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A PHASE ANGLE TEST FOR PERIODIC COMPONENTS IN TIME SERIES

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ABSTRACT

A phase angle test procedure is derived for testing periodicities in a time series. The procedure was applied to monthly data of air temperature of São Paulo and annual rainfall data of Fortaleza, confirming the known 13 years period of the latter and the 12 months period of the former. It was also applied to periodic and white noise simulated series, as limiting cases of a series having a mixed spectrum. Practical and philosophical advantages of the procedure are discussed, showing its properties in situations where tests based on amplitudes are not adequate.

Key words: Phase angle, test, periodicity, time series, mixed spectrum, periodogram, white noise

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1. Introduction

One important problem in the analysis of real time series occurring in diverse fields such as oceanography, meteorology, astronomy, economics, etc, is the determination of periodicities. Formally, this can be formulated as a test of the null hypothesis that the series under analysis has a continuous spectrum against the alternative hypothesis that its spectrum is of a mixed form (a discrete plus a continuous part).

Several tests for periodicities have been proposed in the literature. We mention those by Fisher (1929), Whittle (1952), Hannan (1961), Bartlett (1955), Priestley (1962a, 1962b), Siegel (1980) for the univariate case, and Nicholls (1969), Mac Neill (1974, 1977), for the multivariate case. Most of these tests are based on the periodogram (or cross--periodogram) of the data, except Priestley's, which uses a different approach based on the behavior of the autocorrelation function of the series. Brillinger (1974) proposes a test based on the Whittaker periodogram.

Suppose $\{X_t\ , t=0,\pm1,\pm1,\pm2,\ldots\}$ is a discrete parameter, zero mean, stationary process, with a mixed spectrum, that is, assume that X_t can be written as

$$X_t = Y_t + Z_t \quad , \tag{1}$$

where Y_t and Z_t are uncorrelated processes, such that:

(i) Y_t is a linear process, that is, $Y_t = \sum_{j=0}^{\infty} \beta_j \varepsilon_{t-j}$, where ε_t is a purely (discrete) random process (white noise), with zero mean and variance σ_ε^2 , and β_j are constants satisfying $\sum_{j=0}^{\infty} \beta_j^2 < \infty$.

Usually, one assumes that Y_t follows an autoregressive process of order p, $Y_t = \sum_{j=1}^{p} \alpha_j Y_{t-j} + \varepsilon_t$, or a moving average process of order q, $Y_t = \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j} + \varepsilon_t$, or generally, a mixed autoregressive moving average process of orders p and q,

$$Y_t = \sum_{j=1}^{p} \alpha_j Y_{t-j} + \varepsilon_t + \sum_{j=1}^{q} \gamma_j \varepsilon_{t-j} .$$

(ii) Z_t is a harmonic process

$$Z_t = \sum_{j=1}^{J} A_j \cos(\lambda_j t + \theta_j) \ , \qquad (2)$$

where A_j, λ_j, $j=1,\dots,J$ are constants and θ_j , $j=1,\dots,J$ are independent, identically distributed uniform random variables in $[-\pi,\pi]$. For a given realization of the process, the θ_j will be fixed constants to be estimated together with the A_j, λ_j . The $\{A_j^2 \ , \ j=1,\dots,J\}$ forms the discrete spectrum of X_t and $f_y(\lambda)$ the (purely) continuous part.

We want to test the null hypothesis that $A_j = 0$, for all j. If Y_t is white noise and λ_j are known, then least squares provide efficient estimates for A_j and θ_j, for a given realization of X_t. See Bloomfield (1976), for example. If the frequencies are unknown, we first have to estimate them and afterwards estimate A_j and θ_j. This problem is discussed by Hannan (1973), Walker (1971), Hasan (1982) and Hannan and Quinn (1984).

The values λ_j in (2) are determined from maxima in the <u>periodogram</u>

$$I(\lambda) = \frac{1}{2\pi T} \left| \sum_{t=0}^{T-1} X_t \, e^{-i\lambda t} \right|^2 \qquad (3)$$

for T observations $X_0, X_1, \ldots, X_{T-1}$. The main problem is to test if these peaks in $I(\lambda)$ are significant.

After fitting the harmonic process (2) we fit an ARMA model to the residuals $\hat{Y}_t = X_t - \sum_{j=1}^{J} \hat{A}_j \cos(\hat{\lambda}_j t + \hat{\theta}_j)$ and estimate $\alpha_1, \ldots, \alpha_p$, $\gamma_1, \ldots, \gamma_q$ and σ_ε^2 . This is a two-stage procedure. We might use instead maximum likelihood estimators and estimate all parameters $\lambda_j, \theta_j, A_j, \alpha_j, \gamma_j$ and σ_ε^2 jointly.

In this paper we will not be interested in fitting a model to the series, but only in deriving an heuristic test for periodicidities, based on the phase differences between two series. For an example of fitting model (1) to a series, see Morettin et al. (1984).

2. The Phase Angle Between Two Series

Suppose we have two sets of observations $\{x_t\}, \{y_t\}$, $t=1, \ldots, T$, from two stationary processes. Denote the Fourier frequencies by $\lambda_j = 2\pi j/T$, $j=0,1,\ldots,[T/2]$. The phase angle at frequencey λ_j for the series $\{x_t\}$ is given by

$$\phi_x(\lambda_j) = \tan^{-1} \frac{\sum_{t=1}^{T} x_t \sin(\lambda_j t)}{\sum_{t=1}^{T} x_t \cos(\lambda_j t)} , \quad 0 \le \phi_x(\lambda_j) \le 2\pi , \quad (4)$$

$j=0,1,\ldots,[T/2]$. An analogous definition holds for $\{y_t\}$. The phase difference between the two series is then

$$\phi(\lambda_j) = \phi_x(\lambda_j)-\phi_y(\lambda_j) \ , \qquad (5)$$

for each frequency λ_j .

