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FURTHER DISCUSSION ON THE WHITE-NOISE APPROACH FOR PREDICTING SLOW-DRIFTS IN FAST

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ABSTRACT

Making use of theoretical approximations for the computation of the wave-induced slow-drift forces is a common procedure in the early stages of design of a new floating unit. They can help reducing the computational burden in two different fronts: for generating the QTFs in a frequency domain analysis, and during the subsequent execution of time-domain simulations. In a previous paper, we have discussed a simple procedure for making use of the white-noise approximation in FAST, without the need for any modification of the software. The proposal only requires restricting the computation of the QTFs to pairs of frequencies that are indeed essential to the slow-drift dynamics. For this, however, an additional assumption is made, considering that each motion is decoupled from those in the other dofs. In the present paper, a more detailed analysis of the subject is made, in order to clarify the theoretical aspects of the procedure and supplement the previous analysis. Once again, the results are based on the data available for the OC4 FOWT. The accuracy obtained with the procedure is discussed not only in terms of the resulting motions, but also comparing its effects on the second-order force spectra. A more detailed evaluation of the dynamic couplings is presented, and comparisons with the results obtained with Newman's approximation are made in simulations involving waves only.

INTRODUCTION

One of the key elements of the design of floating units is the specification of the mooring system, which is responsible for keeping the floating structure in position against waves, wind and current. The mooring lines must be designed carefully in order to achieve a solution with minimum weight/cost that is still able to prevent large excursions and to resist extreme environmental conditions. This is no different for Floating Offshore Wind Turbines (FOWT).

Numerical tools for simulating the dynamics of the whole system (floater, turbine and moorings) play an important role in attaining an efficient design. Among these tools, the most widespread are time-domain software that rely on hydrodynamic coefficients pre-calculated in frequency domain by a radiation/diffraction code. These include the difference-frequency force Quadratic Transfer Function (QTF) matrices, which model the second order wave forces responsible for the mean and slow drift of the floating system and, therefore, of great importance for the mooring system design.

Regarding FOWTs, Coulling et al. [1] discussed the relevance of low frequency second order wave forces on the DeepCwind semi-submersible FOWT using Newman's approximation, indicating that they are indeed important in parked/idling turbine scenarios. Roald et al. [2] conducted a similar evaluation for a spar and a tension-leg platform (TLP) in frequency domain, including an analysis of sum-frequency effects on the dynamics of the TLP. Jiawen Li et al. [3] studied the slow-drift of a new hybrid floater FOWT, while Bayati et al. [4] focused on the effects of water depth on the potential flow computations for the OC4 semi-submersible.

In most of these works, the QTF matrices were calculated by a frequency-domain code (WAMIT, WADAM) for a large range of wave frequency pairs (ω_i, ω_j) and imported by the software FAST to conduct time domain analyses [5] [6]. However, this approach is significantly expensive in terms of computational time. Besides, if the second-order potential is included in the analysis, the need for meshing the free-surface largely increases the complexity of calculating the QTFs in frequency domain, and usually makes the analysis of numerical convergence harder.

In order to simplify the computation of the slow-drift forces, some approximations have been proposed. The most well-known is Newman's approximation [7], which considers that the whole difference-frequency QTF matrix can be approximated based on its main diagonal. Besides reducing the computer time considerably, this approximation eliminates the need to obtain the second-order velocity potential, since the main diagonal (responsible for the mean drift) depends exclusively on the first-order solution.

Although widely used for calculating the slow-drifts of offshore systems, it is worth mentioning that the error made in Newman's approximation is of the order of $\Delta\omega^2$, and it is valid only for deep water problems (see, for instance, [8]). Since FOWT are designed for shallower waters than traditional offshore systems, the natural periods induced by the mooring system in surge, sway and yaw are expected to be much lower than those for deep-water systems. Due to the fact that slow-drift is a resonant problem, the most relevant difference frequencies are those close to the natural frequency of motion ($\Delta\omega = \omega_n$), indicating that lower natural periods lead to larger errors in Newman's approximation. Lopez-Pavon et al. [9] and Simos et al. [10] studied the slow-drifts of a semi-submersible FOWT in 100 m water-depth with a resonance period of 75 s, showing that, in this test case, Newman's approximation considerably underestimated the slow-drift motions of the system.

Another option for calculating the slow-drift forces is the white-noise approximation, a classical approach in the field of system dynamics for analyzing weakly damped systems, which have a narrow banded response (see, for example, the classical book of Crandall & Mark [11]; the same idea is also discussed in Faltinsen's book [12]). Under these circumstances, the power spectrum of the exciting force can be considered constant with a value equal to the one for the resonance frequency, producing excellent results.

As offshore systems are typically weakly damped, they fall into the category of systems for which the white-noise approximation is suited. Indeed, Simos et al. [13] and Matos et al. [14] obtained an excellent agreement between the white-noise approximation and experimental results for a large oil & gas semi-submersible, while Lopez-Pavon et al. [9] and Simos et al. [10] showed that the same is true for a semi-submersible FOWT.

When applied to the computation of slow-drift forces, it is important to highlight that the white-noise approximation is not an approximation on the QTF itself. It just indicates the portion of the QTF matrices that contribute the most to the slow-drift dynamics, that is to say the frequencies for which $\Delta\omega = \omega_i - \omega_j \approx \omega_n$. However, all the aforementioned works employed the white-noise approximation in frequency domain, which is natural, since it is an approximation on the force spectrum. Moreover, if the motions can be considered decoupled, the slow-drift motion can be calculated by simply crossing the constant force spectrum with the motion transfer function for unitary force.

In a previous work [15], our group proposed a simple approach for exploiting the white-noise approximation in time domain analyses. The procedure is simply to disregard the diagonals that are far from the most relevant one ($\Delta\omega = \omega_n$), i.e. only the diagonals of the QTF matrices close to the resonance frequency are considered. Strictly speaking, the original assumption of the white-noise approximation is breached, for the force spectrum is no longer constant. Nevertheless, the original idea that the most important QTF terms are those next to the resonance frequency is preserved, and the computer time can be reduced significantly.

