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Idempotents in Rational
Abelian Group Algebras

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1 Introduction

Let G be an abelian group. Given a subgroup H of G , the element $\hat{H} = \frac{1}{|H|} \sum_{h \in H} h$ is an idempotent in $\mathbf{Q}G$. It is well-known [5, prop.VI.1.16] that all idempotents in $\mathbf{Q}G$ are \mathbf{Q} -linear combinations of idempotents of this type, which we shall call *group determined idempotents*.

In this note, we shall show that if G is a p -group, then every primitive idempotent in $\mathbf{Q}G$ is actually a difference of just two idempotents of that type, except for the principal idempotent \hat{G} . As a consequence, we shall give a construction for the primitive idempotents in the general case and shall be able to show that their supports are always subgroups of G . Also, the well-known theorem of S. Perlis and G. L. Walker [5] on the structure of $\mathbf{Q}G$ will follow easily from our results. Our arguments are completely ring-theoretical, in the sense that no representation theory is explicitly applied.

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It is worth noting that these idempotents are useful for the study of other classes of group algebras where the primitive idempotents have their support contained in an abelian subgroup. Examples of this use include certain metacyclic groups [2] and groups leading to R.A.loops [3].

2 Primitive Idempotents

Our first statement appeared in [4] but we shall include its proof here for the sake of completeness, since it will play a key role in the sequel.

Lemma 2.1 *Let $C_{p^m} = S_m \supset S_{m-1} \supset \cdots \supset S_0 = 1$ be the set of all subgroups of a cyclic group C_{p^m} of order p^m . Then, the primitive idempotents of the rational group algebra $\mathbb{Q}C_{p^m}$ are $e_0 = \widehat{S_m}$ and $e_i = \widehat{S_{m-i}} - \widehat{S_{m-i+1}}$, $1 \leq i \leq m$.*

In particular, we have that

$$\mathbb{Q}C_{p^m}(1 - \widehat{S_1}) = \mathbb{Q}(\zeta)$$

where ζ denotes a primitive root of unity of order p^m .

Proof. We have that

$$\mathbb{Q}C_{p^m} \cong \frac{\mathbb{Q}[x]}{(x^{p^m} - 1)} \cong \bigoplus_{i=0}^m \frac{\mathbb{Q}[x]}{\Phi_{p^i}} \cong \bigoplus_{i=0}^m \mathbb{Q}(\zeta_i)$$

where Φ_{p^i} denotes the cyclotomic polynomial of order p^i and ζ_i is a root of Φ_{p^i} , $0 \leq i \leq m$.

Hence, we see that $\mathbb{Q}C_{p^m}$ must contain precisely $\varphi(p^m) + 1$ primitive idempotents. Clearly $\widehat{S_{m-i}} \cdot \widehat{S_{m-j}} = \widehat{S_{m-i}}$ if $i \leq j$ so, the idempotents e_i , $0 \leq i \leq m$ are pairwise orthogonal and $\sum_{i=0}^m e_i = 1$, as desired.

Finally, note that $\mathbf{Q}C_{p^m} \cdot \widehat{S}_i \cong \mathbf{Q}C_{p^m}/S_i$; so $[\mathbf{Q}C_{p^m} \cdot \widehat{S}_i : \mathbf{Q}] = p^{m-1}$. Thus, $[\mathbf{Q}C_{p^m} \cdot (1 - \widehat{S}_i) : \mathbf{Q}] = p^m - p^{m-1}$. Since $\mathbf{Q}C_{p^m} \cdot (1 - \widehat{S}_i)$ is a cyclotomic field, we see that $\mathbf{Q}C_{p^m} \cdot (1 - \widehat{S}_i) \cong \mathbf{Q}(\zeta)$ where ζ is a primitive root of unity of order p^m . \square

Let G be a finite abelian p -group and let $\mathbf{Q}G = \bigoplus_{i=1}^t \mathbf{Q}Ge_i$ be its Wedderburn decomposition as a sum of simple components, where each e_i is a primitive idempotent $1 \leq i \leq t$. Then $\mathbf{Q}Ge_i \cong \mathbf{Q}(\zeta_i)$ where ζ_i is a primitive root of unity of a given order, which is a divisor of $|G|$.

Let $\pi : \mathbf{Q}G \rightarrow \mathbf{Q}Ge$ be the projection onto a given simple component i . e. the mapping given by $\pi(\alpha) = \alpha e, \forall \alpha \in \mathbf{Q}G$. Then $\pi(G)$ is a finite subgroup inside $\mathbf{Q}Ge$ and thus, it is cyclic. Set $H = \text{Ker}(\pi|_G) = \{g \in G \mid ge = e\}$ then G/H is cyclic and, since $he = e$; $\mathbf{Q}Ge = \mathbf{Q}Ge\widehat{H} \subset \mathbf{Q}G \cdot \widehat{H}$.

