RT-MAT 99-35

Representation type of one point extensions of tilted *Euclidean* Algebras

Gladys Chalom,
Héctor Merklen

Dezembro 1999

Esta é uma publicação preliminar ("preprint").

REPRESENTATION TYPE OF ONE POINT EXTENSIONS OF TILTED EUCLIDEAN ALGEBRAS

Gladys Chalom, Hector Merklen

Abstract

We know, after [P1], that, given a tame algebra Λ, the Tits form q_Λ is weakly non negative.

Moreover, the converse has been shown for some families of algebras, but it is not true in general. In the same article [P1], De la Peña proved that if Λ is a tame concealed algebra, not of type $\tilde{A}_n$ and M is an indecomposable Λ-module then $\Lambda[M]$ is tame if and only if $q_{\Lambda[M]}$ is weakly non negative. The purpose of this work is to show the same result for Λ a strongly simply connected tilted algebra of euclidean type.

1 Preliminaries

Throughout this paper, k denotes an algebraically closed field. By an algebra Λ we mean a finite-dimensional, basic and connected k-algebra of the form $\Lambda \cong kQ/I$ where Q is a finite quiver and I an admissible ideal.

We assume that Q has no oriented cycles. Let Λ-mod denote the category of finite-dimensional left Λ-modules, and Λ-ind a full subcategory of Λ-mod consisting of a complete set of non-isomorphic indecomposable objects of Λ-mod.

For each $i \in Q_0$ we denote by S_i (resp. P_i, I_i) the corresponding simple Λ-module (resp. the projective cover, injective envelope of S_i). The dimension-vector of a Λ-module X is the vector $\underline{dim}X = (dim_k Hom_\Lambda(P_i, X))_{i \in Q_0}$ in the Grothendieck group $\mathbb{K}_0(\Lambda)$. The support of a Λ-module X, supp(X) is the full subcategory of Λ defined by the set $\{i \in Q_0 | X(i) \neq 0\}$.

We shall use freely the known properties of the Auslander-Reiten translations, τ and τ^{-1}, and the Auslander-Reiten quiver of Λ-mod, Γ_Λ . For basic notions we refer to [R2] and [ARS]. See also [A] and [CB].

Tame algebras have the Tits form weakly non negative and for some classes of algebras, as for instance tilted or quasi-tilted algebras, this fact is determinant, that is, if Λ is tilted or quasi-tilted, then Λ is tame if and only if the Tits quadratic form is weakly non negative. Also, we have

Theorem 1.1 (De la Peña) [P1] Let $\Lambda = B[M]$ be a one point extension, where B is a tame concealed algebra, not of type $\tilde{A}_n$, and M an indecomposable B-module. Then Λ is tame if and only if q_Λ is weakly non negative.

It is natural to ask when a similar result extends to tilted algebras. In this work we will give a partial answer, that is, we prove the following:

Let B be a strongly simply connected tilted algebra of euclidean type and M an indecomposable B-module, then the one point extension $B[M]$ is tame if and only if $q_{B[M]}$ is weakly non negative.

We begin now to recall the concepts and results that form the background for our work.

Definition 1.2 [R1] A vectorspace category $(\mathbb{K}, |\ |)$ is a pair given by a Krull-Schmidt k-category $\mathbb{K}$ and a faithful functor $|\ | : \mathbb{K} \to \text{mod } k$.

Given a vectorspace category $(\mathbb{K}, |\ |)$, its objects (resp. the morphisms) are usually considered to be the objects (resp. the morphisms) of the image of $|\ |$, and its subspace category $\mathcal{U}(\mathbb{K})$ is defined as follows: the objects are triples (X, U, φ) with $X \in \text{Obj}\mathbb{K}$, U a k-vector space and φ: $U \to |X|$, k-linear. The morphisms $(X, U, \varphi) \to (X', U', \varphi')$ are the pairs (α, β) with β: $X \to X'$ in $\mathbb{K}$, $\alpha : U \to U'$ k-linear and such that $|\beta|\varphi = \varphi'\alpha$. Clearly, any object of $\mathcal{U}(\mathbb{K})$ is isomorphic to a direct sum of a triple (X, U, φ) with $\varphi : U \to |X|$ injective and copies of $(0, k, 0)$.

Definition 1.3 [R1] A k-category $\mathbb{K}$ is *schurian* if $\text{End}_{\mathbb{K}}(X) \cong k$ for any $X \in \text{Obj}\mathbb{K}$, X indecomposable.

Lemma 1.4 [R1] Let $\mathbb{K}$ be a schurian vectorspace category. If $\mathbb{K}$ is of finite representation type then every indecomposable object has dimension 1. If $\mathbb{K}$ is not wild then every indecomposable object has dimension at most 2, and moreover, if X, Y are indecomposables with $dim|X| = 2$ then $Hom(X, Y) \neq 0$ or $Hom(Y, X) \neq 0$.

Modules over a one point extension $B[M]$ can be identified with triples (X, U, φ) where $X \in B$-mod, U is a k-vectorspace and $\varphi : U \to Hom(M, X)$ is k-linear. If B has finite global dimension then $gldim\ B[M] =$

$max\{gldim\ B,\ pd_B M\ +1\}$. Moreover, B-mod is a full, extension closed subcategory of $B[M]$-mod.

It is known ([R1]) that the representation type of $B[M]$ depends on the representation type of B and of $\mathcal{U}(\mathrm{Hom}(M, B-\mathrm{mod}))$.

A schurian vectorspace category whose indecomposable objects have dimension one, corresponds to the additive category of a poset *add kS*.

See [R1] for other notions and notations related to vectorspace categories.

2 Comparing quadratic forms

In this section, we assume that B is such that $gldimB \leq 2$. Then for any B-module M we have $gldimB[M] \leq 3$. Hence we would be able to relate the Euler and the Tits form for $A = B[M]$.

By [R2] the Euler form of $A = B[M]$ can be calculated in terms of χ_B: Let X be a A-module and let:

$\underline{dim}_A(X) = \underline{dim}_B(Y) + n.\underline{dim}_A(S_e)$, where e is the new vertex. Then

$\chi_A(\underline{dim}X) = \chi_B(\underline{dim}Y) + n^2 - n(dim_k Hom_B(M, Y)$
$- dim_k Ext^1_B(M, Y) + dim_k Ext^2_B(M, Y))$

On the other hand, using Bongartz result (see [Bo]) that if $gldimB \leq 2$ then $\chi_B = q_B$, its Tits form is computed in following:

$q_A(x_1, x_2, ..., x_l, n) = q_B(x_1, x_2, ..., x_l) + n^2 -$
$\sum_{j \in Q_0} n.x_j(dim_k Ext^1_A(S_e, S_j) + dim_k Ext^1_A(S_j, S_e)) +$
$\sum_{j \in Q_0} n.x_j(dim_k Ext^2_A(S_e, S_j) + dim_k Ext^2_A(S_j, S_e))$

Comparing, we have:

Proposition 2.1 With the above notation:

$$\chi_A(\underline{dim}X) = q_A(\underline{dim}X) - n.dim_k Ext^2_B(M,Y)$$

Theorem 2.2 (De la Peña)[P1]

If B is a tame algebra, then q_B is weakly non negative.

