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by

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Local Influence in Elliptical Linear Regression Models

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Summary

Influence diagnostic methods are extended in this paper to elliptical linear models. These include several symmetric multivariate distributions such as normal, Student-t, Cauchy, logistic among others. For a particular perturbation scheme and for the likelihood displacement the diagnostics agree with those developed for the normal linear regression model (Cook, 1986) when the coefficients and the scale parameter are treated separately. This result shows the invariance of the diagnostics with respect to the induced model in the elliptical linear family. However, if the coefficients and the scale parameter are treated jointly we have a different diagnostic for each induced model, which makes this approach helpful for selecting the less sensitive model in the elliptical linear family. An example on salinity of water is given for illustration.

key words: Diagnostic; Influence; Likelihood Displacement; Multivariate Symmetric Distributions.

1 Introduction

Diagnostic techniques for normal linear regression models has been extensively studied in the statistical literature. See, for example, Belsley, Kuh and Welsch (1980), Cook and Weisberg (1982) and Chatterjee and Hadi (1988). Several of the diagnostic techniques evaluate the effect of deleting observations on parameter estimates. An alternative approach that assess the influence of small (local) perturbations from the assumed model on key results is considered in Cook (1986). Additional results on local influence and applications can be found in Beckman, Nachtshein and Cook (1987), Lawrance (1988), Thomas and Cook (1990), Tsai and Wu (1992), Paula (1993) and Kim (1995).

The method of local influence was proposed by Cook (1986,1987) as a general tool for assessing the effect of local departures from model assumptions. In this paper, the local influence approach is applied in elliptical linear regression models, that is, when the error vector has an elliptical distribution. The perturbation scheme considered here is the one in which the scale parameter is modified to allow convenient perturbation in the model.

In Section 2, along with the notation, the elliptical linear models are defined. The local influence method is reviewed in Section 3. Section 4 deals with the derivation of the diagnostic procedures for the elliptical linear models. Finally, an illustrative example is given is the last section.

2 Elliptical linear models

The class of elliptical distributions has been of considerable interest in the recent statistical literature. See, for example, Fang, Kotz and Ng (1990), Fang and Zhang (1990) and Fang and Anderson (1990). A $n \times 1$ random vector Y has an elliptical distribution with location vector μ and scale positive definite matrix Λ, if its density takes the form

$$f_Y(y) = |\Lambda|^{-1/2} g[(y-\mu)^t \Lambda^{-1}(y-\mu)], \tag{1}$$

$y \in \mathcal{R}^n$, where the function $g : \mathcal{R} \to [0,\infty)$ is such that $\int_0^\infty u^{n-1} g(u^2)du < \infty$. The function g is typically known as the density generator. For a vector Y distributed according to the density (1) above, we consider the notation $Y \sim El_n(\mu, \Lambda, g)$, or, simply, $Y \sim El_n(\mu, \Lambda)$. In the case where $\mu = 0$ and $\Lambda = I$, the spherical family of densities obtains. This class of symmetric distributions includes the normal, Student-t, contaminated normal and logistic (both, univariate and multivariate), among others, as considered, for example, in Fang et al. (1990). Table 1 below, taken from Fang et al. (1990), reports examples of distributions in the elliptical family. The notation c_1, c_2, c_3 and c_4 is used to denote normalizing constants.

Table 1. Multivariate elliptical distributions.

Distribution	Notation	Generating function
Normal	$N_n(\mu, \Lambda)$	$g(u) = c_1 e^{-u/2}, \quad u \geq 0$
Student-*t*	$t_n(\mu, \Lambda, \nu)$	$g(u) = c_2(1 + u/\nu)^{-(\nu+n)/2}$
Contaminated Normal	$CN_n(\mu, \Lambda, \delta, \tau)$	$g(u) = c_1\{(1-\delta)e^{-u/2} + \delta\tau^{-n/2}e^{-u/(2\tau)}\}$
Cauchy	$C_n(\mu, \Lambda)$	$g(u) = c_3(1 + u)^{-(n+1)/2}$
Logistic	$L_n(\mu, \Lambda)$	$g(u) = c_4 e^{-u}/(1 + e^{-u})^2$

