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Effective field theory of quantum gravity coupled to scalar electrodynamics

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Abstract

In this work, we use the framework of effective field theory to couple Einstein's gravity to scalar electrodynamics and determine the renormalization of the model through the study of physical processes below Planck scale, a realm where quantum mechanics and general relativity are perfectly compatible. We consider the effective field theory up to dimension six operators, corresponding to processes involving one-graviton exchange. Studying the renormalization group functions, we see that the beta function of the electric charge is positive and possesses no contribution coming from gravitational interaction. Our result indicates that gravitational corrections do not alter the running behavior of the gauge coupling constants, even if massive particles are present.

Keywords: quantum gravity, quantum electrodynamics, renormalization in field theory

1. Introduction

Quantum gravity based in Einstein's theory of gravitation has been the subject of several papers over the last 50 years, including the seminal papers by Feynman [1] and DeWitt [2]. The quantum aspects of gravity coupled to scalar electrodynamics have also been discussed, for example, in [3, 4], where the effective action was computed (for a more comprehensive discussion, see [5]).

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The theory is notoriously nonrenormalizable [6–8], since it requires a set of infinite parameters to absorb all the divergences coming from the loop diagrams. The potential harm of a nonrenormalizability, however, can be overcome in the effective field theory (EFT) framework, where there is an unambiguous way to define a well-behaved and reliable quantum theory of general relativity, if only we agree to restrict ourselves to low energies compared to the Planck scale [9, 10]. In the core of the EFT argument is the fact that if we respect all the symmetry of the problem to write down in the Lagrangian all the terms that contribute to a process of energy scale E, all new terms eventually required by the renormalization procedure can be neglected because they must contribute only to higher energy processes. This EFT's methods have been used to compute several gravitational corrections (see [11] and references therein).

Motivated by this modern view of quantum gravity, Robinson and Wilczek [12] considered a non-Abelian gauge field coupled to gravity and found the gravity contributes with a negative term to the beta function of the gauge coupling, meaning that quantum gravity could make gauge theories asymptotically free. The origin of this effect would be the arising of quadratic UV divergences, associated with the one-graviton exchange graph, that could be absorbed in a gauge coupling constant redefinition. This remarkable conclusion motivated a lot of research on the subject. A few months after Robinson-Wilczek's paper, their conclusion was questioned by Pietrykowski, who repeated their calculation for an abelian field and reproduced their result for a particular gauge choice and showed that a different gauge could lead to no gravitational contribution at all [13]. Pietrykowski also suggested at the end of [13] that if dimensional regularization is applied, the quadratic divergence would not be present, and that claim was investigated further by Felipe *et al* in [14], where it is argued that the gravitational correction to the beta function computed in [12] is regularization dependent and therefore ambiguous. The absence of gravitational correction at one-loop order was reinforced by Ebert, Plefka, and Rodigast in a paper where they follow diagrammatic approach using both a cutoff and a dimensional regularization [15]. A detailed study of the use of the Vilkovisky-DeWitt method to this problem was done in [16], where Nielsen shows for the Einstein-Maxwell system that quadratic divergences would break the Ward identities, so the method would guarantee only the gauge invariance of finite and logarithmic divergent parts of the effective action.

The role of the cosmological constant was also investigated, and the result was that it should induce an asymptotic freedom behavior to the electric charge [17–19] and for the $\lambda\phi^4$ model [20]. There have also been some results for other (nongauge) interactions; for example, it was argued that massive particles with Yukawa [21] and ϕ^4 [22] share the same property of asymptotic freedom, an effect that vanishes when the masses are withdrawn.