Nicholls (1969) proves that:

(i) If $\{x_t\}$ and $\{y_t\}$ are both independent, identically distributed $N(0,1)$ and independent of each other, then $\phi(\lambda_j)$ is uniformly distributed on $[-\pi,\pi]$;

(ii) if $\{x_t\}$ and y_t are observations from stationary normal process, independent, and if both have representation as infinite moving averages, then the distribution of (5) is again uniformly distributed on the unit circle.

Suppose now that $\{x_t\}$ and $\{y_t\}$ are observations from any two stationary processes and $\hat{\phi}(\lambda)$ is an estimator of the phase angle between the two process, at frequency λ, obtained through a smoothed periodogram estimator of the cross-spectrum between the processes. Then it can be shown (see Bloomfield, 1976, for example) that $\hat{\phi}(\lambda)$ is asymptotically unbiased, for each λ, with variance $g^2[R^{-2}(\lambda)-1]/2$, where $R^2(\lambda)$ is the true coherence between the two series and g is a factor that depends on the window used. Moreover the distribution of $\hat{\phi}(\lambda)$ is approximately normal, with mean equal to the true phase angle and the above mentioned variance. It is expected that this approximation also holds for the estimatior (5). Since the smoothing procedure reduces the variance and constrains the estimator to be concentrated in a narrower band than $[-\pi,\pi]$, sometimes it is suggested that a transformation of the phase estimator $\hat{\phi}(\lambda)$ be used, namely tg $\hat{\phi}(\lambda)$. See Jenkins and Watts (1969). In this paper (4) and (5) will be used.

3. A Phase Angle Testing Procedure

For simplicity, consider (2) with J=1, observed at times $t=1,2,\ldots,T$. Call $p_1 = 2\pi/\lambda_1$ the corresponding period and let L be an appropriate multiple of p_1. Consider now the two first subseries $X_1,\ldots,X_L$ and $X_{L+1},\ldots,X_{2L}$; the phase angles $\phi_x(\lambda_1)$ and $\phi_y(\lambda_1)$ are calculated via (4) and the phase difference $\phi_1(\lambda_1)$ by (5). Then one moves along the original series by one step considering the series $X_2,\ldots,X_{L+1}$ and $X_{L+2},\ldots,X_{2L+1}$ to compute the next phase angle difference $\phi_2(\lambda_1)$. By proceeding similarly the phase differences $\phi_1(\lambda_1),\ldots,\phi_n(\lambda_1)$ are obtained, where n is the number of such pairs of subseries and λ_1 is the frequency undergoing the test. In general there are T-L+1 subseries and T-2L+1 phase differences.

According to what has been said, the following inferences can be made:

(a) The phase differences $\phi_i(\lambda_1)$, $i=1,2,\dots,n$, will be identically zero, for a trully periodic signal with frequency λ_1 ; thus, in (1), $Y_t = 0$.

(b) The phase differences $\phi_i(\lambda_1)$, $i = 1,\dots,n$, will be uniformly distributed in the interval $[-\pi,\pi]$, with mean zero, for a white noise series.

(c) The phase differences $\phi_i(\lambda_1)$, $i = 1,\dots,n$, will be approximately normally distributed around mean zero, for a series X_t as given in the expression (1).

It is clear that for a null signal/noise ratio, case (c) reduces to (b), if the noise is white. Also, case (c) reduces to (a) if the signal-to-noise ratio is high.

Therefore, to infer that:

(i) The series X_t is a white noise it is necessary to show that the phase differences $\phi_i(\lambda_j)$, $i=1,\dots,n$, $j=1,\dots,[T/2]$ are all uniformly distributed in $[-\pi,\pi]$.

(ii) The series X_t contains periodic components, it is sufficient to show that at least for one j, $1 \le j \le [T/2]$, $\{\phi_i(\lambda_j)$, $i=1,\dots,n\}$ are approximately normally distributed with mean zero.

(iii) The series only contains deterministic periodic components, it is necessary to show that $\{\phi_i(\lambda_j)\}$, for all i and all j , are identically zero.

4. Applications

4.1. A Synthetic Series

Case (iii) of section 3 was tested with T=120 generated values from

$$X_t = 5 \cos(1.2566t) + \cos(0{,}3141t) \tag{6}$$

Here one takes L=20 hence there are $n=T-2L+1 = 81$ phase differences. To apply the test procedure, equation (5) is written as

$$\phi_i(\lambda_j) = \phi_x(\lambda_j) - \phi_y(\lambda_j) , \tag{7}$$

where the index y indicates that the phase angle $\phi_y(\lambda_j)$ is computed from the values $\{X_{L+i},\dots,X_{2L+i-1}\}$ and the index x indicates that $\phi_x(\lambda_j)$ is computed from the values $\{X_i,X_{i+1},\dots,X_{i+L-1}\}$ for $i=1,2,\dots,n$ and $j=1,\dots,[L/2]$.

Table 1 shows the Fourier components of the analysis of the first subseries of the T-L+1 = 101 subseries.

TABLE 1 - Fourier analysis for the first L=20 values of the synthetic series (6)

m	A_m	B_m	C_m	Phases
0	40	0.0	39.992	0.00
1	0.3	-1.0	0.999	288.00
2	0.0	0.0	0.002	305.94
3	-0.0	0.0	0.000	144.00
4	4.8	1.5	4.999	342.02
5	0.0	0.0	0.000	0.00
6	-0.0	-0.0	0.001	197.96
7	0.0	0.0	0.001	36.00
8	-0.0	0.0	0.002	118.52
9	0.0	0.0	0.001	72.00

The analysis of the second subseries of L=20 values, and of all two correspondent pairs, reproduces exactly the same figures of Table 1, so that considering consecutive pairs one obtains that the phase differences (7) are all zeros. Therefore, for $\lambda_1 = 1.2566$, $\lambda_2 = 0.3141$ (corresponding to the periods $p_1 = 5$ and $p_2 = 20$) we obtain $\{\phi_i(\lambda_j)\}$ identically zero, for all i and all j. Therefore it can be concluded that the series (6) only contains deterministic information.