3 Tilted Algebras

An algebra Λ is tilted of type Δ if there exists a *tilting* module T over a path algebra $k\Delta$ such that $\Lambda = End_{k\Delta}(T)$. Tilted algebras are characterized by the existence of *complete slices* in a component of their Auslander-Reiten quiver, called the *connecting component.* This component is standard and directed. A tilted algebra has at most two connecting components, and if it has two, then it is a concealed algebra, that is, it is the endomorphism algebra of a postprojective (or preinjective) tilting module.

Since the A-R-quiver of an hereditary algebra is known, we can get informations about the A-R-quiver of B accordingly with the different possibilities of T and H (see [R2]).

The structure of the Auslander-Reiten quiver of a tilted algebra is given in [R2] and in [K]. One possibility for the tilted algebras of euclidean type is that the connecting component is the preinjective component, and, in this case, the algebra B is a domestic tubular algebra, that is, $B = B_0[E_i, R_i]_{i=1}^t$, for some tame concealed algebra B_0, certain ray-modules E_i in the separating tubular family of B_0, and some non empty branches R_i. The only other possibility is that the connecting component is the postprojective component and the algebra is a domestic cotubular algebra, that is, B is obtained from B_0 by branch coextensions. Other facts about this subject can be seen in the survey of Assem, [A]

Theorem 3.1 [K] Let B be a tilted algebra of infinite representation type. The following conditions are equivalent :

(1) B is tame

(2) χ_B is weakly non negative

4 Modules of the separating tubular family

Let us assume that B is a tilted algebra of euclidean type, and that M is an indecomposable B-module. We begin studying the case that M is not directed.

Let us observe that 4.2 is very similar to [T], but we do not assume that B is a good algebra, but that the preinjective component of B be of tree type.

Lemma 4.1 Let B_0 be a convex subcategory of B such that B is a iterated coextension or a branch coextension of B_0 and assume that $M_0 = M|_{B_0}$, $M_0 \neq 0$. Then $B_0[M_0]$ is a convex subcategory of $B[M]$.

Proof: The proof is done by induction in the number of the coextensions or the length of the branch.□

Let us see the case when B is of domestic tubular type

Let B be a tilted tame algebra of euclidean type with

1) the complete slice in the preinjective component.

2) the preinjective component of tree type. Let M be an indecomposable module, in the separating tubular family.

Proposition 4.2 In the above conditions, if $B[M]$ is wild then $q_{B[M]}$ is strongly indefinite.

To prove this proposition, we need some preliminar results.

Lemma 4.3 [T]

Let $B = End_A(T)$ with T an A-tilting module and $M = Hom(T, X)$ with $X \in \mathcal{G}(T)$. Then there exists a $A[X]$-tilting module T' such that $B[M] = End_{A[X]}(T')$.

Lemma 4.4 Let $B = End_H(T)$ with H an hereditary algebra of euclidean type and T an H-tilting module without preinjective direct summands. Let M be an indecomposable B-module in the separating tubular family. Then $B[M] = End_{H[R]}(T')$ with $M = Hom(T, R)$ and T' splitting.

Proof: Since T does not have preinjective direct summands, B has a complete slice in the preinjective component, and M belongs to the separating tubular family. Then $M \in \mathcal{Y}(T)$ and there exists $R \in \mathcal{G}(T)$ with $M = Hom(T, R)$ and $T' = T \oplus Pw$ is a $H[R]$-tilting module. Let us see that T' is splitting. Since $\mathcal{F}(T') = \{Y\ H[R]\text{-modules such that } Hom_{H[R]}(T', Y) = 0\}$, let $Y \in \mathcal{F}(T')$ so that $Hom(T \oplus Pw, Y) = 0$. Then $Hom(T, Y) = 0$ and $Hom(Pw, Y) = 0$. Therefore Y is an H-module and $Y \in \mathcal{F}(T)$. Since T is splitting $id_H(Y) = 1$.

Consider the $H[R]$-module $(Y, 0, 0)$ and let us see that $id\ (Y, 0, 0) = 1$. Let us consider the minimal injective resolution of Y in H-mod: $0 \longrightarrow Y \xrightarrow{f_1} \overline{I}_0 \xrightarrow{g_1} \overline{I}_1 \longrightarrow 0$ with $\overline{I}_0 = \oplus I_i$, I_i indecomposable injectives and let $\hat{I}_0 = \oplus(I_i,\ Hom(R, I_i),\ 1_{Hom})$ be the corresponding injective in $H[R]$ then: $0 \longrightarrow (Y, 0, 0) \xrightarrow{(f_1, f_2)} \hat{I}_0$ is an injective envelope because

$soc(Y,0,0) = (soc\ Y,0,0)$. By [M2] Coker $(f_1, f_2) = (\overline{I}_1, Hom(R,\overline{I}_0), \zeta) = \oplus(I_i,\ Hom(R,I_i),1) \oplus nSe$ that is again injective, so $id_{H[R]}(Y,0,0) = 1$ therefore, T' is splitting by Hoshino's Lemma.□

Proof of the proposition: Let $B[M]$ be of wild type. Since $B[M] = End_{H[R]}(T')$ and T' is splitting, $H[R]$ is wild.

Since B is tilted of euclidean type and the preinjective component of B is of tree type, H is tame, euclidean and $\tilde{A}_n$-free so, by [P1], there exist $V_1, V_2, \cdots V_n$, preinjective H-modules with $q_{H[R]}(dim(\oplus V_i \oplus n\ S'e)) < 0$ and each $V_i \in \mathcal{G}(T)$, in this case let $W_i = Hom(T,V_i), W_i$ is a preinjective B-module that belongs to $\mathcal{Y}(T)$. So, we have: $\chi_{B[M]}(\underline{dim} \oplus W_i \oplus n\ Se) = \chi_B(\underline{dim} \oplus W_i) + n^2 - n < \underline{dim}\ M, \underline{dim}\ \oplus W_i >_B$

By [R2], pag. 175, there is an isometry $\sigma_T = \mathbb{K}_0(H) \longrightarrow \mathbb{K}_0(B)$ such that: $\sigma_T(\underline{dim}\ V_i) = \underline{dim}\ W_i$ and $\sigma_T(\underline{dim}\ R) = \underline{dim}\ M$ so: $\chi_H(\underline{dim} \oplus V_i) = \chi_B(\underline{dim} \oplus W_i)$ and $< \underline{dim}\ M, \underline{dim} \oplus W_i >_B = < \underline{dim}\ R, \underline{dim} \oplus V_i >_H$ then: $\chi_{H[R]}(\underline{dim}(\oplus V_i \oplus n\ S'e)) = \chi_{B[M]}(\underline{dim}(\oplus W_i \oplus n\ Se)) < 0$ by [P1]. But $q_{B[M]}(\underline{dim}(\oplus W_i + n\ Se)) = \chi_{B[M]}(\underline{dim}(\oplus W_i \oplus n\ Se) + n\ dim_k Ext^2_B(M, \oplus W_i)$ and again, since $Hom(M,W_i) \neq 0\ \forall i$ and W_i is a directed module, we have: $Ext^2(M,\oplus W_i) = 0$ so $q_{B[M]}(\underline{dim}(\oplus W_i \oplus n\ Se)) < 0$. Clearly, $\underline{dim}(\oplus W_i \oplus n\ Se)$ is a vector of positive coordenates.□

5 Tubular extensions

5.1 Patterns

In this section we define an equivalence relation for certain infinite vectorspace categories, see [R1], pag. 229. This method permits the classification of important families of tubular extensions.