Consider now the linear regression model

$$Y = X\beta + \epsilon, \tag{2}$$

where Y is a $n \times 1$ vector of responses, X is a known $n \times p$ matrix of rank p, β is a p-dimensional vector of parameters and ϵ is a p-dimensional error vector with distribution $El_n(0, \phi I)$, where ϕ is the scale parameter. Thus, it follows that $Y \sim El_n(X\beta, \phi I)$. This is typically called the elliptical linear regression model. If g is a continuous and decreasing function then the maximum likelihood estimators of β and ϕ are given by (see Fang and Anderson, 1990)

$$\hat{\beta} = (X^t X)^{-1} X^t Y \quad \text{and} \quad \hat{\phi} = Q(\hat{\beta})/u_g, \tag{3}$$

where $Q(\beta) = (Y - X\beta)^t(Y - X\beta)$ and u_g maximizes the function

$$h(u) = u^{n/2} g(u), \quad u \geq 0. \tag{4}$$

Typically, if g in (4) is continuous and decreasing then its maximum u_g exists, is finite and positive. Moreover, if g is continuous and differentiable then u_g is the solution to (Fang and Anderson, 1990)

$$g'(u) + \frac{n}{2u}g(u) = 0,$$

or, equivalently, the solution to the equation

$$\frac{n}{2u} + W_g(u) = 0, \tag{5}$$

where $W_g(u) = d\log g(u)/du = g'(u)/g(u)$. It is easy to see that for the normal and t distributions, $u_g = n$. However, for the contaminated normal and logistic distributions, u_g has to be obtained numerically. In the case of the logistic distribution, for example, equation (5) becomes

$$\frac{n}{2u} = \tanh\left(\frac{u}{2}\right),$$

where $\tanh(\cdot)$ denotes the hyperbolic tangent.

3 Local influence on the likelihood displacement

Let $L(\theta)$ denote the loglikelihood function from the postulated model (here $\theta = (\beta^t, \phi)^t$), and let ω be a $q \times 1$ vector of perturbations restricted to some open subset $\omega \in I\!R^q$. The perturbations are made on the likelihood function, such that it takes the form $L(\theta|\omega)$. Denoting ω_0 the vector of no perturbation, we assume $L(\theta|\omega_0) = L(\theta)$. To assess the influence of the perturbations on the maximum likelihood estimate $\hat{\theta}$, one may consider the likelihood displacement

$$LD(\omega) = 2\{L(\hat{\theta}) - L(\hat{\theta}_\omega)\},$$

where $\hat{\theta}_\omega$ denotes the maximum likelihood estimate under the model $L(\theta|\omega)$.

Small perturbations on the model may be important, especially to assess weather the sample is robust with respect to the induced model. To assess this kind of robustness, Cook (1986) suggests studying the local influence around ω_0. The idea consists in studying the normal curvature of the surface $\alpha(\omega) = (\omega^t, LD(\omega))^t$ and then taking the direction around ω_0 corresponding to the largest normal curvature.

Cook (1986) has showed that the normal curvature at the direction ℓ, takes the form

$$C_\ell(\theta) = 2|\ell^t \Delta^t (\ddot{L})^{-1} \Delta \ell|, \tag{6}$$

where $\| \ell \| = 1$, $-\ddot{L}$ is the observed information matrix for the postulated model ($\omega = \omega_0$) and Δ is the $(p+1) \times q$ matrix with elements

$$\Delta_{ij} = \frac{\partial^2 L(\theta|\omega)}{\partial\theta_i \partial\omega_i},$$

evaluated at $\theta = \hat{\theta}$ and $\omega = \omega_0$, $i = 1, \ldots, p+1$ and $j = 1, \ldots, q$.

Therefore, the maximization of (6) is equivalent to find the largest eigenvalue C_{max} of the matrix $B = \Delta^t(\ddot{L})\Delta$, and the largest direction around ω_0, denoted by ℓ_{max}, is the corresponding eigenvector. If C_{max} is much greater than the remaining eigenvalues of B, the index plot for ℓ_{max} may be helpful in assessing the influence of small perturbations on $LD(\omega)$. Otherwise, it should be more informative to perform the index plot for the eigenvectors corresponding to the largest eigenvalues.