4.2. The Air Temperature Series of São Paulo

The second example refers to T = 96 monthly observations of the air temperature at São Paulo, Brazil. It has a strongly seasonal period of 12 months (harmonic number j=8 , frequency 0.52), and other weaker components with 6 and 3 months periods.

Taking L=24 to test the 12 months signal, the other weaker signals are also tested. One obtains n=49 phase differences $\phi_i(\lambda_j)$, for j=2,4,6, which are shown in Figure 1. All the values of ϕ_i fall nearly zero for $\lambda_2 = \pi/6$, having mean -2.67^oC and variance $(5.71^oC)^2$. For the frequencies $\lambda_4 = \pi/3$ and $\lambda_6 = \pi/2$ correspond the mean values 10.36^oC and 22.64^oC, respectively, and variances $(18.73^oC)^2$ and $(29.83^oC)^2$, respectively. The histograms of $\phi_i(\lambda_2)$, $\phi_i(\lambda_4)$ and $\phi_i(\lambda_6)$ are shown in Figure 2.

The null hypothesis is that $\phi_i(\lambda_j)$, j=2,4,6, are normally distributed, with mean zero. Using a chi-square test of goodness of fit (see Conover, 1971, for example) at significance level 0.05, the periodicity of 12 months is accepted . The 6 and 4 months periodicities are not significant at the same level.

4.3. The Fortaleza Rainfall Series

The series of atmospheric precipitation at Fortaleza, Brazil, has been analyzed by several authors in the past and by Morettin et al. (1984) recently. Periodicities of 13 and 26 years were detected, according to the tests referred to in the introduction of this paper.

The phase angle test was applied to T=131 annual values of rainfall, using L=26 and resulting n=106 subseries. From the Fourier analysis the n=80 values $\phi_i(\lambda_j)$ were calculated, for j=1 and 2, and are shown in Figure 3. For $\lambda_1 = 2\pi/26$, the average value is 2.64mm, while the variance is $(35.16\text{mm})^2$. For $\lambda_2 = 2\pi/13$, the mean value is 4.46mm and the variance is $(36.06\text{mm})^2$.

The corresponding histograms are shown in Figure 4. The chi-square test of goodness of fit was used again to test the null hypotheses that $\phi_i(\lambda_j) \sim N(0,\sigma^2)$, j=1,2. At the 0.05 significance level the 13 years periodicity is accepted but the 26 years is rejected.

4.4. Series of Random Values

A slightly different approach was used here. One hundred disjoint subseries of L=26 values were considered from T=2600 observations of a white noise, generated by an IMSL subroutine. Figure 5 shows the typical variability of the $\phi_i(\lambda_j)$, for j=3,7 ; the correspondent histograms are given in Figure 6. A chi-square test of goodness of fit accepts the hypothesis that the phase differences $\phi_i(\lambda_3)$, i=1,...,99 , are uniformly hypothesis that the phase differences $\phi_i(\lambda_3)$, i=1,...,99, are uniformly distributed in $[-\pi,\pi]$. This also can be seen for all the λ_j , j=1,...,13.

5. Discussion

Fisher (1929) was the first to look for the aspects of testing periodicities in a time series. He assumed that the observations were normally and independently distributed to derive his test, which is based on the largest periodogram ordinate. Whittle (1952) removed the restriction of independence of the observations, introducing a ratio between the periodogram ordinates and the ordinates of an estimator of the true spectrum (he used the truncated periodogram as an estimator). A similar approach was used by Hannan (1961) and Bartlett (1955). Priestley (1962a, 1962b) proposed a new test based on the behavior of the correlation function of the series. Fisher's test was generalized by Siegel (1980) to cover several periodicities.

Nicholls (1969) derived a test based on the product of the periodogram ordinates and phase angle differences given by (5). The phase angles are calculated using periodogram ordinates and the $\phi_i(\lambda_j)$ are obtained for the λ_j which the test of amplitudes has already accepted. By combining the two test statistics for the case of normal stationary observations he finds a chi-square distribution as the final result for the combined distribution. The phase angles differences $\phi_i(\lambda_j)$, for the case of x_t and y_t normally and independently distributed, with mean zero and unit variance, x_t and y_t independent, have a constant density in $[-\pi,\pi]$. It is also shown that even if the restriction of independence is removed from the observations x_t and y_t, the distribution of the phase differences is asymptofically unchanged and it is the same for the case of normal, stationary, observations, assumed to be generated by process with moving average representations (still assuming x_t and y_t independent).

For the observations x_t and y_t generated by equation (1), the distribution of $\phi_i(\lambda_j)$ will be a function of the contributions of Y_t and Z_t. Asymptotically, a normal distribution for the phase differences is a good aproximation (Bloomfield, 1976).

The philosophical advantages of the method of the phase angle differences test are that the same phase angle is acessed in all subseries via the ordinates of the periodogram, allowing the inspection of their variability along the observational period to be made. This can be seen in Figure 1, where the fundamental $\phi_i(\lambda_2)$ oscilation of the air temperature of São Paulo (12 months) shows apparently a periodic variation around the zero mean. Similar feature is shown in Figure 3 by $\phi_i(\lambda_1)$ and $\phi_i(\lambda_2)$ for the analysis of the rainfall series at Fortaleza.

The good sensivity of the phase angles to the record length is also another favorable aspect of the test. Once the frequency λ_j is chosen to be tested the lenght L of the subseries is determined from a convenient multiple of $2\pi/(\Delta t)(\lambda_j)$, where Δt is the sampling interval. By varying L in steps of plus or minus one unit, the corresponding histograms of $\phi_i(\lambda_j)$ will vary and this may be used to determine the frequency λ_j which best estimates the real frequency of the periodic phenomena, as this is expected to occur for the case when the histogram is the most sharpened among all.

Another testing capability of the procedure is relative to the occurrence of "hidden periodicities" by phase variability. Existing periodicities in a time series which are slowly, continuously or intermitently changing in occurrence may be missed by the periodogram analysis. They are virtually undetectable by the usual methods based on the amplitudes, being the trully "hidden periodicities" of a time series. Also abrupt changes can possibly be detected. As these periodicities do not show up in the periodogram of the complete series, as peaks, all the possible λ_j have to be tested.