Definition 5.1 [[R1], pag. 229] We say that two vectorspace categories $\mathbb{K}_1$ and $\mathbb{K}_2$ belong to the same pattern, if there exist functors that are faithful, full and co-finite $\varphi : \mathbb{K}_1 \to \mathbb{K}_2$ and $\psi : \mathbb{K}_2 \to \mathbb{K}_1$.
If $\mathbb{K}_1$ and $\mathbb{K}_2$ belong to the same pattern, then there exist faithful and full functors $\tilde{\varphi} : \mathcal{U}(\mathbb{K}_1) \to \mathcal{U}(\mathbb{K}_2)$ and $\tilde{\psi} : \mathcal{U}(\mathbb{K}_2) \to \mathcal{U}(\mathbb{K}_1)$.

As in [R1] we are considering only the vectorspace categories such that there exists a faithful, co-finite functor $\varphi : \mathbb{K} \to \mathbb{K}$ such that $\cap\varphi^n(\mathbb{K})$ is finite.

Consider then, an hereditary algebra H of euclidean type and R a regular H-module. We find in [[R1], lemma 2, pag. 232] that for σ a reflection functor, the vectorspace categories $Hom(R, H - mod)$ and $Hom(\sigma R, \sigma H - mod)$ belong to the same pattern. Moreover, if R is a module of period n, the different H-modules of period n can be obtained from R by reflection functors, then $Hom(R, H - mod)$ and $Hom(\tau^i R, H - mod)$ belong to the same pattern.

5.2 Iterated tubular algebras

Under certain conditions, the process of branch extensions may be iterated. We define iterated tubular algebras by means of this iterated process as follows (see [PT]).

Definition 5.2 [PT] Let Λ_0 be a tubular or domestic cotubular algebra, that is, a branch coextension of a tame concealed algebra A_0, and let $E_1, E_2, ..., E_t$ be orthogonal ray-modules in the separating tubular family of Λ_0. The restriction of $E_i's$ to A_0 are again orthogonal ray-modules in the separating tubular family of A_0. Let R_i be a branch and assume that the extension $A = A_0[E_i|_{A_0}, R_i]_{i=1}^t$ is a tubular or domestic tubular algebra. In this case, the extension $\Lambda = \Lambda_0[E_i, R_i]_{i=1}^t$ is called a 1-iterated tubular algebra. If A is tubular, we can repeat the process and we obtain a 2-iterated tubular algebra. Inductively, we define a n-iterated tubular algebra.

Definition 5.3 [PT] We say that an algebra Λ has acceptable projectives if the A-R quiver of Λ has components $\mathcal{P}, \mathcal{C}_1, \mathcal{C}_2, \cdots, \mathcal{C}_n$ with the following properties:

1) Any indecomposable projective Λ-module lies in $\mathcal{P}$ or in some $\mathcal{C}_i$.

2) $\mathcal{P}$ is a postprojective component of Γ_Λ without injective modules.

3) Each $\mathcal{C}_i$ is a standard inserted-coinserted tube.

4) If $Hom(\mathcal{C}_i, \mathcal{C}_j) \neq 0$ then $i \leq j$.

Theorem 5.4 [[PT], 3.4] Let Λ be an algebra with acceptable projectives. The following are equivalent :

a) Λ is an iterated tubular algebra.

b) Λ is tame .

c) q_Λ is weakly non negative.

5.3 Coils

We define the coil algebras, which have been extensively studied in the last years. They are obtained through a construction process, which makes them to appear even more interesting.

Definition 5.5 A coil is a translation quiver constructed inductively from a stable tube, by a sequence of operations called admissible. Let A be an algebra and Γ a standard component of Γ_A. For an indecomposable module X in Γ, called pivot, the admissible operation to apply to Γ depends on the support $S(X)$ of $Hom_A(X,-)|_\Gamma$. For details, we refer to [AS].

We have then

Proposition 5.6 [AS] Corollary 6

Let A be a tame concealed algebra, and $\mathcal{T}$ its separating tubular family. Let $\hat{A}$ be a coil enlargement of A, using modules of $\mathcal{T}$. The following conditions are equivalent :

1) $\hat{A}$ is tame

2) $\hat{A}^+$ and $\hat{A}^-$ are tame.

6 Domestic cotubular algebras

We will see now that the same result see in 4.2 is true for algebras of euclidean type, with a complete slice in the postprojective component.

Theorem 6.1 Let B be a tilted algebra of euclidean type whose preinjective component is of tree type and let M be a indecomposable B-module in the separating tubular family such that the one-point extension $B[M]$ is wild.

Then $q_{B[M]}$ is strongly indefinite.

Proof: Since B is of euclidean type, either B has a complete slice in the preinjective component, and the result follows from 4.2, or B has a complete slice in the postprojective component. Let us see the case when

1) there is a complete slice of B in the postprojective component, and

2) the preinjective component of B is of tree type.

By [R2], B is a branch coextension of a tame concealed algebra B_0 and the preinjective component of B is the same preinjective component of B_0, and so B_0 is $\tilde{A}_n$-free. Assume that $B = {}_{i=1}^{t}[E_i, R_i]B_0$ where E_i is a B_0-ray module and R_i is a branch, for all i. Let us consider separately the following situations: A) $M_0 = M|_{B_0}$ is such that $M_0 = 0$;

B) $M_0 = M|_{B_0}$ is such that $M_0 \neq 0$.

In case A, *suppM* is contained in a branch R and the vectorspace category $Hom(M, B - mod)$ is the same as $Hom(M, R - mod)$. By [MP], if $Hom(M, R - mod)$ is wild then $q_{R[M]}$ is strongly indefinite. As $R[M]$ is a convex subcategory of $B[M]$, if $q_{R[M]}$ is strongly indefinite then $q_{B[M]}$ is strongly indefinite.

In case B, we can distinguish two situations:

B1: $B_0[M_0]$ is wild;

B2: $B_0[M_0]$ is tame.

We begin by B1. If $B_0[M_0]$ is wild, since the preinjective component of B is the same preinjective component of B_0, B_0 is tame concealed and $\tilde{A}_n$-free. So, by [P1], $q_{B_0[M_0]}$ is strongly indefinite. But $B_0[M_0]$ is a convex subcategory of $B[M]$ and so $q_{B[M]}$ is strongly indefinite.

Let us see B2, that is $B_0[M_0]$ is tame , but $B[M]$ wild.

Again, since $B_0[M_0]$ is tame, we have two possibilities:

B2.1 M_0 is a ray module.

