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by

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Asymptotic Efficiency of Null Intercept Measurement Error Regression Models

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KEY WORDS: Maximum likelihood; pretest/posttest data; random effects; asymptotic relative efficiencies; measurement error models.

1 Introduction

Aoki, Bolfarine and Singer (2000) considered null intercept measurement error regression models to reanalyse the data from a pretest/posttest study presented in Singer and Andrade (1997). In that study, designed to compare two types of toothbrushes with respect to the efficacy in removing dental plaque, 26 preschoolers were evaluated with respect to a dental plaque index before (X_{ij}) and after (Y_{ij}) toothbrushing either with a regular or with an experimental (hugger) toothbrush. The basic model is the usual measurement error regression model with null intercepts

$$Y_{ij} = \beta_i x_{ij} + e_{ij}, \tag{1}$$

$$X_{ij} = x_{ij} + u_{ij}, \tag{2}$$

where

$$\begin{pmatrix} e_{ij} \\ u_{ij} \\ x_{ij} \end{pmatrix} \sim N_3 \left[\begin{pmatrix} 0 \\ 0 \\ \mu_x \end{pmatrix}, \begin{pmatrix} \sigma_{ei}^2 & 0 & 0 \\ 0 & \sigma_u^2 & 0 \\ 0 & 0 & \sigma_x^2 \end{pmatrix} \right], \tag{3}$$

are independently distributed $i = 1, 2, j = 1, \ldots, n$. The reason for considering null intercepts is that null pretest dental plaque indices imply null expected posttest values. As the same individuals were evaluated under two different experimental conditions (toothbrushes), an extra term was included in the model to incorporate a possible within subjects correlation structure, leading to

$$x_{ij} = \mu_x + a_j \tag{4}$$

$i = 1, 2, j = 1, \ldots, n$ with the a_j independently distributed as $N(0, \sigma_x^2)$ and independent of the u_{ij}. A more general model where

$$x_{ij} = \mu_x + a_j + \delta_{ij}, \tag{5}$$

with $\delta_{ij} \sim N(0, \sigma_\delta^2)$ independently distributed of a_j and u_{ij} was also considered. Under this formulation, the true value of the pre-toothbrushing dental plaque index varies randomly according to the type of toothbrush as well as with the individual. Under either model, i.e. under (1), (2), (3) and (4) or (5), the likelihood equations are quite involved and iterative procedures are required to obtain the maximum likelihood estimators (MLE) as shown in Aoki et al. (2000). Since in both cases, method of moments estimators (MME) may be easily computed via explicit expressions, they constitute an attractive alternative to the MLE. Given that the main interest in the analysis is to compare the dental plaque index reduction rates for both toothbrushes, namely, β_1 and β_2, we examine the asymptotic relative efficiency (ARE) of the MME of $\Delta = \beta_1 - \beta_2$ with respect to the corresponding MLE in order to evaluate under what conditions, if any, the simpler and explicit alternative may be employed for inferential purposes.

2 Asymptotic Relative Efficiencies

Consider initially, the model defined by (1) and (2) and suppose that

$$\begin{pmatrix} e_{ij} \\ u_{ij} \\ x_{ij} \end{pmatrix} \sim N_3 \left[\begin{pmatrix} 0 \\ 0 \\ \mu_{xi} \end{pmatrix}, \begin{pmatrix} \sigma^2_{ei} & 0 & 0 \\ 0 & \sigma^2_{ui} & 0 \\ 0 & 0 & \sigma^2_{xi} \end{pmatrix} \right], \tag{6}$$

are independently distributed, $i = 1, 2, j = 1, \ldots, n$, i.e., the data obtained under the two experimental conditions are independent and have different variance components, $\sigma^2_{ei}, \sigma^2_{ui}, \sigma^2_{xi}$.