One of the disadvantages of the test is that it requires long series relative to the periodicities being tested, so that a relative large number of subseries can be produced, otherwise the chi-square test (or an alternative one) of goodness of fit will become less effective. Also, a certain degree of statistical dependence is implicitly admited between two consecutive series, to obtain the $\phi_i(\lambda_j)$. In fact random noise values analysed via the same division of subseries, as given in equation (7), produced histograms $\phi_i(\lambda_j)$ which could not be taken as following a uniform distribution. However, by taking the values of the subseries as described in section 4.4 the $\phi_i(\lambda_j)$ values followed a uniform distribution as expected. There was some dependence between the consecutive subseries, which even for large samples, did not give histograms which could be taken as from a uniform distribution. In general, the $\phi_i(\lambda_j)$ determined for the series of sections 4.1., 4.2. and 4.3. may be then overstressing the desired values. On the other hand, the periodo p_1, say chosen to be first tested is in correspondence to the maximum value of the periodogram (3), so some dependence between the subseries is admited before the calculation is done for those cases, being then a sought for quality of the subseries, rather than an unwanted one.

Another aspect which involves the questions of dependence of the subseries and the model given by expression (1) can be seen in Figures 1 to 6 and in the results of section 4.1. In section 4.1. we have subseries with totally dependent elements and in Figure 5 and 6 are shown values $\phi_i(\lambda_j)$ found from the calculation of totally independent elements. A figure showing the $\phi_i(\lambda_j)$ values for series (6) will give a straight horizontal line on the 0^o value and an histogram would give all values concentrated on 0^oC. Opposedely, Figure 6 gives a uniform distribution for $\phi_i(\lambda_7)$ around the zero degree value, and Figure 5 shows the corresponding caotic variability. Usual series represented by the model (1) lie between the above cases and should supposedely show a mixed distribution with the contributions of Y_t and Z_t. But an analysis of Figures 2 and 4 shows that the $\phi_i(\lambda_j)$ values are rather concentrated in an interval roughly from $-\pi/2$ to $\pi/2$.

These points and the apparent periodicities shown by the $\phi_i(\lambda_j)$ values of Figures 1 and 3 are clear subjects for further work in the effort to better understand the procedure capabilities.

6. Conclusions

A phase angle test procedure for detecting periodicities in a time series is suggested. It is applied to two real time series, confirming the existence of known periodicities: the 13 years periodicity of the Fortaleza rainfall series and the 12 months periodicity of the air temperature series of São Paulo. The 26 years "periodicity" of the Fortaleza series was not detected by the test, which is the same conclusion obtained by the Bartlett (grouped periodogram) test. The procedure was also applied to simulated periodic and white noise series, given satisfactory results in the first case, as expected, but not so conclusive in the second case. A raw phase angle estimate was used in the paper and the results show that probably an improvement of the procedure will be obtained using smoothed estimators or working with a transformation ($\tan \phi_i(\lambda_j)$) of the phase differences. This will be the object of a further paper.

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References

Bartlett, M.S. (1955): Stochastic processes: Methods and applications. The Cambridge Univ. Press.

Bloomfield, P. (1976): Fourier analysis of time series: An introduction. New York, Wiley.

Brillinger, D.R. (1974): The saymptotic distribution of the Whittaker periodogram and a related chi-square statistic for stationary processes. Biometrika, 61, 419-422.

Conover, W.J. (1971): Practical nonparametric statistics, New York, Wiley.

Fisher, R.A. (1929): Tests of significance in harmonic analysis. Proc. Roy. Soc. A. 125, 54-59.

Hannan, E.J. (1961): Testing for a jump in the spectral function. J. Roy Statist. Soc., B, 23, 394-404.

Hannan, E.J. (1979): The estimation of frequency. J. Appl. Probab., 10, 510-519

Hannan, E.J. and Quinn, B.G. (1984): Line splitting, forthcoming.

Hasan, T. (1982): Nonlinear time series regression for a class of amplitude modulated cosinusoids. J. Time Series Analysis, 3, 109-122.

Jenkins, G.M. and Watts, D.G. (1968). Spectral analysis and its applications. San Francisco, Holden-Day.

MacNeill, I.B. (1974): Tests for periodic components in multiple time series. Biometrika, 61, 57-70.

MacNeill, I.B. (1977): A test of whether several time series share common periodicities. Biometrika, 64, 495-508

Morettin, P.A., Mesquita, A.R. and Rocha, J.G.C. (1984): Rainfull at Fortaleza in Brazil revisited. Time Series Analysis: Theory and Practice 6 (O.D. Anderson, J.K. Ord, E.A. Robinson, editors), North Holland, forthcoming.

Nicholls, D.F. (1969): Testing for a jump in co-spectra. Austral. J. Statist., 11, 7-13

Priestley, M.B. (1962a): Analysis of stationary processes with mixed spectra - I. J. Roy.Statist.Soc., B, 24, 215-233.

Priestley, M.B. (1962b): Analysis of stationary processes with mixed spectra - II. J. Roy Statist. Soc., B. 24, 511-529

Siegel, A.F. (1980): Testing for periodicity in a time series. J. Amer. Statist. Assoc., 75, 345-348.

