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Representation type of one point extensions of quasitilted algebras

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REPRESENTATION TYPE OF ONE POINT EXTENSIONS OF QUASITILTED ALGEBRAS

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ABSTRACT. It is known that, given a tame algebra Λ, the Tits form q_Λ is weakly non negative. Moreover, the converse has been shown for some families of algebras, but it is not true in general. The purpose of this work is to show that if Λ a strongly simply connected tame quasitilted algebra of canonical type and M is an indecomposable Λ-module then $\Lambda[M]$ is tame if and only if $q_{\Lambda[M]}$ is weakly non negative.

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1 lgebras have the Tits form weakly non negative and for some classes of algebras, as for instance tilted or quasitilted algebras, this fact is determinant, that is, if Λ is tilted or quasitilted, then Λ is tame if and only if the Tits quadratic form is weakly non negative. Also, de la a Peña have shown in [P1] that the result holds in case that Λ is the one point extension of a tame concealed algebra not of type $\tilde{A}_n$ by an indecomposable module. A similar result was obtained by Chalom and Merklen in [CM] in case that Λ is a one-point extension of a tilted algebra of euclidean type not of type $\tilde{A}_n$ by one indecomposable module. Our main result in this work is the following:

Theorem: Let B be a tame strongly simply connected quasitilted algebra of canonical type and M an indecomposable B-module, then the one point extension $B[M]$ is tame if and only if $q_{B[M]}$ is weakly non negative.

1. Preliminaries

Throughout this paper, k denotes an algebraically closed field. By an algebra Λ we mean a finite-dimensional, basic and connected k-algebra of the form $\Lambda \cong kQ/I$ where Q is

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a finite quiver and I an admissible ideal. We assume that Q has no oriented cycles. Let Λ-mod denote the category of finite-dimensional left Λ-modules, and Λ-ind a full subcategory of Λ-mod consisting of a complete set of non-isomorphic indecomposable objects of Λ-mod. For each $i \in Q_0$ we denote by S_i (resp. P_i, I_i) the corresponding simple Λ-module (resp. the projective cover, injective envelope of S_i).

We shall use freely the known properties of the Auslander-Reiten translations, τ and τ^{-1}, and the Auslander-Reiten quiver of Λ-mod, Γ_Λ . For basic notions we refer to [R2] and [ARS]. See also [A] and [CB].

We begin now to recall the concepts and results that form the background for our work.

Definition 1.1. [R1] A vectorspace category $(\mathbb{K}, |\ |)$ is a pair given by a Krull-Schmidt k-category $\mathbb{K}$ and a faithful functor $|\ | : \mathbb{K} \to \text{mod k}$.

Given a vectorspace category $(\mathbb{K}, |\ |)$, its objects (resp. the morphisms) are usually considered to be the objects (resp. the morphisms) of the image of $|\ |$, and its subspace category $\mathcal{U}(\mathbb{K})$ is defined as follows: the objects are triples (X, U, φ) with $X \in \text{Obj}\mathbb{K}$, U a k-vector space and φ: $U \to |X|$, k-linear. The morphisms $(X, U, \varphi) \to (X', U', \varphi')$ are the pairs (α, β) with β: $X \to X'$ in $\mathbb{K}$, $\alpha : U \to U'$ k-linear and such that $|\beta|\varphi = \varphi'\alpha$. Clearly, any object of $\mathcal{U}(\mathbb{K})$ is isomorphic to a direct sum of a triple (X, U, φ) with $\varphi : U \to |X|$ injective and copies of $(0, k, 0)$.

Modules over a one point extension $B[M]$ can be identified with triples (X, U, φ) where $X \in B$-mod, U is a k-vectorspace and $\varphi : U \to Hom(M, X)$ is k-linear.

It is known ([R1]) that the representation type of $B[M]$ depends on the representation type of B and of $\mathcal{U}(\text{Hom}(M, B - \text{mod})$.

See [R1] for other notions and notations related to vectorspace categories.

In this work , we are considering one point extensions of tubular algebras (see [R2]) and one point extensions of quasi-tilted algebras (see [HRS] and [S]). So, in any case, we have that $gldimB \leq 2$ and $gldimB[M] \leq 3$.

Associated to an algebra Λ of finite global dimension, we can define two quadratic forms, in the Grothendieck group of Λ . These forms are very important tools in the study of tame algebras.

