

The Schrödinger Equation, the Zero-Point Electromagnetic Radiation, and the Photoelectric Effect

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Abstract A Schrödinger type equation for a mathematical probability amplitude $\Psi(x, t)$ is derived from the generalized phase space Liouville equation valid for the motion of a microscopic particle, with mass M and charge e , moving in a potential $V(x)$. The particle phase space probability density is denoted $Q(x, p, t)$, and the entire system is immersed in the “vacuum” zero-point electromagnetic radiation. We show, in the first part of the paper, that the generalized Liouville equation is reduced to a simpler Liouville equation in the equilibrium limit where the small radiative corrections cancel each other approximately. This leads us to a simpler Liouville equation that will facilitate the calculations in the second part of the paper. Within this second part, we address ourselves to the following task: Since the Schrödinger equation depends on \hbar , and the zero-point electromagnetic spectral distribution, given by $\rho_0(\omega) = \hbar\omega^3/2\pi^2c^3$, also depends on \hbar , it is interesting to verify the possible dynamical connection between $\rho_0(\omega)$ and the Schrödinger equation. We shall prove that the Planck’s constant, present in the momentum operator of the Schrödinger equation, is deeply related with the ubiquitous zero-point electromagnetic radiation with spectral distribution $\rho_0(\omega)$. For simplicity, we do not use the hypothesis of the exis-

tence of the L. de Broglie matter-waves. The implications of our study for the standard interpretation of the photoelectric effect are discussed by considering the main characteristics of the phenomenon. We also mention, briefly, the effects of the zero-point radiation in the tunneling phenomenon and the Compton’s effect.

Keywords Foundations of quantum mechanics · Schrödinger equation · Zero-point electromagnetic fluctuations · Photoelectric effect

1 Short Introduction

In 1932, E. Wigner published an important paper [1, 2] where he introduced what is called today *the Wigner’s quasi probability function* or simply *Wigner’s phase space function*. We shall denote it by $Q(x, p, t)$ and, according to the Wigner proposal, it is connected with a Schrödinger equation solution $\Psi(x, t)$ by the expression:

$$Q(x, p, t) \equiv \frac{1}{\pi\hbar} \int_{-\infty}^{+\infty} \Psi^*(x-y, t) \Psi(x+y, t) e^{-\frac{2ipy}{\hbar}} dy, \quad (1)$$

where \hbar is the Planck’s constant. Wigner’s intention was to obtain a quantum mechanical description of the phase space microscopic and macroscopic phenomena as demanded by the *correspondence principle* [3–5]. We recall that the Schrödinger equation, namely

$$i\hbar \frac{\partial}{\partial t} \Psi(x, t) = \left[\frac{-\hbar^2}{2M} \frac{\partial^2}{\partial x^2} + V(x) \right] \Psi(x, t), \quad (2)$$

also depends explicitly on the Planck’s constant \hbar [6].

We want to emphasize that neither Wigner nor Schrödinger clarified the relation of the constant \hbar with

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the classical stochastic zero-point electromagnetic radiation. Notice, however, that the existence of the classical stochastic zero-point radiation was considered *seriously* by Albert Einstein and Otto Stern in the period 1911–1913 [7] and by T. W. Marshall in an important paper published in 1963 [8]. Interesting experimental observations of the “vacuum” zero-point fluctuations are described in more recent publications [9].

We also would like to mention that, if applied to an arbitrary solution $\Psi(x, t)$ of the Schrödinger equation (2), the expression (1) may lead to *negative* phase space probability densities. In other words, the relation (1) may be considered as the definition of an *auxiliary* [10] function, which has some properties of a classical distribution, useful to calculate the statistics on the configuration and momentum space. Interesting examples are the excited states of the harmonic oscillator [10–12].

The definition (1), and the Schrödinger equation (2), do not lead to the classical Liouville equation except when $V(x)$ is quadratic in the variable x (see ref. [3]). However, according to the correspondence principle, one should necessarily obtain the Liouville equation in *the case of a macroscopic particle* moving in a *generic* potential $V(x)$.

Starting with the *generalized Liouville equation in phase space*, we shall obtain the mathematical and the physical conditions necessary to relate it with an equation that is formally identical to the Schrödinger equation for a *mathematical probability amplitude* $\Psi(x, t)$ [13, 14].