D. B. Coleman [1] has shown that the isomorphism $\varphi : \mathbf{Q}G \cdot \widehat{H} \rightarrow \mathbf{Q}G/H$ defined by $\varphi(g\widehat{H}) = \bar{g} \in G/H, \forall g \in G$, is an isomorphism. The arguments above show that $\mathbf{Q}Ge$ is actually one of the simple components of $\mathbf{Q}G \cdot \widehat{H}$. Let $s \in G$ be an element such that its image \bar{s} in G/H is a generator of this quotient group and denote the respective orders by $o(s) = p^r, o(\bar{s}) = p^m$, where $m \leq r$.

Then lemma 2.1 shows that $\varphi(e)$ is not the principal idempotent of $\mathbf{Q}G/H$, so it is of the form $\varphi(e) = \widehat{S}_{m-i} - \widehat{S}_{m-i+1}$, where $\widehat{S}_{m-i} = \langle \bar{s}^{p^i} \rangle$ and $\widehat{S}_{m-i+1} = \langle \bar{s}^{p^{i-1}} \rangle$. Now, set $S_{r-i} = \langle s^{p^{i-1}} \rangle$ and $S_{m-i+1} = \langle s^{p^{i-1}} \rangle$. It is easily seen that $\varphi(\widehat{H}(\widehat{S}_{m-i} - \widehat{S}_{m-i+1})) = \widehat{S}_{m-i} - \widehat{S}_{m-i+1} = \varphi(e)$. Since φ is an isomorphism, it follows that $e = \widehat{H}(\widehat{S}_{m-i} - \widehat{S}_{m-i+1})$.

If $\varphi(e)$ is the principal idempotent of $\mathbf{Q}G/H$ it easily seen that $e = \widehat{G}$.

Hence, we have shown:

Theorem 2.2 *Let G be a finite abelian p -group and let $e \neq \widehat{G}$ be a primitive idempotent of $\mathbf{Q}G$. Then, there exist a subgroup H of G and an element $s \in G$ such that $G = \langle H, s \rangle$ and $e = \widehat{H}(S_{m-i} - S_{m-i+1})$ where S_{m-i} and S_{m-i+1} are such that $\bar{S}_{m-i} = \langle \bar{s}^{p^i} \rangle$ and $\bar{S}_{m-i+1} = \langle \bar{s}^{p^{i-1}} \rangle$. Equivalently, if we set $C_0 = \langle H, s^{p^i} \rangle$ and $C_1 = \langle H, s^{p^{i-1}} \rangle$ then*

$$e = \widehat{C_0} - \widehat{C_1}$$

Conversely, it follows easily that any idempotent of this form is primitive.

As a consequence we have:

Corollary 2.3 *Let e be a primitive idempotent in the rational group algebra of an abelian p -group G . Then, $\text{supp}(e)$ is a subgroup of G .*

Proof. Notice that, with the above notation, $C_0 \subset C_1$ and the coefficients of a given element $x \in C_0$ in $\widehat{C_0}$ and in $\widehat{C_1}$ are different. Hence, $\text{supp}(e) = C_1$ is a subgroup. \square

Now, let G be a finite abelian group and let $G = G_1 \times \cdots \times G_l$ be its decomposition as a product of Sylow subgroups, i. e. each G_i is a p_i -group, $1 \leq i \leq l$, where p_1, \dots, p_l denotes pairwise different rational primes.

Then, each group algebra $\mathbf{Q}G_i$ has a Wedderburn decomposition of the form $\mathbf{Q}G_i \cong \bigoplus_{j_i=1}^{s_i} \mathbf{Q}(\zeta_{j_i})$ and we have that:

$$\mathbf{Q}G \cong \mathbf{Q}G_1 \otimes \cdots \otimes \mathbf{Q}G_l \cong (\bigoplus_{j_1=1}^{s_1} \mathbf{Q}(\zeta_{j_1})) \otimes \cdots \otimes (\bigoplus_{j_l=1}^{s_l} \mathbf{Q}(\zeta_{j_l})) \cong$$

$$\bigoplus_{j_1, \dots, j_l} (\mathbf{Q}(\zeta_{j_1}) \otimes \cdots \otimes \mathbf{Q}(\zeta_{j_l})) \cong \bigoplus_{j_1, \dots, j_l} \mathbf{Q}(\zeta_{j_1} \cdots \zeta_{j_l})$$

Hence, the number of simple components in $\mathbf{Q}G$ is $s = s_1 \cdots s_l$.