In order to verify this approach, the OC4 platform was analyzed with FAST in waves only, employing three different sets of QTF matrices: the full matrices provided with FAST; one matrix in which only the main diagonal and the resonance diagonal are considered; and a third one with only the main diagonal and a "strip" around the resonance diagonal (details are given in the following sections). The results showed a good agreement, especially for the QTF matrices with the strip around the resonance diagonal.

However, some questions persisted. Firstly, some large relative discrepancies arose for the motions in pitch, and although small in absolute terms, it was important to clarify the source of this inaccuracy. The main hypothesis was that it might be related to coupling effects, and one of the goals of the present work was to verify it. For doing so, the simulations performed in [15] were repeated considering only the pitch motion, i.e. the calculation of the other degrees of freedom was disabled in FAST. As the results obtained with both the full and the simplified QTF matrix were nearly identical, it was concluded that the differences were indeed due to coupling effects. Another set of simulations was conducted to verify whether the coupling was due to the moorings, but this hypothesis was proven wrong. Hence, the coupling is very probably due to inertial effects.

Moreover, as Newman's approximation is widely used, it is interesting to compare the slow drift it provides with the ones calculated using the white-noise approximation. Although they showed a good agreement for shorter waves, Newman's approximation underpredicted the motion for the longer ones.

Finally, the results are complemented by an analysis of the second-order force spectra calculated for each set of QTF matrices, in order to verify how they compare around the peak of the motion transfer function for unitary force.

The next section brings a brief theoretical background on the white-noise approximation, followed by a summary of the main OC4 platform data and the environmental conditions employed in the analyses. Next, the construction of the QTF matrices for the white-noise approximation is explained, and the second-order force spectra obtained with each of them for a given sea state are discussed. Then, the slow-drift motions calculated with the white-noise approximation and with Newman's approximation are compared. At last, the results of a simulation considering only the pitch motion is presented, indicating that the coupling effects are indeed the source of the discrepancies observed in pitch.

THEORETICAL BACKGROUND

This section briefly presents the application of the white-noise approximation to the calculation of the slow-drift spectrum, and readers interested in a more detailed discussion can refer to [10]. In the following, it is supposed that the motion analysis is conducted in frequency domain and that coupling effects are negligible.[1]

Let $F_\alpha^{(-)}(\omega_i,\omega_j)$ denote the frequency-difference QTF for the motion α $\{\alpha = 1,...,6\}$. Let's then consider the pair of frequencies $(\omega, \omega+\mu)$, with μ a small frequency difference. For a given wave spectrum $S(\omega)$, the low-frequency second-order force spectrum can be calculated using the following relation:

$$S_{F,\alpha}^{(-)}(\mu) = 8\int_0^\infty S(\omega)\, S(\omega+\mu) \left|F_\alpha^{(-)}(\omega,\omega+\mu)\right|^2 \mathrm{d}\omega \quad (1)$$

and the low frequency second order motion for the degree of freedom α is given by:

$$S_\alpha^{(-)}(\mu) = S_{F,\alpha}^{(-)}(\mu)\, |H_\alpha(\mu)|^2 \quad (2)$$

with $H_\alpha(\mu)$ the response function of the floating unit, defined by:

$$H_\alpha(\mu) = \frac{1}{-\mu^2\left[M_{\alpha\alpha}+A_{\alpha\alpha}(\mu)\right] + i\mu\left[B_{\alpha\alpha}(\mu)+B_{\mathrm{ext},\alpha}\right] + \left[C_{\alpha\alpha}+C_{\mathrm{ext},\alpha}\right]} \quad (3)$$

where M is the system's inertia matrix; A and B the added mass and potential damping matrices; C is the hydrostatic restoring matrix; and B_{ext} and C_{ext} the linearized external damping and restoring forces/moments that may be present.

Around the resonance frequency, i.e. $\mu \approx \omega_{n,\alpha}$, the inertial term in $H_\alpha(\mu)$ is nearly canceled by the restoring terms, and the magnitude of the response function is inversely proportional to the damping. Moreover, the band-width around $\mu = \omega_{n,\alpha}$ also depends inversely on damping. Hence, weakly damped systems have a narrow-banded response function with a very pronounced peak at $\mu = \omega_{n,\alpha}$. Based on these characteristics, the white-noise approximation states that:

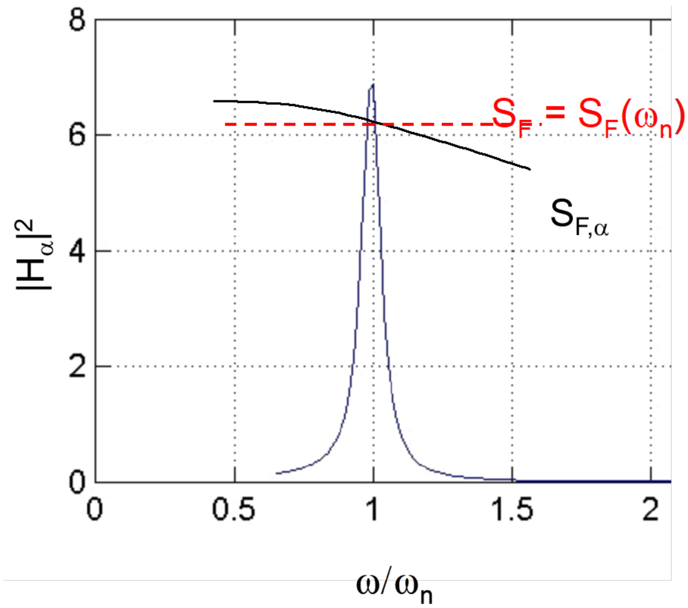

***FIGURE 1**: Illustration of the white-noise approximation.*

1. The only significant contribution of the force spectrum $S_{F,\alpha}^{(-)}$ to the motion spectrum $S_\alpha^{(-)}$ is due to a small range of frequencies close to the resonant frequency $\mu = \omega_{n,\alpha} \pm \delta\omega$;
2. Given that $S_{F,\alpha}^{(-)}$ is expected to be smooth in this range, it can be approximated as a constant spectrum:

$$S_{F,\alpha}^{(-)}(\mu) = S_{F,\alpha}^{(-)}(\omega_{n,\alpha}) = \text{constant} \quad (4)$$

as illustrated in Figure 1. Given the considerations above, it is evident why this approach is referred to as "white-noise".