The controversy over beta function calculations is still unresolved, and some studies have questioned the physical meaning of the definition of running coupling constants [11, 23, 24]. These papers argue that a scattering matrix computation is needed to give a physical definition for the running of the coupling constants. Using S-matrix, it was found in [24] that an attempt to compute the running of the Yukawa coupling can be ambiguous, since it would seem to run in the direction of asymptotic freedom in one process but will increase with energy for another process, and therefore what appears to be asymptotic freedom is not an universal behaviour within the theory. This conclusion is intrinsically related to the construction and meaning of an effective theory: the gravitational corrections will renormalize not the original operator but rather a higher derivative one (because of the dimensional coupling constant), different processes typically involve different combinations of operators, and no universality is to be expected [11]. Through the computation of scattering processes, it was shown that quantum gravitational corrections do not alter the running behavior of the

electric charge in the massless Scalar Quantum Electrodynamics (QED) [25], but the presence of a positive cosmological constant in the massless Einstein-$\lambda\phi^4$ model corroborates earlier proposal [17].

In this work we investigate the renormalization of the EFT of the massive Scalar electrodynamics coupled to Einstein's gravity, showing that gravitational corrections do not alter the running behavior of the electric charge, even in the presence of massive particles. This is done by the computation of scattering amplitudes between charged particles.

2. Massive scalar QED coupled to Einstein's gravity

The model is given by the following action:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left\{\frac{2}{\kappa^2}R - \frac{1}{4}g^{\mu\alpha}g^{\nu\beta}F_{\alpha\beta}F_{\mu\nu} + g^{\mu\nu}(\partial_\mu + \mathrm{i}eA_\mu)\phi_j(\partial_\nu - \mathrm{i}eA_\nu)\phi_j^* \right.$$
$$\left. - m^2(\phi_j^*\phi_j) - \frac{\lambda}{2}(\phi_j^*\phi_j)^2 + \mathcal{L}_{HO} + \mathcal{L}_{GF} + \mathcal{L}_{CT}\right\}, \tag{1}$$

where j assume values a and b according to the flavor of the pion, $\kappa^2 = 32\pi G = 32\pi/M_P^2$, with M_P being the Planck mass and G the Newtonian gravitational constant; e is the electric charge and λ a self-interaction constant. $\mathcal{L}_{GF}$ is the gauge-fixing plus Faddeev-Popov ghost Lagrangian (for the graviton and the photon), and $\mathcal{L}_{CT}$ is the Lagrangian of counterterms Finally, $\mathcal{L}_{HO}$ is the Lagrangian of higher derivatives terms given by

$$\mathcal{L}_{HO} = \lambda_1\partial^\mu(\phi_a\phi_a^*)\partial_\mu(\phi_b\phi_b^*) + \lambda_2(\phi_a\partial^\mu\phi_a^* - \partial^\mu\phi_a\phi_a^*)(\phi_b\partial_\mu\phi_b^* - \partial_\mu\phi_b\phi_b^*)$$
$$+ \lambda_3(\Box\phi_a\phi_a^*\phi_b\phi_b^* + \phi_a\Box\phi_a^*\phi_b\phi_b^* + \phi_a\phi_a^*\Box\phi_b\phi_b^* + \phi_a\phi_a^*\phi_b\Box\phi_b^*) + (\cdots), \tag{2}$$

where $(\cdots)$ stands for omitted higher-order terms, which are not important to our analysis in this paper.

We must keep in mind that for renormalized Lagrangian, we have redefined $\phi_{0j} = Z_\phi^{1/2}\phi_j = \sqrt{1+\delta_\phi}\,\phi_j$ and $A_{0\mu} = Z_A^{1/2}A_\mu = \sqrt{1+\delta_A}A_\mu$. The relation between bare and renormalized coupling constants is given by

$$e_0 = \mu^\epsilon\frac{Z_e}{Z_\phi Z_A^{1/2}}e = \frac{(e+\delta_e)}{Z_\phi Z_A^{1/2}}\mu^\epsilon,$$
$$\lambda_0 = \mu^{2\epsilon}\frac{Z_\lambda}{Z_\phi^2}\lambda = \frac{(\lambda+\delta_\lambda)}{Z_\phi^2}\mu^{2\epsilon}, \tag{3}$$

where μ is a mass scale introduced by the dimensional regularization with $D = 4 - 2\epsilon$.