Walker, A.M. (1971): On the estimation of a harmonic component on a time series with stationary independent residuals. Biometrika, 58, 21-36

Whittle, P. (1952): The simultaneous estimation of a time series harmonic components and covariance structure. Trabajos Estad., 3, 43-57

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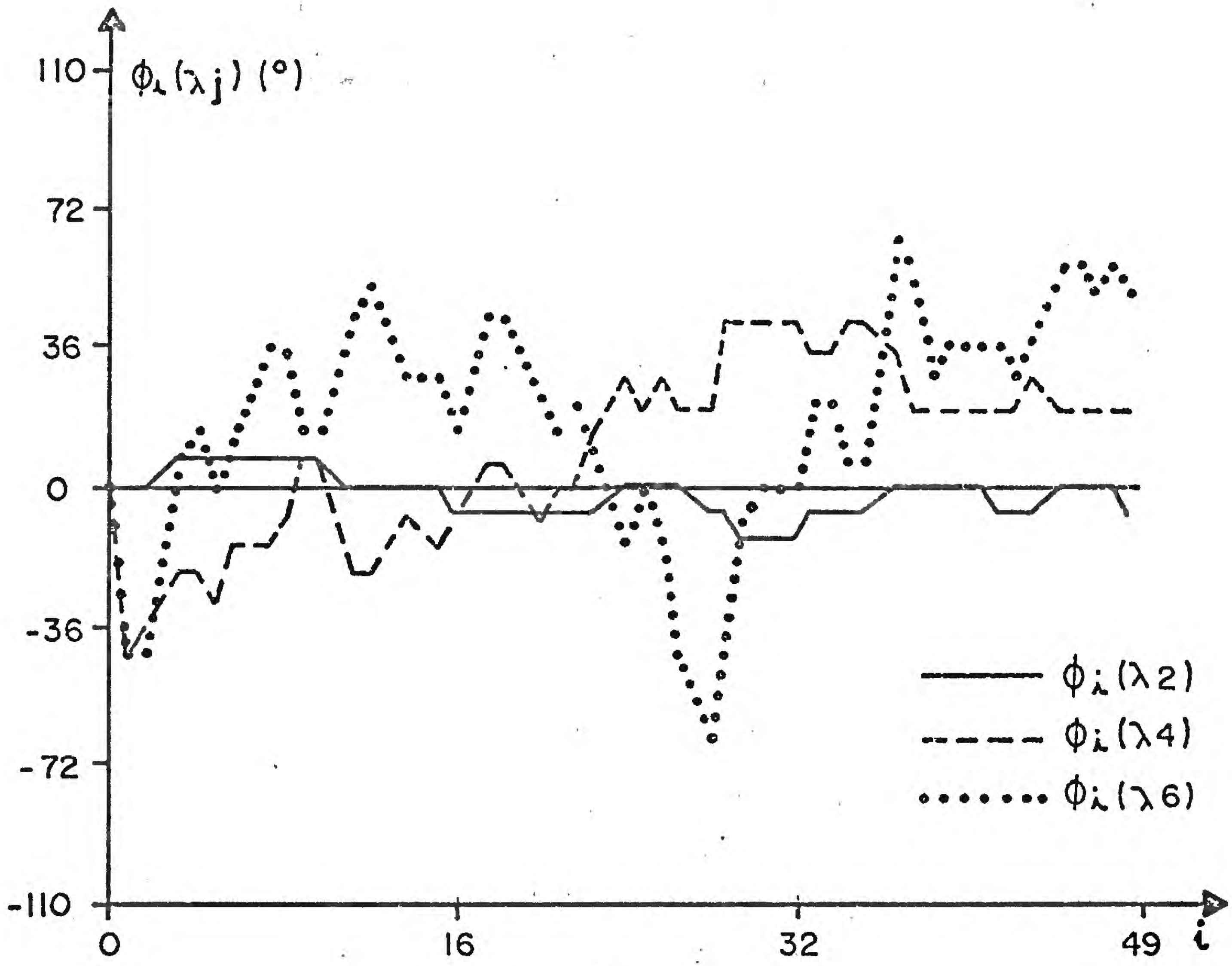

FIG.1- PHASE DIFFERENCES VARIABILITY OF THE TEMPERATURE SERIES OF SÃO PAULO, FREQUENCIES $\lambda_2=\pi/6$, $\lambda_4=\pi/3$ and $\lambda_6=\pi/2$