Definition 1.2. [R2] Let C_B be the Cartan matrix of B and let x and y vectors in $\mathbb{K}_0(\mathrm{B})$. Then we have a bilinear form $<,>= xC_B^{-T}y^T$, where the corresponding quadratic form $\chi_B(x) =< x, x >$ is called the Euler form of B.

Definition 1.3. [Bo] The Tits quadratic form is given by:

$q_B(x_1, x_2, ..., x_l) = \sum_{i \in Q_0} x_i^2 - \sum_{i,j \in Q_0} x_i.x_j.dim_k Ext_B^1(S_i, S_j)$
$+ \sum_{i,j \in Q_0} x_i.x_j.dim_k Ext_B^2(S_i, S_j)$.

If B is such that $gldimB \leq 2$ then for any B-module M we have $gldimB[M] \leq 3$. Hence, using Bongartz result (see [Bo]) that if $gldimB \leq 2$ then $\chi_B = q_B$, we would be able to relate the Euler and the Tits form for $A = B[M]$.

Let X be a A-module and let:

$\underline{dim}_A(X) = \underline{dim}_B(Y) + n.\underline{dim}_A(S_e)$, where e is the new vertex.

Proposition 1.1. *With the above notation:*

$\chi_A(\underline{dim}X) = q_A(\underline{dim}X) - n.dim_k Ext_B^2(M, Y)$

The following important result, that we can see in [P1], was also obtained by Drozd.

Theorem 1.4. If B is a tame algebra, then q_B is weakly non negative.

2. Tubular Algebras

In this section, we consider the class of tubular algebras, as in [R2].

For a tubular algebra B we mean that B is a tubular extension of a tame concealed algebra B_0 with tubular type $(2,2,2,2)$, $(3,3,3)$, $(4,4,2)$ or $(6,3,2)$. Any tubular algebra is also co-tubular, that is $B = B_0[E_i, R_i]_{i=1}^t = [E_i', R_i']_{i=1}^{t'} B_\infty$ with B_0 and B_∞ both tame concealed, E_i and E_i' ray modules of the separating tubular families of the corresponding algebras, and R_i , R_i' branches.

We begin by proving the following theorem:

Theorem 2.1. Let B be a tubular algebra with the directed components of tree type and M an indecomposable B-module. If $B[M]$ is wild then $q_{B[M]}$ is strongly indefinite.

Lemma 2.2. Let B_0 be a convex subcategory of B such that B is a iterated coextension or a branch coextension of B_0 and assume that $M_0 = M|_{B_0}$, $M_0 \neq 0$. Then $B_0[M_0]$ is a convex subcategory of $B[M]$.

Proof : The proof is done by induction in the number of the coextensions or the length of the branch.□

Proof of theorem:

Let us recall the structure of the Auslander-Reiten quiver of a tubular algebra, as in [R2] (pag. 273) As B is a tubular algebra, let $B = B_0[E_i, R_i]_{i=1}^{t} = [E_i', R_i']_{i=1}^{t'} B_\infty$ with B_0 and B_∞ both tame concealed. We have the following pairwise disjoint modulo classes $\mathcal{P}_0, \mathcal{T}_0, \mathcal{Q}_0 \cap \mathcal{P}_\infty, \mathcal{T}_\infty, \mathcal{Q}_\infty$ with $\mathcal{P}_0$ the pos projective component of B_0, and $\mathcal{Q}_\infty$ the preinjective component of B_∞ being the directed components of Γ_B, that we assume that have tree type, so we can assume that B_0 and B_∞ are both not of type $\tilde{\mathrm{A}}_n$, and $\mathcal{T}_0, \mathcal{Q}_0 \cap \mathcal{P}_\infty, \mathcal{T}_\infty$ being tubular families.

Also we know that the projective modules belong to $\mathcal{P}_0$ or $\mathcal{T}_0$ and the injective modules in $\mathcal{T}_\infty$ or $\mathcal{Q}_\infty$.

