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CONTINUOUS - TIME MARKOV MODELS FOR FINITE POPULATIONS

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Summary

Thomsen (1981) and Rodrigues et al. (1985), under a discrete-time Markov chain superpopulation model, considered filtering problems to predict the population total of elements having a specific characteristic. The predictor they suggested makes little real use of the Markov theory. In this paper, procedures for predicting problems to predict the population total, under a continuous-time Markov chain superpopulation model, are considered by using Kalbfleisch and Lawless' (1985) results. A new continous Markov predictor (CMP) and a particular and intuitive version of it, based on the auxiliary variables available and on equilibrium probabilities, are studied. An empirical study for predicting problems of the population total shows that the CMP is substantially better than the one considered by Thomsen (1981).

Key words: continuous-time Markov chain, superpopulation model, equilibrium probability, finite populations.

1. *Introduction*

Methods for filtering problems to predict the population total under a discrete-time Markov chain have been considered by Thomsen (1981) and Rodrigues et al. (1985). An optimal independent-time predictor was obtained.

In this paper, we consider "panel data" in which the observations consist of the states occupied by the individuals under study at a sequence of discrete times $t_0, t_1, \ldots, t_m$. Filtering is predicting the population total at time $t = t_m$, given known auxiliary variables at time $t_0 = 0$ and the panel data at time t_m, while predicting is to predict the population total at time $t > t_m$, given known auxiliary variables at time $t_0 = 0$ and the panel data at times $t_1, t_2, \ldots, t_m$. The predicting problems are concerned with predicting futures values of the population total. In this paper, under a continuous-time Markov chain, a dependent-time predictor is proposed for predicting problems. In this case, as simulation studies shows, the predictor suggested is substantially better than the one considered by Thomsen (1981). In a special case, based on the auxiliary variables available and on equilibrium probabilities, an intuitive version of the CMP is considered.

The population is denoted by $\mathcal{P} = \{1, \ldots, N\}$ and $\mathbf{P} = [0, \infty]$ is the time-support region. Let $\vec{\mathbf{P}} = \mathbf{P} - \{0\}$. Associated with $u \in \mathcal{P}$ at time $t \in \mathbf{P}$, there is a random quantity denoted by $Y_{uj}(t)$, $j = 1, \ldots, k$, where

$$(1) \qquad Y_{uj}(t) = \begin{cases} 1 & \text{iff} \quad Y_u(t) = j \\ 0 & \text{iff} \quad Y_u(t) \neq j \end{cases} \qquad \text{and}$$

$\{Y_u(t),\ t \in \mathbf{P}\}$ denotes the continuous-time Markov chain associated with the state occupied by individual u at time t. Note from (1) that $\sum_{j=1}^{k} Y_{uj}(t) = 1$, $u \in P$, $t \in \mathbf{P}$. As in Thomsen (1981) $\{Y_{uj}(0),\ u \in P,\ j = 1,2,\ldots,k\}$ are known auxiliary variables and $\{Y_{uj}(t),\ t \in \bar{\mathbf{P}},\ u \in \mathbf{P}\}$, $j = 1,2,\ldots,k$, are unknown quantities. Let $Q(t,\theta)$ the kxk transition intensity matrix with entries $q_{ij}(t,\theta)$ (see Cox and Miller, 1965), where $\theta' = (\theta_1,\ldots,\theta_b)$ is an unknown vector of parameters. This paper is concerned with time-homogeneous processes, that is, $q_{ij}(t,\theta) = q_{ij}(\theta)$, $i,j = 1,\ldots, k$. In this case, the process is stationary and the transition probabilities are given by

$$P_{ij}(t,\theta) = P[Y_u(t+v) = j \mid Y_u(v) = i],\quad v,t \in \mathbf{P} \tag{2}$$

and $i,j = 1,\ldots,k$. It is assumed the homogeneity of each transition probability for all members of the population. Estimation of the transition intensity parameters of a continuous-time Markov chain based on panel data has recently been considered by Kalbfleisch and Lawless (1985). In this paper, their approach is used to obtain a maximum likelihood estimator of θ.

2. *Continous-time Markov Superpopulation Model*

If for a given θ, $Q(\theta)$ has distinct eigenvalues $d_1(\theta),\ldots,d_k(\theta)$, it is well known (see Kalbfleisch and Lawless, 1985) that the transition probabilities (2) may be computed as

$$p_{ij}(t,\theta) = A_i'(\theta)\ W(t,\theta)\ \bar{A}_j(\theta), \tag{3}$$

where $A_i'(\theta)$ is the i-th row of $A(\theta)$, the kxk matrix whose columns are the right eigenvectors of $Q(\theta)$, $\bar{A}_j(\theta)$ is the j-th column of $A^{-1}(\theta)$ and $W(t,\theta) = \text{diag}(e^{d_1(\theta)t},\ldots,e^{d_k(\theta)t})$.