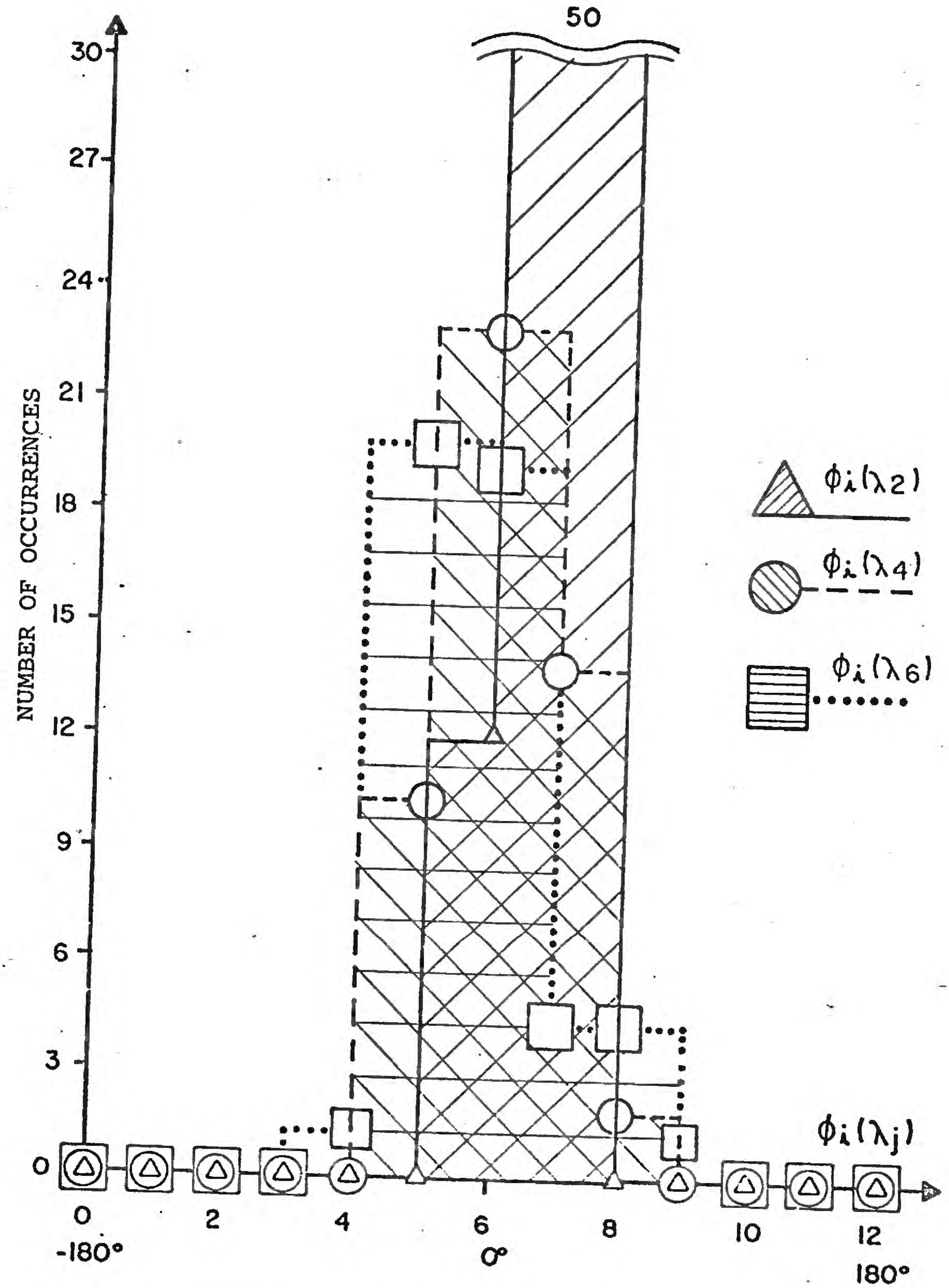

FIG. 2- HISTOGRAMS FOR THE PHASE DIFFERENCES(12 CLASSES), TEMPERATURE SERIES OF SÃO PAULO

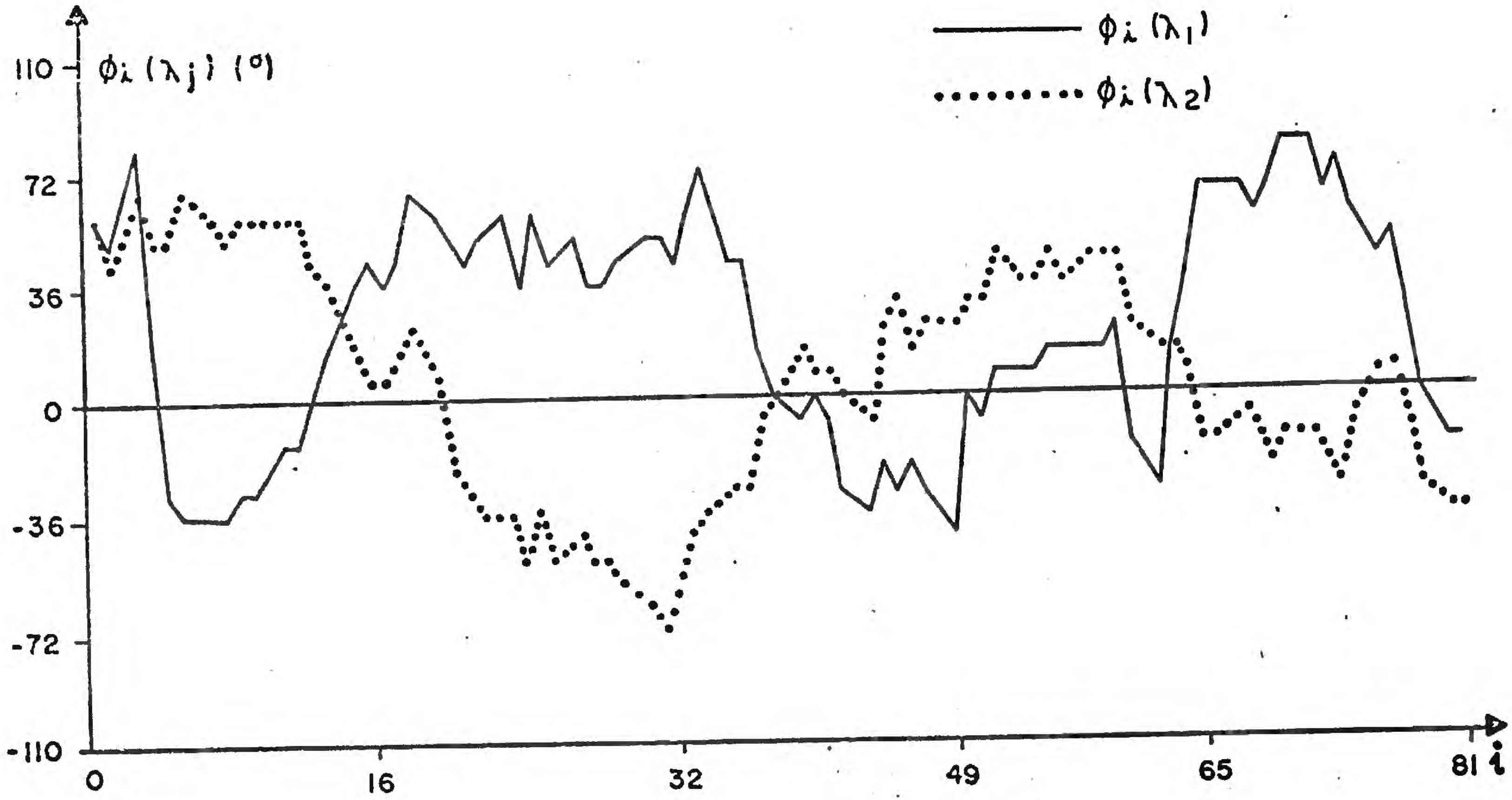

FIG.3- DIFFERENCE VARIABILITY FOR THE RAINFALL SERIES OF FORTALEZA, FREQUENCIES $\lambda_1 = 2\pi/26$ and $\lambda_2 = 2\pi/13$

