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Anchor deployment for deep water floating offshore equipments

O.B. Augusto *, B.L. Andrade

Naval Architecture and Ocean Engineering Department, Escola Politécnica da Universidade de São Paulo, Brazil

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Abstract

In this work a planning methodology for deep water anchor deployment in offshore platforms and floating production systems aiming at operational resources optimization is explored, by minimizing a multi criteria objective function. As an additional advantage provided by the proposed methodology, planning automation is achieved. Planning automation overcomes the traditional way, using a trial error basis. With it, an engineer, using an anchoring software, decides how much work wire and anchoring line must be paid out from both the floating system and the tug boat. Additionally, he decides which horizontal force must be applied to the line, trying to settle the anchor on a previously defined target on the ocean floor.

Keywords: Anchorage; Oil platforms; Optimization; Multiple criteria

1. Introduction

A very common operation for the groups that anchor and up anchor drilling platforms, as well as other equipment concerning oil prospection in the sea, consists in deploying anchors and anchoring lines.

This operation occurs very frequently. However, little attention has been given to developing simulators and applications to help engineers calculate the paying out stages and the resources to be used for such operations. The process consists of

* Corresponding author.
E-mail address: obaugust@usp.br (O.B. Augusto).

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Nomenclature

$\mathbf{X}$	design variables vector
r	fair lead distance to a point of the line
$f_j(\mathbf{X})$	jth objective function
z	catenary point depth
$g_j(\mathbf{X})$	jth inequality constraint function
w_i	linear weight of a line segment
$h_j(\mathbf{X})$	jth homogeneous constraint function
G_i	force concentrated in the ith segment
b_j	jth objective function goal
p_{ref}	seabed reference point
$d_j^{\pm}$	deviation from the jth objective function
α	seabed slope angle
$w_j^{\pm}$	associated weight to deviation of jth goal
U,V	segment forces vertical components
p_j	priority of jth objective function
EA	homogeneous segment axial stiffness
ν	multiple criteria objective function
θ	slope angle at a line point
TDP	touch down point
L	length (work wire, segment, etc)

having a tugboat or supply boat to hoist an anchor and an anchoring line from the platform or ship. It uses a supporting cable, *work wire* and, during the hoisting, the winches, both on the platform side. On the supporting vessel side, limited lengths of lines are released until the anchor reaches the floor, according to a previous plan. This defines how many steps should be performed and how much line should be released in each step, as shown in Fig. 1.

Nowadays, there are systems to calculate the catenary lines for planning these steps. The person in charge must decide how much line should be released from each vessel involved in the operation and what distance one should be from the other, during the attempt to make the anchor go towards the desired target. Otherwise, new values must be tried until the goal is attained. This procedure is not very productive and, although the desired condition may be attained, there is no reliability concerning the optimality of the solution found in relation to the necessary resources to actually carry out the deployment operation.

In Figs. 1–4 there are different situations for the same anchor deployment operation. The intended target was hit; however, the final characteristics significantly differ among themselves, as can be seen in Table 1.

Keeping only the simulations where the target is hit, as shown in Fig. 2, to hit the target, the supply boat should pay out 2679.9 m of work wire and impose a

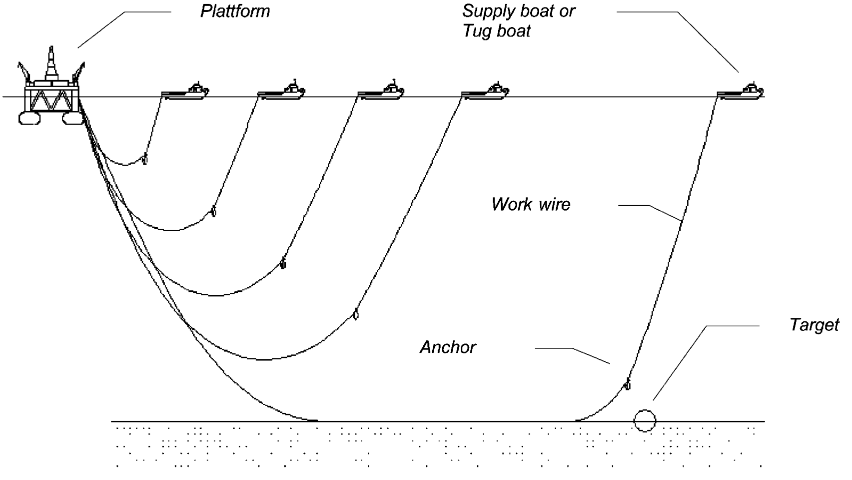

Fig. 1. Simulation of the anchor casting operation.