So, if we consider an indecomposable module M such that there exists an indecomposable injective I such that $Hom(M, I) \neq 0$, and M and I are separated by a separating tubular family, then, the result follows from [PT]. And this would apply to any indecomposable module M in any of the following families : $\mathcal{P}_0, \mathcal{T}_0, \mathcal{Q}_0 \cap \mathcal{P}_\infty$. It remains to consider the cases where M belongs to $\mathcal{T}_\infty$ or $\mathcal{Q}_\infty$. But if M belongs to $\mathcal{Q}_\infty$ that is the preinjective component of the algebra A_∞ (similar to one of tame concealed not of type $\tilde{\mathrm{A}}_n$). Then, the result follows directly from [P1].

By [R2], B is a branch coextension of a tame concealed algebra B_∞ and the preinjective component of B is the same preinjective component of B_∞, and so B_∞ is $\tilde{A}_n$-free. Assume that $B = {}_{i=1}^{t}[E_i, R_i]B_\infty$ where E_i is a B_∞-ray module and R_i is a branch, for all i. Let us consider separately the following situations: A) $M_\infty = M|_{B_\infty}$ is such that $M_\infty = 0$;

B) $M_\infty = M|_{B_\infty}$ is such that $M_\infty \neq 0$.

In case A, $suppM$ is contained in a branch R and the vectorspace category $Hom(M, B-mod)$ is the same as $Hom(M, R-mod)$. By [MP], if $Hom(M, R-mod)$ is wild then $q_{R[M]}$ is strongly indefinite. As $R[M]$ is a convex subcategory of $B[M]$, if $q_{R[M]}$ is strongly indefinite then $q_{B[M]}$ is strongly indefinite.

In case B, we can distinguish two situations:

B1: $B_\infty[M_\infty]$ is wild;

B2: $B_\infty[M_\infty]$ is tame.

We begin by B1. If $B_\infty[M_\infty]$ is wild, since the preinjective component of B is the same preinjective component of B_∞, B_∞ is tame concealed and $\tilde{A}_n$-free. So, by [P1], $q_{B_\infty[M_\infty]}$ is strongly indefinite. But $B_\infty[M_\infty]$ is a convex subcategory of $B[M]$ and so $q_{B[M]}$ is strongly indefinite.

Let us see B2, that is $B_\infty[M_\infty]$ is tame , but $B[M]$ wild. Again, since $B_\infty[M_\infty]$ is tame, we have two possibilities:

B2.1 M_∞ is a ray module.

B2.2 M_∞ is a module of regular length regular two in the tube of rank $n-2$ and B_∞ is tame concealed of type $\tilde{D}_n$. In the case B.2.1, we have that if M is a ray module over B, by [R2] 4.5 and 4.6, the component $\mathcal{T}_e[M]$ is a standard inserted-co-inserted tube. Moreover, all indecomposable projectives of $B[M]$ lie in $\mathcal{P}_0$, the posprojective component, in some tubes of $\mathcal{T}_0$ or in $\mathcal{T}_e[M]$, (calling $\mathcal{T}_e$ the tube of the family $\mathcal{T}_\infty$ where M lies) therefore, $B[M]$ is an algebra with acceptable projectives and in this case, by [PT] $B[M]$, it is wild if and only if $q_{B[M]}$ is strongly indefinite.

On the other hand, if $M = M_\infty$ and therefore, M is a ray module over B_∞, then again $B[M] = B[M_\infty]$ is an algebra with acceptable projectives.

So, we can assume that M is not a ray module over B and moreover that $M \neq M_\infty$ and, therefore, that there exists an indecomposable injective I in $\mathcal{T}_e$, such that $Hom(M, I) \neq 0$ and that there are two arrows starting in M. Also, we can assume that i, the coextension vertex belongs to $supp\ M$, so that there exists a morphism $M \to I_i$.

Let E be the ray module which is the root of the branch. As we can have many branches in the same tube, but *suppM* intercepts only one of them, because it is indecomposable, we are considering that E is the root of the branch that intercepts the support of M.

Let $B_i = [E]B_\infty$ and $M_i = M|_{B_i}$. Then we have: $Hom_{B_i}(M_i, M_\infty) \neq 0$, but $Hom_{B_i}(M_\infty, M_i) = 0$, and again we have two cases:

B.2.1.1 The branch is co-inserted in E, $E \neq M_\infty$;

B.2.1.2 The branch is co-inserted in $E = M_\infty$.

