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NONCONSERVATIVE POSITIONAL SYSTEMS - STABILITY

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1. INTRODUCTION

Consider the differential system

$$\ddot{x} = X(x;y), \quad \ddot{y} = Y(x;y), \quad X, Y : \Omega = \Omega^{\circ} \subset \mathbf{R}^2 \to \mathbf{R},$$
$$X, Y \in C^1, \quad X(0;0) = Y(0;0) = 0. \tag{1}$$

The aim of this work is to study the stability of the origin for (1) when the "force" $F = (X;Y)$ is nonconservative, that is when there is no $\Pi : \Omega \to \mathbf{R}$, $\Pi \in C^2$, such that $\nabla\Pi = -F$.

Central forces are particularly interesting. Namely

$$\ddot{x} = -x f(x;y), \quad \ddot{y} = -y f(x;y), \quad f \in C^1. \tag{2}$$

The origin is an unstable equilibrium for a central force which is repelling, i.e. either $f(0;0) = 0$ and f has a strict maximum at $(0;0)$ or $f(0;0) < 0$. In fact, let us consider the Liapunov function $L = x\dot{x} + y\dot{y}$. This map assumes positive values for some points, in the phase space, which are arbitrarily close to the origin. Furthermore

$$\dot{L} = \dot{x}^2 + \dot{y}^2 - (x^2 + y^2) f(x;y)$$

is positive definite.

Very few results on stability are known when the force is attractive and nonconservative. Let us observe that stability occurs in the conservative attractive case (Dirichlet - Lagrange's theorem).

In Sec.2 we define pseudo-conservative forces in connection with the system (1). Each force of this class is related to a suitable Riemannian metric. Moreover we generalize some well known Teorems of Rational Mechanics.

In the second part of Sec.2 we obtain a necessary and sufficient condition for the system (1) to admit an energy-like first integral (Proposition 4).

In Sec.3 we consider the system

$$\ddot{x} = -x f(x) , \quad \ddot{y} = -y f(x) , \quad f \in C^1 , \tag{3}$$

from the point of view introduced in Sec.2. In this way we obtain a nontrivial stable case (Prop. 5).

In Sec.4 we obtain a new necessary and sufficient condition for the stability of the origin for the system (3) (Prop. 9). This is obtained by the use of the aforementioned nontrivial stable case and some tools borrowed from [Z ; B].

Section 4 ends with the main result of this paper, Proposition 10, where we show that the instability is generic for the system (3) (at least when there exists $f''(0)$). This is obtained by proving a closed condition.

In Sec.5 we generalize the results of Sec.2 for the typical Lagrangean system.

2. PSEUDO-CONSERVATIVE FORCES

In this section we consider the system (1).

Let

$$A = \begin{pmatrix} \alpha & \gamma \\ \gamma & \beta \end{pmatrix} , \quad \alpha, \beta, \gamma \in C^2(\Omega ; R) , \tag{4}$$

be positive definite on the simply connected domain $\Omega = \Omega^o$. Let us define the scalar product $<< \; ; \; >>$ by:

$$<< u ; w >> = < u ; Aw> = \alpha u_1 w_1 + \gamma (u_1 w_2 + u_2 w_1) + \beta u_2 w_2 ,$$

where $u = (u_1 ; u_2)$, $w = (w_1 ; w_2)$ and $< ; >$ is the usual scalar product. The kinetic energy T defined by $<< ; >>$ is

$$T = \frac{1}{2} << v ; v >> = \frac{1}{2}(\alpha \dot{x}^2 + 2\gamma \dot{x}\dot{y} + \beta \dot{y}^2) \qquad (5)$$

(for a material point of unitary mass). Furthermore we define the work of the force $F = (X ; Y)$ (related with $<< ; >>$) by

$$\mathcal{C} = \int_{t_1}^{t_2} << F ; v >> dt = \int_{t_1}^{t_2} [(\alpha X + \gamma Y)\dot{x} + (\gamma X + \beta Y)\dot{y}] dt.$$