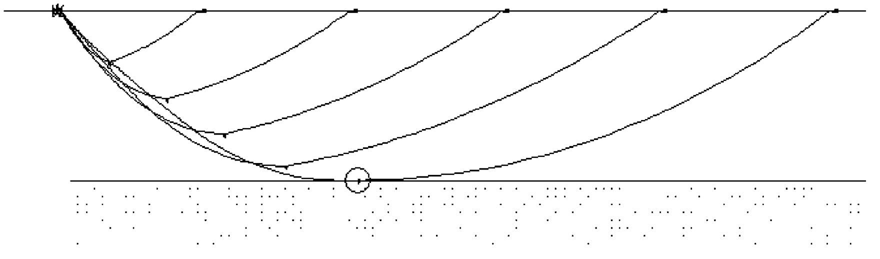

Fig. 2. Simulation. Anchor reaches the target. Horizontal force 798.6 kN, work wire: 2679.9 m.

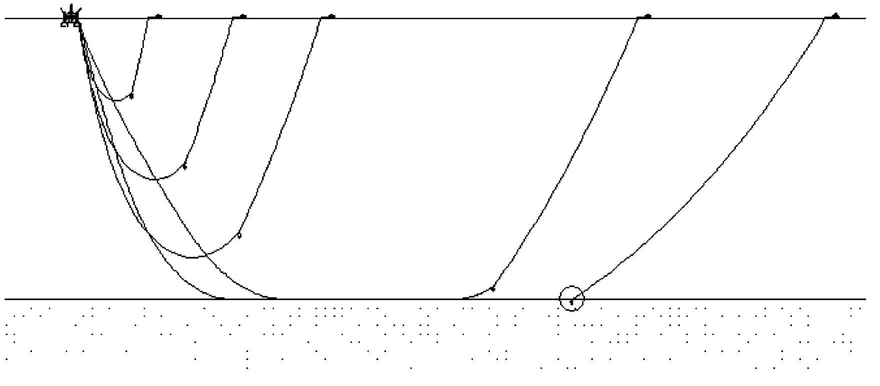

Fig. 3. Simulation. Anchor reaches the target. Horizontal force 266.2 kN; work wire: 1761.6 m.

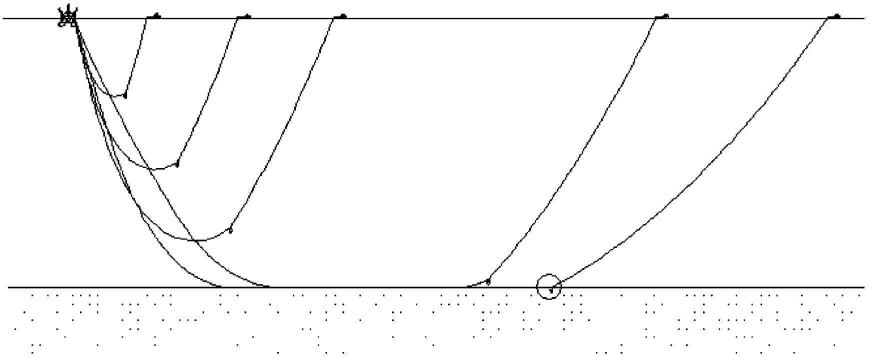

Fig. 4. Simulation. Anchor hits the target. Horizontal force 306.2 kN; work wire: 1285.7 m.

Table 1
Performance of anchor casting procedures

Case Fig.	Platform		Supply boat		Distance (m)	Distance (m)
	Length (m)	Force H (kN)	Length (m)	Force H (kN)	Plat. supply	Flead-anc
1	162.0	169.5	1071.3	18.9	1702.8	1337.7
2	1860.5	1212.8	2679.9	798.6	4068.4	1571.0
3	1761.6	266.2	1209.3	266.2	2373.7	1569.8
4	1682.9	306.2	1285.7	306.2	2490.7	1579.0

798.6 kN horizontal force. For the situation shown in Fig. 3, 1209.3 m work wire should be paid out and a 266.2 kN horizontal force should be imposed. Finally, in Fig. 4, the target is hit with the supply boat paying out 1285.7 m and exerting a 306.2 kN horizontal force on the work wire.