In the first case, since M is not a ray module over B, we can assume that there exists an arrow that start in M and points to the mouth of the tube, say $M \to Y$. Moreover, by [[R2], 4.5] there exists a sectional path $M \to M_t \to M_{t-1} \to \cdots M_\infty$ that does not contain injectives (again, if there were injectives in this sectional path, we will have that M_∞ is the root of another branch, again intercepting the support of M).

So, we can consider that all of these modules $\tau^{-1}M_i$, and in particular $\tau^{-1}M_1$, are non zero.

Since M_∞ is a B_∞-ray module, then $\tau^{-1}M_1$ cannot be a B_∞-module. But in this case, it is a co-ray module and therefore M_∞ is a co-ray module, contradiction. So, the situation B.2.1.1 does not occur.

If the branch is co-inserted in $E = M_\infty, M_\infty = M|_{B_\infty}$, M is not a ray module. Again, we can assume that there exists an arrow starting in M and pointing to the mouth of the tube. Moreover, since the branch is co-inserted in M_∞, there is a sectional path $M \to I$ the injective of the co-insertion. Let us look at the category $Hom(M, B-mod)$. This category has three pieces. Since B is tubular, $Hom(M, X) \neq 0$ only for modules X that are preinjective or in the same tube $\mathcal{T}_e$ where M lies. Let X be a B_∞-module. We have that if $Hom_B(M, X) \neq 0$ then $Hom_{B_\infty}(M_\infty, X) \neq 0$. Since B_∞ is a tame concealed algebra and M_∞ is a ray module over B_∞, $Hom(M, B-mod)$ contains the following subcategories: the ray of $\mathcal{T}_e$ that starts in M_∞, $Hom(M_\infty, \mathcal{Q}_\infty)$ where $\mathcal{Q}_\infty$ is the preinjective component of B_∞ and the subcategory given by the sucessors of M in the tube, that are not B_∞-modules. Since $B_\infty[M_\infty]$ is tame, $Hom(M_\infty, \mathcal{Q}_\infty)$ is given by some of the patterns given in [[R1], pag.. 254]. Let us assume that one of the following two situations occur:

either M is injective and so the vectorspace category restricted to the tube is given by two sectional paths one, finite, pointing to the mouth of the tube and one, infinite, (the ray) or M is not injective but the vectorspace category restricted to the tube is given by two parallel paths. We will see that in this situation, since $B_\infty[M_\infty]$ is tame, also $B[M]$ is tame, in contradiction to the hypothesis. In fact, $A = B[M]$ is a coil enlargement of B_∞, by [AS] because $A^+ = B_\infty[M_\infty]$, $A^- = B$, are both tame. As that $A = B[M]$ is tame.
Let us assume that none of this cases occur, then that M is not injective and that there exists a sectional path $M \to Y_0 \to \cdots \to Y_t$ with $t \geq 1$. In first place, we observe that $Hom_B(Y_i, X)$
$= 0$ for all preinjective X. But Y_i being on the coray, and to the right of M_∞, there does not exist an infinite path coming out of it, and similarly $Hom(\tau^{-1}M, X) = 0$ for all preinjective X.

In particular, $Hom(Y_i, X) = Hom(\tau^{-1}M, X) = 0$ for all X such that $Hom(M_\infty, X) \neq 0$ with X in the preinjective component. Moreover
$Hom(Y_i, \tau^{-1}M) = 0 = Hom(\tau^{-1}M, Y_j)$ for $\forall j \geq 1$. Hence, by [[R1] (3.1)] we can find one of the following path-incomparable (see [Ch]) subcategories in $\mathcal{Q}_\infty$, with the only exception of the case $(\tilde{D}_n, n-2)$. $\mathbb{K}_1 = \{\mathrm{A}, \mathrm{B}, \mathrm{C}\}$, (in cases: $(\tilde{D}_4, 1), (\tilde{D}_6, 2), (\tilde{D}_7, 2), (\tilde{D}_8, 2), (\tilde{E}_6, 2), (\tilde{E}_7, 3), (\tilde{E}_7, 4)$. $(\tilde{E}_8, 5)$) and $\mathbb{K}_2 = \{\mathrm{A}, \mathrm{B} \to \mathrm{C}\}$ in cases $(\tilde{D}_5, 2)$ and $(\tilde{E}_6, 3)$. So, in each case, adding the objects $Y_1, \tau^{-1}M$ to the categories $\mathbb{K}_1$ or $\mathbb{K}_2$ we have that $Hom(M, B-mod)$ is wild and that $q_{B[M]}$ is strongly indefinite.

