0	3.5	0.2
11	1	1.0	-0.1	0.2	43	3	0.6	1.9	0.2	75	4,5	1.0	3.7	0.2
12	1	1.4	-0.1	0.2	44	3	1.0	1.9	0.2	76	4,5	1.4	3.3	0.2
13	1	0.6	0.1	0.2	45	3	1.4	1.9	0.2	77	4,5	1.4	3.5	0.2
14	1	1.0	0.1	0.2	46	3	0.6	2.0	0.2	78	4,5	1.4	3.7	0.2
15	1	1.4	0.1	0.2	47	3	1.0	2.0	0.2	79	5	0.6	3.9	0.2
16	1	0.6	0.3	0.2	48	3	1.4	2.0	0.2	80	5	1.0	3.9	0.2
17	1	0.6	0.5	0.2	49	3	0.6	2.1	0.2	81	5	1.4	3.9	0.2
18	1	0.6	0.7	0.2	50	3	1.0	2.1	0.2	82	5	0.6	4.0	0.2
19	1,2	1.0	0.3	0.2	51	3	1.4	2.1	0.2	83	5	1.0	4.0	0.2
20	1,2	1.0	0.5	0.2	52	3	0.6	2.3	0.2	84	5	1.4	4.0	0.2
21	1,2	1.0	0.7	0.2	53	3	0.6	2.5	0.2	85	5	0.6	4.1	0.2
22	1,2	1.4	0.3	0.2	54	3	0.6	2.7	0.2	86	5	1.0	4.1	0.2
23	1,2	1.4	0.5	0.2	55	3,4	1.0	2.3	0.2	87	5	1.4	4.1	0.2
24	1,2	1.4	0.7	0.2	56	3,4	1.0	2.5	0.2	88	5	0.6	4.3	0.2
25	2	0.6	0.9	0.2	57	3,4	1.0	2.7	0.2	89	5	0.6	4.5	0.2
26	2	1.0	0.9	0.2	58	3,4	1.4	2.3	0.2	90	5	0.6	4.7	0.2
27	2	1.4	0.9	0.2	59	3,4	1.4	2.5	0.2	91	5	1.0	4.3	0.2
28	2	0.6	1.0	0.2	60	3,4	1.4	2.7	0.2	92	5	1.0	4.5	0.2
29	2	1.0	1.0	0.2	61	4	0.6	2.9	0.2	93	5	1.0	4.7	0.2
30	2	1.4	1.0	0.2	62	4	1.0	2.9	0.2	94	5	1.4	4.3	0.2
31	2	0.6	1.1	0.2	63	4	1.4	2.9	0.2	95	5	1.4	4.5	0.2
32	2	1.0	1.1	0.2	64	4	0.6	3.0	0.2	96	5	1.4	4.7	0.2

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Departamento de Estatística
IME-USP
Caixa Postal 66.281
05311-970 - São Paulo, Brasil