Our presentation is organized as follows. We give, within Section 2, the physical background and the mathematical approximations necessary to connect the generalized Liouville equation to the mathematical configuration space probability density $|\Psi(x, t)|^2$. The relation between the approximated Liouville equation and the Schrödinger equation is explained in details within Section 3. The role of the random zero-point radiation with spectral distribution $\rho_0(\omega)$ given in [13, 14].

$$\rho_0(\omega) \equiv \lim_{T \rightarrow 0} \left[\frac{\omega^2}{\pi^2 c^3} \left(\frac{\hbar\omega}{2} + \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \right) \right] = \hbar\omega^3 / 2\pi^2 c^3, \tag{3}$$

is also explained within the Section 3. Some implications of our study for the standard interpretation of the photoelectric effect are presented in Section 4. Section 5 is devoted to a brief discussion in which we mention the role of $\rho_0(\omega)$ in the free particle propagator, in the tunneling phenomenon and in the Compton effect.

2 Physical Background and Mathematical Approximations

Within this section, we shall use important results obtained before by de la Peña et al. [13, 14]. Following these authors, one can say that to study the dynamics of the particle immersed in the random zero-point field (ZPF), we start from the general stochastic Liouville equation for the density of points in the particle’s phase space $R(x, p, t)$. In one dimension, we get

$$\frac{\partial}{\partial t} R + \frac{\partial}{\partial x}(\dot{x}R) + \frac{\partial}{\partial p}(\dot{p}R) = 0, \tag{4}$$

with the time derivatives of the variables x and p determined by the couple of equations of motion:

$$\dot{x} = p/M, \dot{p} = F(x) + M\tau \ddot{x} + eE(t). \tag{5}$$

The electric part of the ZPF, $E(t)$, has been taken in the long-wavelength approximation because the dynamical effect of this field turns out to be of relevance only for wavelengths much larger than the size of typical (atomic) systems, as will become clear below. Further, the contribution of the magnetic force is neglected in the non-relativistic case, and the radiation reaction is approximated, as usual, by $M\tau \ddot{x}$, with $\tau = 2e^2/3Mc^3(10^{-23}$ s for the electron). It is not of our interest to describe the particle motion for a particular (and unknown) realization of the field, but for an ensemble of particles subject to the set of all realizations. Hence, we take the average over these realizations and derive the equation for the phase-space-averaged density $Q(x, p, t) \equiv \overline{R(x, p, t)}$. Using the procedures proposed before by de la Peña et al. [13–15], we get

$$\begin{aligned} \frac{\partial}{\partial t} Q(x, p, t) + \frac{1}{M} \frac{\partial}{\partial x} p Q(x, p, t) + \frac{\partial}{\partial p} \{ [F(x) + M\tau \ddot{x}] Q(x, p, t) \} \\ = e^2 \frac{\partial}{\partial p} \hat{D}(t) Q(x, p, t), \end{aligned} \tag{6}$$

where $\hat{D}(t)$ is an integro-differential operator that can be expressed as the infinite sum, namely

$$\begin{aligned} e^2 \frac{\partial}{\partial p} \hat{D}(t) Q(x, p, t) \\ = e \frac{\partial}{\partial p} \hat{P} E \sum_{k=0}^{\infty} \left[e \hat{G} \frac{\partial}{\partial p} (1 - \hat{P}) E \right]^{2k+1} Q(x, p, t). \end{aligned} \tag{7}$$

Here, \hat{P} stands for the smoothing operator that averages over the realizations of $E(\hat{P}A = \bar{A})$, and \hat{G} represents the inverse of the operator of systematic evolution. In terms

of the Liouville operator for the problem, $\hat{L} = \frac{1}{M} \frac{\partial}{\partial x} p + \frac{\partial}{\partial p} (F + M\tau \ddot{x})$, one can show that [14, 15]

$$\begin{aligned} \hat{G}A(x, p, t) &= \left(\frac{\partial}{\partial t} + \hat{L} \right)^{-1} A(x, p, t) \\ &= \int_0^t e^{-\hat{L}(t-t')} A(x, p, t') dt', \end{aligned} \tag{8}$$

where $A(x, p, t)$ is any phase space function.