Let us consider small fluctuations around the flat metric, i.e.,

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}, \tag{4}$$

$$g^{\mu\nu} = \eta^{\mu\nu} - \kappa h^{\mu\nu} + \kappa^2\, h^{\mu\alpha}h_\alpha^\nu + \mathcal{O}(\kappa^3), \tag{5}$$

$$\sqrt{-g} = 1 + \frac{1}{2}\kappa h - \frac{1}{4}\kappa^2 h_{\alpha\beta}P^{\alpha\beta\mu\nu}h_{\mu\nu} + \mathcal{O}(\kappa^3), \tag{6}$$

where $\eta_{\mu\nu} = (+,-,-,-)$, $P^{\alpha\beta\mu\nu} = \frac{1}{2}(\eta^{\alpha\mu}\eta^{\beta\nu} + \eta^{\alpha\nu}\eta^{\beta\mu} - \eta^{\alpha\beta}\eta^{\mu\nu})$ and $h = \eta^{\mu\nu}h_{\mu\nu}$. For more details, see for instance [26].

Through the harmonic gauge-fixing function, $G_\mu = \partial^\nu h_{\mu\nu} - \frac{1}{2}\partial_\mu h$, the graviton propagator can be cast as

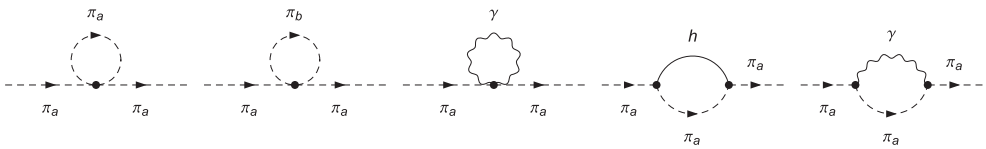

Figure 1. Feynman diagrams for the pions self-energy. Dashed, wavy, and continuous lines represent the scalar, photon, and graviton propagators, respectively.

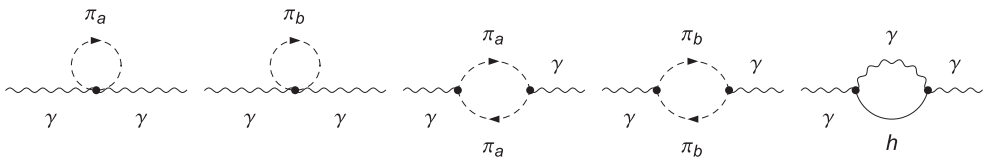

Figure 2. Feynman diagrams for the photon self-energy. Dashed, wavy, and continuous lines represent the scalar, photon, and graviton propagators, respectively.

$$\langle T\ h^{\alpha\beta}(p)h^{\mu\nu}(-p)\rangle = D^{\alpha\beta\mu\nu}(p) = \frac{\mathrm{i}}{p^2}\left[P^{\alpha\beta\mu\nu} + (\xi_h - 1)\frac{Q^{\alpha\beta\mu\nu}(p)}{p^2}\right], \tag{7}$$

where $Q^{\alpha\beta\mu\nu}(p) = (\eta^{\alpha\mu}p^\beta p^\nu + \eta^{\alpha\nu}p^\beta p^\mu + \eta^{\beta\mu}p^\alpha p^\nu + \eta^{\beta\nu}p^\alpha p^\mu)$.

The propagators for the other fields are also obtained by the usual Faddeev-Popov method, resulting in

$$\langle T\ A^\mu(p)A^\nu(-p)\rangle = \Delta^{\mu\nu}(p) = -\frac{i}{p^2}\left[\eta^{\mu\nu} + (1 - \xi_\gamma)\frac{p^\mu p^\nu}{p^2}\right], \tag{8}$$

$$\langle T\ \phi_i^*(p)\phi_j(-p)\rangle = \Delta_{ij}(p) = \frac{i}{p^2 - m^2}\delta_{ij}. \tag{9}$$

The ghost propagators are not useful in the order we are working.The above propagators were written for generic gauge parameters: ξ_h and ξ_γ. Since our interest is in the study of gauge-independent quantities, from now on, we will restrict to the Feynman gauges, $\xi_h = \xi_\gamma = 1$, which simplifies the very long calculations involved. Reproduction of the same results by calculating with generic parameters ξ_h and ξ_γ would be desirable, but we will leave it for future investigations.