Under such an assumption and considering the reparametrization $\lambda_i = \sigma^2_{ei}/\sigma^2_{ui}$, the MME of the elements of the parameter vector $\theta_i = (\beta_i, \mu_{xi}, \sigma^2_{xi}, \sigma^2_{ui}, \lambda_i)$, i=1,2, are

$$\begin{aligned} \tilde{\beta}_i &= \frac{\overline{Y}_i}{\overline{X}_i}, \ \tilde{\lambda}_i = \frac{\overline{Y}_i}{\overline{X}_i} \left(\frac{\overline{X}_i S_{Y_iY_i} - \overline{Y}_i S_{X_iY_i}}{\overline{Y}_i S_{X_iX_i} - \overline{X}_i S_{X_iY_i}} \right), \ \tilde{\mu}_{xi} = \overline{X}_i \\ \tilde{\sigma}^2_{xi} &= \frac{\overline{X}_i S_{X_iY_i}}{\overline{Y}_i}, \ \tilde{\sigma}^2_{ui} = \frac{\overline{Y}_i S_{X_iX_i} - \overline{X}_i S_{X_iY_i}}{\overline{Y}_i}, \end{aligned} \tag{7}$$

where $\overline{Y}_i$ and $S_{X_iY_i}$ are the sample mean and covariance respectively, with similar definitions for the other sample moments. They are also MLE provided that the variance estimators are nonnegative (Chan and Mak, 1979). If we now shift to the more general models under investigation here, defined by (1), (2), (3) and (4), we still have explicit expressions for the MME, i.e.,

$$\beta^*_i = \frac{2\overline{Y}_i}{\overline{X}_1 + \overline{X}_2}, \ \mu^*_x = \frac{\overline{X}_1 + \overline{X}_2}{2}, \ \sigma^{2*}_x = \frac{S_{X_1Y_2}\left(\overline{X}_1 + \overline{X}_2\right)}{2\overline{Y}_2}, \tag{8}$$

$$\sigma_u^{2*} = S_{X_1X_1} - \tilde{\sigma}_x^2; \quad \text{and} \quad \lambda_i^* = \frac{1}{\tilde{\sigma}_u^2}\left(S_{Y_iY_i} - \frac{4\overline{Y}_i^2\tilde{\sigma}_x^2}{\left(\overline{X}_1 + \overline{X}_2\right)^2}\right)$$

where the sample moments $S_{X_1Y_2}$, $S_{Y_1Y_1}$ $S_{Y_2Y_2}$, $\overline{X}_i$ and $\overline{Y}_i$, $i = 1,2$ are defined as before, but the corresponding MLE must be computed via iterative procedures. Similar problems occur if we replace (4) by (5) or even if we consider the simpler model defined by (1), (2) and (3), i.e., a model where the data obtained under both experimental conditions are independent, but have common variance parameters. On the other hand, the MME of the parameters of such models have explicit analytic expressions. In particular, for the model defined by (1), (2), (3) and (5) these estimators are given by

$$\begin{aligned} \sigma_\delta^{2*} &= \frac{S_{X_1Y_1}\left(\overline{X}_1 + \overline{X}_2\right)}{2\overline{Y}_1} - \frac{S_{X_1Y_2}\left(\overline{X}_1 + \overline{X}_2\right)}{2\overline{Y}_2}, \; \sigma_u^{2*} = S_{X_1X_1} - \left(\tilde{\sigma}_x^2 + \tilde{\sigma}_\delta^2\right), \\ \lambda_1^* &= \frac{1}{\tilde{\sigma}_u^2}\left(S_{Y_1Y_1} - \frac{4\overline{Y}_1^2(\tilde{\sigma}_x^2 + \tilde{\sigma}_\delta^2)}{\left(\overline{X}_1 + \overline{X}_2\right)^2}\right), \; \lambda_2^* = \frac{1}{\tilde{\sigma}_u^2}\left(S_{Y_2Y_2} - \frac{4\overline{Y}_2^2(\tilde{\sigma}_x^2 + \tilde{\sigma}_\delta^2)}{\left(\overline{X}_1 + \overline{X}_2\right)^2}\right). \end{aligned} \tag{9}$$

with β_1^*, β_2^*, μ_x^*, and σ_x^{2*} given as in (8).

Besides the computational advantage, the MME are consistent estimators and as a consequence, one is always tempted to employ them in lieu of the corresponding MLE for inferences about $\Delta = \beta_1 - \beta_2$. The question is to what extent we loose efficiency by adopting such a strategy. Our main objective here is to compare method of moments estimators with their maximum likelihood counterparts via their asymptotic relative efficiencies.