It is evident that, besides being a costly procedure, as it demands a difficult iterative learning exercise from the technician, it is unreliable concerning the possibility for success in the simulation. Even in cases in which the target is hit, there is no guarantee to the optimization of the necessary resources for the anchor deployment operation, the minimization of the line horizontal force being pointed out, which allows for the allocation of a smaller supply boat with less bollard pull. Even in other situations, when two supply boats are allocated but, for available thrust force limitations, only one would be used in an optimized calculation.

With the production of Brazilian oil in increasingly deeper waters, there has been a growing need to allocate supply boats with greater bollard pull, or even more than one supply boat which, together, carry out an operation. Under these conditions, it is paramount to have a tool, which, besides automating the process for calculating the deployment steps, also finds solutions to minimize the necessary resources to deploy anchors.

Observing that the operational cost of a 500 kN bollard pull supply boat is approximately US$ 6000.00 per day; supposing that an up anchoring operation and anchoring lasts from 7 to 15 days; supposing that, when deploying an anchor where only one boat would be necessary for the operation and, due to a non-optimized calculus situation, two were allocated, the financial cost waste would be of about US$ 90 000.00. Admitting that about a dozen non-optimized operations as the one mentioned occur within a one-year span, waste would amount to about US$ 1 000 000.00 a year.

Furthermore, process automation would generate other indirect incomes, such as less time for technicians to perform this calculation task and, consequently, higher productivity in the anchoring operations sectors.

2. Multiple criteria optimization

Using the technique called multiple criteria optimization or goal programming can easily solve the problem described in the introduction. This may be considered as a particular class of a more general mathematical programming problem.

Any mathematical programming problem may be formulated in a unique fashion, that is *standard,* which, under an application point of view, is particularly desirable, as its solution methods may be used for different problems without any modifications.

2.1. Design variables

Design variables or decision variables are those characterizing the problem and must be defined by an engineer or designer. In the anchoring problem under study, these include line lengths, positions of vessels, forces on lines, among others. In the structural design of a ship, they may include the stringers section modulus, the shell platting thickness, the frame spacing, etc.

They get grouped in a vector, formed by *n* independent variables, which may be defined as

$$\boldsymbol{X} = (x_1,x_2,x_3,\ldots x_n)^T \tag{1}$$

These variables are generally considered deterministic, that is, they are not subjected to a probability distribution. They may have a continuous spectrum along an interval, as in the case of a web height of a welded beam, or they may be restricted to discrete values, as in the case of the number of stringers on a stiffened panel. The usual procedure for treating discrete variables is to assume they are continuous, in a first approach, and later to research solutions with discrete values closer to the continuous solution formerly found. However, there is a consensus among those researching this subject that, in many cases, the discrete optimum may be far from the continuous optimum rounded for the discrete, as seen in Fu et al. (1991).

2.2. Objective function

The objective function, or cost function, or merit, is a defined scalar function of the design variables generically defined as

$$f(\boldsymbol{X}) = f(x_1,x_2,x_3,\dots x_n)^T \tag{2}$$

and is the function to be optimized.

The objective function may be a simple linear equation involving the x_i decision variables, or any other. For most practical problems, however, the objective function is non linear and not explicit, since the design models involve not only equations, but also tables or other implicit forms for systematization.

2.3. Constraint functions

Practical problems are generally subjected to a series of inequality constraints that may be represented by

$$g_j(\boldsymbol{X})\geq 0 \tag{3}$$

for $j = 1,2,...n_g$. These constraints may be linear or non-linear in the x_i variables. The tension levels in a structure, for example, functions of the geometric variables defining this structure, must be restricted to a maximum admissible value.

2.4. Equality constraints

Practical problems may also be subject of a series of equality or homogeneous constraints

$$h_j(\boldsymbol{X})\geq 0 \tag{4}$$

for j = $1,2,...n_h$. Analogously, such constraints may be linear or non-linear in x_i variables. They may be used to eliminate one or more decision variables once the x_i variables are no longer independent.

In some cases, when working with equality constraints is not desirable, the problem may be overcome by turning an equality constraint into two inequality constraints, that is, by imposing $|h(\mathbf{X})|\leq\epsilon$, where ϵ is a small number.

