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Indifference, Neutrality and Informativeness: Generalizing the Three Prisoners Paradox

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Abstract

The well-known Three Prisoners Paradox has been solved by Bayesian conditioning over the choice made by the Warder when asked to name a(nother) prisoner who will be shot. This paper generalizes the paradox to situations of N prisoners, k executions and m announcements made by the Warder. We also extend the consequences of hierarchically placing uniform and symmetrical priors (for example in the classical $N = 3, k = 2, m = 1$ scenario) for the probability p of the Warder naming Prisoner B, say. We prove that breaks of indifference and neutrality caused by assignment of uniform and symmetrical priors in lieu of degenerate indifference probabilities hold in general. We speculate on the general impossibility of maintaining noninformativeness throughout hierarchization.

Key Words: Three Prisoners Paradox, N-k-m Prisoners Paradox, Indifference probabilities, Uniform priors, Symmetrical distributions, Hierarchization, Bayesian statistics, Bayes-Laplace Postulate, Jensen's Inequality.

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... for this "unknown probability" cannot be defined, and the hypotheses that one would like to introduce in this way have no objective meaning. - Bruno de Finetti, 1937

I regard the use of hierarchical chains as a technique helping you to sharpen your subjective probabilities. - I.J. Good, 1981

Speaking of unknown probabilities or of probability of a probability must be forbidden as meaningless. - Bruno de Finetti, 1977

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1 Introduction

The Three Prisoners Paradox is an old problem which deals with elementary probability, conditional probability and probability updating, and enlargement of sample spaces. It has been called by Richard Jeffrey a "well-known horror story". We start by quoting Jeffrey's statement of the paradox (page 122 of Jeffrey[1992]):

There are three prisoners, A, B, and C. Two are to be shot and the other freed; none is to know his fate until the morning. Prisoner A asks the Warder to confide the name of one other than himself who will be shot, explaining that as there must be at least one, the Warder won't be giving away anything relevant to A's own case. The Warder agrees, and tells him that B will be shot. This cheers A up a little, by making his judgmental probability for being freed rise from 1/3 to 1/2. But that's silly: A knew already that one of the others would be shot, and (as he told the Warder) he is no wiser about his own fate for knowing the name of some other victim.

The paradox appeared in an equivalent version as the "Let's Make a Deal" problem, which draw a lot of attention (outside academia at first) a decade ago (Morgan et al.[1991]).

The solution to the paradox is given by the consideration of a sample space $\Omega = \{Ab, Ac, Bc, Cb\}$, where Xy represents the outcome "prisoner X will live and the Warder informs that prisoner Y will die". The conditional probability of prisoner A being freed, given that the Warder informs that B will die, $P(A|b)$, is then found to be $p/(p+1)$, where p is the conditional probability of the Warder naming B, given that he has a choice, i.e., given that prisoner C will also die. We will use the notation $P_p(A|b)$ to emphasize the dependence of $P(A|b)$ on p, keeping the notation $P(A|b)$ for the expected value $E[P_p(A|b)]$, in the situations in which p is random. The assumptions on which the solution is built are that each prisoner has the same initial probability 1/3 of being freed, that the Warder always tells the truth, and that Prisoner A **plans beforehand** to ask the Warder for the name of one prisoner other than himself who will be shot. (For the situation where Prisoner A does not plan to ask whatsoever and the information is given to him unexpectedly, see Loschi et al.[2001]).

The solution ceases the paradox by making the apparently contradictory probabilities, 1/2 and 1/3, particular cases depending on the value of p. If p equals 1, the information - that B will be shot - presented by the Warder makes the conditional chance of Prisoner A indeed equal to 1/2. Let us consider on the other hand the situation where $p = 1/2$. This would be the noninformative or indifferent prior conditional probability Prisoner A would assign for the event "Warder names B", given that he (the Warder) could have chosen to name C instead. Such an indifferent assignment makes the posterior probability (of A being freed, given that Warder says B will die) indeed equal to 1/3, the initial probability.

The situation above, which we call "classical", has a detailed description in Morgan (1991), for instance. We will now focus on four results of the classical situation, proving only the fourth as the first three are well-known in the literature:

Theorem 1. (A) If $p = 1/2$, then $P(A|b) = P(A)$.

(B) $P_p(A|b) < 1/2$, for every p on $[0, 1)$.