Let us first consider the consistent MME, $\tilde{\Delta} = \tilde{\beta}_1 - \tilde{\beta}_2$ of the parameter Δ. Given that under the models specified by (1), (2), (3) and (4) or by (1), (2), (3) and (5) the vector $(\overline{X}_1, \overline{Y}_1, \overline{X}_2, \overline{Y}_2)^T$ follow normal distributions with covariance matrices given by

$$1/n \begin{bmatrix} \sigma_x^2 + \sigma_u^2 & \beta_1\sigma_x^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \beta_1\sigma_x^2 & \beta_1^2\sigma_x^2 + \lambda_1\sigma_u^2 & \beta_1\sigma_x^2 & \beta_1\beta_2\sigma_x^2 \\ \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2 + \sigma_u^2 & \beta_2\sigma_x^2 \\ \beta_2\sigma_x^2 & \beta_1\beta_2\sigma_x^2 & \beta_2\sigma_x^2 & \beta_2^2\sigma_x^2 + \lambda_2\sigma_u^2 \end{bmatrix}$$

and

$$1/n \begin{bmatrix} \sigma_x^2 + \sigma_u^2 + \sigma_\delta^2 & \beta_1\sigma_x^2 + \beta_1\sigma_\delta^2 & \sigma_x^2 & \beta_2\sigma_x^2 \\ \beta_1\sigma_x^2 + \beta_1\sigma_\delta^2 & \beta_1^2\sigma_x^2 + \beta_1^2\sigma_\delta^2 + \lambda_1\sigma_u^2 & \beta_1\sigma_x^2 & \beta_1\beta_2\sigma_x^2 \\ \sigma_x^2 & \beta_1\sigma_x^2 & \sigma_x^2 + \sigma_u^2 + \sigma_\delta^2 & \beta_2\sigma_x^2 + \beta_2\sigma_\delta^2 \\ \beta_2\sigma_x^2 & \beta_1\beta_2\sigma_x^2 & \beta_2\sigma_x^2 + \beta_2\sigma_\delta^2 & \beta_2^2\sigma_x^2 + \beta_2^2\sigma_\delta^2 + \lambda_2\sigma_u^2 \end{bmatrix}$$

respectively, a direct application of the Delta method may be used to show that the corresponding asymptotic variance is

$$\frac{\sigma_u^2(\beta_1^2 + \lambda_1 + \beta_2^2 + \lambda_2)}{\mu_x^2}. \tag{10}$$

Considering the MME, $\Delta^* = \beta_1^* - \beta_2^* \left(= 2(\overline{Y_1} - \overline{Y_2})/(\overline{X_1} + \overline{X_2})\right)$ of the parameter Δ, we obtained the following expressions for the asymptotic variance of Δ^* under the model defined by (1), (2), (3) and (4)

$$\frac{\sigma_u^2}{\mu_x^2}\left[\frac{(\beta_1^2 + \beta_2^2)}{2} + \lambda_1 + \lambda_2 - \beta_1\beta_2\right], \tag{11}$$

and

$$\frac{1}{\mu_x^2}\left[\frac{(\beta_1^2 + \beta_2^2)(\sigma_\delta^2 + \sigma_u^2)}{2} + \beta_1\beta_2\left(\sigma_\delta^2 - \sigma_u^2\right) + (\lambda_1 + \lambda_2)\sigma_u^2\right], \tag{12}$$

under the model defined by (1), (2), (3) and (5).

On the other hand, the asymptotic variance of the MLE $\hat{\Delta} = \hat{\beta}_1 - \hat{\beta}_2$ under the model (1), (2), (3) and (4) or $\Delta^* = \beta_1^* - \beta_2^*$ under the model (1), (2), (3) and (5) may be obtained by inverting the Fisher information matrix given in Appendices B or D of Aoki et al. (2000) and applying the Delta method. The ARE of $\tilde{\Delta}$ with respect to that of $\hat{\Delta}$ or Δ^* may be obtained from the ratio of (10) to the latter asymptotic variances. Although there are no explicit expressions for such ARE, we can compute them for specific parameter values.