The terms under the sum in (7) contain factors of the form $[e^2 E(t)E(t')]^{2k+1}$ averaged over the realizations of the field. The first of them ($k = 0$) contains the field correlation $\overline{E(t)E(t')}$ and is therefore proportional to the power spectrum of the ZPF, namely

$$\langle E(t)E(t') \rangle = \overline{E(t)E(t')} = \frac{4\pi}{3} \int_0^\infty d\omega \rho_0(\omega) \cos\omega(t - t'), \tag{9}$$

where

$$\rho_0(\omega) = \frac{\hbar\omega^3}{2\pi^2c^3}, \tag{10}$$

corresponding to an energy per normal mode of value $\hbar\omega/2$. According to de la Peña et al., this is the point of entry of \hbar into the theory [14]. We shall show below why this cannot be the only point of entrance of \hbar into the theory.

According to the above analysis, it is possible to conclude that (6) is equivalent to a simpler Liouville equation which can be written as

$$\frac{\partial Q(x, p, t)}{\partial t} + \frac{p}{M} \frac{\partial Q(x, p, t)}{\partial x} + F(x) \frac{\partial Q(x, p, t)}{\partial p} \simeq 0, \tag{11}$$

in the equilibrium limit where the radiation reaction term and the diffusion term in (6) *compensate* each other in an approximate way [13].

We stress that the equation (11) does not depend on the Planck's constant \hbar . Therefore, the dynamical connection with $\rho_0(\omega)$ has to be done in a different manner.

Following Hayakawa [4], we shall consider the following Fourier transform defined by

$$\tilde{Q}(x, y, t) \equiv \int_{-\infty}^\infty dp Q(x, p, t) e^{\frac{2ipy}{\alpha}}, \tag{12}$$

where y is an auxiliary coordinate, α is an auxiliary constant with dimension of action, and $Q(x, p, t)$ is the solution of the equation (11). The constant α will be fixed later with the use of physical and mathematical criteria. We stress that Hayakawa used $\alpha = \hbar$ as an assumption. This is an important observation.

From the classical stochastic point of view, we know that $Q(x, p, t)$ is always real and positive definite and the

classical configuration space probability density can be defined mathematically as

$$P(x, t) \equiv \int dp Q(x, p, t) = \lim_{y \rightarrow 0} \tilde{Q}(x, y, t). \tag{13}$$

Notice that this probability density is the *average* distribution over the realizations of the *fluctuating* electric field indicated by $E(t)$ in the equations (5) and (6).

Since $P(x, t)$ is real and positive, it is *always possible* to introduce a mathematical probability amplitude $\Psi(x, t)$ as

$$P(x, t) \equiv |\Psi(x, t)|^2 = \Psi^*(x, t)\Psi(x, t), \tag{14}$$

where the factorization $\Psi^*(x, t)\Psi(x, t)$ is completely general in the *limit* $y \rightarrow 0$. In other words, the equations (13) and (14) can be summarized as

$$\begin{aligned} \Psi^*(x, t)\Psi(x, t) &\equiv \lim_{y \rightarrow 0} \int_{-\infty}^\infty dp Q(x, p, t) e^{\frac{2ipy}{\alpha}} \\ &\equiv \lim_{y \rightarrow 0} \tilde{Q}(x, y, t), \end{aligned} \tag{15}$$

where the limiting process considered above means that the behavior of $\tilde{Q}(x, y, t)$ when $y \approx 0$ is *very important*. However, the above definition (15) is not enough to determine a differential equation for the amplitude $\Psi(x, t)$. In order to achieve this goal, we shall use the approximate Liouville equation (11). We shall see, in the next section, that the approximate Liouville equation has a *close relation* with the Schrödinger type equation for the mathematical probability amplitude $\Psi(x, t)$.

3 The Approximate Liouville Equation and its Connection with the Schrödinger Equation

It is a widespread belief [3] that the connection between the Wigner quasi probability function (1), the Liouville equation (11), and the Schrödinger equation (2) is only possible for quadratic potentials in the x variable. *We shall extend this connection to a more general potential $V(x)$, that is, beyond the case of the harmonic potential.* In order to achieve this goal, we shall use a procedure similar to that presented by Hayakawa [4] and Dechoum et al. [5]. Dechoum et al. discussed several classical aspects of the *Pauli-Schrödinger equation* written in the spinorial notation. Here, we are treating the spinless case.