4 Curvature derivation for the elliptical linear models

Firstly, we will consider the unperturbated model. The loglikelihood function under the induced model is given by

$$\begin{aligned} L(\theta) &= \log\{|\phi I_n|^{-1/2} g[(y - X\beta)^t (\phi I_n)^{-1} (y - X\beta)]\} \\ &= -\frac{n}{2}\log\phi + \log g(u), \end{aligned}$$

where $u = Q(\beta)/\phi$ and $\theta = (\beta^t, \phi)^t$. Then, the first derivative of $L(\theta)$ with respect to β and ϕ become, respectively, given by

$$\frac{\partial L(\theta)}{\partial\beta} = -(2/\phi) W_g(u) X^t (y - X\beta) \text{ and} \tag{7}$$

$$\frac{\partial L(\theta)}{\partial \phi} = -\frac{n}{2\phi} - \frac{1}{\phi^2} W_g(u) Q(\beta). \tag{8}$$

From (7) and (8) it follows, after some algebraic manipulation, that

$$\begin{aligned}
\frac{\partial^2 L(\theta)}{\partial\beta\partial\beta^t} &= (2/\phi)[W_g(u)(X^tX) + (2/\phi)W_g'(u)X^t(y - X\beta)(y - X\beta)^tX], \\
\frac{\partial^2 L(\theta)}{\partial\phi\partial\beta^t} &= (2/\phi^3)[W_g(u) + W_g'(u)Q(\beta)](y - X\beta)^tX \quad \text{and} \\
\frac{\partial^2 L(\theta)}{\partial\phi^2} &= \frac{n}{2\phi^2} + \frac{Q(\beta)}{\phi^3}[2W_g(u) + W_g'(u)Q(\beta)/\phi],
\end{aligned}$$

where $W_g'(u) = \frac{d}{du}(W_g(u))$. Evaluating these derivatives at $\theta = \hat{\theta}$, given in (3), and by noting that $(y - X\hat{\beta})^tX = 0$ and $Q(\hat{\beta})/\hat{\phi} = u_g$, we have

$$\ddot{L} = \begin{pmatrix} (2/\hat{\phi})W_g(\hat{u})(X^tX) & 0 \\ 0 & \frac{n}{2\hat{\phi}^2} + \frac{u_g}{\hat{\phi}^2}[2W_g(\hat{u}) + u_gW_g'(\hat{u})] \end{pmatrix},$$

where $\hat{u} = Q(\hat{\beta})/\hat{\phi} = u_g$.

Table 2 exhibits the expressions for $W_g(u)$ and $W_g'(u)$ for the distributions given in Table 1, where $f_i(u) = (1-\delta)e^{-u/2} + \delta\tau^{-(n/2)-i}e^{-u/2\tau}$, $i = 0, 1, 2$.

Table 2. Expressions for $W_g(u)$ and $W_g'(u)$ for some elliptical distributions.

Distribution	$W_g(u)$	$W_g'(u)$
Normal	-1/2	0
Student-t	$-((\nu+n)/2\nu)(1+u/\nu)^{-1}$	$((n+\nu)/2\nu^2)(1+u/\nu)^{-2}$
Cauchy	$-((1+n)/2)(1+u)^{-1}$	$((1+n)/2)(1+u)^{-2}$
Contaminated Normal	$-\frac{1}{2}\frac{f_1(u)}{f_0(u)}$	$\frac{1}{4}\left\{\frac{f_2(u)}{f_0(u)} - \left[\frac{f_1(u)}{f_0(u)}\right]^2\right\}$
Logistic	$-\tanh(u/2)$	$-2/(e^{u/2} + e^{-u/2})^2$

Note that for the normal case ($u_g = n, W_g(u) = -1/2$ and $W_g'(u) = 0$) the matrix $\ddot{L}$ reduces to

$$\ddot{L} = \begin{pmatrix} -(1/\hat{\phi})(X^tX) & 0 \\ 0 & -(n/2\hat{\phi}^2) \end{pmatrix}.$$

Consider now the model (2), with the assumption that $\epsilon \sim El_n(0, \phi D^{-1}(\omega))$, where $D(\omega) = \mathrm{diag}\{\omega_1, \ldots, \omega_n\}$ and $D^{-1}(\omega)$ denotes the inverse of $D(\omega)$. Here $q = n$ and ω_i is the weight corresponding to the ith case, $i = 1, \ldots, n$. When $\omega = \omega_0 = 1$, the perturbed model reduces to the postulated model. Under the perturbed model we will denote $Y \sim El_n(X\beta, \phi D^{-1}(\omega))$. Thus, the loglikelihood function becomes given by

$$L(\theta|\omega) = -\frac{n}{2}\log\phi + \frac{1}{2}\log|D(\omega)| + \log g(u),$$

where $u_\omega = Q_\omega(\beta)/\phi$ and $Q_\omega = (y - X\beta)^t D(\omega)(y - X\beta)$.

