INFINITE WORDS AND
A THEOREM OF HINDMAN

Imre Simon

ABRIL 1987

Infinite Words and a Theorem of Hindman

Imre Simon[*]

Instituto de Matemática e Estatística

Universidade de São Paulo

01498 São Paulo, SP, Brasil

Abstract

It is shown that every infinite word s over a finite alphabet is a weak sesquipower, i.e. it admits a factorization

$$s = v_1' v_2' v_1 v_3' v_1 v_2 v_1 v_4' v_1 v_2 v_1 v_3 v_1 v_2 v_1 v_5' \cdots,$$

where $|v_1| > 0$ and, for each i, v_i and v_i' are finite words of equal length. This implies a strong version of a theorem of Hindman.

1 Weak sesquipowers

Let A^* be the free monoid generated by the finite set A and let $A^{\mathbb{N}}$ be the set of (one-sided) infinite words over A. An element s of $A^{\mathbb{N}}$ will be considered as a function $s \colon \mathbb{N} \to A$.

Let $\mathbf{v} = (v_1, v_2, \ldots)$ and $\mathbf{v}' = (v_1', v_2', \ldots)$ be sequences of words in A^*, such that $|v_1| > 0$ and, for each $i \geq 1$, $|v_i| = |v_i'|$. We recursively define two sequences $\mathbf{x} = (x_1, x_2, \ldots)$ and $\mathbf{y} = (y_1, y_2, \ldots)$ as follows:

$$x_1 = v_1 \quad \text{and} \quad x_{n+1} = x_n v_{n+1} x_n \quad (n \geq 1),$$

$$y_1 = v_1' \quad \text{and} \quad y_{n+1} = y_n v_{n+1}' x_n \quad (n \geq 1).$$

For each $n \geq 1$, x_n and y_n are left factors of x_{n+1} and y_{n+1}, respectively, hence $\mathbf{x}$ and $\mathbf{y}$ define words x and y in $A^{\mathbb{N}}$. We say that a word s in A^* is a *sesquipower* (*weak sesquipower*) *of order* n if there exist $\mathbf{v}$ ($\mathbf{v}$ and $\mathbf{v}'$) such

that $s = x_n$ $(s = y_n)$. We say that a word s in $A^{I\!N}$ is a *sesquipower* (*weak sesquipower*) if there exist v (v and v') such that $s = x$ $(s = y)$. Note that a weak sesquipower is obtained from a sesquipower by substituting the first occurrence (from left to right) of each variable by a word of equal length. Sesquipowers constitute an unavoidable regularity in words and were used by Coudrain and Schützenberger [2].

Our main result can be stated as follows:

Theorem 1 *Every infinite word is a weak sesquipower.*

The proof uses dynamical techniques first applied to Ramsey theory by H. Furstenberg [4]. It is based on the following theorem of Auslander [1] and Ellis [3]: "In a dynamical system on a compact metric space every point is proximal to a uniformly recurrent point in its orbit closure." [4, p. 160].

Proof. We consider the cyclic dynamical system (X, T), where $X = A^{I\!N}$ is a compact metric space induced by the distance

$$d(s, t) = \inf\{\, 1/(1 + n) \mid s(i) = t(i) \text{ for all } i, 0 \le i < n \,\},$$

and T is the shift transformation, i.e. $[Ts](n) = s(n + 1)$, for $s \in X$ and $n \in I\!N$. Clearly, $T : X \to X$ is continuous.

Let s be a given word in X. By the theorem of Auslander and Ellis there exists a uniformly recurrent word t in the orbit closure of s, such that s and t are proximal.