To obtain the differential equation for $\Psi(x, t)$, we shall use the Wigner type transformation defined previously in the equation (12). Regarded as a simple mathematical definition, we stress that this Fourier transform contains the same dynamical information carried by the phase space density $Q(x, p, t)$.

We observe that, due to the definitions (12), (13), (14), and (15), the Wigner type transform $\tilde{Q}(x, y, t)$ is a complex

function that has a simple mathematical meaning in the limit $|y| \rightarrow 0$, that is,

$$\lim_{y \rightarrow 0} \tilde{Q}(x, y, t) = |\Psi(x, t)|^2 = \Psi(x, t)\Psi^*(x, t). \tag{16}$$

For this reason, we shall consider, in what follows, the definition (12) only for small values of y . Our goal is to obtain the differential equation for $\Psi(x, t)$, from the differential equation for $\tilde{Q}(x, y, t)$, in the limit of small y .

Our first step is to consider the equation

$$\frac{\partial}{\partial t} \tilde{Q}(x, y, t) = \int_{-\infty}^{\infty} \frac{\partial Q}{\partial t} e^{\frac{2ipy}{\alpha}} dp. \tag{17}$$

Using the Liouville equation (11) and after an integration by parts, we obtain

$$\begin{aligned} \frac{\partial \tilde{Q}}{\partial t} &= - \int_{-\infty}^{\infty} \left[\frac{p}{M} \frac{\partial Q}{\partial x} + F(x) \frac{\partial Q}{\partial p} \right] e^{\frac{2ipy}{\alpha}} dp \\ &= \left[-\frac{i\alpha}{2M} \frac{\partial^2}{\partial y \partial x} + F(x) \frac{2iy}{\alpha} \right] \int_{-\infty}^{\infty} Q(x, p, t) e^{\frac{2ipy}{\alpha}} dp. \end{aligned} \tag{18}$$

Substituting (12) into (18), we get the Hayakawa [4] equation for $\tilde{Q}(x, y, t)$, namely

$$i\alpha \frac{\partial}{\partial t} \tilde{Q}(x, y, t) = \left[\frac{(-i\alpha)^2}{2M} \frac{\partial^2}{\partial y \partial x} - 2yF(x) \right] \tilde{Q}(x, y, t). \tag{19}$$

In order to facilitate the calculations, it is convenient to use new variables, namely $s = x - y$ and $r = x + y$, so that the equation (19) can be written as

$$i\alpha \frac{\partial}{\partial t} \tilde{Q}(r, s, t) = \left[\frac{(-i\alpha)^2}{2M} \left(\frac{\partial^2}{\partial r^2} - \frac{\partial^2}{\partial s^2} \right) - (r - s)F\left(\frac{r+s}{2}\right) \right] \tilde{Q}(r, s, t). \tag{20}$$

According to (16), we want an equation for $\tilde{Q}(r, s, t)$, when $y = (r - s)/2 \rightarrow 0$, that is, when $r \rightarrow s$. Consequently, we must consider that the points r and s are arbitrarily close in the equation (20). Therefore, in accordance to the mean value theorem, one can consider that

$$-(r - s)F\left(\frac{r+s}{2}\right) \simeq - \int_s^r F(\xi) d\xi \simeq V(r) - V(s), \tag{21}$$

is a very good approximation [4]. Notice that the mean value theorem requires only that the integrand function (force $F(x) = -V'(x)$ in this case) is a continuous function in the small integration interval under consideration.

The use of equation (21) will imply in a great simplification of the problem of finding the differential equation for the mathematical probability amplitude $\Psi(x, t)$, because

(20) becomes separable in the variables r and s as we shall see below. Substituting (21) into (20), we get the equation

$$i\alpha \frac{\partial}{\partial t} \tilde{Q}(r, s, t) = \left[\frac{(-i\alpha)^2}{2M} \left(\frac{\partial}{\partial r^2} - \frac{\partial}{\partial s^2} \right) + V(r) - V(s) \right] \tilde{Q}(r, s, t), \tag{22}$$

which was obtained by Hayakawa in 1965. One can verify that the above equation is separable in the variables r and s if we introduce a function $\tilde{Q}(r, s, t)$ such that