Let $\underset{\sim}{Y}'_u(0) = (Y_{u1}(0),\ldots,Y_{uk}(0))$ the known vector of auxiliary variables for all $u \in \mathcal{P}$. Under the model (1), the continuous-time Markov superpopulation model is defined by

$$E[Y_{uh}(t) \mid \underset{\sim}{Y}_u(0)] = \sum_{i=1}^{k} Y_{ui}(0)A'_i(\theta)W(t,\theta)\bar{A}_h(\theta),$$

(4) and

$$V[Y_{uh}(t) \mid \underset{\sim}{Y}_u(0)] = \sum_{i=1}^{k} Y_{ui}(0)A'_i(\theta)W(t,\theta)\bar{A}_h(\theta).[1-A'_i(\theta)W(t,\theta)\bar{A}_h(\theta)]$$

for a fixed h, $u \in \mathcal{P}$ and $t \in \bar{\mathcal{P}}$. The model (4) is an extension of Thomsen's model for continuous-time Markov chain. (For more details of the above model for the filtering problems, see Thomsen, 1981). We now select a random sample of n elements, of $\mathcal{P}$, $s(t) = s$, and observe it at times $0,t_1,\ldots,t_m$. Our problem consists, based on s and the model (4), in predicting the number of elements having a specific characteristic, $T_h(t) = \sum_{u=1}^{N} Y_{uh}(t)$, at a future time $t > t_m$. Sometimes, it may be of interest to predict $T_h = \int_0^T T_h(t)dt$, (T known), as considered by Bartlett (1985).

3. *Continuous-time Markov Predictors (CMP).*

For predicting $T_h(t)$, $t > t_m$, we suggest the following simple CMP:

$$\hat{T}_{CMP,h}(t) = \sum_{u=1}^{k} \hat{Y}_{uh}(t), \text{ where}$$

(5)

$$\hat{Y}_{uh}(t) = \sum_{i=1}^{k} Y_{ui}(0)A'_i(\hat{\theta})W(t,\hat{\theta})\bar{A}_h(\hat{\theta}),$$ and $\hat{\theta}$ is the

MLE of θ obtained by using Kalbfleisch and Lawless' procedure (1985). Observe that $\hat{T}_{CMP,h}(t)$ may be written as

$$(6) \qquad \hat{T}_{CMP,h}(t) = N \sum_{i=1}^{k} \frac{N_i(0)}{N} A_i'(\hat{\theta})W(t,\hat{\theta})\bar{A}_j(\hat{\theta}), \quad \text{where}$$

$N_i(0) = \sum_{u=1}^{N} Y_{ui}(0)$. In the filtering problem the predictor considered by Thomsen (1981) at time t_m is given by

$$(7) \qquad \hat{T}_h(t_m) = N \sum_{i=1}^{k} \left[\frac{N_i(0)}{N} \frac{\sum_{u\in s} Y_{ui}(0)Y_{uh}(t_m)}{\sum_{u\in s} Y_{ui}(0)}\right].$$

The predictor (7) predicts the total number of elements having the characteristic h at time t_m based on the data $\{Y_{uh}(t_m),\ u \in s\}$ and $\{Y_{ui}(0),\ u \in P,\ i = 1,\dots,k\}$. It is optimal in Royal's sense (1970).

In explaining the predictor $\hat{T}_{CMP,h}(t)$ to a client, it sometimes helps if the predictor has an intuitive and simple form. This motivate us to explore in this paper the possibility of creating a simple and intuitive version of the CMP at time t. Following this idea, we now introduce two intuitive definitions:

Definition 3.1. A state h is <u>equally accessible</u> iff $q_{ih}(\theta) = \lambda$, for all $i \neq h$, where λ is a positive component of the vector θ. Let π' a 1xk equilibrium probability vector, that is, $\pi'Q(\theta) = 0$, where $\pi' = (\pi_1,\dots,\pi_k)$ (see Cox and Miller, 1965).

Theorem 3.1. If a state h is <u>equally accessible</u>, then the h-th component of π' is given by

$$\pi_h = \frac{\lambda}{\lambda - q_{hh}(\theta)}, \quad \text{for any } \pi.$$

Proof: Since $\pi' Q(\theta) = 0$, it follows that $\lambda \sum_{j \neq h} \pi_j + \pi_h q_{hh} = 0$. From the above equation the result follows trivially.