Similarly to the unperturbated case one has

$$\begin{aligned}\frac{\partial L(\theta|\omega)}{\partial\beta} &= -(2/\phi)W_g(u_\omega)[X^t D(\omega)y - (X^t D(\omega)X)/\beta] \quad \text{and} \\ \frac{\partial L(\theta|\omega)}{\partial\phi} &= -\frac{n}{2\phi} - \frac{1}{\phi^2}W_g(u_\omega)Q_\omega(\beta).\end{aligned}$$

From these equations it follows that

$$\begin{aligned}\frac{\partial^2 L(\theta|\omega)}{\partial\beta\partial\omega^t} &= -\frac{2}{\phi}\left\{W_g(u_\omega)X^t D(\epsilon) + \frac{1}{\phi}W_g'(u_\omega)X^t D(\omega)\epsilon^t D(\epsilon)\right\} \quad \text{and} \\ \frac{\partial^2 L(\theta|\omega)}{\partial\phi\partial\omega^t} &= -\frac{1}{\phi^2}\left\{W_g(u_\omega)\epsilon^t D(\epsilon) + \frac{1}{\phi}W_g'(u_\omega)Q_\omega(\beta)\epsilon^t D(\epsilon)\right\},\end{aligned}$$

since $\partial(X^t D(\omega)\epsilon)/\partial\omega^t = X^t D(\omega)$ and $\partial u_\omega/\partial\omega^t = (1/\phi)\epsilon^t D(\epsilon)$, where $\epsilon = y - X\beta$. Evaluating the matrix Δ at $(\theta, \omega) = (\hat\theta, \omega_0)$ we find

$$\Delta = \begin{pmatrix} -(2/\hat\phi)W_g(\hat u)X^t D(e) \\ -\frac{1}{\hat\phi^2}[W_g(\hat u) + u_g W_g'(\hat u)]e^t D(e)\end{pmatrix},$$

where $e = y - X\hat\beta$. Note that for the normal linear case the matrix Δ reduces to

$$\Delta = \begin{pmatrix} X^t D(e)/\hat\phi \\ e^t D(e)/2\hat\phi^2 \end{pmatrix},$$

that is in agreement with the expression obtained by Cook (1986). Therefore, one may write

$$B = \Delta^t \ddot{L}^{-1}\Delta = B_1 + B_2,$$

where

$$B_1 = (2/\hat{\phi})W_g(\hat{u})D(e)PD(e)$$

and

$$B_2 = \frac{1}{\hat{\phi}^2}\frac{[W_g(\hat{u}) + u_g W_g'(\hat{u})]^2}{[\frac{n}{2} + u_g\{2W_g(\hat{u}) + u_g W_g'(\hat{u})\}]}D(e)ee^t D(e),$$

with $P = X(X^tX)^{-1}X^t$. Also, for the normal linear case, the matrix B reduces to the one obtained by Cook (1986), equation (31). Then, the normal curvature at the direction ℓ takes the form

$$C_\ell(\theta) = 2|\ell^t[B_1 + B_2]\ell|.$$

In particular, if we are interested in the vector β, the normal curvature at the direction ℓ yields

$$\begin{aligned} C_\ell(\beta) &= 2|\ell^t B_1 \ell| \\ &= 4|W_g(\hat{u})\ell^t D(e)PD(e)\ell|/\hat{\phi} \\ &= \frac{4}{\hat{\phi}}|W_g(\hat{u})||\ell^t D(e)PD(e)\ell|. \end{aligned}$$

Thus, the index plot for ℓ_{max} of the matrix $D(e)PD(e)$ may reveal how to perturb the scale parameter in order to get larger changes in the regression coefficients.