(C) If p has a uniform distribution on the interval $[0, 1]$, then $P(A|b) < P(A)$.

(D) If p has a non-degenerate symmetrical distribution around 1/2, then $P(A|b) < P(A)$.

(A) is discussed above, (B) says that - in the "Let's Make a Deal" version of the problem - it is always wise for the Player to switch doors, (C) states that by adopting the so-called Bayes-Laplace Postulate for p, Prisoner A loses the noninformativeness he had when his indifference entailed $p = 1/2$, and (D) says that this loss holds not only for a uniform on $[0, 1]$ density, but for **any** symmetrical (and non-degenerate) distribution around the indifference point 1/2. We will now prove (D):

Proof. As $P_p(A|b) = p/(p + 1)$ is a strictly concave function of p on the interval $[0, 1]$ and p has a non-degenerate distribution, Jensen's Inequality yields $P(A|b) = E[p/(p + 1)] < E[p]/(E[p] + 1)$. As p has a symmetrical around 1/2 distribution on $[0, 1]$, we have $E[p] = 1/2$. □

One should notice that the proof above works for any distribution for p having expected value 1/2. This strengthening of the result does not have immediate interest at this point of the paper, though.

This paper initially generalizes the problem for N prisoners, k executions, and m announcements made by the Warder. We then proceed to extend the four results of Theorem 1 to such a generalized scenario. We discuss the loss of noninformativeness caused by a reiterative assignment of indifferent or neutral probabilities and its interpretation for Bayesian statistical inference. In the conclusion, we also speculate on the impossibility of keeping noninformativeness throughout hierarchization (or conditioning, in a less statistical jargon) being general.

The paper is organized as follows:

Section 2 establishes the generalized Prisoners Paradox, the N-k-m Prisoners Paradox. The usual solution to the paradox is developed, under the proviso that Prisoner A had assigned positive probability for what the Warder says. Parts (A) and (B) of Theorem 1 are extended in this Section. Section 3 deals with uniformity (or hierarchical indifference) in the generalized problem. Beta densities as the marginal distributions of the generalized uniform multivariate densities yield the generalization of part (C) of Theorem 1. Section 3 generalizes also the notion of symmetry around indifference points and presents the generalization of part (D) of Theorem 1. Finally, in Section 4 we present the discussion and conclusions.

2 N-k-m Prisoners Problems

We start by defining an appropriate probability space. N is to be interpreted as the number of prisoners, k as the number of executions, and m as the number of announcements made by the Warder. All probabilities are supposed to be computed by Prisoner A (or "You" , or "Prisoner 1"), without loss of generality. An element of the sample space is a list having the names of the k prisoners who will be shot and another list having the names of the m prisoners (among those k) disclosed by the Warder. As he never lies nor tells you (Prisoner A) that You will be shot, the sample space is defined as follows:

Definition 1. Let N, k, and m be integer numbers satisfying $N \geq 3$, $2 \leq k \leq N-1$, and $1 \leq m \leq k-1$. The elements of the sample space Ω are the matrices $\omega_{2\times N}$ that satisfy:

$\omega_{ij} \in \{0,1\}, i=1,2; j=1,2,...,N$

$\omega_{21}=0$

$\sum_{j=1}^{N} \omega_{1j} = k$

$\sum_{j=2}^{N} \omega_{2j}\omega_{1j} = m$ and $\sum_{j=2}^{N} \omega_{2j} = m$.

We interpret ω_{1j} as the indicator of condemnation of Prisoner j, $(j = 1, 2, ..., N)$, while ω_{2j} is the indicator of the Warder saying that Prisoner j will be shot. The number of points in Ω is easily seen to be $\binom{N-1}{k-1}\binom{k-1}{m} + \binom{N-1}{k}\binom{k}{m} = (1 - mN^{-1})\binom{N}{k}\binom{k}{m}$.

Definition 2. An N-k-m **Prisoners Problem** is a discrete probability space (Ω, P), where the probability measure P defined for every subset of Ω satisfies the marginal equiprobability condition $P(\omega_1) = 1/\binom{N}{k}$, for every $\omega_1 \in \Omega$, i.e., every row ω_1 such that there is a matrix $\omega \in \Omega$, the first row of which is ω_1.