Now we claim that for every factorization $t = ut'$ of t, with u in A^*, there exist v and v' in A^* such that

$$|v| = |v'| \ge |u|, \qquad t = vut'' \qquad \text{and} \qquad s = v'us''.$$

Indeed, since t is uniformly recurrent, the word u occurs in t syndetically, that is, there exists a $d \in I\!N$, such that for every $n \in I\!N$, u is a segment of $t(n)t(n+1)\cdots t(n+d)$ [4, p. 31]. On the other hand, s and t are proximal, hence, given d, there exists n, such that $s(n+i) = t(n+i)$ for every i, $0 \le i \le d$ [4, p. 158]. We can clearly assume that $n \ge |u|$. Since u is a segment of $t(n)\cdots t(n+d)$, it follows that there exist v, v', t'' and s'', such that $|v| = |v'| \ge n \ge |u|$, $t = vut''$ and $s = v'us''$, as claimed.

Let us define $v_1' = s(0)$ and $v_1 = t(0)$. Assume that $v_1, \ldots, v_n, v_1', \ldots v_n'$ have been defined, such that $s = y_n s'$ and $t = x_n t'$. Applying the above property for $u = x_n$ we have that $t = vx_n t''$ and $s = v'x_n s''$, with $|v| = |v'| \ge |x_n| = |y_n|$. There exist then v_{n+1} and v_{n+1}', such that $v = x_n v_{n+1}$

and $v' = y_n v'_{n+1}$, with $|v_{n+1}| = |v'_{n+1}|$. Thus, $t = x_n v_{n+1} x_n t'' = x_{n+1} t''$ and $s = y_n v'_{n+1} x_n s'' = y_{n+1} s''$. Note that $|v_1| = 1$ implies that $|x_{n+1}| = |y_{n+1}| > |x_n| = |y_n|$; hence, the sequence $(y_i)_{i \geq 1}$ converges to s. This completes the proof of the theorem. ∎

REMARKS -

(1) By way of an example, consider the infinite word s in $\{a, b\}^{\mathbb{N}}$ defined by
$$s = aba^2b^2a^3b^3 \cdots a^nb^n \cdots .$$

Word s has the desired factorization (1 denotes the empty word) with
$$\mathbf{v} = (b, 1, b^2, b^{30}, b^{1722}, \dots)$$

and

$$\mathbf{v'} = (a, 1, a^2, a^3b^3a^4b^4a^5b^5a^6, a^7b^7 \cdots a^{41}b^{41}a^{42}, \dots).$$

In general, having determined $v'_1, v_1, v'_2, \dots, v_{n-1}, v'_n$ word v_n will be $b^{|v'_n|}$ and v'_{n+1} will be chosen so that $y_n v'_{n+1}$ is the word preceding the first occurrence of $x_n \in b^*$ in s.

(2) In the proof we have chosen v_1 to be of length one, but clearly v_1 could have been chosen to be of any positive length.

We state now the finite version of Theorem 1, which follows from it by the compactness of X.

Corollary 2 *Given integers $n, k \geq 1$, there exists an integer $m = m(n, k)$, such that every word of length m over an alphabet of cardinality k has a left factor which is a weak sesquipower of order n.*

Note that the proof of Corollary 2 is non-constructive and, in particular, it does not give any information about the function $m(n, k)$. We do not know of a constructive proof neither for Theorem 1 nor for Corollary 2.

2 Combinatorial consequences

In this section we first derive a Theorem of Hindman [7] as a consequence of Theorem 1. Later, we show that even a stronger version of Hindman's Theorem can be derived from Theorem 1.

Let $p_1, p_2, \ldots$ be a sequence of naturals. The *Hindman set generated by* the sequence $(p_i)_{i \geq 1}$ is the subset H of $\mathbb{N}$ which consists of all finite sums

$$p_{i_1} + p_{i_2} + \cdots + p_{i_k},$$

for $k \geq 1$, and $1 \leq i_1 < i_2 < \cdots < i_k$. A subset H of $\mathbb{N}$ is a *Hindman set* if it is a Hindman set generated by some sequence $(p_i)_{i \geq 1}$.