Now, let us find conditions on A in order that the Theorem of the Kinetic Energy holds, that is

$$\frac{d}{dt} T = << F ; v >> .$$

We have

$$\frac{d}{dt} T = \alpha \dot{x}\ddot{x} + \gamma(\dot{x}\ddot{y} + \dot{y}\ddot{x}) + \beta \dot{y}\ddot{y} + \frac{1}{2}\alpha_x \dot{x}^3 + \frac{1}{2}\beta_y \dot{y}^3 +$$

$$+ (\frac{1}{2}\alpha_y + \gamma_x)\dot{y}\dot{x}^2 + (\frac{1}{2}\beta_x + \gamma_y)\dot{x}\dot{y}^2 .$$

The theorem above holds if and only if

$$\frac{d}{dt} T = << \dot{v} ; v >>$$

or, equivalently,

$$\alpha_x = 0 , \ \beta_y = 0 , \ \frac{1}{2}\alpha_y + \gamma_x = 0 , \ \frac{1}{2}\beta_x + \gamma_y = 0 .$$

This yields

$$\alpha = a + by + cy^2 , \ \beta = d + ex + cx^2 , \ \gamma = k - \frac{b}{2}x - \frac{e}{2}y - cxy \qquad (6)$$

where a, b, c, d, e, k are constants.

Definition - If there exists $\Pi : \Omega \to R$, $\Pi \in C^2$, such that

$$-\Pi_x = \alpha X + \gamma Y \, , \qquad -\Pi_y = \gamma X + \beta Y \tag{7}$$

then we say that the force F is pseudo-conservative.

Proposition 1 - The force $F = (X\,;Y)$ is pseudo-conservative (for some metric) iff there exist a, b, c, d, e, k $\in$ R such that A defined by (4) and (6) is positive definite and

$$\frac{3}{2}(b+2cy)X - \frac{3}{2}(e+2cx)Y + (k - \frac{b}{2}x - \frac{e}{2}y - cxy)(Y_y - X_x) +$$

$$+ (a + by + cy^2)X_y - (d + ex + cx^2)Y_x = 0 \tag{8}$$

Proposition 2 - Under the preceding conditions, the function $E = T + \Pi$ — see (5) and (7) — is a first integral for the system (1). (Conservation of the Mechanical Energy with respect to the new metric).

Proposition 3 - If the aforementioned Π has a strict minimum at the origin, then the latter is Liapunov-stable. (Dirichlet - Lagrange's Theorem with respect to the new metric).

Prop. 2 shows that, in some cases, the system (1) has a first integral of the following form:

$$V = \frac{1}{2}(\alpha\dot{x}^2 + 2\gamma\dot{x}\dot{y} + \beta\dot{y}^2) + \Pi(x,y) \tag{9}$$

where $\Pi \in C^2$. Conversely, if V in (9) is a first integral, then (6) and (7) hold. This yields (8). Therefore

Proposition 4 - The map V in (9) is a first integral of (1) iff (8) holds for some a, b, c, d, e, k $\in$ R.

Remark - The matrix A, defined by (4) and (6), is not necessary positive definite. Observe that it was positive definite in Proposition 1. This is the essential difference between Propositions 1 and 4.

3. A NONTRIVIAL STABLE CASE FOR $\ddot{x} = -x f(x)$, $\ddot{y} = -y f(x)$

Let us consider the case

$$\ddot{x} = -x f(x)\,,\ \ddot{y} = -y f(x)\,,\ f : I = I^{\circ} \to \mathbb{R},\ f \in C^1,\ 0 \in I. \tag{3}$$

This trivially admits the first integrals:

$$\frac{1}{2}\dot{x}^2 + \int_0^x z f(z)\,dz \quad \text{and} \quad (x\dot{y} - y\dot{x})^2. \tag{10}$$

These have the form (9). Now, let us look for the conditions on f in order that the system (3) admits some first integral of the form (9) (independent of these in (10)). The condition in (8) yields

$$\left[\frac{3}{2}\,ef + (d + \frac{1}{2}\,ex)\,f'\right]y + \left[-\frac{3}{2}\,bf + (k - \frac{1}{2}\,bx)\,f'\right]x = 0.$$