Technical remark: Strictly speaking, the definition above fixes (for example in the classical $N = 3, k = 2, m = 1$ scenario) p, which is derived from the probability measure P. We will nevertheless allow hierarchization on p, i.e., assignments of probability measures for p. This would call for appropriate measurability considerations in the definition (and would allow the usual Bayesian notation $P(A|b, p)$ instead of $P_p(A|b)$).

Let W, W_{ij}, W_i represent **random** $\omega, \omega_{ij}, \omega_i$, respectively. The probability of Prisoner A being freed is of course $P(W_{11} = 0) = 1 - k/N$. As the Warder will announce to You information W_2, the above probability $P(W_{11} = 0)$ is a prior probability and we will in the sequel obtain the posterior probability $P(W_{11} = 0|W_2 = \omega_2)$.

Let ω_2 be a row with positive P-probability, i.e., ω_2 is such that there is a matrix $\omega \in \Omega$, the second row of which is ω_2, with $P(\omega) > 0$. The case where ω_2 has P-probability zero can not be dealt by Bayesian conditioning and calls for solutions which use other than Bayes's rules for probability updating, such as, for example, Jeffrey's rule (see Hacking[1967], de Finetti[1972,1975], Howson and Urbach[1993], Howson[1996] for general probability updating, Jeffrey[1965] and Diaconis and Zabell [1982] for Jeffrey's rule, and Loschi et. al[2001] for probability updating in the prisoners classical scenario). In this paper, only the case where ω_2 has positive P-probability is being considered.

Theorem 2. Consider an N-k-m Prisoners Problem and a fixed point ω_2 with positive P-probability. Let $C_{\omega_2} = \{\ \omega_1 :\ \sum_{j=1}^{N} \omega_{1j} = k, \omega_{2j} = 1$ implies $\omega_{1j} = 1\}$ be the set of condemnations that are compatible with ω_2 and let $C_{\omega_2,0} = \{\ \omega_1 \in C_{\omega_2} :\ \omega_{11} = 0\}$ be the set of condemnations that are compatible with ω_2 and with You being freed. We then have

$$P(W_{11}=0|W_2=\omega_2)=\frac{\sum_{\omega_1\in C_{\omega_2,0}}P(\omega_2|\omega_1)}{\sum_{\omega_1\in C_{\omega_2}}P(\omega_2|\omega_1)} \tag{2.1}$$

Proof. Definition 1 points to the construction of the sets C_{ω_2} and $C_{\omega_2,0}$. The proof then follows straightforwardly from the definition of conditional probability and the equiprobability of W_1.

□

Theorem 2 obtains the posterior probability $P(W_{11}=0|W_2=\omega_2)$ of You being freed, given that the Warder revealed ω_2 to You, in a general N-k-m prisoners problem. We now state without the easy proof a useful lemma:

Lemma 2.1. *C_{ω_2} has $\binom{N-m}{k-m}$ rows, $C_{\omega_2,0}$ has $\binom{N-m-1}{k-m}$ rows, and there are $\binom{N-m-1}{k-m-1}$ rows which belong to C_{ω_2} and do not belong to $C_{\omega_2,0}$.*

Let us now consider the very important case $m=k-1$. In this situation, the posterior probability (2.1) reduces to

$$P(W_{11}=0|W_2=\omega_2)=\frac{\sum_{\omega_1\in C_{\omega_2,0}}P(\omega_2|\omega_1)}{\sum_{\omega_1\in C_{\omega_2,0}}P(\omega_2|\omega_1)+1} \tag{2.2}$$

with the numerator in (2.2) having $(N-k)$ terms.

We are now able to extend the results (A) and (B) of Theorem 1 to the general N-k-m prisoners problem. The next theorem generalizes (B):

Theorem 3. Consider an N-k-m Prisoners Problem and a fixed point ω_2 with positive P-probability.

(i) For $m=k-1$, we have $P(W_{11}=0|W_2=\omega_2)\leq 1-(N-k+1)^{-1}$, regardless of the values of $P(\omega_2|\omega_1)$ of $\omega_1\in C_{\omega_2,0}$.

(ii) For $m<k-1$, we can have $P(W_{11}=0|W_2=\omega_2)$ arbitrarily close (or equal) to 1, depending on the values of $P(\omega_2|\omega_1)$ of ω_1 of C_{ω_2} that are not in $C_{\omega_2,0}$.