The benefits of applying this approximation to the computation of the slow-drift motions are clear: instead of having to compute the full QTF matrix $F_\alpha(\omega_i,\omega_j)$, it is only necessary to calculate the terms at the diagonal corresponding to $\omega_i = \omega_j + \omega_{n,\alpha}$, thus saving considerable computational effort. Furthermore, it highlights the part of the QTF matrix that contributes the most to the slow-drift response, hence also making evident the parts that do not. By our own experience, this is particularly important, since the most troublesome frequency pairs for numerical convergence are often outside the range that dominates the second-order motions.

The white-noise approximation, as stated above, showed

[1] In some cases, coupling effects may be significant, and the test case analyzed in this work is one of them. Nevertheless, in many situations an approximate uncoupled model is acceptable, especially during preliminary stages of design.

good results in previous studies of the low-frequency wave induced motions of large oil & gas semi-submersible units [13] [14]. In a more recent work, it also provided good results in the analysis of a typical semi-submersible FOWT [10].

As the accuracy of the white-noise approximation depends on the narrowness of the response function, the motions must be weakly damped. In the aforementioned works, the linearized damping factors were between 3% and 15% of the critical damping for the respective motion, typical figures for semi-submersible structures in a wide range of sea conditions.

Besides, the dynamic analysis conducted in these works was linear and in frequency domain. As it is about the force spectrum, the application of the white-noise approximation in frequency domain is straightforward, since the motion spectra can be directly calculated with the white-noise force spectra and Eqs. 2 and 4.

However, common practice for the design of floating structures (including FOWTs) is to perform non-linear time-domain simulations with software that import hydrodynamic coefficients precomputed in frequency-domain, including the QTF matrices. This is the approach adopted by FAST, the software employed to conduct the simulations presented in this work. In this kind of numerical method, the white-noise approximation can not be applied as originally proposed. Nevertheless, an alternative approach can be adopted, following the same principles that guide the original white-noise approximation, as explained below.

Application in Time-Domain

In order to extend the application of the white-noise approximation to time-domain codes, our group suggested an alternative approach in a previous work [15]. Considering that the second order force spectrum is significant only in a narrow range of frequencies around the resonance frequency, it is reasonable to think that only the QTF values around the resonance diagonal (i.e. the diagonal for which $\mu = \omega_{n,\alpha}$) are relevant for calculating the slow-drift motions. Hence, the proposal is to compute the slow-drift motions considering QTF matrices without the diagonals that are far from the resonance diagonal. Alternatively, this can be seen as setting to zero the elements outside the vicinity of the referred diagonal. In the studied conducted in [15], QTF matrices with only 4 diagonals (the main diagonal + the resonance diagonal + the diagonals around the resonance diagonal) showed very good results.

Adopting this approach, only a reduced number of QTF values need to be computed, saving considerable effort in obtaining these matrices in frequency-domain. Besides, this approximation could reduce the time required for calculating the difference-frequency wave forces in time-domain, since less frequency pairs would need to be considered. It is worth mentioning that the original assumption of the white-noise approximation is no longer followed, as the force spectrum is no longer constant. However, the main idea is preserved.

CASE-STUDY: THE OC4 MODEL

To test the approximation presented above, simulations with the OC4 semi-submersible model were performed with the software FAST (version 8) in time domain, considering different wave conditions and waves only. The input files were the ones provided with the software, and four different groups of simulations were conducted:

1. A group in which the second-order motions are calculated using the full difference-frequency QTF matrices, which is one of the input files distributed with FAST;
2. Simulations using QTF matrices in which only the main diagonal and the resonance diagonal for the respective motions are retained, while the other elements are set to zero. For simplicity, these matrices will be referred to as "WN diagonal QTF";
3. Another group employing QTF matrices with only the main diagonal and a "strip" around the resonance diagonal, i.e. the resonance diagonal, the diagonal immediately above it ($\mu = \omega_{n,\alpha} + \delta\omega$) and the diagonal immediately below it ($\mu = \omega_{n,\alpha} - \delta\omega$). These matrices will be referred to as "WN strip QTF"; and
4. Simulations employing Newman's approximation.

The three different sets of QTFs are presented in the next section. The rest of this section presents the main characteristic of the floater and the mooring system of the OC4 semi-submersible, extracted from [16] (where further details can be found), as well as the wave conditions analyzed.

Floater

The floater of the OC4 project is the same as the semi-submersible tested in a wave basin in the DeepCwind project [17]. It comprises a central column, for sustaining the turbine tower, connected to three offset columns through an assemblage of slender pontoons and cross braces. To help suppress vertical motions, a larger cylinder is attached to the base of each offset column. The main characteristics of the floater are provided in Table 1, while Figure 2 gives an illustration.

