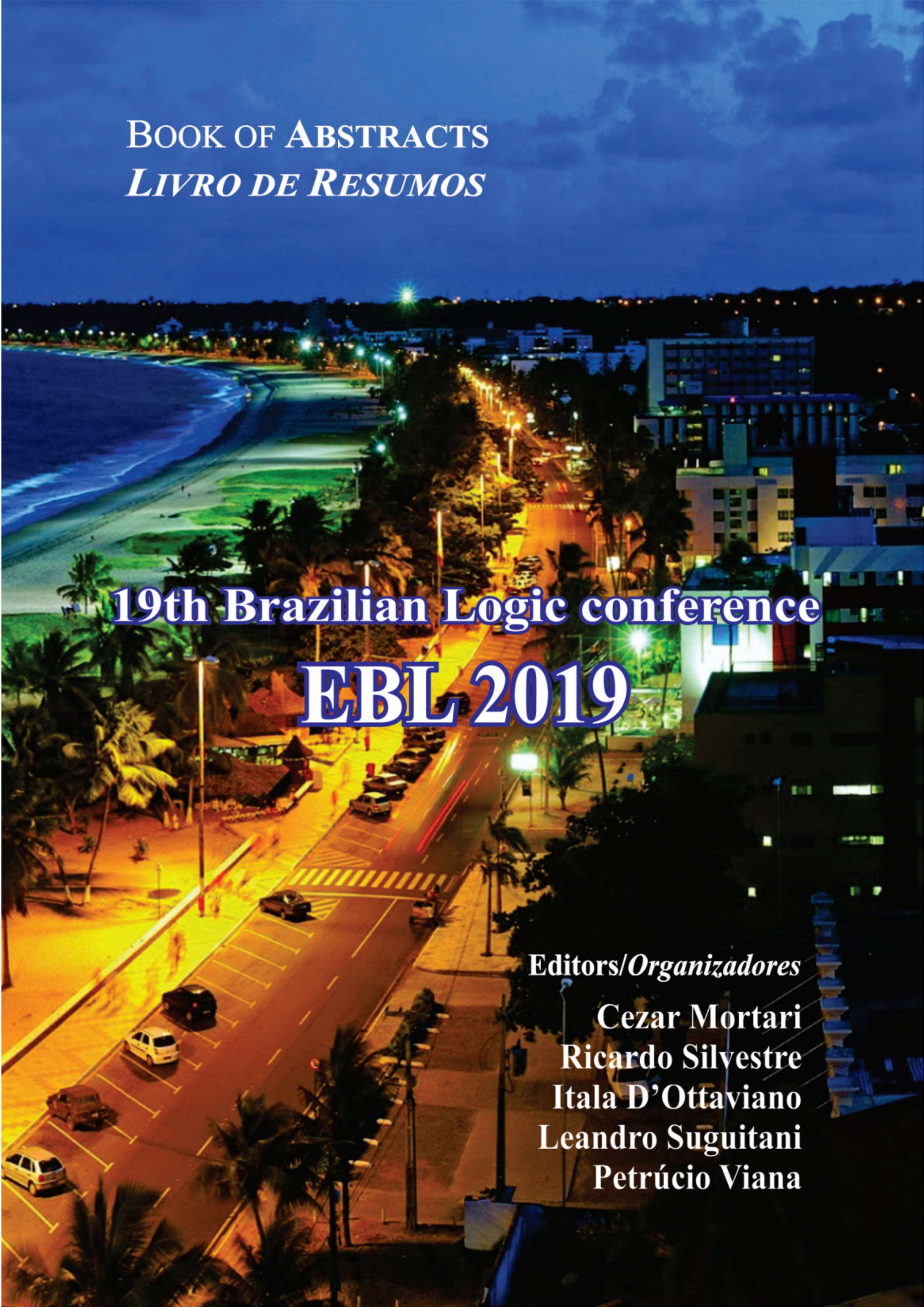

BOOK OF ABSTRACTS
LIVRO DE RESUMOS

19th Brazilian Logic conference
EBL 2019

Editors/*Organizadores*

Cezar Mortari
Ricardo Silvestre
Itala D'Ottaviano
Leandro Suguitani
Petrúcio Viana

19TH BRAZILIAN LOGIC CONFERENCE

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XIX ENCONTRO BRASILEIRO DE LÓGICA

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XIX EBL
19th Brazilian Logic Conference

May 6-10th, 2019 — João Pessoa, Brazil

https://ebl2019.ci.ufpb.br

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EDUFCG

MÍDIA GRÁFICA E EDITORA LTDA

Editoração eletrônica:

Cezar Augusto Mortari
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B794 19th Brazilian Logic Conference EBL 2019: Book of Abstracts / Cezar Augusto Mortari, Ricardo Silvestre, Itala Maria Loffredo D'Ottaviano, Leandro Suguitani, Petrucio Viana (Organizadores) – João Pessoa : Mídia Gráfica e Editora Ltda, Co-Editora UFCG – EDUFCG, Campina Grande – 2019.
226 p.