Proof. (i) The upper bound $1-(N-k+1)^{-1}$ is obtained immediately by recalling that $C_{\omega_2,0}$ has $\binom{N-m-1}{k-m}$ elements.

(ii) By having the values $P(\omega_2|\omega_1)$ of $\omega_1 \notin C_{\omega_2,0}$ arbitrarily close to zero, the value of the posterior $P(W_{11}=0|W_2=\omega_2)$ stays arbitrarily close to 1, as shown by (2.1).

□

When $m=k-1$, Part (i) of theorem 3 implies that there will be always at least one prisoner other than You among the $(N-m)$ not named by the Warder with whom it will be wise to switch doors (cells), in the "Let's Make a Deal" equivalent version of the problem. One will notice that the prior equiprobability of those $(N-m)$ prisoners does **not** necessarily hold posterior to the Warder's announcement. It will be wise to switch doors (switch cells) only with prisoners having posterior probability of being freed larger than yours (the larger, the better).

When $m<k-1$, nevertheless, the above extension of the conclusion from the classical scenario does not hold: as shown by Part(ii), there might not be any prisoner to switch doors with advantageously. This is the first qualitative difference between the cases $m=k-1$ and $m<k-1$.

We are now ready to extend result (A) of Theorem 1. It will be shown that $P(W_{11}=0|W_2=\omega_2)=P(W_{11}=0)$ whenever all the relevant conditional distributions $P(\omega_2|\omega_1)$ are chosen (discrete) uniform or of *indifference*, in the flavour of the so-called "objectivistic" (or "reference") Bayesian school of inference (Bernardo and Smith[1994], section 5.6.2). It is intuitive that such uniform conditional distributions, by expressing the absolute indifference the Warder has relative to the m-lists he may reveal (for every ω_1), lead to the situation of coincidence between prior and posterior values, as there is no real "information" given out by the Warder.

Theorem 4. Consider an N-k-m Prisoners Problem and a fixed point ω_2 with positive P-probability. If for every $\omega_1 \in C_{\omega_2}$ we place

$$P(\omega_2|\omega_1)=\binom{k-\omega_{11}}{m}^{-1}, \tag{2.3}$$

then $P(W_{11}=0|W_2=\omega_2)=P(W_{11}=0)$.

Proof. We have, respectively by Theorem 2, the hypothesis, and Lemma 2.1,

$$P(W_{11}=0|W_2=\omega_2)=\frac{\sum_{\omega_1\in C_{\omega_2,0}}P(\omega_2|\omega_1)}{\sum_{\omega_1\in C_{\omega_2}}P(\omega_2|\omega_1)}=$$

$$=\frac{\sum_{\omega_1\in C_{\omega_2,0}}\binom{k-0}{m}^{-1}}{\sum_{\omega_1\in C_{\omega_2}}\binom{k-\omega_{11}}{m}^{-1}}=$$

$$=\frac{\binom{k}{m}^{-1}\binom{N-m-1}{k-m}}{\binom{k}{m}^{-1}\binom{N-m-1}{k-m}+\binom{k-1}{m}^{-1}\binom{N-m-1}{N-k}}=1-k/N$$

□

The upper bound in Part(i) of Theorem 3, $1-(N-k+1)^{-1}$, is the value of $P(W_{11}=0|W_2=\omega_2)$ obtained by the (possibly fallacious) argument of posterior equiprobability of the $N-k+1$ prisoners not named by the Warder. Comparison of values of $P(W_{11}=0|W_2=\omega_2)$ given by Theorem 4 and by the upper value in Part(i) of Theorem 3 gives rise to the generalized "paradox". The next section will extend result (C) of Theorem 1 to the generalized N-k-m scenario.

3 Uniformity

Result (C) of Theorem 1 states that the posterior probability $P(A|b)$ is strictly smaller than the initial probability $P(A)$ whenever p has a uniform distribution on the interval [0, 1]. Assignment of such a uniform distribution for p is in accordance with the so-called Bayes (or Bayes-Laplace) Postulate. This postulate - which says that an **unknown probability** ought to have assigned to it a uniform density on the interval [0, 1] - and its justification have been central and polemical in the history and the philosophy of Statistics (e.g., Stigler[1982 and 1986], Dale[1991]). We will discuss this polemic in detail in Section 4.