In particular, for $y = 0$, we have

$$-\frac{3}{2}\,bf + (k - \frac{1}{2}\,bx)\,f' = 0\,. \tag{11}$$

This implies

$$\frac{3}{2}\,ef + (d + \frac{1}{2}\,ex)\,f' = 0\,. \tag{12}$$

The equation (11) gives

$$f(x) = \frac{c_1^3}{(2k - bx)^3}\,.$$

Similary (12) yields

$$f(x) = \frac{c_2^3}{(2d + ex)^3} \cdot$$

The last two expressions imply

$$0 \neq p = \frac{2k}{c_1} = \frac{2d}{c_2} \quad \text{and} \quad q = -\frac{b}{c_1} = \frac{e}{c_2} .$$

Thus

$$bp + 2qk = 0 , \quad ep - 2dq = 0 , \quad f(x) = \frac{1}{(p + qx)^3} \cdot \tag{13}$$

In this case the system (3) has the following first integrals:

$$\frac{1}{2}\dot{x}^2 + \frac{x^2}{2p(p + qx)^2} \tag{14}$$

(this corresponds to, $a = 1, b = c = d = e = k = 0$);

$$(x\dot{y} - y\dot{x})^2 \tag{15}$$

($c = 1, a = b = d = e = k = 0$);

$$-\frac{q}{p} y\dot{x}\dot{y} + \left(\frac{1}{2} + \frac{q}{p} x\right)\dot{y}^2 + \frac{y^2}{2p(p + qx)^2} \tag{16}$$

($d = 1, e = \frac{2q}{p}, a = b = c = k = 0$);

$$-\frac{q}{p} y\dot{x}^2 + \left(1 + \frac{q}{p} x\right)\dot{x}\dot{y} + \frac{xy}{p(p + qx)^2} \tag{17}$$

($k = 1, b = -\frac{2q}{p}, a = c = d = e = 0$).

Obviously, only three of these first integrals are functionally independent. We can arbitrarily consider three of them. However in the sequel we use all of them.

Proposition 5 - The origin is stable for the system (3), with f as in (13), for any positive p.

Proof - Add the maps in (14) and (16) and use Prop. 3. □

Finally let us obtain, in a different way, a result which is already known - see [D] and [H].

Proposition 6 - Any trajectory of the system (3), with f as in (13), is a conic.

Proof - By the use of (14), (15) and (17) we can prove that

$$y^2 = \frac{(p + qx_o)^2}{x_o^2} (x_o - x)(p x_o + p x + 2q x_o x) \tag{18}$$

when x_o, $\dot{x}_o = 0$, $y_o = 0$, $\dot{y}_o = 1$, are the initial conditions. This corresponds to the solution $\psi(t)$ of the equation (7) in [Z ; B]. The equation (18) represents a conic. Now, formula (10) in [Z ; B] yields the result. □

4. FOR $\ddot{x} = -x\,y(x)$, $\ddot{y} = -y\,f(x)$, THE INSTABILITY IS GENERIC

Let us consider

$$\ddot{x} = -x f(x), \ \ddot{y} = -y f(x), \ f \in C^\circ(I = I^\circ; R), \ f(0) > 0 \tag{19}$$

where f guarantees uniqueness. Let us fix $x_o > 0$ small enough, and let $\mathcal{F}_{x_o}$ be the family of the solutions of (19) with x_o, $\dot{x}_o = 0$, y_o, $\dot{y}_o$ as initial conditions. Furthermore, let

$$x_o^- = \max\{x \in R_- : V(x) = V(x_o)\}, \quad V(x) = \int_0^x z f(z)\,dz .$$

We have:

Proposition 7 - The solution in $\mathcal{F}_{x_o}$ are all periodic iff

$$\lim_{x\to 0+}\left\{\int_{x_o}^{x}\frac{dz}{z^2\sqrt{1-\frac{V(z)}{V(x_o)}}}+\frac{1}{x}\right\} = \lim_{x\to 0-}\left\{\int_{x_o^-}^{x}\frac{dz}{z^2\sqrt{1-\frac{V(z)}{V(x_o^-)}}}+\frac{1}{x}\right\} \tag{20}$$

This was proved in [Z ; B] .