Mooring

The mooring system comprises three catenary lines spread symmetrically around the floater, so that each line is 120° apart from the other two, as illustrated in Figure 3. The fairleads are located at the top of each base column, 14 m below the waterline. The anchors are situated at a water depth of 200 m. The main characteristics of the mooring system are given in Table 2

***TABLE 1**: Main characteristics of the floater. Data extracted from [16].*

Total draft	20 m
Total displacement	14 265 t
Vertical position of center of mass (CM)	20 m
Roll/pitch inertia about CM	$6.8 \cdot 10^9$ kg $\cdot$ m^2
Spacing between offset columns	50 m
Length of upper columns	26 m
Length of base columns	6 m
Diameter of main column	6.5 m
Diameter of offset (upper) columns	12 m
Diameter of base columns	24 m
Diameter of pontoons and cross braces	1.6 m

***FIGURE 2**: Picture of the model-scale floater analyzed in the DeepCwind project. Extracted from [16].*

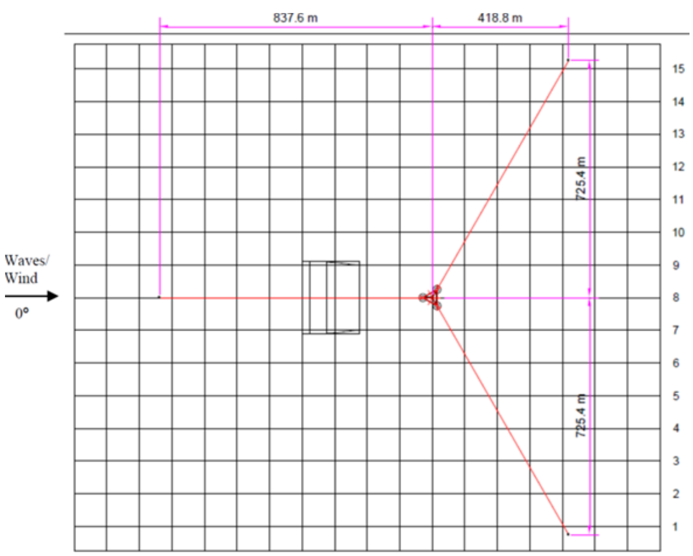

***FIGURE 3**: Mooring configuration. Extracted from [16].*

***TABLE 2**: Mooring properties. Data extracted from [16].*

Number of mooring lines	3
Angle of adjacent lines	120°
Depth to anchors below SWL	200 m
Depth to fairleads below SWL	14 m
Radius to anchors from platform centerline	837.6 m
Radius to fairleads from platform centerline	40.9 m
Unstretched mooring line length	835.5 m
Mooring line diameter	0.0766 m
Equivalent mooring line mass density	113.35 kg/m
Hydrodynamic drag coefficient	1.1
Hydrodynamic added-mass coefficient	1.0

Environmental conditions

As this work focuses on the second order wave induced forces, all the simulations are performed with waves only, hence neither current nor wind/air damping effects are considered. The only FAST modules employed are HydroDyn, MoorDyn and ElastoDyn. The sea state is represented by a JONSWAP spectrum, specified by a significant wave height H_s and a peak period T_p.

Nine different sea states are considered, with peak periods $T_p = 4$ s, 6 s, 8 s, 10 s, 12 s, 14 s, 16 s, 18 s, and 20 s. Focus is given to the effect of the wave period, hence the same wave height is considered in all cases, $H_s = 2$ m. For all the sea states, the wave direction is 0°, i.e. the waves propagate from -x to +x (see Figure 3). Thus, the only degree of freedom that show significant motions are surge, heave and pitch.

SIMPLIFYING THE ORIGINAL QTF MATRICES

As mentioned in the previous section, simulations are performed with three different sets of QTF matrices: the original full QTF matrices distributed with FAST and two other matrices derived from the original one, denoted as "WN diagonal QTF" and "WN strip QTF".

The "WN diagonal QTF" is obtained simply by setting to zero the elements outside the resonance diagonal, i.e. the diagonal where $\mu = \omega_i - \omega_j = \omega_{n,\alpha}$. For the "WN strip QTF", the elements in the diagonals immediately below and above the resonance diagonal are also retained, representing a strip of width $2\delta\omega$ around $\mu = \omega_{n,\alpha}$. For both matrices, the main diagonal is kept unchanged, since it is only responsible for the mean drift.

It must be mentioned that this approach does not reduce the computer time required for each simulation, since the zero elements are still included in the computation, and it was chosen only because FAST requires full QTF matrices as input. Never-