Corollary 3 (Hindman [7]) *In a finite partition of* $\mathbb{N}$,

$$\mathbb{N} = B_1 \cup B_2 \cup \cdots \cup B_q,$$

one of the sets B_i contains a Hindman set.

Proof. The given partition defines an infinite word s in $A^{\mathbb{N}}$, with $A = \{1, \ldots, q\}$, where $s(n) = j$ if and only if $n \in B_j$. Since s is a weak sesquipower there exist $\mathbf{v}$ and $\mathbf{v}'$, such that $s = y$. We may as well assume that $|v_1| = 1$. We define $p_i = |y_i v'_{i+1}|$, for $i \geq 1$, and claim that the sum $p_{i_1} + p_{i_2} + \cdots + p_{i_k}$, for $k \geq 1$, and $1 \leq i_1 < i_2 < \cdots < i_k$, belongs to B_{v_1}. Indeed, since x_p is a left factor of $x_{p'}$, whenever $p \leq p'$, it follows from the construction of the x_n's that

$$x_{i_k} = x_{i_{k-1}} v_{i_{k-1}+1} \cdots x_{i_2} v_{i_2+1} x_{i_1} v_{i_1+1} v_1 u,$$

for some u in A^*. Hence,

$$s = y_{i_k+1} s' = y_{i_k} v'_{i_k+1} x_{i_k} s' = y_{i_k} v'_{i_k+1} x_{i_{k-1}} v_{i_{k-1}+1} \cdots x_{i_2} v_{i_2+1} x_{i_1} v_{i_1+1} v_1 u s',$$

and $s(p_{i_1} + p_{i_2} + \cdots + p_{i_k}) = v_1$. Thus, $p_{i_1} + p_{i_2} + \cdots + p_{i_k} \in B_{v_1}$, as claimed. This completes the proof. ∎

REMARKS -

(3) Hindman's original proof of Corollary 3 was combinatorial. The first proof using topological dynamics was given by Furstenberg and Weiss [5].

(4) Just as Hindman's theorem was deduced from Theorem 1, its finite version, the Folkman-Rado-Sanders theorem (see [6, p. 65]) can be obtained from Corollary 2.

Now we use Theorem 1 to establish a strong version of Hindman's theorem.

Let H be the Hindman set generated by the sequence $(p_i)_{i \geq 1}$. For given $m \geq 1$, let H_m be the Hindman set generated by $(p_i)_{i \geq m}$; then we say that H_m is a *restriction* of H. These Hindman sets are related as follows:

$$H = H_1 \supseteq H_2 \supseteq H_3 \supseteq \cdots .$$

For $a \in \mathbb{N}$ and $H \subseteq \mathbb{N}$ we shall denote by $a + H$ the set

$$a + H = \{ a + h \mid h \in H \}.$$

Thus, if H is a Hindman set generated by $(p_i)_{i \geq 1}$, $a \geq 0$ and $m \geq 1$ are naturals, then $a + H_m$ consists of all finite sums

$$a + p_{i_1} + p_{i_2} + \cdots + p_{i_k},$$

for $k \geq 1$, and $1 \leq i_1 < i_2 < \cdots < i_k$.

Theorem 4 *For every finite partition of* $\mathbb{N}$,

$$\mathbb{N} = B_1 \cup B_2 \cup \cdots \cup B_q,$$

there exists a Hindman set H generated by $(p_i)_{i \geq 1}$, such that for every a in $\mathbb{N}$, H has a restriction H_m $(m \geq 1)$, for which the set $a + H_m$ is monochromatic.

Proof. Consider the alphabet $A = \{1, \ldots, q\}$ and define s in $A^{\mathbb{N}}$ by puting $s(n) = j$ if and only if $n \in B_j$. Since s is a weak sesquipower there exist $\mathbf{v}$ and $\mathbf{v}'$, such that $s = y$. Without loss of generality we may assume that $|v_1| = 1$ and let us define $p_i = |y_i v'_{i+1}|$, for $i \geq 1$.

