

SOLVING $Ay = b$ VIA ORDINARY DIFFERENTIAL EQUATIONS

Marcelo P. Klein

Department of Mathematics - UFCE and UECE

Cyro de C. Patarra*

Leon Sinay

Instituto de Pesquisas Espaciais - INPE/MCT
Laboratório Associado de Computação e Matemática Aplicada
Caixa Postal 515, 12201, São José dos Campos, SP

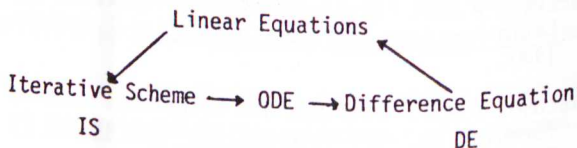
We analyse in this paper some relations between iterative methods for solving the linear system

$$Ay = b \quad (1)$$

difference equations, B-extrapolations [2] and ordinary differential equations.

We consider, through this work only $N \times N$ real matrices, and $N \times 1$ real vectors. The matrix A in (1) is assumed to be nonsingular.

We establish the diagram



and show that: the stability of the static solution of the ODE is related to the convergence of both, the IS and the DE; if the DE is obtained by the Euler discretization, then the IS and the DE coincide; the static solution of the ODE is stable if and only if the DE obtained by a numerical Taylor integration method is convergent for

* on leave from the Department of App. Math., IME/USP.

some step h and, finally, the limit of $y(t)$, solution of the ODE, as $t \rightarrow \infty$ is equal to the solution of (1), thus providing an alternative method for solving the linear algebraic equation. The restriction that the eigenvalues of the iteration matrix for the IS have modulus less than 1 is improved to only have real part smaller than 1. In fact, it is known that, in this case, it is possible to find a β -extrapolation which converges, the difficulty resides in finding the extrapolating parameter, a problem which we avoid obtaining the DE. Furthermore, there is no reason for restricting the integration of the ODE to one-step integration methods, any good numerical one may yield an approximate solution of (1).

We believe that further investigation may lead to some significant results such as better computing times, better approximations for the same time, etc. Furthermore, other relations between ODE's and iterative schemes may appear, for example between conditional convergence and initial manifolds.

REFERENCES

- [1] David Young, On the consistency of linear stationary iterative methods, SIAM J. NUMER. ANAL., 9, 1, (1972), pp. 89-96.
- [2] P. Albrecht and M.P. Klein, Extrapolated iterative methods for linear systems, SIAM J. NUMER. ANAL., 21, 1, (1984).
- [3] P. Albrecht, Métodos de discretização, 9 Colóquio Brasileiro de Matemática, IMPA, 1973.