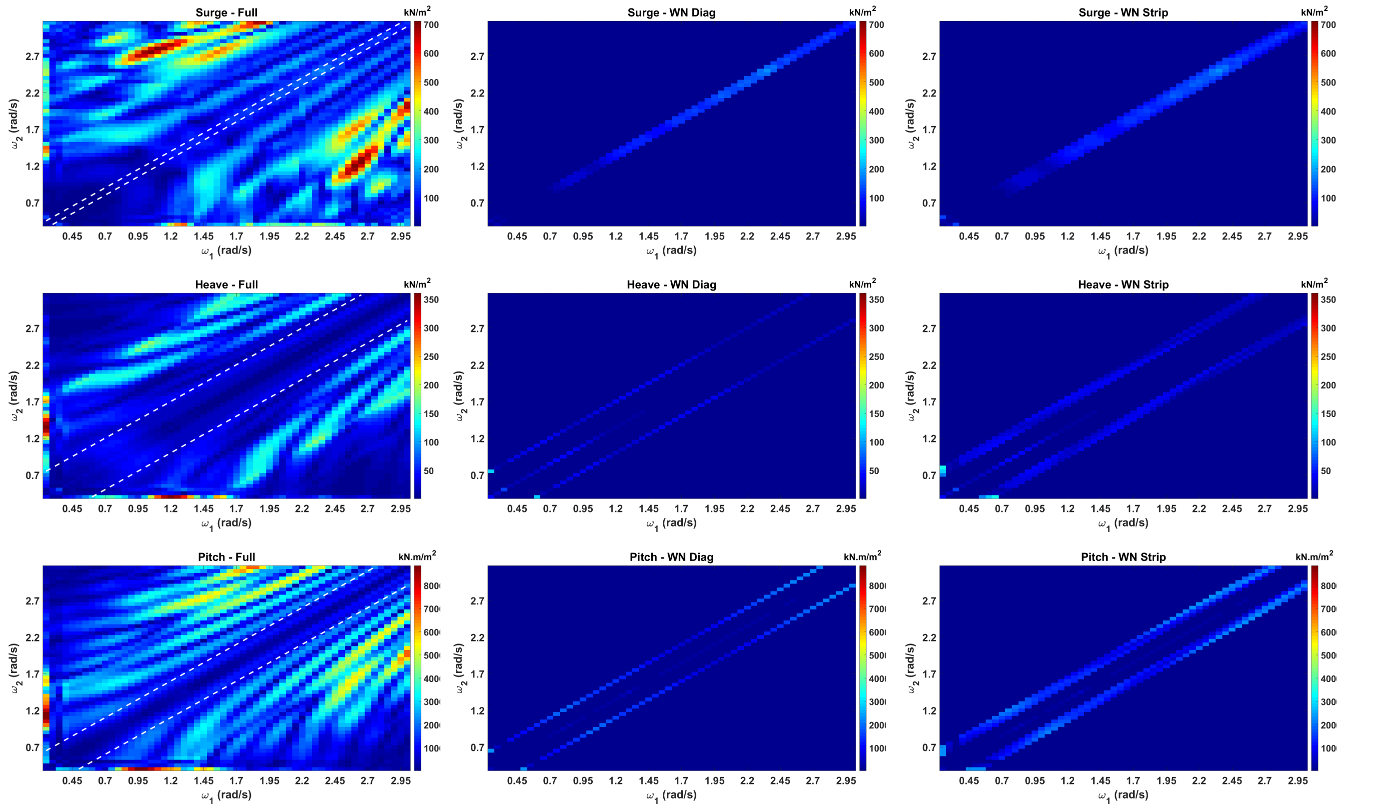

***FIGURE 4**: QTF matrices (amplitude) originally distributed with FAST (left), the simplified "WN diagonal QTF" matrices (middle), and the simplified "WN strip QTF" matrices (right).*

theless, this procedure allows verifying the white-noise approximation, which could motivate the use of algorithms that support the inclusion of only selected frequency pairs (ω_i, ω_j).

Table 3 lists the resonance periods/frequencies of the system. As the QTF matrices distributed with FAST have a discretization of 0.05 rad/s, the diagonals for which the frequency difference μ is closest to the resonance frequencies in surge, heave and pitch are, respectively, the first, the seventh, and the fifth ones below/above the main diagonal (the matrices are symmetric).

The three different sets of QTF matrices in surge, heave and pitch, the only degrees of freedom that show significant motion (only following seas are considered), are illustrated in Figure 4. The resonance diagonals are indicated by white dashed lines in the left column of Figure 4, which corresponds to the full QTF matrices distributed with FAST. It is important to mention that the resonance diagonal in surge is the first one below the main diagonal, hence one of the adjacent diagonals is the main diagonal itself. Thus, the surge "WN strip QTF" contains only five diagonals, instead of the seven diagonals present in the heave and pitch "WN strip QTFs" (if we consider that the matrix is symmetric, the number reduces to three distinct diagonals in surge and four in heave and pitch).

***TABLE 3**: Resonance period and frequency in surge, heave and pitch*

	T_n	f_n	ω_n
Surge	109 s	0.0092 Hz	0.058 rad/s
Heave	17.5 s	0.057 Hz	0.36 rad/s
Pitch	25.6 s	0.039 Hz	0.25 rad/s

Figure 5 presents the low-frequency second-order force spectra calculated for each of the QTF matrices using Equation 1 and considering a JONSWAP with $H_s = 2$ m and $T_p = 8$ s. In the same graphs, the response function of each of the considered degrees of freedom is also included. They were obtained using the added mass and potential damping matrices provided with FAST, with an additional damping of 5% of the critical damping to account for viscous effects, and neglecting any coupling effects.

It can be seen that the spectrum in surge is nearly flat around the resonance frequency, and the utilization of the simplified matrices approaches the original white-noise approximation [10]. On the other hand, the spectra in heave and pitch show a more pronounced variation around $\mu = \omega_{n,\alpha}$, but the peak of the response function is so narrow that it should not make a difference. More important, the force spectra make clear that most of the QTF matrices do not make any difference for the resonant motions, and only a very small part of them is actually important.