For a particular coefficient, namely β_1, rearranging the columns of $X = (X_1, X_2)$, such that X_1 and X_2 are matrices with dimensions $n \times 1$ and $n \times (p-1)$, respectively, it follows from expression (33) of Cook (1986), that

$$C_\ell(\beta_1) = 4|W_g(\hat{u})||\ell^t D(e)rr^t D(e)\ell| / \parallel r \parallel^2 \hat{\phi},$$

where $r = (I - P_2)X_1$, $P_2 = X_2(X_2^tX_2)^{-1}X_2^t$ and $\parallel a \parallel$ denotes norm of the vector a. Thus, the maximum curvature occurs at the direction

$$\ell_{max} \propto D(e)r.$$

Accordingly, the cases with $|r_i e_i|$ large are locally most influential on the estimate $\hat{\beta}_1$.

Similarly, the normal curvature for the scale parameter ϕ at the direction ℓ becomes given by

$$\begin{aligned} C_\ell(\phi) &= 2|\ell^t B_2 \ell| \\ &= \frac{2}{\hat{\phi}^2}|C_\omega||\ell^t D(e)ee^t D(e)\ell|, \end{aligned}$$

where $C_\omega = [W_g(\hat{u}) + u_g W_g'(\hat{u})]^2/[\frac{n}{2} + u_g\{2W_g(\hat{u}) + u_g W_g'(\hat{u})\}]$. Here, one has for the largest curvature

$$\ell_{max} \propto D(e)e,$$

which means that the observations with large values for e_i^2 are most influential on $\hat{\phi}$.

Therefore, at least for the perturbation scheme defined in Section 4 and for the likelihood displacement, we may conclude that the diagnostics for the elliptical linear models are equivalent to those deduced by Cook (1986) for the normal linear model when β and ϕ are treated separately; that is, the index plots do not change with the induced model in the elliptical linear family. However, if β and ϕ are treated jointly, the ℓ_{max} vector may change from one model to other, which suggests a helpful way of revealing those observations that are most locally influential under each model.

5 Water salinity

In order to illustrate the methodology described in this paper we will consider the data set reported by Rupper and Carrol (1980) on the salinity of water during the spring in Pamlico Sound, North Carolina. The response Y is bi-weekly salinity, and the explanatory variables are salinity lagged two weeks x_1, a dummy variable x_2 for the time period and river discharge x_3. The value of x_{1i} may differ from y_{i-1}, once the data are not a contiguous sequence. This data set has been analyzed, for instance, by Atkinson (1985), Carrol and Ruppert (1985) and Davinson and Tsai (1992). Atkinson assumes a normal distribution for the response while Davinson and Tsai consider a Student-t distribution with 3 degrees of freedom to allow the possibility of the data having tails longer than normal. In both cases it is assumed the linear model

$$Y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \epsilon_i,$$

where ϵ_i follows either a normal or a Student-t distribution with 3 degrees of freedom.

Figure 1. Index plot of $|\ell_{max}|$ for $\hat{\beta}$ ((a)) and for $\hat{\phi}$ ((b)).

Atkinson (1985) and Davinson and Tsai (1992) use deleting diagnostic methods to assess the individual influence of the observations on $\hat{\beta} = (\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3)^t$ and $\hat{\phi}$. Atkinson, assuming errors normally distributed, found cases 16 and 5 as the most influential. Case 5 was shown to be influential after a correction was made for case 16. In Davinson and Tsai's analysis, where a Student-t distribution with 3 degrees of freedom is used, cases 16, 5 and 3 appear as the most influential ones. Figures 1a and 1b present the index plot of $|\ell_{max}|$ for $\hat{\beta}$ and $\hat{\phi}$ separately. We can notice from Figure 1a an outstanding local influence of case 16, while from Figure 1b it follows that cases 9, 15, 16 and 17 present the highest local influences. On the other hand, when we use the global loglikelihood $L(\theta)$ in the Cook's approach instead of the profiles $L(\beta \mid \phi)$ or $L(\phi \mid \beta)$, the local influence of the observations on $\hat{\theta}$ is not invariant in the elliptical linear family anymore. Figure 2 illustrates this behaviour. Moreover, this figure presents case 16 as the most locally influential for the normal, Cauchy and Student-t models. However for the logistic model, cases 9, 15 and 17 also appear with a high local influence. Therefore, we may conclude from this example that the index plot of $|\ell_{max}|$ for $\hat{\theta}$ may be helpful in the sense of selecting the less sensitive model with respect to local perturbations in the elliptical linear family, especially when one has interest in both $\hat{\beta}$ and $\hat{\phi}$.