$$\tilde{Q}(r, s, t) \equiv \Psi^*(s, t)\Psi(r, t). \tag{23}$$

Notice that equation (23) is in accordance with our previous equation (14). Thus, (22) is decomposed into the equation

$$i\alpha \frac{\partial}{\partial t} \Psi(x, t) = \left[\frac{1}{2M} \left(-i\alpha \frac{\partial}{\partial x} \right)^2 + V(x) \right] \Psi(x, t), \tag{24}$$

and its complex conjugate. The equation (24) is formally identical to the Schrödinger equation provided that the operator $-i\alpha \partial/\partial x$ is identified with the Schrödinger momentum operator $p = -i\hbar \partial/\partial x$, that is, the auxiliary constant α is identified with \hbar . See the paragraphs containing our equations (28), (29), and (30), where we discuss this point using the Heisenberg picture.

Using a more explicit notation, we get, from the equation (24), the result

$$i\hbar \frac{\partial \Psi}{\partial t}(x, t) = \left[\frac{1}{2M} \left(-i\hbar \frac{\partial}{\partial x} \right)^2 + V(x) \right] \Psi(x, t) \equiv \hat{H} \Psi(x, t), \tag{25}$$

which has exactly the form of the Schrödinger equation for probability amplitude $\Psi(x, t)$. Notice that \hat{H} is the Hamiltonian operator of the system and the equation (25) may possess a complete set of solutions, $\Psi_n(x, t) \equiv \phi_n(x) \exp(-i\epsilon_n t/\hbar)$, where the functions $\phi_n(x)$ are such that

$$\hat{H} \phi_n(x) \equiv \epsilon_n \phi_n(x). \tag{26}$$

The constants ϵ_n are the “energy levels” and the functions $\phi_n(x)$ are the “eigenfunctions” of the system [20].

We stress that, due to our approximations, the excited states of the set $\{\epsilon_n, \phi_n(x)\}$ do not decay (see the references [16–18] for the physical explanation of the decay processes).

We know that the Planck’s constant \hbar is related with the electromagnetic fluctuation phenomena characteristic of Stochastic Electrodynamics (SED) [11, 13] and Quantum Electrodynamics (QED) [17, 18]. The most important of these fluctuations are associated with the zero temperature electromagnetic radiation which has a spectral distribution such that

$$\rho_0(\omega) = \frac{\hbar \omega^3}{2\pi^2 c^3}, \tag{27}$$

in both QED and SED. We recall that these two theories have many features in common [11, 17, 18]. It is also interesting to recall that the *classical* effects of the zero-point radiation were discovered very early by M. Planck in the period 1911–1912. The quantum zero-point radiation was discovered later, in 1927, by Paul Dirac [12].

Some properties of the spectral distribution $\rho_0(\omega)$ will be explicitly used at this point. For the moment, we would like to say that, using the *Heisenberg picture*, Sokolov and Tumanov [19] and Milonni [21, 22] were able to show that the commutation relation between the position operator $x(t)$ and the momentum operator $p(t)$ are strongly related to $\rho_0(\omega)$ given in the equation (27). Moreover, according to de la Peña et al. [23], who use a *classical* system as a starting point, the action of the electric zero-point field on matter is essential and ultimately leads the matrix (or Heisenberg) formulation of quantum mechanics for the motion in an *arbitrary* potential $V(x)$. This is an important conclusion. Therefore, according to these authors, a “free” electron (mass M and charge e) has a non-relativistic equation of motion for the position operator $x(t)$ such that

$$M\ddot{x}(t) = \frac{2e^2}{3c^3} \ddot{\ddot{x}}(t) + eE(t), \tag{28}$$

within the Heisenberg picture [19, 21–23]. Recall that, according to the authors quoted in ref. [23], $E(t)$ can be a *fluctuating classical electric field*. We shall use the Heisenberg approach in what follows.

Within the Heisenberg picture, the equation (28) can be easily solved. We get the position operator $x(t)$ and the canonical momentum operator $p(t) \equiv M\dot{x} + \frac{e}{c}A(t)$, where $A(t)$ is the quantized vector potential ($E = -\frac{1}{c}\frac{\partial A}{\partial t}$).