B2.2 M_0 is a module of regular length regular two in the tube of rank $n-2$ and B_0 is tame concealed of type $\tilde{D}_n$. In the case B.2.1, we have that if M is a ray module over B, by [R2] 4.5 and 4.6, the component $\mathcal{T}[M]$ is a standard inserted-co-inserted tube. Moreover, all indecomposable projectives of $B[M]$ lie in $\mathcal{P}$, the postprojective component, or on $\mathcal{T}[M]$ (where is the unique projective that is outside of $\mathcal{P}$) therefore, $B[M]$ is an algebra with acceptable projectives and in this case, by 5.4 $B[M]$, it is wild if and only if $q_{B[M]}$ is strongly indefinite. On the other hand, if $M = M_0$ and therefore, M is a ray module over B_0, then $B[M] = B[M_0]$ is an iterated tubular algebra and in this case, $B[M]$ is tame, a contradiction. So, we can assume that M is not a ray module over B and moreover that $M \neq M_0$ and, therefore, that there exists an indecomposable injective I in $\mathcal{T}$, the tube where M lies, such that $Hom(M, I) \neq 0$ and that there are two arrows starting in M. Also, we can assume that i, the coextension vertex belongs to *supp* M, so that there exists a morphism $M \to I_i$.

Let E be the ray module which is the root of the branch.

Let $B_i = [E]B_0$ and $M_i = M|_{B_i}$. Then we have: $Hom_{B_i}(M_i, M_0) \neq 0$, but $Hom_{B_i}(M_0, M_i) = 0$, and again we have two cases:

B.2.1.1 The branch is co-inserted in E, $E \neq M_0$;

B.2.1.2 The branch is co-inserted in $E = M_0$.

In the first case, since M is not a ray module over B, we can assume that there exists an arrow that start in M and points to the mouth of the tube, say $M \to Y$. Moreover, by [[R2], 4.5] there exists a sectional path $M \to$

$M_t \to M_{t-1} \to \cdots M_0$ that does not contain injectives. So, we can consider that all of these modules $\tau^{-1}M_i$, and in particular $\tau^{-1}M_1$, are non zero.

Since M_0 is a B_0-ray module, then $\tau^{-1}M_1$ cannot be a B_0-module. But in this case, it is a co-ray module and therefore M_0 is a co-ray module, contradiction. So, the situation B.2.1.1 does not occur.

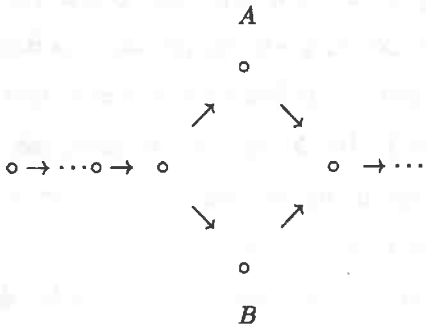
If the branch is co-inserted in $E = M_0, M_0 = M|_{B_0}$, M is not a ray module. Again, we can assume that there exists an arrow starting in M and pointing to the mouth of the tube. Moreover, since the branch is co-inserted in M_0, there is a sectional path $M \to I$ the injective of the co-insertion. Let us look at the category $Hom(M, B - mod)$. This category has three pieces. Since B is tilted, $Hom(M, X) \neq 0$ only for modules X that are preinjective or in the same tube $\mathcal{T}$ where M lies. Let X be a B_0-module. Since M is a co-inserted module, $Hom_B(M, X) \neq 0$ and, hence, $Hom_{B_0}(M_0, X) \neq 0$. Since B_0 is a tame concealed algebra and M_0 is a ray module over B_0, $Hom(M, B - mod)$ contains the following subcategories: the ray of $\mathcal{T}$ that starts in M_0, $Hom(M_0, \mathcal{I}(B_0)$ where $\mathcal{I}(B_0)$ is the preinjective component of B_0 and the subcategory given by the sucessors of M in the tube, that are not B_0-modules. Since $B_0[M_0]$ is tame, $Hom(M_0, \mathcal{I}(B_0))$ is given by some of the patterns given in [[R1], pag. 254]. Let us assume that one of the following two situations occur:

either M is injective and so the vectorspace category restricted to the tube is given by two sectional paths one, finite, pointing to the mouth of the tube and one, infinite, (the ray) or M is not injective but the vectorspace category restricted to the tube is given by two parallel paths. We will see that in this situation, since $B_0[M_0]$ is tame, $B[M]$ is tame, in contradiction to the hypothesis, because $A = B[M]$ is a coil enlargement of B_0, by 5.6 because $A^+ = B_0[M_0]$, $A^- = B$, are both tame. As that $A = B[M]$ is tame.

Let us assume then that M is not injective and that there exists a sectional path $M \to Y_t$ with $t \geq 1$. In first place, we observe that $Hom_B(Y_i, X)$ $= 0$ for all preinjective X. But Y_i being on the coray, and to the right of M_0, there does not exist an infinite path coming out of it, and similarly $Hom(\tau^{-1}M, X) = 0$ for all preinjective X.

In particular, $Hom(Y_i, X) = Hom(\tau^{-1}M, X) = 0$ for all X such that $Hom(M_0, X) \neq 0$ with X in the preinjective component. Moreover $Hom(Y_i, \tau^{-1}M) = 0 = Hom(\tau^{-1}M, Y_j)$ for $\forall j \geq 1$. Hence, by [[R1] (3.1)] we can find one of the following path-incomparable (see [Ch]) subcategories in $\mathcal{I}(B_0)$, with the only exception of the case $(\tilde{D}_n, n-2)$. $\mathbb{K}_1 = \{A, B, C\}$, (in cases: $(\tilde{D}_4, 1), (\tilde{D}_6, 2), (\tilde{D}_7, 2), (\tilde{D}_8, 2), (\tilde{E}_6, 2), (\tilde{E}_7, 3), (\tilde{E}_7, 4)$, $(\tilde{E}_8, 5)$ and $\mathbb{K}_2 = \{A, B \to C\}$ in cases $(\tilde{D}_5, 2)$ and $(\tilde{E}_6, 3)$ So, in each case, adding the objects $Y_1, \tau^{-1}M$ to the categories $\mathbb{K}_1$ or $\mathbb{K}_2$ we have that $Hom(M, B-mod)$ is wild and that $q_{B[M]}$ is strongly indefinite.