Figure 2. Index plot of $|\ell_{max}|$ for $\hat{\theta}$ in normal ((a)), Cauchy ((b)), Student-t with 3 d.f. ((c)) and logistic ((d)) elliptical linear models.

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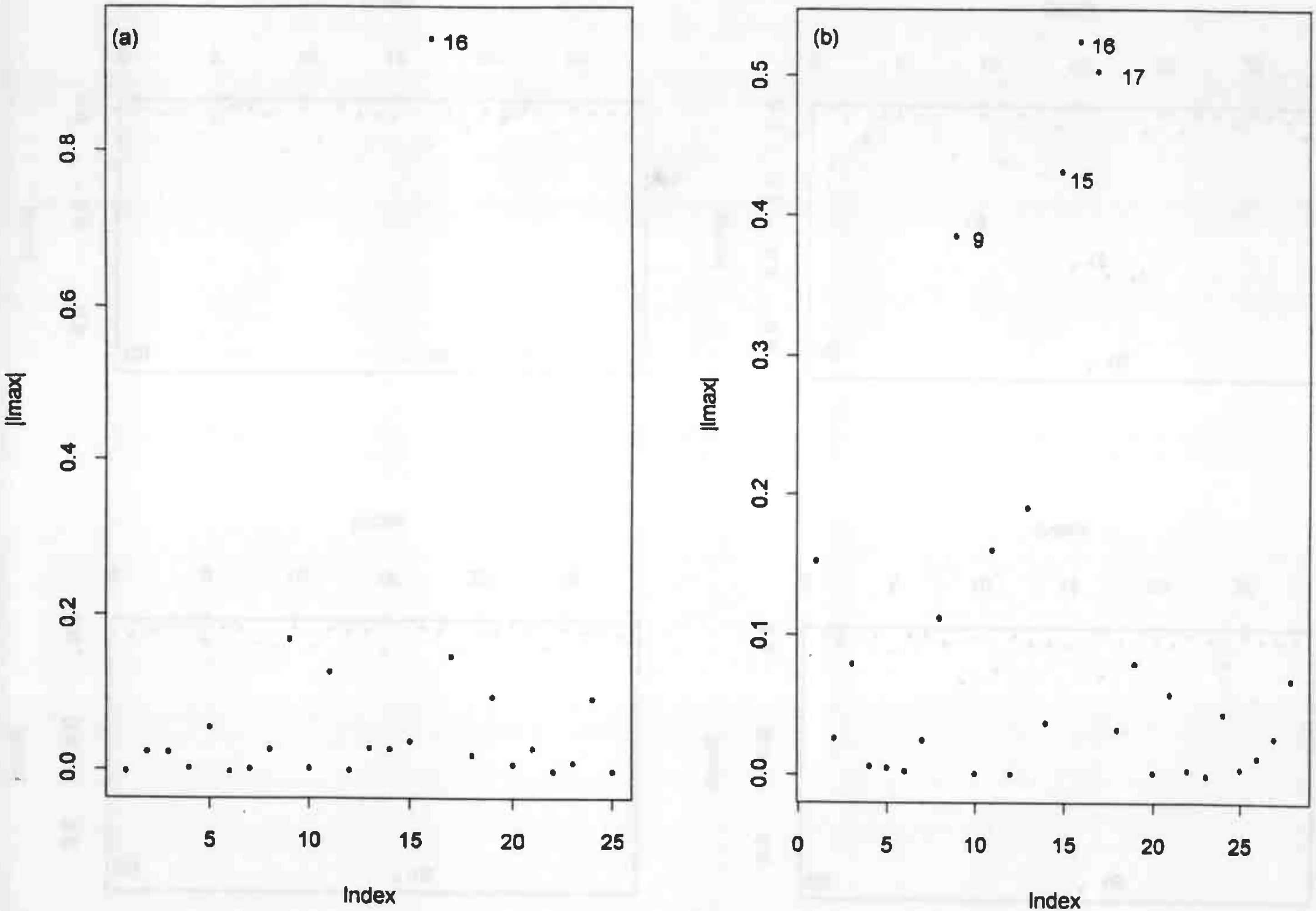