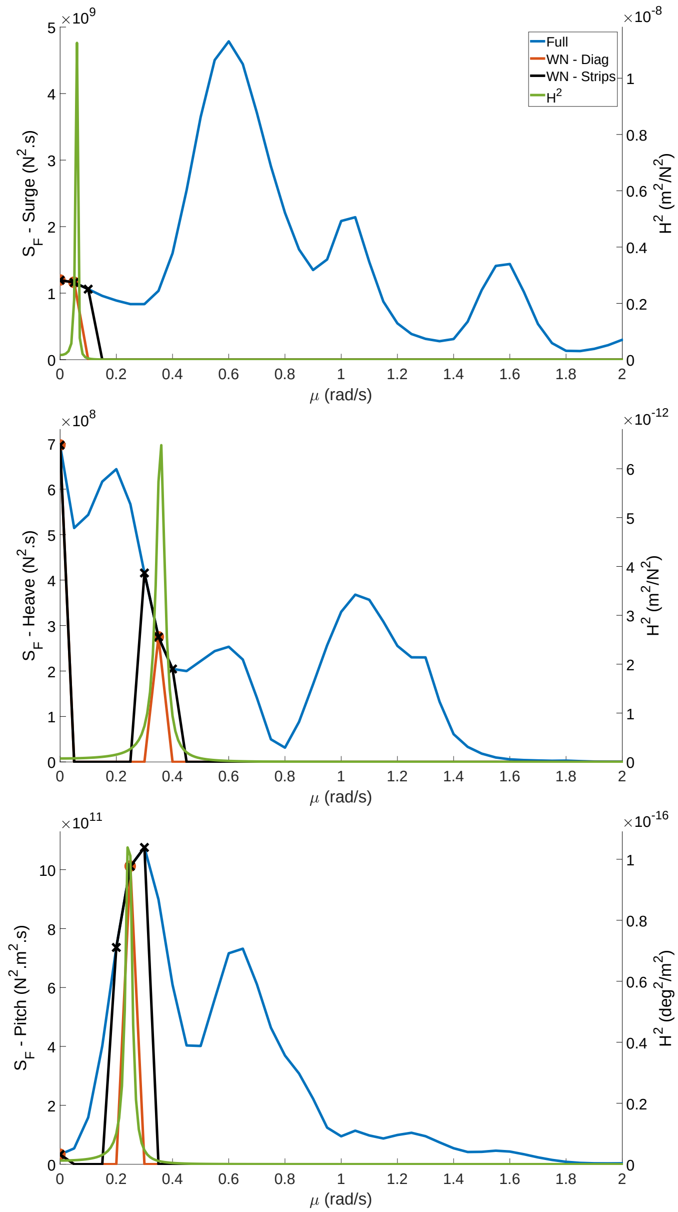

***FIGURE 5**: Second order force spectra calculated with the different QTF matrices for $T_p = 8\,s$.*

In the procedure explained in this section, effects of dynamic coupling among the different low-frequency motions were completely disregarded. However, it is known that they may play a significant role on the rotational degrees of freedom, and coupling effects are observed in the results discussed in the next section. Nevertheless, they may often be neglected, specially in early design stages.

RESULTS

The simulations conducted in this work were performed in time domain with FAST for the nine different sea states (T_p ranging from 4 s to 20 s) for a duration of three hours, thus ensuring statistical significance. In order to compare the results obtained with the four different approaches (full QTF matrix, Newman's approximation, and the two white-noise QTF matrices), the amplitude of the slow motion for each degree of freedom α is calculated as:

$$A_\alpha = 2\sqrt{\int_a^b S_\alpha^{(-)}(\omega)\,\mathrm{d}\omega} \tag{5}$$

with a and b the frequencies that delimit the slow-motion peak of the motion spectra $S_\alpha^{(-)}(\omega)$, which is extracted from the time series of motion using Welch's overlapped segment averaging estimator.

For illustration purposes, the time series and motion spectra of surge, heave and pitch for $T_p = 8$ s are provided in Figure 6, while the amplitudes calculated for each sea state are compared in Tables 4, 5 and 6. It is important to note that Newman's approximation is not supposed to be applied to heave and pitch, since the hypothesis that the resonance frequency is low does not apply to these degrees of freedom, and the results obtained are plotted in the heave and pitch graphs only to show that they are indeed discrepant. For this reason, only the surge amplitudes calculated with Newman's approximation are compared in the tables. It is worth mentioning that some results adhere so well that they may be indistinguishable in Figure 6.

***TABLE 4**: Slow-drift surge motion amplitudes (in meters). Relative differences larger than 5% are highlighted.*

T_p (s)	Full QTF	Newm.	Diff.	WN – Diag.	Diff.	WN – Strip	Diff.
4	0.88	0.93	5.3%	0.71	-20.3%	0.88	-0.1%
6	0.75	0.77	2.5%	0.59	-21.0%	0.75	-0.4%
8	0.64	0.62	-3.9%	0.49	-23.8%	0.64	-0.3%
10	0.45	0.43	-4.9%	0.35	-23.6%	0.45	0.0%
12	0.29	0.28	-4.4%	0.23	-21.5%	0.29	0.0%
14	0.20	0.19	-6.3%	0.16	-17.0%	0.20	0.2%
16	0.16	0.13	-14.7%	0.14	-11.4%	0.16	0.6%
18	0.14	0.11	-22.4%	0.13	-7.6%	0.14	0.9%
20	0.12	0.09	-25.4%	0.12	-2.3%	0.13	1.1%

TABLE 5: *Slow-drift heave motion amplitudes (in meters). Relative differences larger than 5% are highlighted.*

T_p	Full QTF	WN – Diag	Diff.	WN – Strip	Diff.
4	0.02	0.01	-39.3%	0.02	-3.1%
6	0.03	0.02	-42.9%	0.03	-3.9%
8	0.03	0.02	-38.6%	0.03	-2.4%
10	0.05	0.04	-17.6%	0.04	-1.0%
12	0.29*	0.29*	-0.1%	0.29*	0.0%
14	0.63*	0.63*	0.0%	0.63*	0.0%
16	0.86*	0.86*	0.0%	0.86*	0.0%
18	0.97*	0.97*	0.0%	0.97*	0.0%
20	0.98*	0.98*	0.0%	0.98*	0.0%

TABLE 6: *Slow-drift pitch motion amplitudes (in meters). Relative differences larger than 5% are highlighted.*

Tp	Full QTF	WN Diag.	Diff.	WN Strip	Diff.
4	0.12	0.13	4.6%	0.18	44.3%
6	0.27	0.25	-9.8%	0.35	26.4%
8	0.24	0.20	-16.2%	0.29	20.8%
10	0.20	0.16	-19.3%	0.24	20.0%
12	0.15	0.12	-23.2%	0.18	15.4%
14	0.13	0.10	-17.4%	0.14	7.9%
16	0.12	0.10	-11.4%	0.12	3.2%
18	0.11	0.10	-8.2%	0.11	1.3%
20	0.09	0.09	-6.4%	0.09	0.4%

In surge, the simulations using the "WN diagonal QTF" provided relative errors up to around 25%, which may be acceptable depending on the application, specially if one considers all the other inaccuracies and approximations present in the computation of the second order forces. Employing the "WN strip QTF", the motions are practically identical to the ones obtained with the full QTF matrix, confirming that, at least in this test case, the only significant part of the QTF is indeed the region around the resonance diagonal. Finally, the calculation of the slow-drift surge motion using Newman's approximation showed errors of the same order of magnitude as the "WN diagonal QTF", but once again, they may be acceptable depending on the application. It is important to note that the surge resonance period is large, $Tn,1 = 109\,s$, so it was expected that Newman's approximation would provide fair results. However, this may not be the case for FOWT with shorter resonance periods.