Let us calculate the quadratic form for the case $(\tilde{D}_5, 2)$, the other cases are similar. Let $\tilde{L}$ be the B-module $\tilde{L} = 2Y_1 \oplus 2\tau^{-1}M \oplus 2A \oplus B \oplus C$ and $L = \tilde{L} \oplus 4S_e$, then $q_{B[M]}(\underline{dim}L) = \chi_{B[M]}(\underline{dim}L) + 4dim_k Ext^2(M, \tilde{L}) = \chi_{B[M]}(\underline{dim}L) = \chi_{B[M]}(\underline{dim}\tilde{L}) + 4^2 - 4(8) = 15 + 16 - 32 = -1$.

Let us see the case $(\tilde{D}_n, n-2)$. In this case, the pattern is given by:

$$\begin{tikzcd}[row sep=small, column sep=small]
 & & & \overset{A}{\circ} \arrow[dr] & & \\
\circ \arrow[r] & \cdots\circ \arrow[r] & \circ \arrow[ur] \arrow[dr] & & \circ \arrow[r] & \cdots \\
 & & & \underset{B}{\circ} \arrow[ur] & &
\end{tikzcd}$$

If $t > 1$, considering that $\mathbb{K} = \{\mathrm{A}, \mathrm{B}, \tau^{-1}\mathrm{M}, \mathrm{Y}_1 \to \mathrm{Y}_2\}$ is wild, again the quadratic form is strongly indefinite. On the other hand, if $t = 1$ we have two possibilities:

Case 1

$$\begin{tikzcd}[row sep=small, column sep=small]
 & & Y_1 & & \\
 & Y_0 \arrow[ur] \arrow[dr] & & & \\
M \arrow[ur] \arrow[dr] & & \tau^{-1}M \arrow[dr] & & \\
 & Z_1 \arrow[ur] \arrow[dr] & & \tau^{-1}Z_1 & \\
 & & Z_2 \arrow[ur] \arrow[dr] & & \\
 & & & \cdots & \\
 & & & I \arrow[dr] & \\
 & & & & M_\infty
\end{tikzcd}$$

and case 2

$$\begin{tikzcd}[row sep=small, column sep=small]
 & & Y_1 \\
 & Y_0 \arrow[ur] \arrow[dr] & \\
M \arrow[ur] \arrow[dr] & & \tau^{-1}M \\
 & I \arrow[ur] \arrow[dr] & \\
 & & M_\infty
\end{tikzcd}$$

In case 1, we can again consider the wild subcategory

$\{Y_1, \tau^{-1}M \to \tau^{-1}Z_1, A, B\}$ and the quadratic form is strongly indefinite. On the other hand, in case 2, we have a vectorspace category which is in fact tame, by Nazarova Theorem, so $B[M]$ is tame .

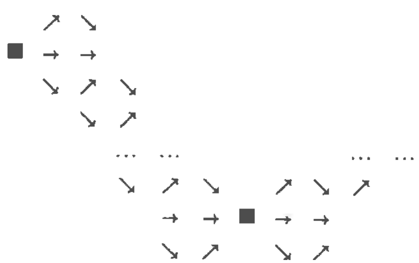
Let us examine now B.2.2, M_∞ is a module of regular length 2 in a tube of rank $n-2$ and B_∞ is tame concealed of type $\tilde{D}_n$. If $M = M_\infty$ lies in a stable tube, then $Hom(M, B-mod) = Hom(M_\infty, B_\infty - mod)$ and therefore both are tame or wild simultaneosly. So, we can assume that M belongs to a co-inserted tube. Since M_∞ has regular length 2, there exist E_1 and E_0 ray-modules over B_∞ such that $\tau E_0 = E_1 \to M_\infty \to E_0$ is the ARS for E_0. Let $E_0, E_1, \cdots E_{n-3}$ be the ray-modules over B_∞ of the tube where M lies. Again, we divide in possibilities.

B.2.2.1 The branch is co-inserted in E_0.

B.2.2.2 The branch is co-inserted in E_1.

B.2.2.3 The branch is co-inserted in E_j for $j \neq 0$ or 1.