In the case where $f(x) = (1 + qx)^{-3}$, the origin is a stable equilibriun and each trajectory is a conic - see Prop. 5 and 6. Thus, in this case we have

$$\lim_{x\to 0+}\left\{\int_{x_o}^{x}\frac{dz}{z^2\sqrt{1-\frac{\bar{V}(z)}{\bar{V}(x_o)}}}+\frac{1}{x}\right\} = \lim_{x\to 0-}\left\{\int_{\bar{x}_o^-}^{x}\frac{dz}{z^2\sqrt{1-\frac{\bar{V}(z)}{\bar{V}(\bar{x}_o^-)}}}+\frac{1}{x}\right\} \tag{21}$$

where

$$\bar{x}_o^- = \max\{x \in R_- : \bar{V}(x) = \bar{V}(x_o)\}, \quad \bar{V}(x) = \frac{x^2}{2(1+qx)^2} \tag{22}$$

By setting $q = q(x_o)$ in (22) with

$$q(x_o) = -\frac{x_o + x_o^-}{2x_o\, x_o^-} \tag{23}$$

we have $\bar{x}_o^- = x_o^-$. Let us observe that (23) can be obtained by solving the equation $\bar{V}(x_o^-) = \bar{V}(x_o)$ i. e.

$$\frac{(x_o^-)^2}{(1+q(x_o)\,x_o^-)^2} = \frac{(x_o)^2}{(1+q(x_o)\,x_o)^2} .$$

Now, let us consider (21) with $\bar{V}(x) = \dfrac{x^2}{2(1+q(x_o)\,x)^2}$. If we subtract the resulting equality from equality (20), we obtain

$$\int_{x_o}^{x_o^-} \frac{\dfrac{V(x_o)}{\left(\dfrac{x_o}{1+q(x_o)x_o}\right)^2} - \dfrac{V(z)}{\left(\dfrac{z}{1+q(x_o)z}\right)^2}}{(1+q(x_o)\,z)^2\sqrt{\left(1-\dfrac{\bar{V}(z)}{\bar{V}(x_o)}\right)\left(1-\dfrac{V(z)}{V(x_o)}\right)}\left(\sqrt{1-\dfrac{\bar{V}(z)}{\bar{V}(x_o)}}+\sqrt{1-\dfrac{V(z)}{V(x_o)}}\right)}\,dz = 0. \quad (24)$$

Summing up, we have proved:

<u>Proposition</u> 8 - <u>The equality in</u> (24) <u>holds iff all the solutions in</u> $\mathcal{F}_{x_o}$ <u>are periodic</u>.

Since $V''(0) = f(0) > 0$, there exists a map

$$h : J \to J\,,\quad 0 \in J = J^o \subset I\,,\quad V(x) = V(h(x))\,,\quad xh(x) < 0\,. \quad (25)$$

This map is C^1 because V is C^1. Moreover $h'(0) = -1$. Observe that $x_o^- = h(x_o)$.

Now, let us define the C^o maps g and G and the C^1 map q by:

$$g(0) = \frac{f(0)}{2}\,,\quad g(x) = \frac{V(x)}{x^2}\,,\quad x \in J\,,\quad x \neq 0\,; \quad (26)$$

$$q(\mu) = -\frac{\mu + h(\mu)}{2\mu\, h(\mu)}\,,\quad \mu \in J\,,\quad \mu \neq 0\,; \quad (27)$$

and

$$G(x\,;\mu) = (1 + q(\mu)\,x)^2\, g(x)\,,\quad x, \mu \in J\,,\quad \mu \neq 0\,. \quad (28)$$

The typical solution of the equation (19) can be obtained from a solution with $\dot{x}(t=0) = 0$ by a suitable time translation.