3. Scattering amplitudes and the running of the coupling constants

The self-energy process of the scalar particle, figure 1, is

$$\tau_2 = (1 + \delta_\phi)(p^2 - m^2 - \delta_{m^2}) - \Sigma_1(p, m, \lambda, \kappa, e), \tag{10}$$

where $\Sigma_1(p, m, \lambda, \kappa, e)$ is the one-loop correction given by

$$\Sigma_1 = -(p^2 - m^2)\left(\frac{e^2}{8\pi^2\epsilon} - \frac{\kappa^2 m^2}{16\pi^2\epsilon}\right) + \frac{3(\lambda - e^2)m^2}{16\pi^2\epsilon}. \tag{11}$$

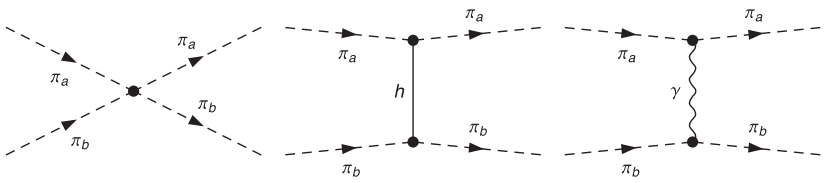

Figure 3. Feynman diagrams for the pion scattering amplitude ($\pi_a^+ + \pi_b^+ \to \pi_a^+ + \pi_b^+$) at tree level.

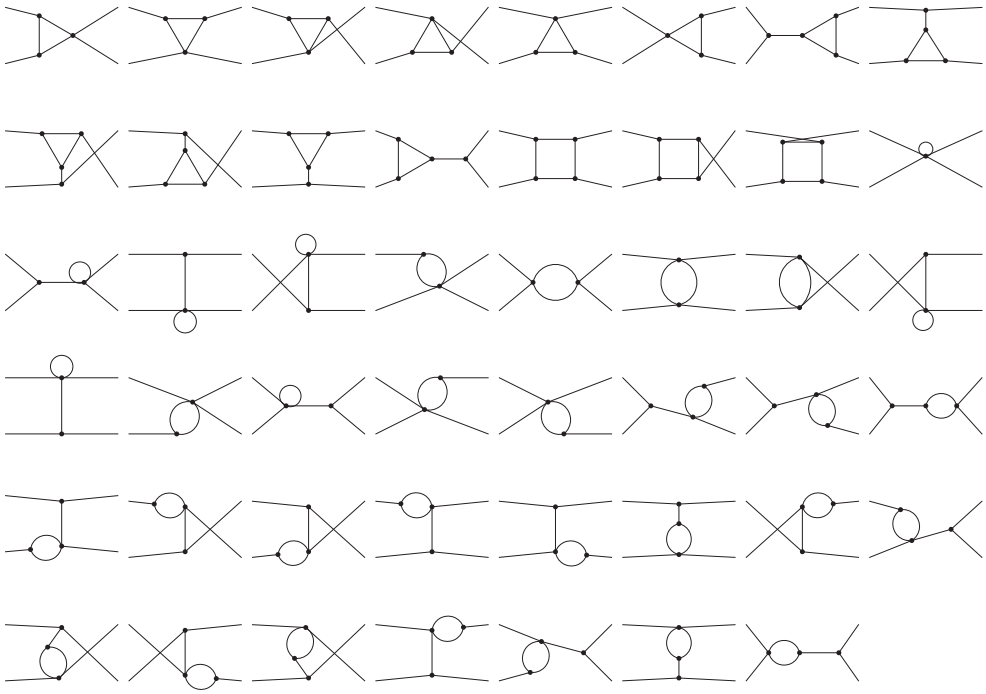

Figure 4. Topologies to the ($\pi_a^+ + \pi_b^+ \to \pi_a^+ + \pi_b^+$) scattering amplitude at one-loop order. After considering the vertices, we have more than 80 Feynman diagrams to this reaction up to $\mathcal{O}(\kappa^2)$.