Let us calculate the quadratic form for the case $(\tilde{D}_5, 2)$, the other cases are similar. Let $\tilde{L}$ be the B-module $\tilde{L} = 2Y_1 \oplus 2\tau^{-1}M \oplus 2A \oplus B \oplus C$ and $L = \tilde{L} \oplus 4S_e$, then $q_{B[M]}(\underline{dim}L) = \chi_{B[M]}(\underline{dim}L) + 4dim_k Ext^2(M, \tilde{L})$ $= \chi_{B[M]}(\underline{dim}L) = \chi_{B[M]}(\underline{dim}\tilde{L}) + 4^2 - 4(8) = 15 + 16 - 32 = -1$. Let us see the case $(\tilde{D}_n, n-2)$. In this case, the pattern is given by:



If $t > 1$, considering that $\mathbb{K} = \{A, B, \tau^{-1}M, Y_1 \to Y_2\}$ is wild, again the quadratic form is strongly indefinite. On the other hand, if $t = 1$ we have two possibilities:

Case 1

$$
\begin{array}{cccccccc}
 & & & & Y_1 & & & \\
 & & & \nearrow & & & & \\
 & & Y_0 & & & & & \\
 & \nearrow & & \searrow & & & & \\
M & & & & \tau^{-1}M & & & \\
 & \searrow & & \nearrow & \searrow & & & \\
 & & Z_1 & & \tau^{-1}Z_1 & & & \\
 & & & \searrow & \nearrow & & & \\
 & & & Z_2 & & & & \\
 & & & & \searrow & & & \\
 & & & & & \cdots & & \\
 & & & & & I & & \\
 & & & & & & \searrow & \\
 & & & & & & & M_0
\end{array}
$$

and case 2

$$
\begin{array}{ccccc}
 & & & & Y_1 \\
 & & & \nearrow & \\
 & & Y_0 & & \\
 & \nearrow & & \searrow & \\
M & & & & \tau^{-1}M \\
 & \searrow & & \nearrow & \\
 & & I & & \\
 & & & \searrow & \\
 & & & & M_0
\end{array}
$$

In case 1, we can again consider the wild subcategory $\{Y_1, \tau^{-1}M \to \tau^{-1}Z_1, A, B\}$ and the quadratic form is strongly indefinite. On the other hand, in case 2, we have a vectorspace category which is in fact tame, by Nazarova Theorem, so that $B[M]$ is tame .

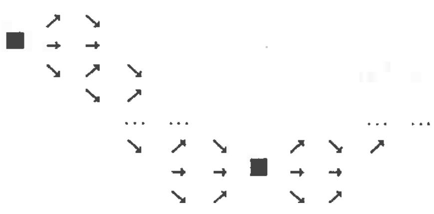
Let us examine now B.2.2, M_0 is a module of regular length 2 in a tube of rank $n-2$ and B_0 is tame concealed of type $\tilde{D}_n$. If $M = M_0$ lies in a stable tube, then $Hom(M, B - mod) = Hom(M_0, B_0 - mod)$ and therefore both are tame or wild simultaneosly. So, we can assume that M belongs to a co-inserted tube. Since M_0 has regular length 2, there exist E_1 and E_0 ray-modules over B_0 such that $\tau E_0 = E_1 \to M_0 \to E_0$ is the ARS for E_0. Let $E_0, E_1, \cdots E_{n-3}$ be the ray-modules over B_0 of the tube where M lies. Again, we divide in possibilities.

B.2.2.1 The branch is co-inserted in E_0.

B.2.2.2 The branch is co-inserted in E_1.

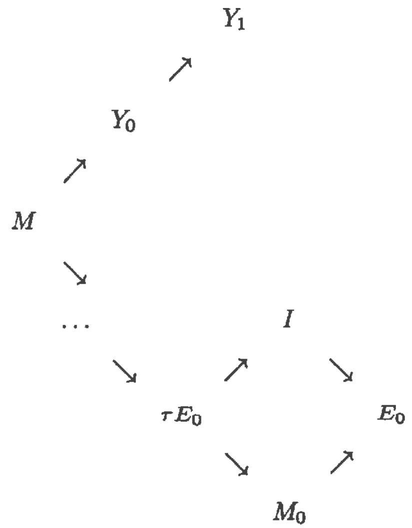
B.2.2.3 The branch is co-inserted in E_j for $j \neq 0$ or 1.

Observe that if $M = M_0$, then $Hom(M, B - mod)$ has the same pattern as $Hom(M_0, B_0 - mod)$. If M is a B_0-module, then $Hom_B(M, N) \neq 0$ for modules N in the same tube as M or for modules N in the preinjective component. Hence, being $Hom(M, N) = Hom(M_0, N_0)$ it has the following pattern



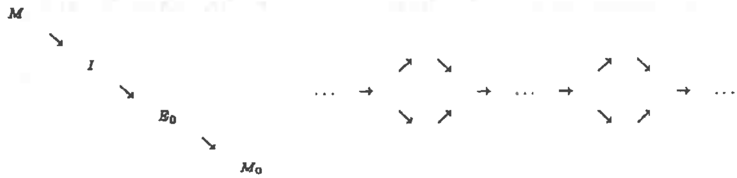
which is tame , by [R1]. (In this picture we indicate the non zero modules in the category with ■ indicating the objects of dimension 2.) We can assume that M belongs to the co-ray and that there exists an injective I in the tube $\mathcal{T}$ such that $Hom(M, I) \neq 0$.

Let us consider B.2.2.1. We have a co-inserted branch in E_0, and



If there exists a sectional path $M \to Y_0 \to Y_1$, then, $Hom(M, Y_1) \neq 0$. Let us observe that $Y_1|_{B_0} = 0$ and $Hom(Y_1, X) = 0$ for all preinjective module X and in particular, $Hom(Y_1, X_i) = 0$ for each of the preinjective $X_i's$ such that $Hom(M_0, X_i)$ has dimension 2. Hence $q_{B[M]}$ is strongly indefinite. Let us assume that the longest sectional path starting at M in the direction of the mouth of the tube has length 1. In this case, again, $Hom(M, B-mod)$ has the same pattern than $Hom(M_0, B_0 - mod)$ and so it is tame.

Let us consider B.2.2.2 Since $Hom(E_1, E_0) = 0$, the morphisms from M to X, for X preinjective, are just the ones that factor through the successor of M_0, M_1, and those that factor through E_0 are equal to zero and the vectorspace category $Hom(M, B-mod)$ is of the form:

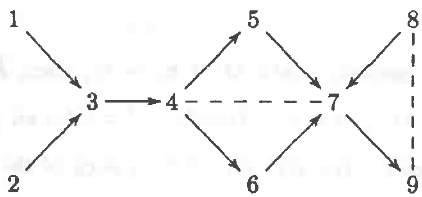


and we can repeat the arguments of the case B.2.1.2.

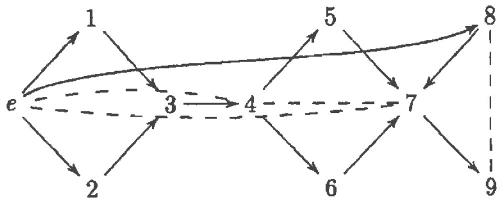
Finally, let us look at B.2.2.3. The branch is inserted in E_j with $j \neq 0$ or 1. But, in this case, $M = M_0$, $Hom(M_0, I) = 0$ for any I injective in $\mathcal{T}$ and we fall again in a already examined case.□

Example 6.2 Let us see an example

Let B be given by:



B is tilted of type $\tilde{D}_8$, with a complete slice in the postprojective component. Let us consider M_1 a module of the separating tubular family, such that the ordinary quiver of $\Lambda_1 = B[M_1]$, is given below. Then Λ_1 is wild and $q_{\Lambda_1}(I_3 \oplus I_3 \oplus I_8 \oplus 2S_e) = -1$.