Let us observe that if $M = M_\infty$, then $Hom(M, B - mod)$ has the same pattern as $Hom(M_\infty, B_\infty - mod)$. If M is a B_∞-module, then $Hom_B(M, N) \neq 0$ for modules N in the same tube as M or for modules N in the preinjective component. Hence, being $Hom(M, N) = Hom(M_\infty, N_\infty)$ it has the following pattern



which is tame , by [R1]. (In this picture we indicate the non zero modules in the category with ■ indicating the objects of dimension 2.) We can assume that M belongs to the co-ray and that there exists an injective I in the tube $\mathcal{T}_e$ such that $Hom(M, I) \neq 0$.

Let us consider B.2.2.1. We have a co-inserted branch in E_0, and

$$
\begin{tikzcd}[row sep=small, column sep=small]
 & & Y_1 & & \\
 & Y_0 \arrow[ur] & & & \\
M \arrow[ur] \arrow[dr] & & & & \\
 & \cdots \arrow[dr] & & I \arrow[dr] & \\
 & & \tau E_0 \arrow[ur] \arrow[dr] & & E_0 \\
 & & & M_\infty \arrow[ur] &
\end{tikzcd}
$$

If there exists a sectional path $M \to Y_0 \to Y_1$, then, $Hom(M, Y_1) \neq 0$. Let us observe that $Y_1|_{B_\infty} = 0$ and $Hom(Y_1, X) = 0$ for all preinjective module X and in particular, $Hom(Y_1, X_i) = 0$ for each of the preinjective $X_i's$ such that $Hom(M_\infty, X_i)$ has dimension 2. Hence $q_{B[M]}$ is strongly indefinite. Let us assume that the longest sectional path starting at M in the direction of the mouth of the tube has length 1. In this case, again, $Hom(M, B-mod)$ has the same pattern than $Hom(M_\infty, B_\infty - mod)$ and so it is tame.

Let us consider B.2.2.2 Since $Hom(E_1, E_0) = 0$, the morphisms from M to X, for X preinjective, are just the ones that factor through the successor of M_∞, M_1, since those that factor through E_0 are equal to zero, the vectorspace category $Hom(M, B-mod)$ is of the form:

$$
\begin{tikzcd}[row sep=small, column sep=small]
M \arrow[dr] & & & & & & & & & & & & \\
 & I \arrow[dr] & & & & & {} \arrow[dr] & & & & {} \arrow[dr] & & \\
 & & E_0 \arrow[dr] & & \cdots \arrow[r] & {} \arrow[ur] \arrow[dr] & & {} \arrow[r] & \cdots \arrow[r] & {} \arrow[ur] \arrow[dr] & & {} \arrow[r] & \cdots \\
 & & & M_\infty & & & {} \arrow[ur] & & & & {} \arrow[ur] & &
\end{tikzcd}
$$

and we can repeat the arguments of the case B.2.1.2.

Finally, let us look at B.2.2.3. The branch is inserted in E_j with $j \neq 0$ or 1. But, in this case, $M = M_\infty$, $Hom(M_\infty, I) = 0$ for any I injective in $\mathcal{T}$ and we fall again in a already examined case.□

The condition that B has all directed components of tree type, is for tubular algebras in fact equivalent to the condition of strongly simply connectedness. For tame quasitilted algebras of canonical type simply connectedness is equivalent to the condition that A^+, C and A^- all have directed components of tree type.

Definition 2.3. ([S1]) An algebra A is called strongly simply connected when every full and convex subcategory of A is simply connected, or equivalently, every full convex subcategory of A satisfies the separation condition.

We recall the following results:

Theorem 2.4. [ALP] Let B be a tame tilted algebra. Then B is strongly simply connected if and only if the orbit quiver of each directed component of $\Gamma(modB)$ is a tree.

Theorem 2.5. [A1] Let B be a tubular algebra. Then B is strongly simply connected if and only if the orbit quiver of each directed component of $\Gamma(modB)$ is a tree.

Theorem 2.6. [ACT] Let B be a tame quasitilted algebra that is not tilted. Then B is strongly simply connected if and only if B^+ and B^- are strongly simply connected.

3. One point extensions of tame quasitilted algebras

In this section, we consider now the case where A is a quasitilted algebra. The more relevant characterization, in this work, is that A is a semiregular iterated tubular algebra, (see [S],[PT] and [ACT]).