It is important to highlight that it was not possible to calculate the amplitude of the low-frequency heave motion for the largest wave periods, since the resonant motion is mainly excited by the first order forces. Taking the spectrum illustrated in Figure 6 as an example, the peak that corresponds to the first order motion (the motion for higher frequencies appearing to the right of the spectrum) gets closer to the slow-drift peak as the wave period gets larger, and it becomes difficult to distinguish the first from the second order motion. This behavior is relevant

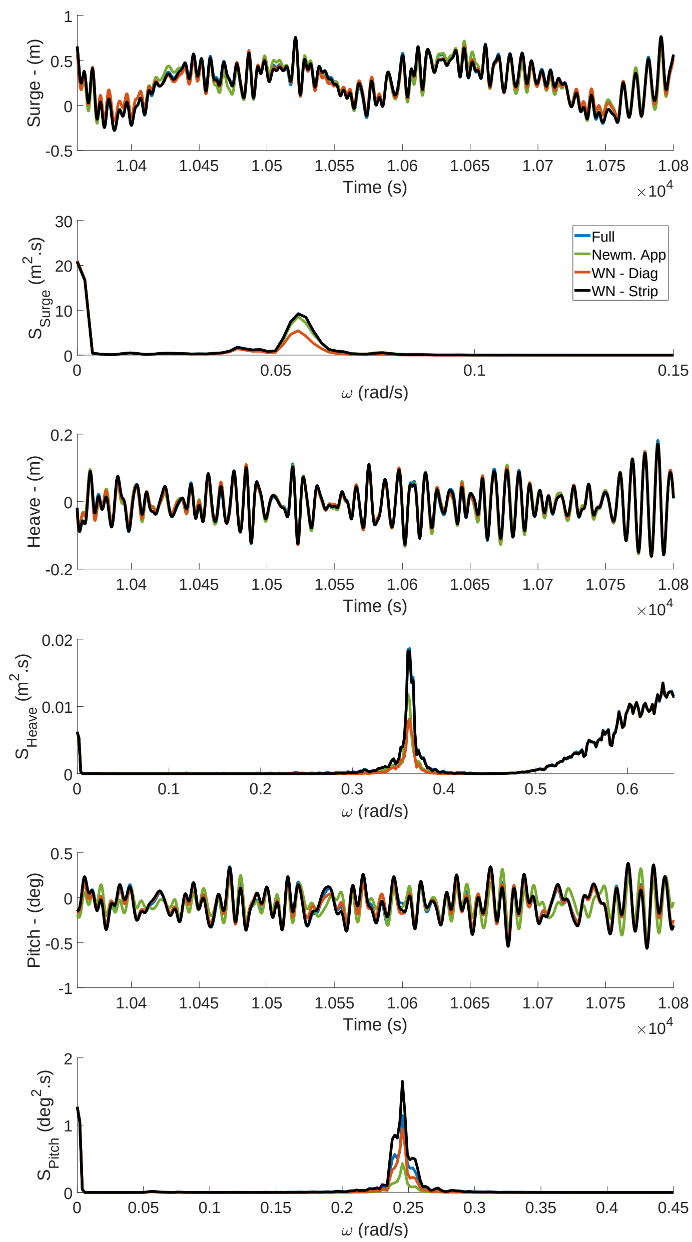

FIGURE 6: *Time series and spectral density functions of surge, heave and pitch obtained with the full QTF matrices (in blue), with the simplified matrices (in red and black) and with Newman's approximation (in green) for $T_p = 8\,s$.*

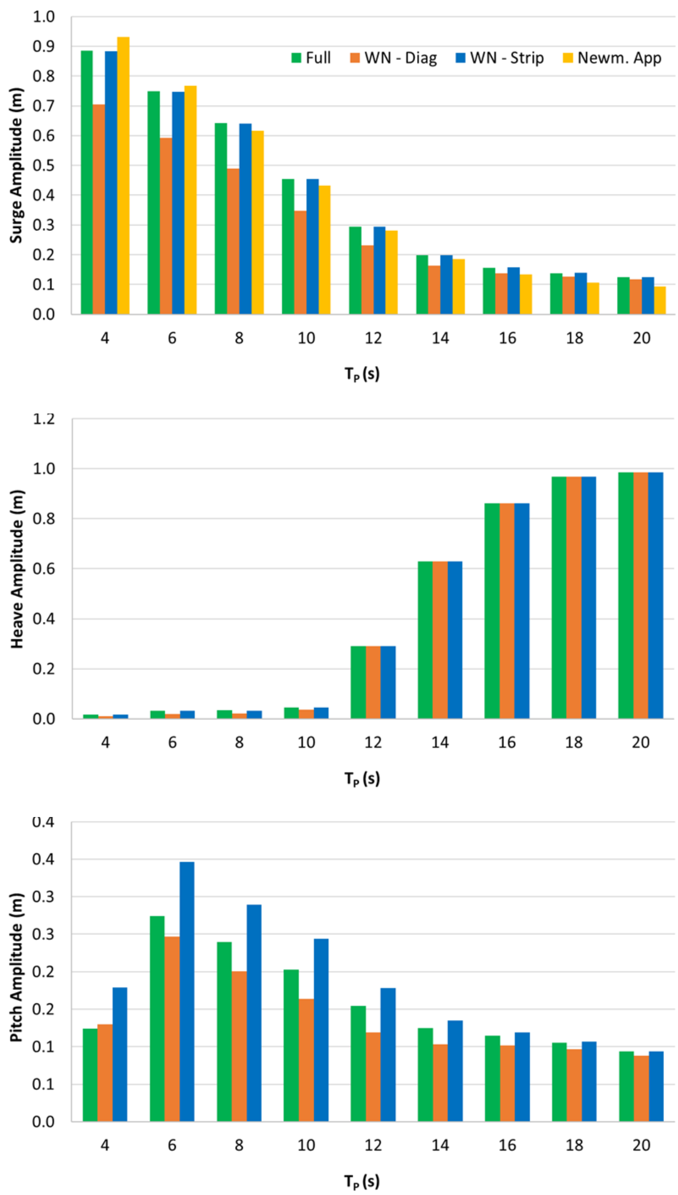

***FIGURE 7**: Amplitude of the slow motion obtained with the full QTF matrices and using the different approximations.*

for $T_p \geq 12$ s, and explains why the heave amplitudes presented in Table 5 are so large for these periods. The amplitudes calculated for $T_p \geq 12$ s are marked with an asterisk, and the comparisons among the different approximations should be disregarded. For the other wave periods, the motion amplitude is so small that the comparison is not relevant.