Proposition 6.3 Let B be a tilted algebra of euclidean type, with the postprojective component of tree type and M an indecomposable B-module in this component. Then, if $B[M]$ is wild, the Tits form $q_{B[M]}$ is strongly indefinite.

To prove this results we need the following definition and facts

Definition 6.4 [Co] A component $\mathcal{C}$ of the Auslander-Reiten quiver of an algebra is called a π-component if:

i) almost all modules in $\mathcal{C}$ lie in the τ-orbit of a projective module; and

ii) at most finitely many modules in $\mathcal{C}$ belong to oriented cycles.

Theorem 6.5 [Co] The following conditions are equivalent for an algebra A:

i) $l(Hom_A(\_, A)) < \infty$

ii) $l(Hom_A(\_, M)) < \infty$ for all postprojective modules M.

iii) Any component $\mathcal{C}$ of $\Gamma(modA)$ containing a projective module is a π-component.

Proposition 6.6 [Co] If $\mathcal{C}$ is a π-component and contains no injective module, then $\mathcal{C}$ is a postprojective component.

Now we can prove the proposition

Proof Since B is of euclidean type we have two possibilities

1) B has a complete slice in the preinjective component, or

2) B has a complete slice in the postprojective component.

In the first case, all injectives are in the preinjective component, so for any I such that $Hom(M,I) \neq 0$, M and I are separated by a separating tubular family and the result follows from [PT].

In case 2 all projectives are in the postprojective component and again we divide in two cases

2A) If $M \in \mathcal{Y}(T)$ or

2B) If $M \in \mathcal{X}(T)$.

Let us see 2A, if $M \in \mathcal{Y}(T)$ then $M = Hom(T,X)$ for some $X \in \mathcal{G}$ and there exists (by 4.3 and 4.4) a splitting tilting $H[X]$-module T' such that $B[M] = End_{H[X]}(T')$, so using the same argument used in 4.2 we have that $q_{B[M]}$ is strongly indefinite.

Let us see 2B, that is $M \in \mathcal{X}$. Let us consider $\mathcal{C}'$ the component in the Auslander-Reiten quiver of $B[M]$ that contains the new projective module P_e, we will see that $\mathcal{C}'$ is a π-component. By the above theorem, it is enough to prove that $l(Hom(\_, B[M]) < \infty$, but as $B[M] = B \oplus P_e$ and the number of indecomposable modules that are predecessors of $B[M]$ is finite, so, $\mathcal{C}'$ is a π-component. Again two situations can occur:

1) The new simple injective I_e belongs to $\mathcal{C}'$, or

2) The new simple injective I_e does not belong to $\mathcal{C}'$.

Recall that the $B[M]$-indecomposable injectives are of the form $\overline{I}_i = (I_i, Hom(M, I_i), id.)$ when $Hom(M, I_i) \neq 0$, $(I_i, 0, 0)$ when $Hom(M, I_i) = 0$, where I_i are the indecomposable injectives of B and the new injective I_e is equal to $(0, k, 0)$.

Let us consider 1), so $I_e \in C'$, again by the above theorem ([Co]), since C' contains a projective module then $l(Hom(\_, I_e)) < \infty$. But in this case the number of $B[M]$-modules that are not B-modules is finite and so $B[M]$ is tame.

Let us consider 2). The new injective I_e does not belong to C'. If no other injective belongs to C', by [Co] C' is a postprojective component that contains all projectives and no injectives. In this case $B[M]$ is a tilted algebra and the representation type is given by the corresponding quadratic form.

Let us see that no injective belongs to C'. Let I be a B-indecomposable injective, if $Hom(M, I) \neq 0$, there exists a non zero morphism $(I, 0, 0) \to (I, Hom(M, I), id.)$ Consider P the B- indecomposable projective associated to I, then $(P, 0, 0)$ is the $B[M]$-projective associated to $(I, Hom(M, I), id.)$ and $Hom((P, 0, 0), (I, 0, 0)) \neq 0$. As in B-mod, P and I are in different components, there exists infinite B-modules X_i such that $Hom(X_i, I) \neq 0$ but in this case, $Hom_{B[M]}((X_i, 0, 0), (I, 0, 0)) \neq 0$ for infinite modules, a contradiction to the fact that $(l(Hom(\_, (I, 0, 0)) < \infty$. So C does not contain any injective.□

We have been assuming that some of the directed components of B are of tree type. In general these hypothesis does not imply that the algebra is a good algebra or is strongly simply connected. But for tilted tame algebras, this is the case.

Definition 6.7 ([S3]) An algebra A is called strongly simply connected when every full and convex subcategory of A is simply connected, or every full convex subcategory of A satisfies the separation condition.

Theorem 6.8 [ALP] Let B be a tame tilted algebra. Then B is strongly simply connected if and only if the orbit quiver of each directed component of $\Gamma(modB)$ is a tree.

Corollary 6.9 Let B be a strongly simply connected tilted algebra of euclidean type and M an indecomposable B-module. If $B[M]$ is wild then $q_{B[M]}$ is strongly indefinite.

Proof: If M is a postprojective module, we have the result by 6.6. If M is a module of the tubular family, the result follows by 6.1. Let us assume that M is preinjective. If B has a complete slice in the postprojective component the result follows from [P1]. Let us assume that B has a complete slice in the preinjective component, we are going to use the same argument used by De la Peña in [P4]. Let $\mathcal{S}(M \rightarrow) = \{Y \in B - mod$ such that there exist a sectional path $M \rightarrow Y\}$ and let P_e denote the new projective in $B[M]$. Let us call $\mathcal{S} = \mathcal{S}(M \rightarrow) \cup \{P_e\}$. Then $\mathcal{S}$ is a slice (in general not complete) in $B[M]$, and we can consider C the full subcategory of $B[M]$ determined by the vertices i such that $Y(i) \neq 0$ for $Y \in \mathcal{S}$. In this case, C is a convex subcategory of $B[M]$, and $\mathcal{S}$ is a complete slice in C, so C is tilted. Moreover all $B[M]$-modules are B-modules or are C-modules. If $B[M]$ is wild, then C is wild, and as C is convex in $B[M]$ $q_{B[M]}$ is strongly indefinite.□

References

[A] - Assem, I.; Tilting theory - an introduction; Topics in Algebra, Banach Center Publications, vol 26 (1990) 127-180.

[AC] - Assem, I.; Castonguay, D.; Strongly simply connected one-point extensions of tame hereditary algebras; Rapport n 207 (1997) Sherbrooke, Canada.

[AL1] - Assem, I.; Liu, S.; Strongly simply connected algebras, Rapport n 179 (1996) Sherbrooke, Canada.

[AL2] - Assem, I.; Liu, S.; Strongly simply connected tilted algebras, Rapport n 180 (1996) Sherbrooke, Canada.

[ALP] - Assem, I.; Liu, S.; Peña, J.A.; The strong simple connectedness of a tame tilted algebra, Rapport n 214 (1998) Sherbrooke, Canada.

[ARS] - Auslander, M.; Reiten, I.; Smalo, S.; Representation theory of Artin algebras; Cambridge Studies in Advanced Mathematics 36,1995.