The optimization problem may now be expressed as selecting the $\mathbf{X_o}$ vector from design variables, which will minimize $f(\mathbf{X})$ subject to constraints, resulting in an optimum value for the objective function, $f(\mathbf{X_o})$. Minimization is used as an optimum for the merit as max(f) can always be treated as min($-f$).

2.5. Local solutions×global solutions

A minimum of function $f(\mathbf{X})$ may be a global minimum, that is, the smallest value of $f(\mathbf{X})$ for any $\mathbf{X}$ satisfying the constraint functions, that is, which is feasible. A minimum may also be a local minimum, that is, the smallest value of $f(\mathbf{X})$ in some feasible local region of $\mathbf{X}$ vectors.

2.6. Classification of optimization methods

Some authors, Novaes (1978) and Parsons (1975), classify optimization methods as direct or indirect. They are indirect, or gradient, if independent of direct comparison of numerical values of merit function calculated in two or more points. Indirect methods make use of necessary conditions for a point to be either minimum or maximum. These conditions are expressed by mathematical relations, which are indirect by their own nature. The direct or search methods presuppose the determination and comparison of function values to be optimized in several points located within the dominion of independent variables.

Other authors, Gill et al. (1981) prefer to separate them as methods for treating smooth functions and for treating non smooth functions.

Independently of classification, optimization methods may be free of constraints, if there are no constraints in the process, or with constraints, if otherwise.

Some important features of this approach are:

(a) all constraints are rigid and, therefore, of equal importance. There is no way to consider a greater or smaller flexibility in approaching constraints;
(b) all constraints must be consistent; that is, the existence of a viable region for solutions is always admitted. It is not possible to consider the existence of multiple and conflicting requirements and goals.

These features evidently limit the application of this approach to several engineering problems. They invariably involve multiple and conflicting requirements and goals which make the solution essentially a compromise solution, where such requirements and goals are more or less attended, according to the relative importance given to each.

To overcome such limitations, some generalizations of the classical approach to the optimization problem were developed, which allow the consideration of multiple and conflicting requirements and goals. One of these approaches is *Goal Programming*, which is an optimization model whose formulation guides the search for solutions so that certain predefined goals are attained. These goals are associated to requirements and attributes specified to assess the merit of solutions. Once goals are set for the design, the aim of goal programming is to find solutions whose performance is as close as possible to the set goals. That is, the aim is to minimize deviations or distances from goals. In this process, some goals, or even all of them, may not be attained. The classical minimization (or maximization) goal may be introduced in this model by defining goal values smaller than the smallest value that could be expected to the attribute or requirement assessed.

The design solutions produced by the use of such a technique do not always manage to satisfy all the conflicting requirements and goals. Therefore, they are essentially compromise solutions, in which deviations or distances to the more important goals are smaller than the ones to the less important ones. It is evident, therefore, that these solutions do not carry the meaning of “optimum” according to the conventional use of the term. In this approach, the mathematical representation of the goals is as follows:

$$\frac{f_j(\mathbf{X})}{b_j} + d_j^- - d_j^+ = 1 \quad (5)$$

where: $f_j(\mathbf{X}) =$ the jth goal function, function of the **X** design variables, with b_j aspiration level; $d_j^-, d_j^+ =$ deviation variables, representing respectively sub or super attainment to the b_j aspiration level, and always with values ≥ 0.

$b_j =$

aspiration value of the j goal.

The optimization problem thus constitutes in the following: if $f_j(\mathbf{X})$ has to be $\geq b_j$, then d_j^- is minimized; if $f_j(\mathbf{X})$ has to be $\leq b_j$, then d_j^+ is minimized; if $f_j(\mathbf{X})$ has to be $= b_j$, then both d_j^- and d_j^+ are minimized; $d_j^- \cdot d_j^+ = 0$, since there $f_j(\mathbf{X})$ can be no simultaneous sub and super attainment of the goal.

Now considering that with this new formulation goals and objectives start to define an aspiration subspace which must be attained as much as possible, and that the constraints define a subspace within which solutions must necessarily be inserted to satisfy feasibility requirements, the optimization problem may be expressed as a generalized goal programming problem, in which the weighed sum of deviation variables must be minimized, considering the existence of goals and constraints. The mathematical representation of this problem is the following: minimize

$$\psi = \sum_{j=1}^{N} p_j \cdot (w_j^- \cdot d_j^- + w_j^+ \cdot d_j^+) \quad (6)$$

subject to:

$$\frac{f_j(\mathbf{X})}{b_j} + d_j^- - d_j^+ = 1, \ j = 1,2,...,n_f \quad (7)$$

$$g_i(\mathbf{X}) \geq 0, \quad i = 1,2,...,n_g \quad (8)$$

$$h_i(\mathbf{X}) = 0, \quad i = 1,2,...,n_h \quad (9)$$

where: $w_j^-, w_j^+ =$ weights associated to the jth goal deviations; $p_j =$ priority associated to the jth goal.