Therefore, Prop. 8 implies:

Proposition 9 - The origin is a stable equilibrium for (19) iff

$$\int_{\mu}^{h(\mu)} \frac{G(\mu;\mu) - G(x;\mu)}{(1+q(\mu)\,x)^2 \sqrt{\left(1-\frac{\bar{V}(x)}{\bar{V}(\mu)}\right)\left(1-\frac{V(x)}{V(\mu)}\right)}\left(\sqrt{1-\frac{\bar{V}(x)}{\bar{V}(\mu)}}+\sqrt{1-\frac{V(x)}{V(\mu)}}\right)}\,dx = 0 \qquad (29)$$

for any μ in some neighbourhood of zero.

Let us consider the system

$$\ddot{x} = -x\,f(x),\ \ddot{y} = -y\,f(x)\ ,\ f: I = I^{\circ} \subset R \to R\ ,\ F \in C^1,\ \exists f''(0). \qquad (30)$$

In this case, the map g defined by (26) is C^2 and we have

$$g(0) = \frac{f(0)}{2},\quad g'(0) = \frac{f'(0)}{3}\ ,\quad g''(0) = \frac{f''(0)}{4}\ . \qquad (31)$$

Lemma - There exists the limit as $\mu \to 0$ of $q(\mu)$ defined by (27).

Proof - We have

$$V(h(\mu)) = V(\mu)\ ,\quad V'(h(\mu))\,h'(\mu) = V'(\mu),$$

and

$$\frac{h(\mu)}{\mu}\,h'(\mu) = \frac{f(\mu)}{f(h(\mu))}\ . \qquad (32)$$

Thus

$$\frac{V(h(\mu))}{h^2(\mu)}\ \frac{h^2(\mu)}{\mu^2} = \frac{V(\mu)}{\mu^2}\ .$$

This yields

$$g(h(\mu))\,\frac{h^2(\mu)}{\mu^2} = g(\mu).$$

Now, (32) yields

$$h'(\mu) = -\frac{f(\mu)}{f(h(\mu))}\sqrt{\frac{g(h(\mu))}{g(\mu)}} .$$

By differentiating this expression at $\mu = 0$ and by using (31), we have

$$h''(0) = -\frac{4}{3}\,\frac{f'(0)}{f(0)} .$$

On the other hand

$$-2\lim_{\mu\to o} q(\mu) = \lim_{\mu\to o}\frac{\dfrac{\mu + h(\mu)}{\mu^2}}{\dfrac{h(\mu)}{\mu}} = \frac{2}{3}\,\frac{f'(0)}{f(0)} . \qquad \square$$

Therefore we can extend the function G in (28) by defining $q(0) = -\frac{f'(0)}{3f(0)}$. In this way we have a continuous map on a neighbourhood of the origin in R^2.

<u>Proposition</u> 10 - <u>The condition</u>

$$3f(0)\, f''(0) = 4\,[f'(0)]^2$$

<u>is necessary for the stability of the origin for the system in</u> (30).

<u>Proof</u> - For

$$G(x\,;\mu) = (1 + q(\mu)\,x)^2\, g(x) , \quad x, \mu \in J$$

we have

$$G_x(x\,;\mu) = (1 + q(\mu)\,x)^2\, g'(x) + 2q(\mu)\,(1 + q(\mu)\,x)\,g(x),$$

and

$$G_{xx}(x\,;\mu) = (1 + q(\mu)x)^2\, g''(x) + 4q(\mu)(1 + q(\mu)x)\,g'(x) + 2q^2(\mu)\,g(x) .$$

The preceding Lemma and (31) yield

$$G_x(0\,;0) = g'(0) + 2\,q(0)\,g(0) = 0$$

$$G_{xx}(0\,;0) = g''(0) + 4\,q(0)\,g'(0) + 2\,q^2(0)\,g(0) = \frac{3\,f(0)\,f''(0) - 4\,[f'(0)]^2}{12\,f(0)}\,.$$

Now, if $3f(0)\,f''(0) \neq 4\,[f'(0)]^2$ then the function $G(x\,;0)$ has an extremum at $x = 0$, which is guaranteed by the second derivative. Since G_{xx} is continuous then there exists a neighbourhood of the origin in $\mathbb{R}^2$ where it never vanishes. Moreover $G(\mu\,;\mu) = G(\mu\,;h(\mu))$ because $q(\mu) = q\,(h(\mu))$. Therefore $G(\mu\,;\mu) + - G(x\,;\mu) \neq 0$ for any positive μ which is small enough, and any x satisfying $h(\mu) < x < \mu$. Hence the integral in (29) is nonvanishing. – Therefore the origin is unstable fcr (30). ☐