From the solution for $x(t)$, we get $p(t)$. Therefore, it is possible to calculate the commutation relation between the position operator $x(t)$ and the canonical momentum operator $p(t)$. The result is

$$[x(t), p(t)] = [x, M\dot{x}] = 4i\pi^2 c^3 \tau \int_0^\infty d\omega \frac{\rho_0(\omega)}{\omega^3(1+\tau^2\omega^2)} = i\hbar, \tag{29}$$

where $\tau = 2e^2/3Mc^3$. This is an interesting and relevant result because only $\rho_0(\omega)$ depends on \hbar in the above integral. Moreover, the result (29) is independent of the charge of the particle. Since the Heisenberg and the Schrödinger pictures are equivalent, one concludes that the momentum operator used in the Schrödinger type equation (25), namely

$$p = -i\hbar\partial/\partial x, \tag{30}$$

is *deeply related with the “vacuum” electromagnetic fluctuations* with spectral distribution $\rho_0 = \hbar\omega^3/2\pi^2c^3$.

4 The Photoelectric Effect and the Zero-Point Radiation

A three-dimensional generalization of the one-dimensional Schrödinger type equation (25) is

$$i\hbar\frac{\partial\Psi(\vec{r}, t)}{\partial t} = \left[\frac{-\hbar^2}{2M}\nabla^2 + V(\vec{r}) - e\vec{r} \cdot \vec{E}_c(\vec{r}, t) \right] \Psi(\vec{r}, t), \tag{31}$$

where we have used the *dipole* approximation. The term $-e\vec{r} \cdot \vec{E}_c(\vec{r}, t)$ is the interaction energy, e is the electron charge, and M is the electron mass.

In the above equation, $\vec{E}_c(\vec{r}, t)$ is a *classical deterministic* electric field, associated with the *incident* radiation, and $V(\vec{r})$ is the atomic Coulomb potential for instance. We are not explicitly considering the stochastic “vacuum” fields in the equation (31) because this may generate double counting. *The operator $-i\hbar\vec{\nabla}$ already contains the effects of the zero-point electromagnetic fields.*

An interesting physical feature of the above three-dimensional generalization of the mathematical Schrödinger type equation (25) is that it leads to a novel interpretation of the photoelectric effect.

The reader can find a good discussion of this point in the subsection entitled “*The photoelectric effect*” (pgs. 40, 41 and 42) of the article by Scully and Sargent III published in 1972 [24]. Within this paper, it is used as a classical deterministic electric field, of a monochromatic plane wave, polarized in the z direction, which is denoted $\vec{E}_c(\vec{r}, t) = \hat{z}E_0 \cos(\omega t - ky)$, where E_0 is the amplitude of the electric field and ω is the corresponding angular frequency ($\omega = kc$).

According to an standard analysis, based on the equation (31) and on the “Fermi Golden Rule”, these authors obtained the expression [24, 25]

$$\frac{dP_f}{dt} = 2\pi \left| \langle f | e\vec{r} \cdot \hat{z} | g \rangle \right|^2 \hbar^2 E_0^2 t \delta \left(\omega - \frac{\epsilon_f - \epsilon_g}{\hbar} \right), \tag{32}$$

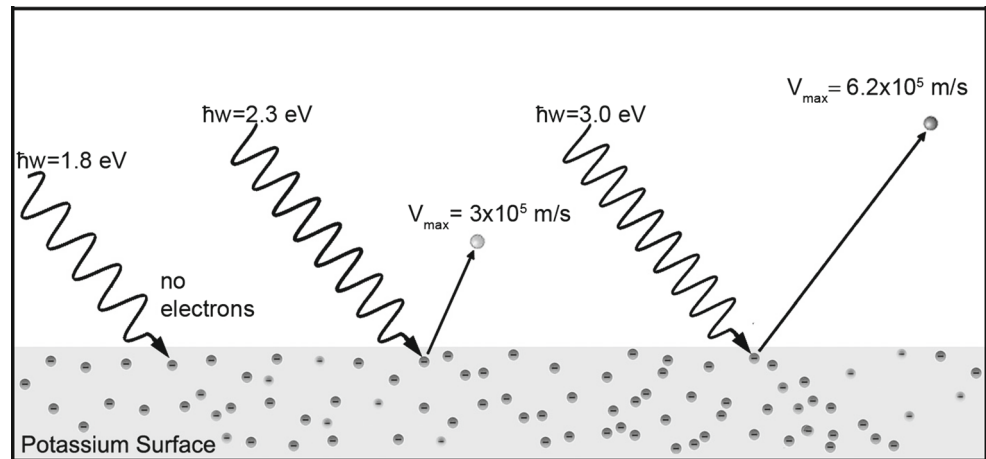
for the probabilistic rate of emission of photo-electrons from a bounded initial state of energy ϵ_g to the final state of energy $\epsilon_f = Mv^2/2$, where ϵ_f is the continuous energy of the ejected electron and $e\vec{r}$ is the electric dipole of the atom. Notice, however, that the authors mentioned in references [24] and [25] do not mention the role of the zero-point radiation.