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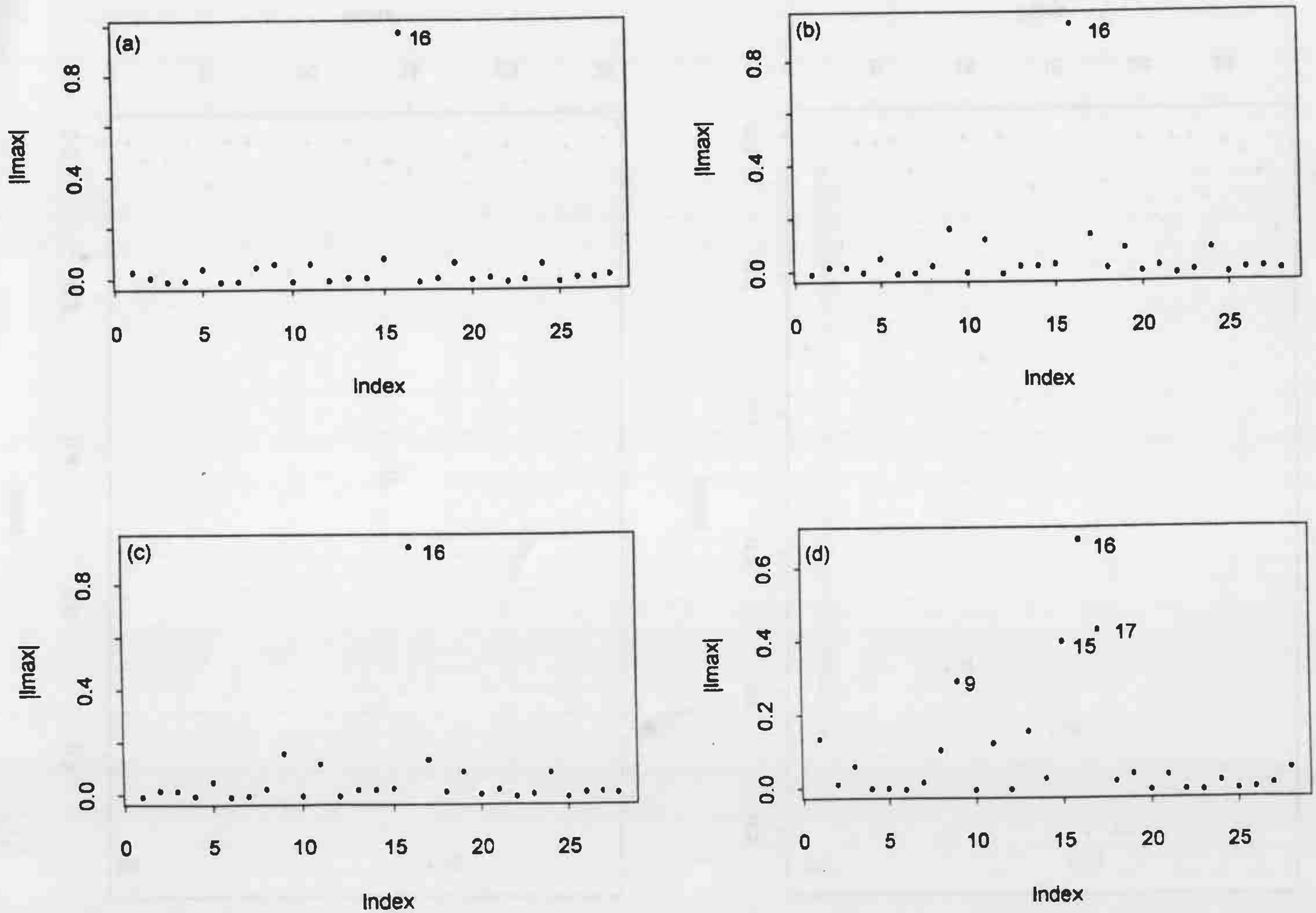
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