Definition 3.2. If a state h is equally accessible, a Simple Markov Predictor (SMP) of $T_h(t)$ is defined by

$$(8) \qquad \hat{T}_{SMP,h}(t) = N[a(t,\hat{\theta}) \frac{N_h(0)}{N} + (1 - a(t,\hat{\theta}))\, \hat{\pi}_h],$$

where $\hat{\pi}_h = \dfrac{\hat{\lambda}}{\hat{\lambda} - q_{hh}(\hat{\theta})}$ and $a(t,\theta)$ is a known weighted function of $t \in \bar{P}$ for each fixed θ such that

(9)

(i) $0 < a(t,\theta) \leq 1, \quad \forall\, t \in \bar{P}, \quad \forall\, \theta,$

(ii) $a(0,\theta) = 1, \quad \forall\, \theta,$

(iii) $\lim_{t \to \infty} a(t,\theta) = 0, \quad \forall\, \theta.$

The conditions (i), (ii) and (iii) emphasize very clearly the role of the Markov theory in our problem.

Theorem 3.2. If a state h is equally accessible, then

(10)

(i) $\hat{T}_{CMP,h}(t) = \hat{T}_{SMP,h}(t)$, and

(ii) $a(t,\theta) = e^{-(\lambda - q_{hh}(\theta))t}$.

Proof: If a state h is equally accessible, it follows from the Kolmogorov's equations that

$$(11)\qquad \frac{dp_{ih}(t)}{dt} = p_{ih}(t)\, q_{hh} + \lambda \sum_{\substack{r=1\\ r\neq h}}^{k} p_{ir}(t) = (q_{hh} - \lambda)p_{ih}(t) + \lambda,\ i = 1,2,\ldots,k.$$

It can be shown that the differential equations (11) have solutions given by

$$(12)\qquad p_{ih}(t) = \pi_h(1 - e^{-(\lambda - q_{hh})t}),\ i \neq h$$
$$= \pi_h + (1 - \pi_h)e^{-(\lambda - q_{hh})t},\ i = h.$$

Let $p_0(i) = P[Y_u(0) = i] = \frac{N_i(0)}{N}$. Then, after some algebric simplifications it follows that

$$(13)\qquad P[Y_u(t) = h] = \sum_{i=1}^{N} p_0(i)p_{ih}(t) = p_0(h)e^{-(\lambda - q_{hh})t} + \pi_h(1 - e^{-(\lambda - q_{hh})t}),$$

from where (10) follows.

Remarks.

1. If $q_{ih}(\theta) = \lambda = 0$, for all $i \neq h$ (h fixed), it is not difficult to see from (11) that

$$(14)\qquad \hat{T}_{SMP,h}(t) = N_h(0)\, e^{q_{hh}(\hat{\theta})t}.$$ In this case, it is interesting to observe two particular situations:

(i) If $q_{hh}(\theta) = 0$, then $\hat{T}_{SMP,h}(t) = N_h(0)$ for $\forall\, t \in \bar{\mathbf{P}}$.

(ii) If $q_{hh}(\theta) < 0$, then $\lim_{t\to\infty} \hat{T}_{SMP,h}(t) = 0$.

2. Let P(t) be the kxk transition probability matrix and $\hat{P}(t_m)$ the kxk empirical transition probability matrix with entries

$$\hat{p}_{ij}(t_m) = \frac{\sum_{u\in s} Y_{ui}(0)Y_{uj}(t_m)}{\sum_{u\in s} Y_{ui}(0)} .$$

Consider the filtering problem, $t = t_m$. If $\hat{P}(t_m) = e^{Qt_m}$ admits a solution $\hat{Q} = Q(\hat{\theta})$ (see Kalbfleisch and Lawless, (1985)), then the CMP (6) coincides with Thomsen's predictor (7).

4. *Some Illustrative Examples*

In this section we discuss two examples to illustrate the performance of the CMP considered in Section 3.

Example 4.1. Consider a two-state continuous Markov process discussed by Kalbfleisch and Lawless (1985) with

$$Q = \begin{pmatrix} -\alpha & \alpha \\ \beta & -\beta \end{pmatrix}, \text{ where } \alpha,\ \beta > 0 \text{ and } \theta' = (\alpha,\beta).$$

Let n_{ij} the number of individuals observed in state i at $t_{\ell-1}$ and j at t_ℓ, $\ell = 1,\dots,m$, $n_{ij.} = \sum_{\ell=1}^{m} n_{ij\ell}$, $n_{i..} = \sum_{j=1}^{k} n_{ij.}$, and