We will argue in this section that extension of Bayes Postulate to the generalized N-k-m problem entails the assignment of a uniform density for each simplex set of probabilities generated by a fixed ω_1. Theorem 1(C) will then be generalized into Theorem 5. Either one states that neutrality is lost after integration of $P_p(A|b)$ (or of $P(W_{11}=0|W_2=\omega_2)$). In particular, adoption of Bayes Postulate leads to a posterior probability for

Prisoner A, which is different than the prior $P(A)$. This was not the case, for example, in Theorem 4, where degenerate equiprobability points were assigned, yielding equality of prior and posterior values.

Example: $N = 5, k = 3, m = 2$

Making, for example, $\omega_2 = bc$ and recalling expression (2.2), we obtain, with obvious notation,

$$P(W_{11} = 0|W_2 = \omega_2) = P(A|W_2 = bc) = \frac{P(bc|AD)+P(bc|AE)}{P(bc|AD)+P(bc|AE)+1}$$

Theorem 3(i) gives the upper bound $P(A|W_2 = bc) \leq 2/3$ and Theorem 4 gives $P(A|W_2 = bc) = P(A)$ if $P(bc|AD) = P(bc|AE) = 1/3$. The value 1/3 for these two probabilities, $P(bc|AD)$ and $P(bc|AE)$, expresses the indifference of the Warder about what he may say, for every given list ω_1 of C_{ω_2}(or your indifference about what the Warder may say, for every given list ω_1 of C_{ω_2}). If, for example, $\omega_1 = AD$ [or, more formally, if $\omega_1 = (0, 1, 1, 0, 1)$] the Warder may say bc , be, or ce. Indifference about this choice brings the discrete uniform distribution $P(bc|AD) = P(be|AD) = P(ce|AD) = 1/3$. The same holds given $\omega_1 = AE$.

We claim that the rôle of the hierarchical uniform density for p in the classical scenario is played in the example above by hierarchical uniform densities for the random vectors $[P(bc|AD), P(be|AD), P(ce|AD)]$ and $[P(bc|AE), P(bd|AE), P(cd|AE)]$. As both trivariate random vectors have the sum of their non-negative components equal to 1, their uniform densities are over the resultant simplex sets, analogously to the classical scenario, where, strictly speaking, the uniform density for p is a uniform density over the simplex set $\{ (p, 1-p) : 0 \leq p \leq 1\}$.

In general, we can now prove the extension of result (C) of Theorem 1:

Theorem 5. Consider an N-k-m Prisoners Problem and a fixed point $\omega_2{}^*$ with positive P-probability. Suppose also that $m = k-1$. If, for every fixed $\omega_1 \in C_{\omega_2{}^*,0}$, the joint distribution of the values of $P(\omega_2|\omega_1)$ is uniform over the simplex set

$S_{\omega_1} = \{ P(\omega_2|\omega_1) \geq 0 : \omega_{21} = 0, \omega_{2j} \in \{ 0, 1\}$ for $j > 1, \sum_{j=2}^{N} \omega_{2j}\omega_{1j} = \sum_{j=2}^{N} \omega_{2j} = k-1$ and $\sum_{\omega_2} P(\omega_2|\omega_1) = 1\}$,

then $P(W_{11} = 0|W_2 = \omega_2{}^*) \leq P(W_{11} = 0)$.

Proof. Representing $\sum_{\omega_1 \in C_{\omega_2^*,0}} P(\omega_2|\omega_1)$ by $T_{\omega_2^*}$, we will have, by (2.2),

$$P_{T_{\omega_2^*}}(W_{11} = 0|W_2 = \omega_2^*) = \frac{T_{\omega_2^*}}{T_{\omega_2^*} + 1} \qquad (3.1)$$

As $t/(t+1)$ is a concave function of t on the set of non-negative real numbers, Jensen's Inequality yields

$$P(W_{11} = 0|W_2 = \omega_2^*) = E[T_{\omega_2^*}/(T_{\omega_2^*} + 1)] \leq E[T_{\omega_2^*}]/(E[T_{\omega_2^*}] + 1).$$

Now, each $P(\omega_2|\omega_1)$ in $C_{\omega_2^*}$ is distributed as the marginal of a uniform density over the simplex set S_{ω_1}. We can look such a uniform density as a Dirichlet density over S_{ω_1} with k parameters, all equal to 1. A well-known property (e.g., DeGroot 1970) of the Dirichlet density give us the density of each marginal $P(\omega_2|\omega_1)$ as beta(1,$k-1$). In other words, $T_{\omega_2^*}$ is a sum of $N-k$ random variables identically distributed as beta(1,$k-1$). Since their common expected value is k^{-1}, we obtain the result.