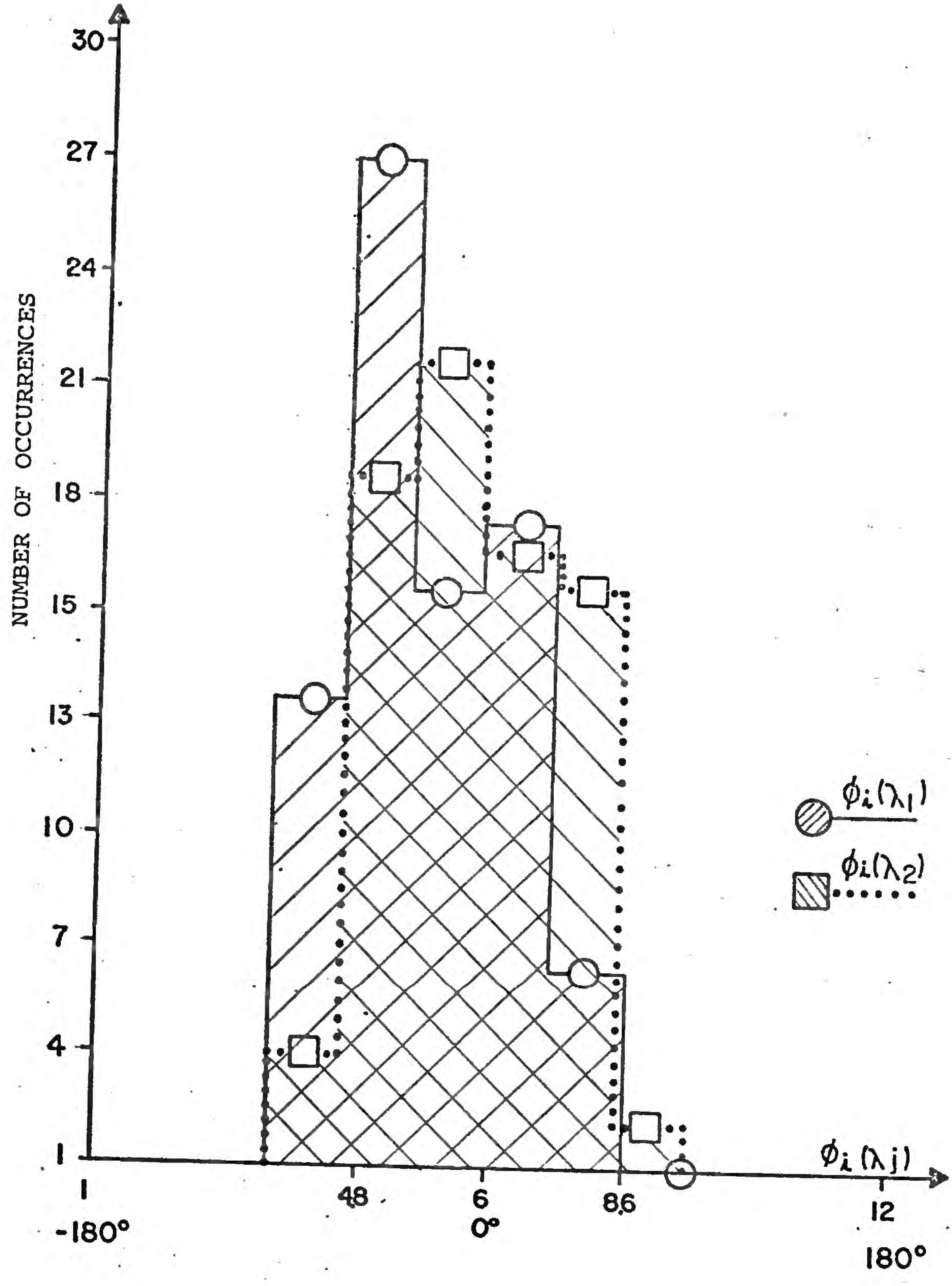

FIG.4- HISTOGRAMS FOR THE PHASE DIFFERENCES (12 CLASSES), RAINFALL SERIES OF FORTALEZA

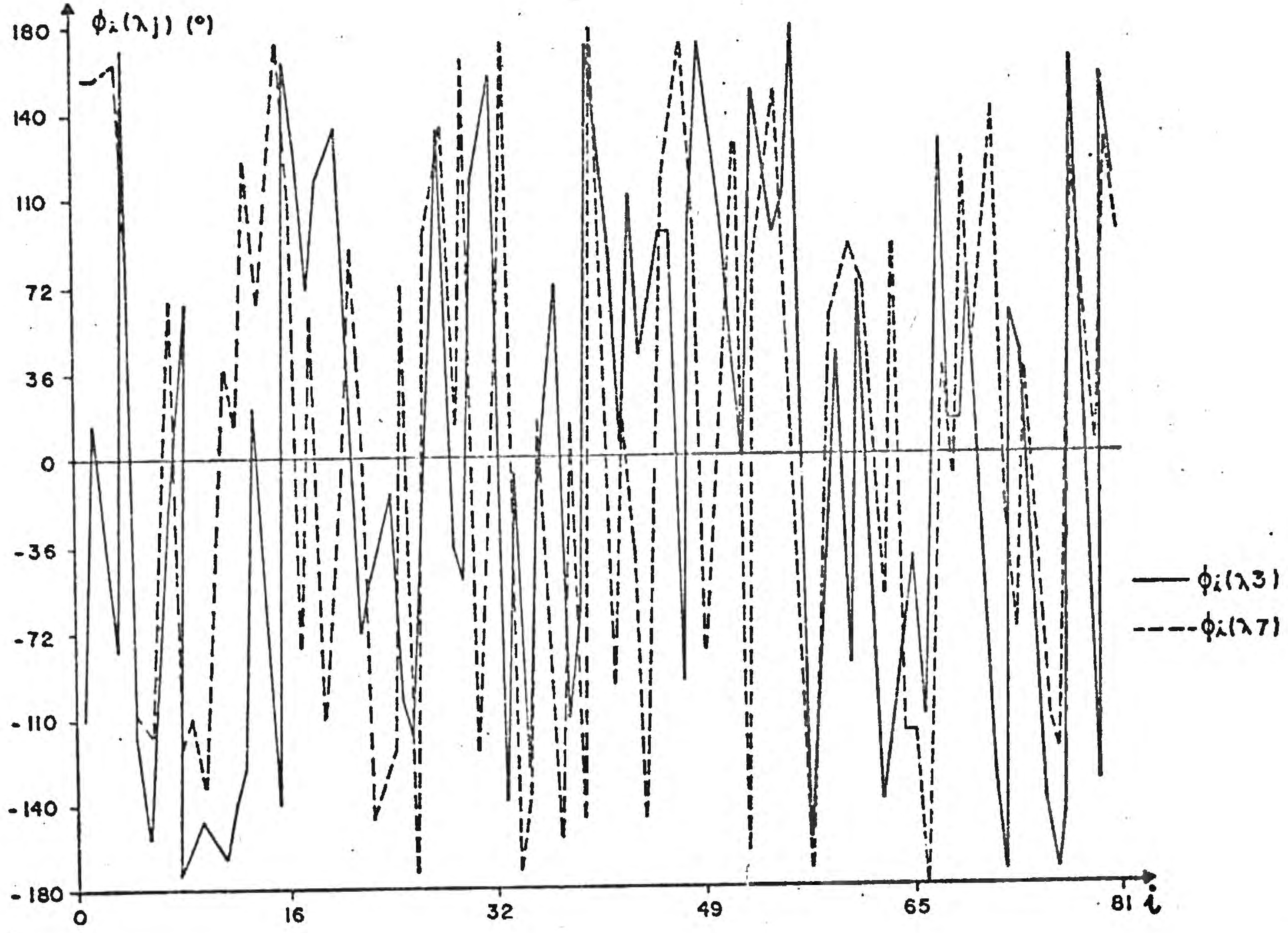

FIG.5- PHASE DIFFERENCES VARIABILITY FOR THE SERIES OF RANDOM