Theorem 3.1. Suppose that A is a strongly simply connected tame quasitilted algebra of canonical type, and M is an indecomposable A-module. Then $A[M]$ is tame if and only if the corresponding Tits form is weakly non negative.

Proof: If A is a tilted algebra, then A is a tilted algebra of euclidean type an the result follows from [CM]. Then we consider the case when A is not tilted. As we are considering A non tilted, we know (by [S]) that $A = {}^{t}_{i=1}[E_i, R_i]C[E'_j, R'_j]^{s}_{j=1}$, where C is a tame concealed algebra, E_i and E'_i are ray modules of the separating tubular family of C, and R_i , R'_i branches. Also the E_i and E'_i do not lie in the same tube. We call

$A^+ = [E_i, R_i]C$ and $A^- = C[E'_i, R'_i]$

The proof is done considering the Auslander-Reiten quiver of A and the possibilities for M. In the case that M is in the preinjective component of A, that is the the preinjective component of A^-, as A^- is tilted of euclidean type or a tubular algebra, the result follows from [CM] or the previous theorem for tubular algebras.

We consider two possibilities:

A)- A is domestic and so A^+ and A^- are tilted of euclidean type or

B)- A is non domestic and then A^+ or A^- are of tubular type.

If M is in the posprojective component of A, we know that $suppM \subset A^+$ and so $A[M] = (A^+[E'_j, R'_j]_{j=1}^s)[M] = (A^+[M])[E'_j, R'_j]_{j=1}^s$. and $A^+[M]$ is a full convex subcategory of $A[M]$.

Now we consider the two different cases:

If A^+ is of tubular type, we have that $A^+[M]$ is wild, then $q_{A^+[M]}$ is strongly indefinite and the same is true for $A[M]$.

If A^+ is tilted of euclidean type, then A^+ has a complete slice in the posprojective component. If the vectorspace category $Hom(M, A^+ - mod)$ is finite we will have that $A^+[M]$ and $A[M]$ are tame.

If A^+ is tilted and the category $Hom(M, A^+ - mod)$ is not finite, the algebra $A^+[M]$ has a component in the Auslander-Reiten quiver that contains all the projectives and is a *pi*-component (see [Co]). Using the same argument that in [CM], this component does not contain injectives, so $A^+[M]$ is again a tilted algebra.

Consider now that M is a module in a tube.

Assume that A^+ and A^- are tilted of euclidean type. Then the Auslander-Reiten quiver of A has a tubular family, where some tubes contain projectives (support in A^-) and some tubes contain injectives (support in A^+). Also, it is easy to see that, by construction, there is no morphisms between tubes. In this case, if M belongs to a tube that contains projectives, or a stable tube, then, the vectorspace categories $Hom(M, A - mod) = Hom(M, A^- - mod)$ (because the modules X, such that $Hom(M, X) \neq 0$ are all A^--modules) . As $A^-[M]$ is a convex subcategory of $A[M]$ by 2.2, if A^- has the preinjective component of tree type, the result follows from [CM].

We consider now the case where M belongs to a tube with injectives (so, $suppM \subset A^+$). Let $M_0 = M|_C$, by 2.2 $C[M_0]$ is a convex subcategory in $A^+[M]$, also, as $A = A^+[E'_j, R'_j]$ then, $A[M] = (A^+[M])[E'_j, R'_j]$ and $A^+[M]$ is a convex subcategory of $A[M]$. So, if $C[M_0]$ is wild and C is of tree type (and similarly for $A^+[M]$) then $q_{A[M]}$ is strongly indefinite. Then, assume that $C[M_0]$ is tame. If $M = M_0$, and M is a ray-module, then $C[M_0]$ is domestic or tubular and $A^+[M] = ([E_i, R_i]C)[M]$ is iterated tubular and, more over $A[M] = A^+[M][E'_j, R'_j]$ is iterated tubular, and so is tame. If $M \neq M_0$, but M is a ray-module, by similar arguments $A[M]$ is an algebra with acceptable projectives and so, the representation type is determined by the quadratic form. And by the other hand, if the support of M is contained in the branch, the result follows from [MP].