Using the minimal subtraction (MS) renormalization scheme [27, 28], the counterterms for mass (δ_{m^2}) and wave function (δ_ϕ) are given by

$$\delta_{m^2} = \frac{3(e^2 - \lambda)m^2}{16\pi^2\epsilon}, \tag{12}$$

$$\delta_\phi = \frac{e^2}{8\pi^2\epsilon} - \frac{m^2\kappa^2}{16\pi^2\epsilon}. \tag{13}$$

The one-loop correction to the polarization tensor of the photon field, figure 2, is

$$\Pi^{\mu\nu} = -\frac{e^2}{24\pi^2\epsilon}(\eta^{\mu\nu}p^2 - p^\mu p^\nu) + H.O., \tag{14}$$

where $H.O. = -\frac{e^2\kappa^2}{96\pi^2\epsilon}(\eta^{\mu\nu}p^2 - p^\mu p^\nu)p^2$ (see for instance [15]) is a high-order gravitational correction that only contributes to the renormalization of a high-order operator. From the above expression, we find the photon wave-function counterterm as

$$\delta_A = -\frac{e^2}{24\pi^2\epsilon}. \tag{15}$$

The scattering amplitude $\pi_a^+ + \pi_b^+ \to \pi_a^+ + \pi_b^+$ (figures 3 and 4), is given by

$$\begin{aligned}\mathcal{M} &= \mathcal{M}_{tree} + \mathcal{M}_{CT} + \mathcal{M}_{1\,l}\\ &= -\lambda + \frac{e^2(S-U)}{T} + \frac{\kappa^2}{4}\frac{SU}{T} + \frac{m^2\kappa^2}{2} - \frac{m^4\kappa^2}{2\,T} + \mathcal{M}_{CT} + \mathcal{M}_{1\,l},\end{aligned} \tag{16}$$

For convenience, we have set $\lambda_1 = \lambda_2 = \lambda_3 = 0$ at tree level. In the above expression, U, T and S are the Mandelstam variables, $\mathcal{M}_{CT}$ is the expression for the counterterms

$$\begin{aligned}\mathcal{M}_{CT} = &-\delta_\lambda + \frac{2e\delta_e(S-U)}{T} + \frac{\delta_{\kappa^2}}{4}\frac{SU}{T} + \frac{\delta_{m^2}\kappa^2}{2} + \frac{m^2}{2}\delta_{\kappa^2}\\ &-\frac{m^2\delta_{m^2}\kappa^2}{T} - \frac{m^4\delta_{\kappa^2}}{2T} + \delta_{\lambda_1}T + \delta_{\lambda_2}(S-U) - 4m^2\delta_{\lambda_3},\end{aligned} \tag{17}$$

and $\mathcal{M}_{1l}$ the one-loop correction. To compute $\mathcal{M}_{1l}$ up to order of κ^2 (one-graviton exchange), we used a set of Mathematica© packages [29–31] and found

$$\begin{aligned}\mathcal{M}_{1l} = &\frac{3\lambda^2}{8\pi^2\epsilon} + \frac{3e^4}{8\pi^2\epsilon} - \frac{e^4S}{4\pi^2T\epsilon} + \frac{e^4U}{4\pi^2T\epsilon} - \frac{11e^2S\kappa^2}{384\pi^2\epsilon} + \frac{e^2\,m^4\kappa^2}{48\pi^2\,T\epsilon} + \frac{7e^2m^2S\kappa^2}{48\pi^2T\epsilon}\\ &+ \frac{5e^2S^2\kappa^2}{384\pi^2T\epsilon} + \frac{11e^2T\kappa^2}{192\pi^2\epsilon} + \frac{11e^2U\kappa^2}{384\pi^2\epsilon} + \frac{5e^2m^2U\kappa^2}{24\pi^2T\epsilon} - \frac{11e^2SU\kappa^2}{96\pi^2T\epsilon} - \frac{25e^2U^2\kappa^2}{384\pi^2T\epsilon}\\ &- \frac{e^2\lambda}{8\pi^2\epsilon} - \frac{m^2\kappa^2\lambda}{8\pi^2\epsilon} - \frac{S\kappa^2\lambda}{32\pi^2\epsilon} - \frac{3\,m^4\kappa^2\lambda}{16\pi^2\,T\epsilon} - \frac{3\,T\kappa^2\lambda}{64\pi^2\epsilon} - \frac{U\kappa^2\lambda}{32\pi^2\epsilon} + \text{finite terms}\end{aligned} \tag{18}$$