In pitch, the amplitudes computed with both the "WN diag QTF" and the "WN strip QTF" showed significant discrepancies. As noted in [15], the errors in pitch increase as the surge slow-drift motion gets larger, indicating that they are related to coupling effects between surge and pitch. These disparities are due to the fact that the slow-drift surge force is not calculated for the pitch resonance frequency, leading to an error in the surge force that, through the dynamic coupling, results in an error in the pitch motion. The first hypothesis was that the coupling could be related to the mooring lines. But this hypothesis was discarded, after the results from a set of simulations replacing the mooring lines by an equivalent spring in surge produced errors of the same magnitude.

Then, another group of simulations was conducted, this time considering only the pitch motion, i.e. the calculation of the other degrees of freedom was disabled in FAST. Only the full QTF and the "WN strip QTF" were tested. Figure 8 presents the results obtained for $T_p = 4$ s, the one that originally showed the largest errors, and one can see that they are nearly identical. Hence, it can be concluded that the discrepancies observed in the adoption of the white-noise approach are indeed due to coupling effects, most probably related to inertial effects.

Alternatively, the coupling effects could be added in the analysis, but this would require including additional QTF diagonals to account for the couplings among all the dofs. By doing this, the approach that was supposed to be simple and straightforward would become somewhat cumbersome and complicated. Therefore, it was chosen to keep the approach as initially proposed, and one should keep in mind that coupling effects are neglected when using it.

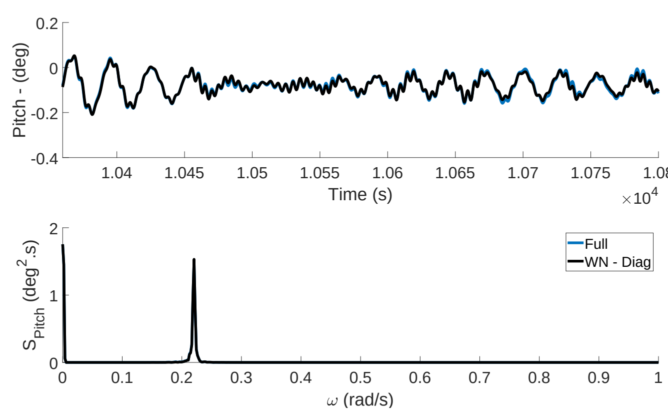

***FIGURE 8**: Time series and spectral density functions of pitch obtained with the full QTF matrix (in blue) and with the "WN strip QTF" (in black) for a simulation with pitch only and $T_p = 4$ s.*

CONCLUSION

An approach for calculating the slow-drifts of a FOWT, based on the white-noise approximation, was proposed in a previous work [15]. It states that, since the motions are weakly damped, the only elements of the QTF matrices that are relevant for computing the slow-drift motions are the ones close to the resonance diagonal, i.e. the diagonal in which the difference frequency is equal to the natural frequency of motion for a given degree of freedom. This approximation implies that QTF matrices with only a reduced number of diagonals may be precise enough. The objective is to save time in early design stages, since time domain simulations would need to consider much less frequency pairs. Besides, these QTF matrices would be much easier to compute in frequency domain codes, as the convergence analysis would be simplified.

In order to verify the approach, two different sets of QTF matrices were constructed, derived from the full QTF matrices of the OC4 platform distributed with the software FAST. The first one comprised only the main diagonal and the resonance diagonal, while the rest of the matrix was set to zero ("WN diagonal QTF"). The other set consisted of matrices with the main diagonal and a strip around the resonance diagonal, i.e. the resonance diagonal, the diagonal immediately above it, and the diagonal immediately below it ("WN strip QTF"), while all the other elements were set to zero. One should be aware that, by doing this, the second order forces are not considered in the couplings between different degrees of freedom.

The OC4 semi-submersible was taken as a case study and, as the objective was to study the wave-induced slow-drift motions, simulations were conducted with FAST considering waves only, thus neither current nor wind effects were included. The second-order force spectra calculated with these matrices were not constant, disregarding what is proposed in the original white-noise approximation; however, the peak of the response function is so narrow that this is not important.

The results obtained with these two sets of QTFs were compared with the ones calculated with the original full QTF matrices and with Newman's approximation (for surge only). In surge, the simulations with the "WN strip QTF" showed an excellent agreement with the ones employing the full QTF matrix, while the "WN diagonal QTF" could be considered acceptable depending on the application. The results with Newman's approximation showed errors of the same order as the "WN diagonal QTF", which could also be considered satisfactory. This was expected, as the surge resonance period is large, which may not be the case for other FOWTs, specially in shallower water.

In heave, the slow drift amplitudes calculated for the shortest waves were very small, in such a way that the comparison is not significant. For longer waves, it is difficult to distinguish the first and second order motions, so these comparisons are discarded as well.

For the second order pitch motion, both the "WN diag QTF" and the "WN strip QTF" compared poorly with the results obtained with the full matrix. This discrepancy is due to coupling effects, as demonstrated by simulations performed considering only the pitch motion (i.e. the calculation of the other degrees of freedom was disabled), and it is related to neglecting the surge QTF diagonal corresponding to the pitch resonance frequency. Another group of simulations was conducted to verify whether the coupling effects were due to the mooring system, but this was not the case, meaning that the dynamic coupling is very probably due to inertial effects. As the inclusion of coupling effects would complicate an approach that is intended to be simple, it was chosen to keep it as initially proposed.

Therefore, it can be concluded that the elements of the QTF matrices that contribute the most for the slow-drift motions are indeed the ones close to the resonance diagonal, so they require special attention when calculating the QTFs in frequency domain. Furthermore, if coupling effects can be neglected, good results can be obtained considering only a narrow strip around the resonance diagonal, which should allow faster simulations and reduce computer time.

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