□

One will notice that the proof above does not ask for any restriction on the joint distributions of the values $P(\omega_2|\omega_1)$, other than each set of the marginal densities being suitable betas. The result therefore is more general in this sense and holds, for example, for joint densities with dependent beta marginal densities. As a matter of fact, the result is even stronger: it is enough that $E[T_{\omega_2^*}] \leq (N-k)/k$. We have nevertheless emphasized the beta distribution as it is a (non-uniform!) marginal consequence of the assumption of "not knowing anything" about the Warder options. On the other hand, the result **is not proved yet for the case** $m < k-1$.

4 Conclusion

What would Bayes himself have done ? More precisely, were he in place of Prisoner A and thinking that **nothing at all is known antecedently to any trials made or observed concerning it** about p ? (boldface are Bayes's own words. Bayes[1763], page 392). We quote from Stigler (1982, page 254): " Bayes made no comment that could be taken as extending beyond the immediate binomial situation he considered, although posterity (starting with Laplace) had no such scruple...... It is tempting to ask how

Bayes would have addressed such problems". Our question deals with a situation where p, if seen as a parameter, is transformed to determine a Bernoulli $(p/(p+1))$ random observable, while as a probability it is *conditional* on b. In trying to answer such a question, and bearing in mind Stigler's reading of Bayes's Scholium, we again are puzzled by the Paradox ! Bayes could demand a discrete uniform distribution for the indicator of Prisoner A being freed, this meaning a Bernoulli(1/2) random observable as Prisoner B is dead. On the other hand, Bayes could think that the Warder naming one of the other prisoners who will be shot constitutes irrelevant information and demand a Bernoulli(1/3) random observable. The former situation implies a degenerate on 1 distribution for p, while the latter implies any distribution on $[0,1]$ making 1/3 the expected value of $p/(p+1))$ (no such distribution is symmetric around 1/2). These speculations will remain open. To fix $p = 1$ would make Bayes an even more radical predictivistic Bayesian ? (Stigler [1982], Wechsler[1993]). Such an assignment would follow from the sole observation made - the Warder's information - but the conditioning event A had never been verified. On the other hand, asymmetric distributions on $[0,1]$ playing the role of *noninformative* priors for p would strengthen Stigler's point on Bayes's scope and aim of Bayes postulate.

What can be more safely thought of is that Bayes himself would **not** advocate the use of the uniform density on $[0,1]$ (let alone on the simplex sets of the general scenario) because of the consequent loss of neutrality shown by Theorem 5.

What would Bruno de Finetti have done ? It seems much easier to address this question, not only because de Finetti left us much more written works than Bayes did, but mainly from the unquestionable and direct style he used to express his radical uncompromising definition of probability. The two De Finetti epigraphs of this paper, for example, leave little doubt about his reasoning were he in place of Prisoner A: for De Finetti, $P(A|b)$ could only be $p'/(p'+1)$, with p' being the sharp value of his personal conditional probability $P(b|A)$. This is all that can be said (and there is no reason to believe he would necessary let anyone know the value p'). De Finetti would never think of a probability for p : "Speaking of unknown probabilities or of probability of a probability must be forbidden as meaningless" (De Finetti, 1977). Nor would he "not know" his OWN probability. We believe that not even the understanding of p as the integration variable in the integral representation of the law of a suitable sequence of exchangeable

$0-1$ random variables would change his point of view, as the whole Prison situation is unique, without any replication (let alone an *infinite* sequence of replications). It is interesting to compare De Finetti's position with Bayes's, or even with the position of the contemporary statisticians that use invariably hierarchical models and/or reference priors: De Finetti would never find himself deprived of neutrality nor of indifference, if it hapened that his $p' = 1/2$. In fact, he would think that Theorems 1.C, 1.D, 5, and 6 in this paper are meaningless, to say the least.

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