The design problem is then to find the **X** vector of the decision variables so that constraints $g_i(\mathbf{X}) \geq 0$e $h_i(\mathbf{X}) = 0$ are satisfied and the$f_j(\mathbf{X})$ goals, or objectives, attained, within the best approximation possible. Constraints must evidently be consistent, as, otherwise, there will be no solution to the problem. Goals, however, do not have to be, and generally are not, consistent.

There are two typical approaches to define a ν objective function in terms of priority values and weights:

1. if all the $p_j = 1$ and if $\sum_{j=1}^{N} (w_j^- + w_j^+) = 1$, an objective function is called Archimedean;

2. if all weights $w_j^- = w_j^+ = 1$ and if $p_1 >> p_2 >> p_3 >> ... >> p_{nf}$, the objective function is called Preemptive.

The Archimedean form is used when the relative importance among goals is well defined and it is of interest to have them interacted. The Preemptive form is employed when only the order of importance of goals is available or when it is desired that priority goals be fully attained before the others. Generally, the same problem approached in the two forms will not present the same solution and the variation of weight values or priorities will also lead to different solutions. This feature carries the most interest in the problem formulation by means of goal programming technique, as it allows wider freedom for generating and comparing solutions than traditional optimization methods. Nevertheless, it also introduces difficulties associated to defining weights and priorities.

3. Solution of the catenary problem with multiple segments and with the two ends on the ocean surface

To model the physical problem of the line, with one end fixed on the vessel to be anchored and the other on the tug boat, the methodology proposed by Oppenheim and Wilson (1982) was used, where, for each segment of homogeneous material integrating the anchoring line, the gravitational force may be admitted as being the only field force present in the system, and thus, the classical catenary equation for a homogeneous segment may be used.

Being:

- w_i unit weight in the water for the ith homogeneous line segment ($w_i > 0$ for floating segments);
- G_i, suspended concentrated weight in the upper end of the ith segment, (negative represents a submersed buoy, positive, a clamp weight);
- L_i, length of the ith line homogeneous segment;
- $f_B(r,p_{ref},\alpha)$, the function that describes the ocean floor in the line plane, supposing a straight line with α inclination.

The problem thus proposed is iteratively solved. Admitting T_t the tension at the top of the line to be known, the following steps are followed:

1. the line inclination angle at the top, θ_0, close to the end that belongs to the vessel to be anchored is admitted and, with this, the vertical projection of T_T, at the upper end, U, as

$$U = T_T\cos(\theta_0) \tag{10}$$

and

$$H_P = T_T\sin(\theta_0) \tag{11}$$

2. as from the top, the problem of a homogeneous catenary segment is solved by calculating the geometry and the force at the lower end of the segment, V_i,

$$V_i = g_i(G_i,r_i,L_p,w_p,EA_p) + L_i w_i - U_i \tag{12}$$

where g_i is the force concentrated at the upper end of the segment, generated by the buoy or clamp weight, immersion function of the buoy, in case this is not completely submersed, or the concentrated force generated by the clamp weight, in case this is touching the floor; L_p is the length of the pendant, admitted a single homogeneous segment, with w_p submersed weight and EA_p axial stiffness; r_i is the abscissa of the upper end of the ith segment. For a V_T volume cylindrical buoy, partially immersed, with draft,[1] h, one can obtain

$$g_i = \frac{\pi D^2 h}{4V_T} G_i \tag{13}$$

3. by the balance of the point at the segment lower end, the projection of the U_{i+1} vertical force is calculated, acting on the next segment upper end; and thus successively up to the last segment.
4. the vertical distance of the opposite end of the line—the one belonging to the tug boat—is calculated on the surface of the sea. As this end must be on the surface, the distance function must be zeroed.

$$\delta_E = z_E \tag{14}$$

5. by an iterative method, Newton for example,[2] the angle at the top, θ_0, is adjusted until the function previously described is zeroed, that is, the function is minimized,

$$\delta_E = |z_E| \tag{15}$$

6. at this point, it must be observed whether some point of the line touches the ocean floor.
7. in case the touch does not occur, the configuration of the line is determined.
8. otherwise, another procedure is started where stages 1, 2 and 3 are the same as previously defined.
9. as the line is previously known to touch the floor, for a certain θ_0 top angle, the point of the line whose tangent is identical to the floor inclination is searched. Such a point is liable to be the contact point of the line with the floor, the TDP (*Touch Down Point*).
10. the distance from this point to the floor is verified. This is then the new distance function for which the θ_0 angle is iteratively searched so that the distance is zeroed.