Clearly, if we denote by $\{e_{j_1}\}_{j_1=1}^{s_1}, \dots, \{e_{j_l}\}_{j_l=1}^{s_l}$ the corresponding sets of primitive idempotents, then the elements $\{e_{i_1} \cdots e_{i_l}\}_{i_1, \dots, i_l}$ form a set of orthogonal idempotents in $\mathbf{Q}G$ and are, thus, the primitive idempotents of $\mathbf{Q}G$.

We are now read to prove:

Theorem 2.4 *Let G be a finite abelian group with l Sylow subgroups and let $e = \widehat{G}$ be a primitive idempotent in $\mathbf{Q}G$. Then, there exist 2^w subgroups D_1, \dots, D_{2^w} , $w \leq l$ in G such that e is a linear combination of the elements \widehat{D}_i , $1 \leq i \leq 2^w$ with coefficients equal to ± 1 . Furthermore, one of these subgroups, say D_1 contains all others and $\text{supp}(e) = D_1$.*

Proof. We shall proceed by induction on l , the number of Sylow subgroups of G . If $l = 1$, our statement reduces to theorem 2.2 e Corollary 2.3.

So, assume that the theorem holds for groups with $l-1$ Sylow subgroups and write $G = G_1 \times \cdots \times G_l$, as before. A primitive idempotent $e \in \mathbf{Q}G$ can be written as a product $e = e_1 e_2$ where e_1 is a primitive idempotent in $\mathbf{Q}(G_1 \times \cdots \times G_{l-1})$ and e_2 is a primitive idempotent in $\mathbf{Q}G_l$.

From the induction hypothesis, there exist subgroups D_i , $1 \leq i \leq 2^{w-1}$, $w \leq l$, in $G_1 \times \cdots \times G_{l-1}$ such that:

$e_1 = \sum_{i=1}^{2^{w-1}} a_i \widehat{D}_i$ where $a_i = \pm 1$, $1 \leq i \leq 2^{w-1}$, and we may assume that $D_i \subset D_1$, $\forall i$. Theorem 2.2 shows that there exist subgroups C_0, C_1 of G_l such that:

$$e_2 = \widehat{C}_0 - \widehat{C}_1, \text{ with } C_0 \subset C_1$$

$$e = \sum_{i=1}^{2^w-1} a_i \widehat{D}_i \widehat{C}_0 - \sum_{i=1}^{2^w-1} a_i \widehat{D}_i \widehat{C}_1$$

Since C_0, C_1 are p_i -groups and p_i is relatively prime to $|D_i|$, the products $D_i C_0$ and $D_i C_1$ are actually direct products and $\widehat{D}_i \widehat{C}_0 = D_i \widehat{\times} C_0, \widehat{D}_i \widehat{C}_1 = D_i \widehat{\times} C_1, 1 \leq i \leq 2^w-1$.

It is easy to see that these groups are pairwise different and, since $D_i \subset D_1, C_0 \subset C_1$ it follows that $D_i \times C_j \subset D_1 \times C_1, 1 \leq i \leq 2^w-1, j = 0, 1$. Hence, e has been written as a linear combination of 2^w group determined idempotents with coefficients that are equal to ± 1 .

Furthermore, the induction hypothesis also states that $\text{supp}(e_1) = D_1$ and we have seen in corollary 2.3 that $\text{supp}(e_2) = C_1$ so, we write e_1, e_2 in the form

$$e_1 = \sum_{x \in D_1} a(x)x$$

$$e_2 = \sum_{y \in C_1} b(y)y$$

Then, $e = \sum_{x, y} a(x)b(y)xy$. Since $D_1 C_1$ is a direct product no two group elements in this expression coincide; hence, $\text{supp}(e) = D_1 \times C_1$ as stated. \square

As an easy consequence, we obtain a slightly refined version of [5, prop. VI.1.16].

Corollary 2.5 *Every idempotent in \mathbf{QG} is a \mathbf{Z} -linear combination of group determined idempotents.*

3 The Structure of \mathbf{QG}

We saw, in the previous section, that simple components of \mathbf{QG} can be obtained as components of group algebras of cyclic factors of G . We shall look more carefully at this correspondence and establish a 1 - 1 correspondence between simple components and cyclic factors.