ISBN 978-85-2370-166-6

1. Lógica. 2. Matemática. 3. Ciências da Computação.
4. Filosofia.

CDU: 510:6

Algebraizing Higher-Order Logics

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At first, the objective of this work is to show a way to extend (naturally) a category of propositional logics [1] to a category of logics with quantifiers, following some desired conditions:

- Generalize results of algebraizable logics, in particular for Blok-Pigozzi algebraizable logics [2] in a uniform way;
- Generalize some traditional proposals of algebrization of First-Order Logic (FOL), like Tarski's Cylindrical Algebra, Halmos's Polyadic Algebra, Quine's Predicative Functor Logic and Relation Algebra.
- Have connections with existing categorical logic approach ("quantifiers are adjoints") [3].
- Have definitions of abstract quantifier which are compatible with some of prescriptions with a quantifier *should be*, for example, for Frege, Russell or Quine.
- Generalize notions of "generalized quantifiers", for example, for Henkin, Mostowski, Curry and Lindström.

Based on Voutsadakis's approach of "sets with hierarchy" [4], [5], we propose definitions of formula functor and generalized quantifier:

Lets **var** the category of variables $(Parts(V), \subseteq)$ and **clo** the category of (inflationary, increasing and idempotent) closure operators and morphisms that preserves indiscernibleness ($x \sim y \Leftrightarrow \forall A \subseteq X(x \in \overline{A} \Leftrightarrow y \in \overline{A})$). F : **var** $\to$ **clo** is a formula functor iff:

- (VA) F Preserves Intersections (Voutsadakis's Axiom)
- (CT) Images of monomorphisms are conservative translations (Conservative Translation)
- (PI) If $\sigma \in Aut(V)$ and $N \subseteq V$, then $F_N \cong F_{\sigma[N]}$ (Permutation Invariance)
- (DL) If $M \subseteq N$ and L is disjoint of M and N, then the inclusion $i : F_M \to F_N$ can be "lifted" to $\hat{i} : F_{M\cup L} \to F_{N\cup L}$ (Disjoint Lifting)

A family $\mathcal{Q}$ of quantifier instances is a family of **clo** morphisms, $Q_{N,M} : F_N \to F_M$, associated with inclusion morphisms $F_i : F_M \to F_N$, such that:

- (RD) $Q_{N,M} \circ F_{i_{M,N}} \dashv\vdash id_{F_M}$ (Deformation Retration)

- (PI) If $\sigma \in Aut(V)$ e $M, L \subseteq V$, so for all $Q_{M\cup L,M} : F_{M\cup L} \to F_M \in \mathscr{Q}$ exists $Q_{\sigma[M\cup L],\sigma[M]} : X_{\sigma[M\cup L]} \to F_{\sigma[M]} \in \mathscr{Q}$ such that the diagrams commutes: (Permutation Invariance)

$$\begin{tikzcd}
F_M \arrow[r, "\eta^{\sigma}_{M}"] & F_{\sigma[M]} \\
F_{M\cup L} \arrow[u, "Q_{M\cup L,M}"] \arrow[r, "\eta^{\sigma}_{M\cup L}"'] & F_{\sigma[M\cup L]} \arrow[u, "Q_{\sigma[M\cup L],\sigma[M]}"']
\end{tikzcd}$$

- (DL) If $M \subseteq N$ and L are disjoint of M and N, the diagram commutes: (Disjoint Lifting)

$$\begin{tikzcd}
F_{N\cup L} \arrow[r, "Q_{N\cup L,N}"] & F_N \\
F_{M\cup L} \arrow[u, "F_{i\cup L}"] \arrow[r, "Q_{M\cup L,M}"'] & F_M \arrow[u, "F_i"']
\end{tikzcd}$$

A quantifier is a family of instances of quantifiers $\mathscr{Q} = \{A_{M,N} : M \subseteq N \subseteq V\}$ that is closed under composition.

A quantifier $\mathscr{A} = \{A_{M,N} : M \subseteq N \subseteq V\}$ is universal if:

$F_{i_{M,N}} \circ A_{N,M} \vdash id_{F_N}$ and for all $\mathscr{Q} = \{Q_{M,N} : M \subseteq N \subseteq V\}$, if $F_{i_{M,N}} \circ Q_{N,M} \vdash id_{F_N}$, then $Q_{N,M} \vdash A_{N,M}$ (Infimum)

A quantifier $\mathscr{E} = \{E_{M,N} : M \subseteq N \subseteq V\}$ is existential if:

$id_N \vdash i_{M,N} \circ E_{N,M}$ and for all $\mathscr{Q} = \{Q_{M,N} : M \subseteq N \subseteq V\}$, if $id_{F_N} \vdash F_{i_{M,N}} \circ Q_{N,M}$, then $E_{N,M} \vdash Q_{N,M}$ (Supremum)

Besides that, we want to show a covariant sheaf-like structure of algebras that works as a first-order algebraization of logics. In some cases, this structure can be extended to higher-order logics, generalizes the notions of topos, and have connections with monoidal categories and combinatory logic.

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