[1] The buoy draft will depend on the axial stiffness of the pendant, therefore, locally, there is an iterative process to define, as from the connection point of the pendant, which the buoy draft must be, its buoyancy force, the pendant elongation, which in turn results in a new buoy draft. A successive approximations method ensures a fast convergence in this process.

[2] As it is a search for the zero of a function, any optimization algorithm may alternatively be used to solve the problem.

$$\delta_{TDP} = z_{TDP} - f_B(r_{TDP}, p_{ref}, \alpha) \quad (16)$$

11. when the θ_0 angle that zeroes this function is found, a new iterative process is begun where the geometry of the suspended catenary on the side of the vessel to be anchored is already defined, from the fair lead up to the TDP. A certain length of the line that lies at the floor, L_f, is then arbitrated, as from the TDP, and the second TDP_2 is defined.
12. as from the second TDP, TDP_2 , the geometry of the suspended catenary is set, up to the end of the supporting vessel.
13. the vertical distance of the opposite end of the line—the one belonging to the tug boat – to the sea surface is calculated. As this end must be on the surface, the distance function must be zeroed.

$$\delta_E = |z_E| \quad (17)$$

14. by an iterative method, Newton for example, the length on the floor, L_f, is adjusted until the function previously described, eq. (17), is zeroed.

In several points of the algorithm for calculating the catenary, an iterative method to calculate a root of an *error* function was suggested. Different methods may be used for this end. In the work, the Newton Method was used, as it allows for the convergence of the process in few iterations. The derivatives of the *error* functions, however, had to be obtained through finite differences in the cases mentioned.

4. Optimization by multiple objectives for the anchor deployment problem

For an anchor deployment situation, decision variables can be defined as:

(a) the work wire length, L_{ww}, to be paid out by the tug boat;
(b) the line length, L_p, to be paid out by the vessel to be anchored; and
(c) the horizontal force, H_p, to be exerted by the vessel to be anchored.

Once these variables are defined, the geometry of the line in catenary can be found, which satisfies the established conditions. In this configuration, the anchor reaches position $P_{Anc} = P_{Anc}(r_{Anc}, z_{Anc})$, at the floor or not, from which the distance function, δ_a, can be defined between the target, $P_{\mathrm{Target}} = P_{\mathrm{Target}}(r_{\mathrm{Target}}, z_{\mathrm{Target}})$, and the anchor,

$$\delta_a = dist(P_{Anc}(H_P, L_{ww}, L_P), P_{\mathrm{Target}}) \quad (18)$$

the horizontal force in the tug boat or supply boat,

$$H_S = H_S(H_P, L_{ww}, L_L) \quad (19)$$

As objectives to be minimized, there should be:

$$H_S + d_H^- - d_H^+ = H_S \min \quad (20)$$

$$L_{ww} + d_{Lw}^{-} - d_{Lw}^{+} = L_{ww}\text{min} \tag{21}$$

$$L_P + d_P^{-} - d_P^{+} = L_P\text{min} \tag{22}$$

$$\delta_a + d_a^{-} = 0 \tag{23}$$

It should be noted that the horizontal force to be applied to the line, on the side of the supply boat, may be different from the horizontal force applied to the side of the vessel to be anchored in case the line, composed by the work wire and anchoring line, touches the ocean floor and the friction between it and the floor is not null.

With this, the objective function to be minimized is obtained

$$\min(d_H^{-} + d_H^{+} + d_{Lw}^{-} + d_{Lw}^{+} + d_P^{-} + d_P^{+} + d_a^{-}) \tag{23}$$

As H_r, L_{ww} and L_P are to be minimized, the following values are adopted as target

$$H_R\text{min} = 0, \quad L_{ww}\text{min} = 0, \quad L_P\text{min} = 0 \tag{24}$$

To solve the multiple objectives minimization problem, the direct method for optimization, developed by Augusto and Kawano(1998), was used, believing that for the anchoring problem, even with a small number of variables, the derivative functions, be they from tensions, be they from displacements, are not trivially obtained. Therefore, its calculation must be numerically processed by some finite difference algorithm, which, per se, would harm the performance of the indirect optimization methods.