First, let G be an abelian p -group. We consider a subgroup H of G such that G/H is cyclic and let $s \in G$ be an element such that $G/H = \langle \bar{s} \rangle$, as before. Set $\alpha(\bar{s}) = p^n$ in G/H and set $C = \langle H, s^{p^{n-1}} \rangle$. Clearly, C is independent of the choice of s since it is the pre-image of the subgroup of order p of G/H . Now, to the subgroup H we assign the element $e = \hat{H} - \hat{C}$ which is one of the primitive idempotents described in theorem 2.2.

Conversely, let $e \in \mathbf{QG}$ be a primitive idempotent. We know that there exist subgroups C_0, C_1 of G such that $e = \sum_{g \in C_1} e(g)g$ and it is easy to recognize C_0 ; namely:

$$C_0 = \{ g \in C_1 \mid |e(g)| \text{ is minimum} \}.$$

Clearly, G/C_0 is cyclic. So, to the idempotent e we assign the subgroup C_0 .

Since both correspondences are inverses to one another, we have just shown:

Lemma 3.1 *Let G be a finite abelian p -group. Then the number of simple components in \mathbf{QG} is equal to the number of cyclic factors of G .*

Now, given a finite abelian group G write, as before, its decomposition as a product of Sylow subgroups, $G = G_1 \times \cdots \times G_l$. Given a subgroup H of G ,

it can be written in the form $H = H_1 \times \cdots \times H_l$, where $H_i \subset G_i$, $1 \leq i \leq l$; hence $G/H \cong G_1/H_1 \times \cdots \times G_l/H_l$. Thus G/H is cyclic if and only if each factor G_i/H_i is cyclic, $1 \leq i \leq l$. This remark, together with the fact that the number of simple components in $\mathbf{Q}G$ is the product of the number of simple components in each algebra $\mathbf{Q}G_i$, $1 \leq i \leq l$, shows that, also in this case, the number of simple components of $\mathbf{Q}G$ is equal to the number of cyclic factor of G .

Now, we are read to prove the following.

Theorem 3.2 (*S. Perlis and G. L. Walker [5]*) *Let G be a finite abelian group. Then*

$$\mathbf{Q}G \cong \bigoplus_{d|n} a_d \mathbf{Q}(\zeta_d)$$

where ζ_d denotes a primitive root of unity of order d and a_d stands for the number of cyclic factors of G of order d .

Proof. Assume first that G is a p -group. Notice that the description of a primitive idempotent e allows us to describe the simple component $\mathbf{Q}Ge$. In fact, if $e = \widehat{H} - \widehat{C}$ as above, then its image in $\mathbf{Q}G/H$ is $1 - \widehat{S}$ and lemma 2.1 shows that $\mathbf{Q}G/H \cdot (1 - \widehat{S}) \cong \mathbf{Q}(\zeta)$ where ζ is a primitive p^n -root of unity.

So, the number of simple components in $\mathbf{Q}G$ isomorphic to $\mathbf{Q}(\zeta)$ is equal to the number of cyclic factors of G of order p^n .

The statement for arbitrary finite abelian groups follows by working with the group algebras of its Sylows subgroups, as above. \square

As a final remark, we would like to observe that the fact that $\text{supp}(e)$ is a subgroup holds only when assuming that e is primitive. In fact if G is a

cyclic p -group this will still hold for every idempotent since in this case the supports of primitive idempotents form a chain and, when adding some of them, it is easily seen that cancellations will not occur.

However, as soon as we take groups which involve more than one prime, the property will not hold, even in the cyclic case. Take as an example $G = \langle a \rangle \times \langle b \rangle$ with $o(a) = 2$ and $o(b) = 3$. Then the primitive idempotents of $\mathbf{Q} \langle a \rangle$ are $e_1 = \frac{1+a}{2}$ and $e_2 = \frac{1-a}{2}$ while the primitive idempotents of $\mathbf{Q} \langle b \rangle$ are $f_1 = \frac{1+b+b^2}{3}$ and $f_2 = \frac{2-b-b^2}{3}$.

Hence, $e = e_1 f_1 + e_2 f_2 = \frac{1}{6}(3 - a + 2ab + 2ab^2)$ is an idempotent in $\mathbf{Q}(\langle a \rangle \times \langle b \rangle)$ and $\text{supp}(e) = \{ 1, a, ab, ab^2 \}$ is not a subgroup.

Also, we would like to remark that idempotents of abelian group algebras over cyclotomic extensions of the rationals can be exhibited in a similar fashion since they can be obtained as homomorphic images of rational group algebras of enlarged groups.

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