[AS] - Assem, I.; Skowroński, A.; Multicoil Algebras; Rapport n 99 (1992) Sherbrooke, Canada.

[BB] - Brenner, S.; Butler, M.; Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors, Proc. ICRA II (Ottawa,1979), Lecture Notes in Math. 832, Springer, Berlin (1980), 103-169.

[B1] - Bekkert, V.; Schurian vector space categories of polynomial growth; Preprint (1995).

[B2] - Bekkert, V.; Non-domestic schurian vector space categories of polynomial growth; Preprint (1997).

[B3] - Bekkert, V.; Sincere cycle-finite schurian vector space categories; Preprint (1997).

[Bo] - Bongartz, K.; Algebras and quadratic forms; J. London Math. Soc.(2) 28 (1983) 461-469.

[Co] - Coelho, F. U.; Components of Auslander-Reiten quivers containing only preprojective modules; J. Algebra (157) (1993) 472-488.

[CB] - Crawley-Boevey, W. W.; On Tame algebras and Bocses; Proc. London Math. Soc. (3) 56 (1988) 451-483.

[Ch] - Chalom, G.; Vectorspace Categories Immersed in Directed Components; to appear in Comm. in Algebra.

[D] - Draxler, P.; Completely separating algebras; Journal of Algebra, vol 165, n 3 (1994) 550-565.

[DR] - Dlab, V.; Ringel, C.M.; Indecomposable representations of graphs and algebras; Memoirs Amer. Math. Soc. 173 (1976).

[HR] - Happel, D.; Ringel, C.M.; Tilted Algebras, Trans. Amer. Math. Soc. 274 (1982), N 2, 399-443.

[K] - Kerner, O.; Tilting wild algebras; J. London Math. Soc.(2) 39 (1989) 29-47.

[L] - Liu, S.; Tilted algebras and generalized standard Auslander Reiten components ; Arch. Math. vol 61(1993) 12-19.

[L1] - Liu, S.; Infinite radicals in standard Auslander Reiten components; Journal of Algebra 166 (1994) 245-254.

[M1] - Marmaridis, N.; Strongly Indefinite Quadratic Forms and Wild Algebras; Topics in Algebra, Banach Center Publications, vol 26 (1990) 341-351.

[M2] - Marmaridis, N.; Comma categories in representation theory; Communications in Algebra 11(17),1983,1919-1943.

[MP] - Marmaridis, N.; Peña, J.A.; Quadratic Forms and Preinjective Modules; Journal of Algebra 134 (1990) 326-343.

[P1] - Peña, J.A.; On the Representation Type of One Point Extensions of Tame Concealed Algebras; Manuscripta Math. 61, (1988) 183-194.

[P2] - Peña, J.A.; Tame algebras with sincere directing modules; Journal of Algebra 161,(1993) 171-185.

[P3] - Peña, J.A.; Algebras with hypercritical Tits form; Topics in Algebra, Banach Center Publications, vol 26 (1990) 353-369.

[P4] - Peña, J.A.; Tame Algebras - Some Fundamental Notions; Sonderforschungbereich Diskrete Strukturen in der Mathematik, Ergãnzungsreihe 343, 95-010. Bielefeld (1995).

[PT] - Peña, J.A.; Tomé, B.; Iterated Tubular Algebras; Journal of Pure and Applied Algebra 64 (1990) North Holand, 303-314.

[R1] - Ringel, C.M.; Tame Algebras-on Algorithms for Solving Vector Space Problems II; Springer Lecture Notes in Mathematics 831 (1980) 137-287.

[R2] - Ringel, C.M.; Tame Algebras and Integral Quadratic Forms; Springer Lecture Notes in Mathematics 1099.

[R3] - Ringel, C.M.; The regular components of the Auslander-Reiten quiver of a tilted algebras; Chin. Ann. of Math. 9B (1) (1988) 1-18.

[Ro] - Roiter, A.V.; Representations of posets and tame matrix problems; London Math. Soc. L.N.M. 116 (1986) 91-107.

[S] - Skowroński, A.; Tame quasitilted algebras; preprint (1996)

[S2] - Skowroński, A.; Simply connected algebras of polynomial growth; preprint

[S3] - Skowroński, A.; Simply connected algebras and Hochschild Cohomologies; preprint

[T] - Tomé, B.; One point extensions of algebras with complete preprojective components having non negative Tits forms; Comm. in Algebra 22(5) (1994) 1531-1549.

[U] - Unger, L.; Preinjective components of trees; Springer Lecture Notes in Mathematics 1177 (1984) 328-339.

Instituto de Matemática e Estatística

Universidade de São Paulo

e-mail: agchalom@ime.usp.br, merklen@ime.usp.br

TRABALHOS DO DEPARTAMENTO DE MATEMÁTICA

TÍTULOS PUBLICADOS

98-01 ASSEM, I. and COELHO, F.U. On postprojective partitions for torsion pairs induced by tilting modules. 16p.

98-02 HÜGEL, L.A. and COELHO, F.U. On the Auslander-Reiten-quiver of a $\mathcal{P}_1$-hereditary artin algebra. 27p.

98-03 BENAVIDES, R. and COSTA, R. Some remarks on genetic algebras. 11p.

98-04 COSTA, R., IKEMOTO, L.S. and SUAZO, A. On the multiplication algebra of a Bernstein algebra, 11p.

98-05 GONÇALVES, D. L. Fixed point free homotopies and Wecken homotopies. 4p.

98-06 POLCINO MILIES, C. and SEHGAL, S. K. Central Units of Integral Group Rings. 9p.

98-07 BOVDI, V. and DOKUCHAEV. M., Group algebras whose involuntary units commute. 15p.

98-08 FALBEL, E. and GORODSKI, C. Some Remarks on the Spectrum of Sub-Riemannian Symmetric Spaces. 16p.

98-09 FUTORNY, V.M., GRISHKOV. A.N. and MELVILLE, D.J. Imaginary Verma Modules for Quantum Affine Lie Algebras. 24p.

98-10 BARBANTI, L. Simply regulated functions and semivariation in uniformly convex spaces. 5p.

98-11 BARBANTI, L. Exponential Solution for Infinite Dimensional Volterra-Stieltjes Linear Integral Equation of Type (K). 10p.

98-12 BARBANTI, L. Linear Stieltjes Equation with Generalized Riemann Integral and Existence of Regulated Solutions. 10p.

98-13 GIULIANI, M.L.M. and POLCINO MILIES, C. Linear Moufang Loops. 15p.

98-14 COSTA, E. A. A note on a theorem of Lawson and Simons on compact submanifolds of spheres. 8p.

98-15 GIANNONI, F., MASIELLO, A. and PICCIONE, P. Convexity and the Finiteness of the Number of Geodesics. Applications to the Gravitational Lensing Effect. 26p.

98-16 PICCIONE, P. A Variational Characterization of Geodesics in Static Lorentzian Manifolds. Existence of Geodesics in Manifolds with Convex Boundary. 39p.

98-17 POLCINO MILIES, C. The Torsion Product Property in Alternative Algebras II. 7p.