From the argument of the delta function in (32), we see that

$$\omega = \frac{\epsilon_f - \epsilon_g}{\hbar}. \tag{33}$$

The Planck’s constant \hbar appearing above was introduced within the mathematical Schrödinger type equation (31),

Fig. 1 Illustration of the photoelectric effect from a potassium surface (2.0 eV needed to eject electrons). In the figure, we indicate the characteristic energy $\hbar\omega$ of the “photons”. For instance $\hbar\omega = 1.8$ eV, $\hbar\omega = 2.3$ eV and $\hbar\omega = 3.1$ eV. The maximum velocity V_{max} of the ejected electrons is indicated for each frequency ω (see the Einstein equation (34)). Notice that the electrons inside the metal move with different energies



without using the concept of *photon*. Moreover, according to the interpretation of the Schrödinger equation presented within our paper (see the Section 3), the origin of \hbar can be traced back to the expressions (29) and (30) which involves $\rho_0(\omega)$, that is, *the Planck’s constant, in the equation (33), has its origin in the zero-point radiation with spectral distribution $\rho_0 = \hbar\omega^3 / (2\pi^2 c^3)$. Therefore, the famous Einstein equation for the photoelectric phenomenon follows from (33) and can be put in the form [24, 25].*

$$\hbar\omega = \epsilon_f - \epsilon_g \geq \frac{Mv^2}{2} + \phi, \tag{34}$$

where ϕ is the work function characteristic of the material and $Mv^2/2$ is the continuous kinetic energy of the ejected electron. See Fig. 1 for an illustration.

In order to better describe the “collision” (or scattering) process, we need the equations for the *momentum conservation*. We recall that Einstein introduced the *classical concept* of “needle radiation” [12] which has “momentum and energy”. See Fig. 2 for an illustration.

The role of the *momentum conservation equations* is very well illustrated in the section III of the paper by Kidd et al. [26]. These authors gave very interesting examples relating to the *velocity variation* of the recoiling electron and the *direction of the recoil* of the parent atom. In other words, it is shown that for some values of $\hbar\omega$ and ϕ , the value of the momentum of the recoiling electron can be *much larger* than the momentum of the incident “photon” due to the recoil of the parent atom (see Fig. 2 in Section 3 of the reference [26]).

Consequently, the Einstein’s equation for the photoelectric effect can be interpreted in a classical manner where the zero-point electromagnetic field plays a very important role. This opens a new way of understanding the photoelectric effect. It is our intention to explore this interpretation in future calculations, by considering the atoms in *special physical conditions* (flying between very close to conducting Casimir plates [27], localized close to some element of

an electric circuit [28] or moving inside a solid material [29]). In these cases, there are important modifications in the spectral distribution $\rho_0(\omega)$.

5 Brief Discussion

We want to recall that within this paper, we have clarified the deep relationship between the Schrödinger equation, the zero-point electromagnetic radiation, and the photoelectric effect. Another important observation is that the kinetic energy operator present in the unidimensional equation (25), namely