Tables 1 to 4 corresponds to such ARE for $\beta_1 = \beta_2 = \beta = 0.10,\ 0.25,\ 0.50,\ 0.75,$ $\lambda_1 = \lambda_2 = \lambda = 0.1,\ 1.0,\ 10,\ \mu_x = 1,\ 5,$ and $\rho_{12} = 0.167,\ 0.330,\ 0.670$. The ARE of $\tilde{\Delta}$ with respect to $\hat{\Delta}$ and the ARE of Δ^* with respect to $\hat{\Delta}$ are displayed in Tables 1 and 2, respectively, in which case $\rho_{12} = \sigma_x^2/(\sigma_x^2 + \sigma_u^2)$ and $\sigma_u^2 = 0.5,\ 2.0,\ 5.0$. Tables 3 and 4 corresponds to the ARE of $\tilde{\Delta}$ with respect to Δ^* and the ARE of Δ^* with respect to Δ^*, respectively. In this case, $\rho_{12} = \sigma_x^2/(\sigma_x^2 + \sigma_u^2 + \sigma_\delta^2)$ with $\sigma_u^2 = 0.2,\ 1.2,\ 3.0$ and $\sigma_\delta^2 = 0.3,\ 0.8,\ 2.0$.

Under the model specified by (1), (2), (3) and (4) we have considered the MME $\tilde{\Delta}$ and Δ^* (Table 1 and Table 2, respectively). Notice that in Table 1 the values of ARE decrease as we increase the value of β and decrease the value of λ and μ_x, in which case the ARE are extremely low, indicating the lack of efficiency of the consistent estimator $\tilde{\Delta}$. For $\mu_x = 5$ and large values of λ, these values are greater and in connection with low values of ρ_{12} and small values of β it could be an alternative estimator for the MLE of Δ. In Table 2 the values of the AREs are the same as we varied the values of $\beta_1 = \beta_2 = \beta$ and $\lambda_1 = \lambda_2 = \lambda$. These values are somewhat greater than those evaluated in Table 1. For $\mu_x = 5$ and low values of ρ_{12} it could be a good alternative estimator too.

Considering the estimator $\tilde{\Delta}$ and Δ^* of Δ under the model defined by (1), (2), (3)

Table 1: ARE of $\tilde{\Delta}$, MME of Δ under the model defined by (1), (2) and (6) with respect to $\hat{\Delta}$, MLE of Δ under the model defined by (1), (2), (3) and (4)

		$\mu_x = 1$			$\mu_x = 5$		
			ρ_{12}			ρ_{12}	
λ	β	0.167	0.33	0.67	0.167	0.33	0.67
0.1	0.1	0.7071	0.6061	0.5050	0.8988	0.8913	0.8809
	0.25	0.4786	0.4103	0.3419	0.6084	0.6033	0.5963
	0.5	0.2222	0.1905	0.1587	0.2825	0.2801	0.2769
	0.75	0.1174	0.0101	0.0839	0.1492	0.1480	0.1463
1	0.1	0.7701	0.6601	0.5500	0.9789	0.9707	0.9594
	0.25	0.7320	0.6275	0.5229	0.9305	0.9227	0.9120
	0.5	0.6222	0.5333	0.4444	0.7910	0.7843	0.7752
	0.75	0.4978	0.4267	0.3556	0.6328	0.6275	0.6202
10	0.1	0.7770	0.6660	0.5550	0.9877	0.9794	0.9680
	0.25	0.7729	0.6625	0.5521	0.9826	0.9743	0.9630
	0.5	0.7588	0.6504	0.5420	0.9646	0.9565	0.9454
	0.75	0.7364	0.6312	0.5260	0.9360	0.9282	0.9174

Table 2: ARE of Δ^*, MME of Δ under the model defined by (1), (2), (3) and (4) with respect to $\hat{\Delta}$, MLE of Δ under the model defined by (1), (2), (3) and (4)

$\mu_x = 1$			$\mu_x = 5$		
	ρ_{12}			ρ_{12}	
0.167	0.33	0.67	0.167	0.33	0.67
0.7778	0.6667	0.5556	0.9888	0.980	0.969

Table 3: ARE of $\tilde{\Delta}$, MME of Δ under the model defined by (1), (2) and (6) with respect to Δ^*, MLE of Δ under the model defined by (1), (2), (3) and (5)