$\tilde{p}_{ij} = \frac{n_{ij.}}{n_{i..}}$. If $t_\ell - t_{\ell-1} = \omega$, $\ell = 1,\ldots,m$ and $0 < \tilde{p}_{12} + \tilde{p}_{21} < 1$, the SMP(10) with h = 1 can be written as

$$(15) \qquad \hat{T}_{SMP,1}(t) = N\{e^{-\hat{\Psi}t} \frac{N_1(0)}{N} + (1 - e^{-\hat{\Psi}t})\,\hat{\pi}_1\},$$

where,

$$\Psi = \alpha + \beta, \qquad \hat{\Psi} = -\frac{1}{\omega}\log[1 - \tilde{p}_{12} - \tilde{p}_{21}]$$

$$\pi_1 = \frac{\beta}{\alpha + \beta}, \qquad \hat{\pi}_1 = \frac{\tilde{p}_{21}}{\tilde{p}_{12} + \tilde{p}_{21}}$$

(see Kalbfleisch and Lawless, (1985), for more details). If $\tilde{p}_{12} + \tilde{p}_{21} \geq 1$, then $\hat{\Psi} = \infty$ and $\hat{\pi}_1 = \frac{n_{12.} + n_{22.}}{n_{...}}$, where $n_{...} = \sum_{i,j} n_{ij.}$. So, $\hat{T}_{SMP,1}(t) = N.\frac{n_{12.}+n_{22.}}{n_{...}}$.

To illustrate the performance of the SMP(15), we consider data simulated from an ergodic two-state Markov chain with transition probabilities given by $p_{11} = 0.7$ and $p_{21} = 0.3$. The inicial probabilities were taken to be $p_0(1) = 0.7$ and $p_0(2) = 0.3$. From a population with N = 500, 200 were observed in consecutive times, $t_0 = 0$, t_1 and t_2. It is intended to predict the number of individuals in state 1 at some specified time $t > t_2$. Table 1 presents estimates of the MSE of $\hat{T}_{SMP,1}(t)$ and $\hat{T}_1(t_2)$, based on 100 samples for several values of t and ω.

Table 1. Estimated mean squared error under the continuous-time Markov chain superpopulation model based on 100 random samples with m = 2.

Table 1

t		$\omega = 0.45$	$\omega = 0.75$
2.0	$\hat{T}_{SMP,1}(t)$	13	94
	$\hat{T}_1(t_2)$	132	132
3.0	$\hat{T}_{SMP,1}(t)$	142	196
	$\hat{T}_1(t_2)$	445	445
3.75	$\hat{T}_{SMP,1}(t)$	136	159
	$\hat{T}_1(t_2)$	445	445

Note that $\hat{T}_{SMP,1}(t)$ is substantially better than $\hat{T}_1(t_2)$ if one is interested in predicting $\hat{T}_1(t)$ for $t > t_2$ ($t_2 = 0.9; 1.5$).

Example 4.2. In this study, a homogeneous Markov process was generated according to the transition intensity matrix

$$Q = \begin{bmatrix} -.3 & .3 & 0 \\ 0 & -.6 & .6 \\ 0 & .7 & -.7 \end{bmatrix}$$

The initial probabilities were taken to be $p_0(1) = .6$, $p_0(2) = .25$ and $p_0(3) = .15$. In the case where $t_\ell - t_{\ell-1} = 1.0$, it follows that the transition probabilities corresponding to Q above are given by $p_{11} = 0.74$, $p_{12} = 0.21$, $p_{13} = 0.05$, $p_{21} = 0.0$, $p_{22} = 0.66$, $p_{23} = 0.34$, $p_{31} = 0.0$, $p_{32} = 0.39$ and $p_{33} = .61$. Estimates of the MSE of $\hat{T}_{CMP,h}(t)$ and $\hat{T}_h(t)$, $h = 1,2$, based on 100 samples of size 300, $\sum_1^{100} (\hat{T}_{CMP,h}(t) - T_h(t))^2/100$, $\sum_1^{100} (\hat{T}_h(t_2)-T_h(t))^2/100$, are presented in Table 2 below for $t = 3,4$, and 5. The samples were generated at times $t_0 = 0$, $t_1 = 1.0$ and $t_2 = 2.0$, and the population size was taken to be 600.

Table 2. Estimated mean squared error under the continuous-time Markov chain superpopulation model based on 100 random samples with $m = 2$.

h		$t = 3.0$	$t = 4.0$	$t = 5.0$
1	$\hat{T}_{SMP,1}(t)$	207	203	175
	$\hat{T}_1(t_2)$	2893	8290	14144
2	$\hat{T}_{CMP,2}(t)$	363	489	486
	$\hat{T}_2(t_2)$	767	1698	2596

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