98-18 BASSO, I., COSTA, R. Invariance of p-Subspaces in Algebras Satisfying the Identity $x^3=\lambda(x)x^2$. 12p.

98-19 GORODSKI, C. Delaunay-type surfaces in the 2×2 real unimodular group. 12p.

98-20 JURIAANS, S.O. and PERESI, L.A. Polynomial Identities of RA2 Loop Algebras. 12p.

98-21 GREEN, E.L. and MARCOS, E.N. Self-Dual Hopf Algebras. 10p.

98-22 COSTA, R. and PICANÇO, J. Invariance of Dimension of p-Subspaces in Bernstein Algebras. 18p.

98-23 COELHO, F.U. and LIU, S.X. Generalized path algebras. 14p.

98-24 CATALAN, A. and COSTA, R. E-ideals in train algebras. 13p.

98-25 COELHO, F.U. and LANZILOTTA, M.A. Algebras with small homological dimensions.15p.

98-26 COSTA, R. and LELIS, M.L. Recurrent linear forms in Bernstein algebras. 29p.

98-27 GIANNONI, F., PERLICK, V., PICCIONE, P. and VERDERESI, J.A. Time minimizing trajectories in lorentzian geometry. The general-relativistic brachistochrone problem. 28p.

98-28 BARDZELL, M.J., LOCATELI, A.C. and MARCOS, E.N. On the hochschild cohomology of truncated cycle algebras. 24p.

98-29 ALMEIDA, D. M. Sub-Riemannian Symmetric Spaces in Dimension 4. 16p.

98-30 COELHO, F. U., MARTINS, M. I. R., DE LA PEÑA, J. A. Projective dimensions for one point extension algebras. 7p.

98-31 BOVDI, V. On a filtered multiplicative basis of group algebras. 10p.

98-32 BOVDI, V., ROSA, A. L. On the order of the unitary subgroup of modular group algebra. 8p.

98-33 JURIAANS, O. S. and POLCINO MILIES, C. Units of integral group rings of Frobenius groups. 9p.

99-01 FERNANDES, J.D., GROISMAN, J. and MELO, S.T. Harnack inequality for a class of degenerate elliptc operators. 19p.

99-02 GIULIANI, O. F. and PERESI, A.L., Minimal identities of algebras of rank 3. 9p.

99-03 FARKAS, D. R., GEISS, C., GREEN, E.L., MARCOS, E.N. Diagonalizable Derivations of Finite-Dimensional Algebras I. 25p.

99-04 FARKAS, D. R., GEISS, E.L., MARCOS, E.N. Diagonalizable Derivations of Finite-Dimensional Algebras II. 13p.

99-05 LOBÃO, T. P. and MILIES, C. P. The normalizer property for integral group rings of Frobenius groups. 7p.

99-06 PICCIONE, P. and TAUSK, D.V. A note on the Morse index theorem for geodesics between submanifolds in semi-Riemannian geometry. 15p.

99-07 DOKUCHAEV, M., EXEL, R. and PICCIONE, P. Partial representations and partial group algebras. 32p.

99-08 MERCURI, F., PICCIONE, P. and TAUSK, D.V. Stability of the focal and geometric index in semi-Riemannian geometry via the Maslov index. 72.

99-09 BARBANTI, L. Periodic solution for Volterra-Stiltjes integral linear equations of type (K). 9p.

99-10 GALINDO, P., LOURENÇO, M.L. and MORAES, L.A. Compact and weakly compact homomorphisms on Fréchet algebras of holomorphic functions. 10p.

99-11 MARCOS, E.N, MERKLEN, H.A. and PLATZECK, M.I. The Grothendiek group of the category of modules of finite projective dimension over certain weakly triangular algebras. 18p.

99-12 CHALOM, G. Vectorspace Categories Immersed in Directed Components. 32p.

99-13 COELHO, F.U. Directing components for quasitilted algebras. 5p.

99-14 GOODAIRE, E.G. and POLCINO MILIES, C. Alternative Loop Rings with Solvable Unit Loops. 13p.

99-15 GOODAIRE, E.G. and POLCINO MILIES, C. A Normal Complement for an Ra Loop in its Integral Loop Ring. 9p.

99-16 LOURENÇO, M. L. and MORAES, L.A. A class of polynomials. 9p.

99-17 GRISHKOV, A. N. The automorphisms group of the multiplicative Cartan decomposition of Lie algebra E_8. 18p.

99-18 GRISHKOV, A. N. Representations of Lie Algebras over rings. 14p.

99-19 FUTORNY, V., KÖNIG, S. and MAZORCHUK, V. $\mathcal{S}$-subcategories in $\mathcal{O}$ 16p.

99-20 BASSO, I., COSTA, R., GUTIÉRREZ, J. C. and GUZZO JR., H. Cubic algebras of exponent 2: basic properties. 14p.

99-21 GORODSKI, C. Constant curvature 2-spheres in CP^2. 5p.

99-22 CARDONA, F.S.P and WONG, P.N.S, On the computation of the relative Nielsen number. 15p

99-23 GARCÍA, D., LOURENÇO, M.L., MAESTRE, M. and MORAES, L.A. de, The spectrum of analytic mappings of bounded type. 19p.

99-24 ARAGONA, J. and JURIAANS, S.O. Some structural properties of the topological ring of Colombeau's generalized numbers. 35p.

99-25 GIULIANI, M.L.M. and POLCINO MILIES, C. The smallest simple Moufang loop. 27p.

99-26 ASPERTI, A. C. and COSTA, E. A. Vanishing of homology groups, Ricci estimate for submanifolds and applications. 21p.

99-27 COELHO, F.U., MARTINS, M.I.R. and DE LA PEÑA, J.A. Quasitilted Extensions of Algebras I. 11p.

99-28 COELHO, F.U., MARTINS, M.I.R. and DE LA PENA, J.A. Quasitilted Extensions of Algebras II. 17p.

99-29 FARKAS, D. R., GEISS, C. and MARCOS, E.N. Smooth Automorphism Group Schemes. 23p.

99-30 BOVDI, A. A. and POLCINO MILIES, C. Units in Group Rings of Torsion Groups. 15p.

99-31 BARDZELL. M.J. and MARCOS, E.N. $H^1(A)$ and presentations of finite dimensional algebras. 8p.

99-32 GRISHKOV, A. N. and SHESTAKOV, I. P. Speciality of Lie-Jordan algrbras. 17p.

99-33 ANGELERI-HÜGEL, L. and COELHO, F. U. Infinitely generated tilting modules of finite projective dimension. 14p.

99-34 AQUINO, R. M. and MARCOS, E. N. Koszul Tilted Algebras. 19p.

99-35 CHALOM, G. and MERKLEN, H. Representation Type of One Point Extensions of Tilted Euclidean Algebras. 28p.

Nota: Os títulos publicados nos Relatórios Técnicos dos anos de 1980 a 1997 estão à disposição no Departamento de Matemática do IME-USP.
Cidade Universitária "Armando de Salles Oliveira"
Rua do Matão, 1010 - Cidade Universitária
Caixa Postal 66281 - CEP 05315-970