So, we assume that M is not a ray module, that $M_0 \neq 0$ and that $C[M_0]$ is tame , so M_0 is a ray-module or is a module of level two in a tube of rank $n-2$, and C is a $\tilde{\mathrm{D}}_n$ concealed algebra.

In the case that M_0 is a ray module, we see that, with the same arguments used for the similar situation for tubular algebras, we have the same result.

Consider now that C is a concealed algebra of $\tilde{D}_n$-type, and that $C[M_0]$ is a 2-tubular algebra. In this case there is two possibilities

If $A[M]$ is a tree algebra, the result follows directly from Brustle theorem (see also [DP]).

If $A[M]$ is not a tree, up to orientations we may assume that C is of the form

$$\begin{tikzcd}[column sep=small, row sep=small] r+1 \arrow[dr] & & & & & r+3 \\ & 1 \arrow[r] & \cdots \arrow[r] & {} \arrow[urr] \arrow[drr] & & \\ r+2 \arrow[ur] & & & & & r+4 \end{tikzcd}$$

then M_0 has the dimension vector

$$\begin{tikzcd}[column sep=small, row sep=small] 1 \arrow[dr] & & & & & 1 \\ & 2 \arrow[r] & \cdots \arrow[r] & {} \arrow[urr] \arrow[drr] & & \\ 1 \arrow[ur] & & & & & 1 \end{tikzcd}$$

or the simmetric one, and since $A^- \neq A$

there is a extension in a different tube, since each simple regular in a different tube has $r+1$ or $r+2$ in its support, then there exists in A an arrow $w \to r+1$ or to $r+2$.

But, in this case $dimHom(M, S_{r+1}) = 1$, because $S_{r+1} \subset topM$, $dimHom(M, I_1) = 2$, and is easy to see that S_{r+1} and I_1 are path incomparable. So, in this case, we have that $A[M]$ is wild and that $q_{A[M]}$ is strongly indefinite.

We consider now the non domestic case. We have the following possibilities:

A) If A^+ is tubular and A^- is domestic, as A is tame, the Auslander-Reiten quiver of A^+ is given by $\mathcal{P}_0, \mathcal{T}_0, \cup_{\gamma \in P^1(k)} \mathcal{T}, \mathcal{T}_\infty, \mathcal{Q}_\infty$ and the Auslander-Reiten quiver of A is $\mathcal{P}_0, \mathcal{T}_0, \cup_{\gamma \in P^1(k)} \mathcal{T}_\gamma, \mathcal{T}_\infty^-, \mathcal{Q}$ where $\mathcal{Q}_\infty^-$ is the preinjective component of A and of A^- and the new projectives are inserted in stable tubes belonging to $\mathcal{T}_\infty$ (we denote $\mathcal{T}^-{}_\infty$ the new family of tubes). But, in this case, considering all the possibilities for M (shortly saying, if $suppM \subset A^+$ *or* A^-) the result follows from the previous cases.

B) If If A^- is tubular and A^+ is domestic, as A is tame, the Auslander-Reiten quiver of A is given by (with the above notation)

$\mathcal{P}_0^+, \mathcal{T}_0^+, \cup_{\gamma \in P^1(k)} \mathcal{T}_\gamma, \mathcal{T}_\infty, \mathcal{Q}_\infty$ and the new injectives (that have support in A^+) are inserted in the $\mathcal{T}_0$. Let M be an indecomposable module in a tube in $\mathcal{T}_0^+$ a tube containing injectives (and so $suppM \subset A^+$), then the vectorspace category $Hom(M, \_)$ is finite (for those modules such that the support is contained in the branch) and the result follows from [MP] or $Hom(M, \_)$ is infinite and there exists an injective module I (outside of the tube) such that $Hom(M, I) \neq 0$, but again this morphism factors through the following families and the result follows.

C) Finally we consider that A^+ and A^- are tubular, in this case the Auslander-Reiten quiver of A is $\mathcal{P}_0^+, \mathcal{T}_0^+, \cup_{\gamma \in P^1(k)} \mathcal{T}_\gamma^+, \mathcal{T}_\infty^+ = \mathcal{T}_0^-, \cup_{\gamma \in P^1(k)} \mathcal{T}_\gamma^-, \mathcal{Q}_\infty^-$ but considering that $suppM \subset A^+$ *or* A^- we can see that all cases were already considered.

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