5. Results

As an application of the proposed methodology, let it be taken as an example the anchor deployment operation of an anchoring line from a FPSO (Floating Production, Storage and Offloading) ship located at 200 m water depth, in a continental slope with a 5° inclination, the line composed of 2 chain segments, their proprieties shown in Table 2. By the design of the anchoring system, such line must have its anchor

Table 2
Proprieties of the line to be installed

Seg.	Material	Diameter (mm)	Linear weight (kN/m)	Length (m)	Axial stiffness (kN)	Friction coefficient	Break strength (kN)	Anchor	
								Type	Stevpris MK III (kN)
1	Chain	76.0	107.66	100.0	372250,00	1.000	6001	Holding power	3413
2	Chain	105.0	205.49	1000.0	710468,00	1.000	10754	Weight	150.0

Table 3
Results of the use of the optimization algorithm

Ship				Supply				Distance ship-sup	Anchor coords	
Length (m)	Traction (kN)	Angle hor. (°)	Tract (kN)	Length (m)	Traction (kN)	Angle hor. (°)	Tract (kN)	(m)	R (m)	Z (m)
174.5	272.1	0.15	29.3	79.1	268.9	-0.15	29.3	76.6	67.7	78.7
349.1	539.5	0.15	58.7	158.1	386.9	-0.14	58.7	160.4	135.4	156.5
523.6	594.7	0.15	88.0	238.2	337.1	-0.14	64.1	572.9	524.5	232.6
698.1	533.4	0.14	117.3	316.2	276.4	-0.12	109.6	811.1	671.3	284.0

set at a 622 m anchoring radius, at the 142° azimuth, in relation to the north. For the deployment operation, to this value a 50 m operational margin is added, so that, when hoisted, the anchor line drags and penetrates on the floor in the desired position. The target, for anchor deployment operation is thus modified to 672 m, with 1 m tolerance. Supply boats with maximum bollard pull of 500 kN and 1000 m work wire are available. It may be observed, through the results shown in Table 3, that the objectives are fully attained.

6. Conclusions

This work presented a methodology for optimizing procedures for deploying drag anchors for ocean equipment to explore and produce oil in the ocean. As decision variables to be established by the technical body responsible for deploying the anchor, there is an attempt to minimize the work wire length to be paid out by the supporting vessel, its horizontal force and, at the same time, to make the anchor hit a pre-determined target for deployment. To deal with these distinct objectives, multiple objectives optimization technique was used, where the merit function, due to decision variables, was set with non dimensional deviations of the objectives previously described. To solve the minimization problem of the merit function, without harming the choice for any other, a sequential minimization algorithm was used with external penalty, to deal with continuous and discrete variables. Besides optimizing resources for the anchor deployment operation, as an additional result to the process, an automation of the calculation of all intermediary anchor deployment steps is obtained. Such tool is believed to become a powerful ally to technicians in charge of anchoring offshore equipment for the deep water oil production industry.

References

Augusto, O.B., Kawano, A., 1998. A mixed continuous and discrete nonlinear constrained algorithm for optimizing ship hull structural design. Ocean Engineering, an International Journal of Research and Development 25 (9), 793–811.

Fu, J., Fenton, R.G., Cleghorn, W.L., 1991. A mixed integer-discrete-continuous programming method and its application to engineering design optimization. Engineering Optimization 17, 263–280.

Gill, P.E., Murray, W., Wright, M.H., 1981. Practical Optimization. Academic Press Limited.

Novaes, A.N., 1978. Métodos de Otimização, Aplicação aos Transportes. Editora Edgar Blücher Ltda.

Oppenheim, B.W., Wilson, P.A., 1982. Static 2-D solution of a mooring line of arbitrary composition in the vertical and horizontal operating modes. International Shipbuilding Progress 29 (334), 142–153 June.

Parsons, M.G., 1975. Optimization Methods for Use in Computer-Aided Ship Design. The Society of Naval Architects and Marine Engineers First Ship Technology and Research (STAR) Symposium, Washington, DC August.