We use also equation (12) to write the amplitude (16) as

$$\begin{aligned}\mathcal{M} = &-\lambda + \frac{e^2(S-U)}{T} + \frac{\kappa^2}{4}\frac{SU}{T} + \frac{m^2\kappa^2}{2} - \frac{m^4\kappa^2}{2T} - \delta_\lambda + \frac{2e\delta_e(S-U)}{T}\\ &+ \frac{\delta_{\kappa^2}}{2}\frac{SU}{T} + \frac{m^2}{2}\delta_{\kappa^2} - \frac{m^4\delta_{\kappa^2}}{2T} + \delta_{\lambda_1}T + \delta_{\lambda_2}(S-U) - 4m^2\delta_{\lambda_3}\\ &+ \frac{(3\lambda^2 + 3e^4 - \lambda e^2)}{8\pi^2\epsilon} - \frac{e^4(S-U)}{4\pi^2T\epsilon} - \frac{11e^2\kappa^2(S-U)}{384\pi^2\epsilon} + \frac{(e^2 - 9\lambda)m^4\kappa^2}{48\pi^2T\epsilon}\\ &+ \frac{7e^2\,m^2\kappa^2S}{48\pi^2T\epsilon} + \frac{5e^2\kappa^2(S^2 - 5U^2)}{384\pi^2T\epsilon} + \frac{11e^2\kappa^2\,T}{192\pi^2\epsilon} + \frac{5e^2m^2\kappa^2U}{24\pi^2T\epsilon}\\ &- \frac{11e^2\kappa^2SU}{96\pi^2T\epsilon} - \frac{m^2\kappa^2\lambda}{8\pi^2\epsilon} - \frac{\kappa^2\lambda(S+U+T)}{32\pi^2\epsilon} - \frac{\kappa^2\lambda T}{64\pi^2\epsilon} + \text{finite terms.}\end{aligned} \tag{19}$$

The kinematical identity $S + T + U = 4m^2$ can then be used to eliminate U from $\mathcal{M}$ in order to simplify the analysis. Collecting terms with the same kinematical factor, we have

$$\begin{aligned}\mathcal{M} = &-\lambda + e^2 + \frac{m^2\kappa^2}{2} + 2e\delta_e + \frac{m^2\delta_{\kappa^2}}{2} - \delta_\lambda - 4m^2\delta_{\lambda_2} - 4m^2\delta_{\lambda_3} + \frac{e^4}{8\pi^2\epsilon} \\ &+ \frac{25e^2\, m^2\kappa^2}{48\pi^2\epsilon} - \frac{e^2\lambda}{8\pi^2\epsilon} - \frac{11m^2\kappa^2\lambda}{32\pi^2\epsilon} + \frac{3\lambda^2}{8\pi^2\epsilon} + \left(-\frac{\delta_{\kappa^2}}{4} - \frac{\kappa^2}{4} + \frac{e^2\kappa^2}{16\pi^2\epsilon}\right)\frac{S^2}{T} \\ &+ \left(-4e^2m^2 - 8em^2\delta_e - \frac{m^4\delta_{\kappa^2}}{2} + \frac{2e^4\, m^2}{\pi^2\epsilon} - \frac{m^4\kappa^2}{2} - \frac{3e^2m^4\kappa^2}{8\pi^2\epsilon}\right)\frac{1}{T} \\ &+ \left(-\frac{\delta_{\kappa^2}}{4} + 2\delta_{\lambda_2} - \frac{\kappa^2}{4} - \frac{7e^2\kappa^2}{96\pi^2\epsilon}\right)S + \left(\delta_{\lambda_1} + \delta_{\lambda_2} - \frac{7e^2\kappa^2}{192\pi^2\epsilon} - \frac{\kappa^2\lambda}{64\pi^2\epsilon}\right)T \\ &+ \left(2e^2 + 4e\delta_e + m^2\delta_{\kappa^2} - \frac{e^4}{2\pi^2\epsilon} + m^2\kappa^2\right)\frac{S}{T} + \text{finite terms}. \end{aligned} \tag{20}$$