NUMBERS,FREQUENCIES $\lambda_3 = 3\pi/13$ and $\lambda_7 = 7\pi/13$

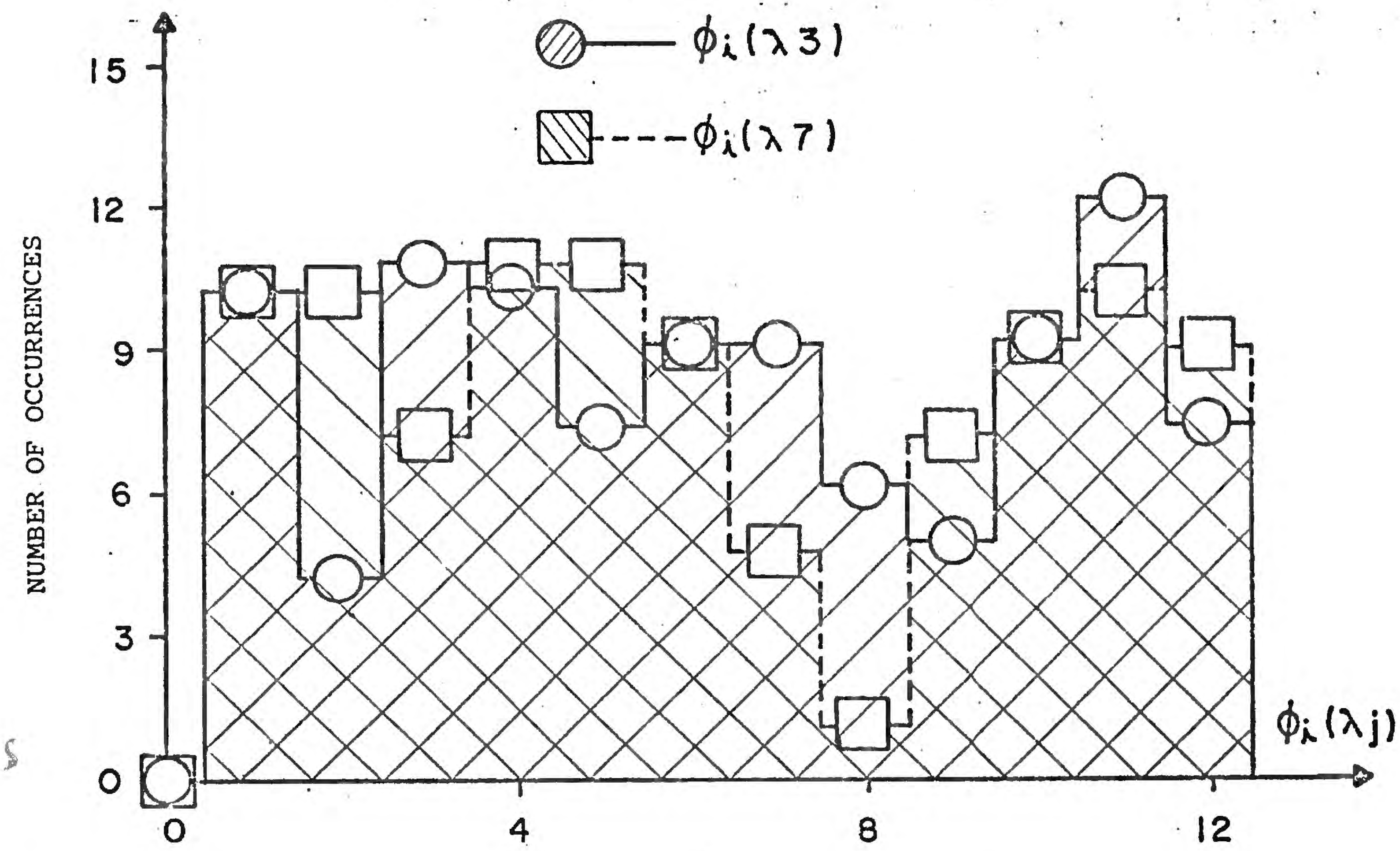

FIG.6- HISTOGRAMS FOR THE PHASE DIFFERENCES (12 CLASSES), SERIES OF RANDOM NUMBERS

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