$$\frac{1}{2M} \left(-i\hbar \frac{\partial}{\partial x} \right)^2, \tag{35}$$

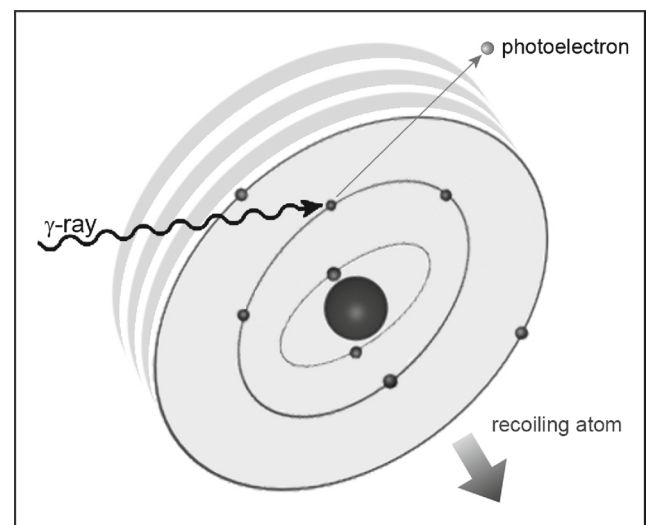


Fig. 2 Illustration of the momentum conservation in the photoelectric phenomenon. In this case, the atom will suffer a recoil in the indicated direction. In other words, the parent atom (or crystal) plays an important role in the photoelectric effect (see the Section 3 of the reference [26])

generates the spreading of the mathematical probability amplitude $\Psi(x, t)$. In other words, if $V(x) = 0$ we get from the equation (25) the “free” particle propagator [30]

$$K(x, t|x_0, t_0) = \sqrt{\frac{M}{2\pi i\hbar(t-t_0)}} \exp\left[\frac{iM(x-x_0)^2}{2\hbar(t-t_0)}\right], \quad (36)$$

which connects $\Psi(x_0, t_0)$ with $\Psi(x, t)$ in the time interval $t - t_0$, that is

$$\Psi(x, t) = \int_{-\infty}^{\infty} dx_0 \Psi(x_0, t_0) K(x, t|x_0, t_0). \quad (37)$$

This solution was obtained from the mathematical Schrödinger equation (25) assuming that $V(x) = 0$. The above equations mean that, even in the far “empty” space, for example, the matter particles are always interacting with the vacuum zero-point radiation. The inevitable conclusion is that the fluctuating electromagnetic background with spectral density $\rho_0(\omega) = \hbar\omega^3/2\pi c^3$ is the source of the *zero temperature diffusion*. In other words, the constant \hbar present in the propagator $K(x, t|x_0, t_0)$ has its origin in the spectral density $\rho_0(\omega)$. Notice that the equations (36) and (37), and the *superposition principle* suggest that the interference phenomena of electrons, and other matter particles, may be *explained without the use of de Broglie waves*.

We would also like to say that, from the connection between the approximate Liouville equation (11) and the Schrödinger type equation (25) presented here, one can conclude that the mathematical probability density $|\Psi(x, t)|^2$ also may be not necessarily associated with real waves [31]. Moreover, according to the *Copenhagen Interpretation*, the amplitude $\Psi(x, t)$ is not a physical wave but a “probability wave” [32]. As a matter of fact, the existence of de Broglie waves was recently questioned by Sulcs et al. [33] in their analysis of the experiments, by Arndt et al. [34], on the interference of *massive* particles as the C_{60} molecules (fullerenes). According to M. Arndt et al. [34], “the de Broglie wavelength of the interfering fullerenes is already smaller than the diameter of a single molecule by a factor of almost 400”! This is an important observation which deserves further attention.

It is equally important to mention that, recently, a *tunneling* phenomenon was also interpreted in classical terms, that is, without using *the concepts of de Broglie waves* and the *wave-particle duality hypothesis* [35]. The important concept used was, again, the existence of the zero-point radiation with spectral distribution given by $\rho_0(\omega) = \hbar\omega^3/2\pi^2c^3$.

Finally, we want to say a few words concerning the relationship between the Compton effect and the zero-point radiation. This relationship was established by França and Barranco [36] in 1992, by studying a system of

free molecules in equilibrium with thermal and zero-point radiation.

In their work Barranco and França, using a statistical approach, replaced the Einstein concept of *random spontaneous emission* by the concept of *stimulated emission by the random zero-point electromagnetic fields* with spectral distribution $\rho_0(\omega)$. As a result, Compton and Debye’s kinematic relations were obtained within the realm of a completely classical theory, that is, without having to consider the wave-particle duality hypothesis for the molecules or the radiation bath. This is another important dynamical effect of the zero-point radiation. We recall that many other relevant effects of the zero-point electromagnetic fields are presented in the excellent books by de la Peña et al. [12, 15].

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