Imposing finiteness, we find

$$\delta_e = \frac{e^3}{8\pi^2\epsilon} - \frac{e\kappa^2m^2}{16\pi^2\epsilon}, \tag{21}$$

$$\delta_{\kappa^2} = \frac{\kappa^2e^2}{4\pi^2\epsilon}, \tag{22}$$

$$\delta_{\lambda_1} = \frac{\kappa^2(\lambda - 2e^2)}{64\pi^2\epsilon}, \tag{23}$$

$$\delta_{\lambda_2} = \frac{13\kappa^2e^2}{192\pi^2\epsilon}. \tag{24}$$

Since $e_0 = \mu^\epsilon e\frac{Z_e}{Z_\phi Z_A^{1/2}} = \mu^\epsilon\frac{(e+\delta_e)}{(1+\delta_\phi)(1+\delta_A)^{1/2}}$, and $\delta_\phi = \frac{\delta_e}{e} = \frac{e^2}{8\pi^2\epsilon} - \frac{\kappa^2\, m^2}{16\pi^2\epsilon}$, we have $e_0 = e\mu^\epsilon Z_A^{-1/2}$. Therefore, the beta function of the electric charge is

$$\beta(e) = \mu\frac{\mathrm{d}e}{\mathrm{d}\mu} = \frac{e^3}{48\pi^2}, \tag{25}$$

which is simply the usual beta function in absence of gravity, so we can say that quantum gravitational corrections do not alter the running behavior of the electric charge.

On the other hand, for the renormalization of λ, we find

$$-\delta_\lambda - 4m^2\delta_{\lambda_3} + \frac{(3\lambda^2 + 3e^4 - \lambda e^2)}{8\pi^2\epsilon} + \frac{e^2\kappa^2\, m^2}{4\pi^2\epsilon} - \frac{11\lambda\kappa^2\, m^2}{32\pi^2\epsilon} = 0, \tag{26}$$

and we see that we cannot separate the contributions for δ_λ and δ_{λ_3}.

This arbitrariness in the definition of the coupling constants is due to the mixing of operators, typical in EFTs, as discussed in [24]. The separation of the renormalization of λ and λ_3 would require the study of off-shell processes where these two parameters (or at least one of them) are involved. For the moment, we will not pursue this analysis.

4. Final remarks

In summary, we have shown that massive scalar QED coupled to gravity is renormalizable within the framework of EFT. The appropriate counterterms were set by imposing finiteness of the scattering amplitude at one-loop order.

Our attention was focused on the gravitational correction to the renormalization of the coupling constants, since previous works have indicated that the presence of a dimensionful parameter

(the cosmological constant in [17]) might render a non-vanishing new term. We found that the counterterm for the electric charge does receive a gravitational contribution in the presence of a dimensionful parameter (the mass of the scalars). However, this dependence on κ^2 is exactly canceled when we compute $\beta(e)$, such that the running of the electric charge is not altered by gravitational effects, similarly to what was observed in massless QED coupled to gravity [25].

On the other hand, the counterterms for the scalar coupling constants λ and λ_3 will depend on the gravitational coupling κ^2 in the case considered ($m \neq 0$) and cannot be separated, a manifestation of what is called mixing of operators (typical of EFTs, as discussed in [24]). Their separation would requires the study of off-shell processes involving these coupling constants. For the moment, we will not pursue this analysis.

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