5. GENERALIZATION OF SECTION 2 FOR LAGRANGEAN SYSTEMS

Consider Lagrange's equations:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q^i} - \frac{\partial T}{\partial q^i} = Q_i \qquad (i = 1, \ldots, n)\,, \tag{33}$$

in the case of positional forces $Q_i(q)$ and

$$T = \frac{1}{2}\,a_{ij}(q)\,\dot q^i\,\dot q^j$$

where Einstein's convention on dummy indices is used. These equations are equivalent to

$$\ddot q^i + \Gamma^i_{j\ell}\,\dot q^j\,\dot q^\ell = Q^i$$

where $Q^i = a^{ij}\,Q_j$ – the matrix (a^{ij}) is the inverse of (a_{ij}) – and

$$\Gamma^h_{ij} = \frac{1}{2} a^{h\ell}(\partial_j a_{i\ell} + \partial_i a_{\ell j} - \partial_\ell a_{ij})$$

are the Christoffel's symbols –

Briefly

$$\frac{\nabla \dot{q}^i}{dt} = Q^i . \qquad (34)$$

Furthermore, let us consider another symmetric tensor $b_{ij}(q)$ and let $\nabla_r b_{ik}$ be its covariant derivative, that is

$$\nabla_r b_{ij} = \partial_r b_{ij} - b_{i\ell} \Gamma^\ell_{jr} - b_{\ell j} \Gamma^\ell_{ir} .$$

Proposition 11 - The map

$$V(q ; \dot{q}) = \frac{1}{2} b_{ij}(q) \dot{q}^i \dot{q}^j + \Pi(q)$$

is a first integral for the system (34) iff the following two conditions hold for any q:

(i) the symmetrization of $\nabla_r b_{i\ell}$ vanishes, and

(ii) $b_{ij} Q^j = - \partial_i \Pi$ for some map $\Pi(q)$.

Proof - We have

$$\frac{d}{dt}(\frac{1}{2} b_{ij} \dot{q}^i \dot{q}^j + \Pi) = \frac{1}{2} \partial_r b_{ij} \dot{q}^i \dot{q}^j \dot{q}^r + b_{\ell i} \ddot{q}^\ell \dot{q}^i + \dot{q}^i \partial_i \Pi =$$

$$= \frac{1}{2} (\partial_r b_{ij} + b_{i\ell} \Gamma^\ell_{jr} - b_{\ell j} \Gamma^\ell_{ir}) \dot{q}^i \dot{q}^j \dot{q}^r + b_{\ell i} (\ddot{q}^\ell + \Gamma^\ell_{rj} \dot{q}^r \dot{q}^j) \dot{q}^i +$$

$$+ \dot{q}^\ell \partial_\ell \Pi = \frac{1}{2} \nabla_r b_{ij} \dot{q}^i \dot{q}^j \dot{q}^r + (b_{i\ell} \frac{\nabla \dot{q}^i}{dt} + \partial_\ell \Pi) \dot{q}^\ell .$$

This expression vanishes identically iff the condition (i) and (ii) hold. □

In particular, if the metric $a_{ij}\, dq^i\, dq^j$ is Euclidean, and the coordinates are Cartesian, then the condition (i) becomes

$$\partial_r b_{ij} + \partial_i b_{jr} + \partial_j b_{ri} = 0.$$

For the two dimensional case, we have

$$\partial_1 b_{11} = 0\,, \quad \partial_2 b_{22} = 0\,, \quad 2\partial_1 b_{12} + \partial_2 b_{11} = 0\,, \quad 2\partial_2 b_{12} + \partial_1 b_{22} = 0.$$

This yields (6) with $\alpha = b_{11}$, $\beta = b_{22}$, and $\gamma = b_{12}$.

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