		$\mu_x = 1$			$\mu_x = 5$		
			ρ_{12}			ρ_{12}	
λ	β	0.167	0.33	0.67	0.167	0.33	0.67
0.1	0.1	0.3728	0.4427	0.4492	0.8907	0.9044	0.9214
	0.25	0.3063	0.3630	0.3960	0.7254	0.7363	0.8094
	0.5	0.2311	0.2726	0.3359	0.5396	0.5474	0.6835
	0.75	0.2001	0.2352	0.3112	0.4636	0.4702	0.6320
1	0.1	0.3910	0.4646	0.4638	0.9363	0.9507	0.9523
	0.25	0.3800	0.4514	0.4550	0.9088	0.9227	0.9337
	0.5	0.3481	0.4132	0.4295	0.8293	0.8420	0.8798
	0.75	0.3119	0.3697	0.4005	0.7392	0.7504	0.8188
10	0.1	0.3930	0.4670	0.4654	0.9413	0.9558	0.9557
	0.25	0.3919	0.4656	0.4644	0.9384	0.9528	0.9537
	0.5	0.3878	0.4607	0.4612	0.9281	0.9424	0.9468
	0.75	0.3813	0.4529	0.4560	0.9119	0.9258	0.9358

Table 4: ARE of Δ^*, MME of Δ under the model defined by (1), (2), (3) and (5) with respect to Δ^*, MLE of Δ under the model defined by (1), (2), (3) and (5)

		$\mu_x = 1$			$\mu_x = 5$		
			ρ_{12}			ρ_{12}	
λ	β	0.167	0.33	0.67	0.167	0.33	0.67
0.1	0.1	0.3844	0.4566	0.4296	0.9186	0.9326	0.8814
	0.25	0.3514	0.4164	0.3322	0.8321	0.8445	0.6788
	0.5	0.3034	0.3578	0.2475	0.7083	0.7185	0.5036
	0.75	0.2791	0.3281	0.2185	0.6467	0.6557	0.4437
1	0.1	0.3923	0.4662	0.4615	0.9394	0.9539	0.9476
	0.25	0.3876	0.4604	0.4420	0.9269	0.9412	0.9070
	0.5	0.3730	0.4427	0.3904	0.8886	0.9021	0.7998
	0.75	0.3544	0.4201	0.3394	0.8900	0.8527	0.6939
10	0.1	0.3932	0.4672	0.4651	0.9416	0.9561	0.9552
	0.25	0.3927	0.4666	0.4630	0.9405	0.9548	0.9508
	0.5	0.3909	0.4645	0.4556	0.9358	0.9502	0.9354
	0.75	0.3881	0.4611	0.4441	0.9284	0.9427	0.9115

and (5) (Table 3 and Table 4, respectively). we notice that these values are not much different in these two tables. The values of ARE in this case increase as we decrease the value of β and increase the value of λ and μ_x, as we have observed in Table 1.

3 Discussion

In Aoki et al. (2000) it was fitted the model defined by (1), (2), (3) and (4) to the data set presented in Singer and Andrade (1997), obtaining the following values for the MLE of the parameter vector θ=(β_1, β_2, μ_x, σ_x^2, σ_u^2, λ_1, λ_2) as well as the corresponding standard errors (within parentheses)

Table 5: Maximum Likelihood Estimator under the model defined by (1), (2), (3) and (4)

	Parameter						
	β_1	β_2	μ	σ_x^2	σ_u^2	λ_1	λ_2
MLE	0.147	0.454	1.758	0.539	0.482	0.102	0.267
	(0.025)	(0.045)	(0.172)	(0.200)	(0.123)	(0.040)	(0.123)

Using these parameter values we obtained the ARE of $\tilde{\Delta}$, MME under the model defined by (1), (2), (6), with respect to $\hat{\Delta}$, MLE of $\Delta = \beta_1 - \beta_2$, which value was given by 0.357. Considering the MME of Δ using the dependent model defined by (1), (2), (3) and (4), with respect to the MLE of Δ the ARE were 0.512. We observe that in this case either values of ARE are low showing the necessity of the use of the MLE approach developed in Aoki et al. (2000). Nevertheless there are situations as the one pointed out in Tables 1 and 2 where the use of the moment estimators could be a good approach.

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