Let $m, k \geq 1$ be naturals and let i_j be such that $m \leq i_1 < i_2 < \cdots < i_k$. We claim that, for some $s' \in A^{\mathbb{N}}$, s has a factorization

$$s = u x_m s' \qquad \text{with} \qquad |u| = p_{i_1} + p_{i_2} + \cdots + p_{i_k}.$$

To see the claim we fix m and begin proving, by induction on k, that, for some $g \in A^*$,

$$x_{i_k} = x_{i_{k-1}} v_{i_{k-1}+1} \cdots x_{i_2} v_{i_2+1} x_{i_1} v_{i_1+1} x_{i_1} g.$$

Indeed, for $k = 1$, puting $g = 1$, we have $x_{i_1} = x_{i_1}$. Assuming that

$$x_{i_{k-1}} = x_{i_{k-2}} v_{i_{k-2}+1} \cdots x_{i_2} v_{i_2+1} x_{i_1} v_{i_1+1} x_{i_1} g,$$

5

we have, since x_p is a left factor of $x_{p'}$ whenever $p \leq p'$, and $i_k \geq i_{k-1} + 1$,

$$x_{i_k} = x_{i_{k-1}+1}g' = x_{i_{k-1}}v_{i_{k-1}+1}x_{i_{k-1}}g';$$

hence,

$$x_{i_k} = x_{i_{k-1}}v_{i_{k-1}+1}x_{i_{k-2}}v_{i_{k-2}+1} \cdots x_{i_2}v_{i_2+1}x_{i_1}v_{i_1+1}x_{i_1}gg'.$$

Now, it suffices to observe that y_{i_k+1} is a left factor of s, hence, for an appropriate $s'' \in A^{\mathbb{II}}$,

$$s = y_{i_k+1}s'' = y_{i_k}v'_{i_k+1}x_{i_k}s''.$$

Substituting x_{i_k} we get

$$s = y_{i_k}v'_{i_k+1}x_{i_{k-1}}v_{i_{k-1}+1}x_{i_{k-2}}v_{i_{k-2}+1} \cdots x_{i_2}v_{i_2+1}x_{i_1}v_{i_1+1}x_{i_1}gg's''.$$

Observing that x_m is a left factor of x_{i_1} because $m \leq i_1$ and puting

$$u = y_{i_k}v'_{i_k+1}x_{i_{k-1}}v_{i_{k-1}+1}x_{i_{k-2}}v_{i_{k-2}+1} \cdots x_{i_2}v_{i_2+1}x_{i_1}v_{i_1+1},$$

we have, for an appropriate $s' \in A^{\mathbb{II}}$,

$$s = ux_ms' \quad \text{and} \quad |u| = p_{i_1} + p_{i_2} + \cdots + p_{i_k}.$$

This proves the claim.

Now, to prove the theorem let us assume that $a \geq 0$ is given. Let us choose $m \geq 1$, such that $|x_m| > a$, and let j, $1 \leq j \leq q$, be the $(a+1)$-th letter of x_m, that is, $x_m = g_1 j g_2$, with $|g_1| = a$, $j \in A$ and $g_2 \in A^*$. Now, using the claim, it follows that whenever $k \geq 1$ and $m \leq i_1 < i_2 < \cdots < i_k$, $s(a + p_{i_1} + p_{i_2} + \cdots + p_{i_k}) = j$, that is, by the construction of s, $a + p_{i_1} + p_{i_2} + \cdots + p_{i_k} \in B_j$. Thus, $a + H_m \subseteq B_j$; hence $a + H_m$ is monochromatic. This completes the proof. ∎

REMARK -

(5) Observe that $|x_i| \geq 2^i$, hence, for a given $a \geq 0$, one can take $m = 1 + \lceil \log_2(a+1) \rceil$ in the statement of Theorem 4, since for this m one already has $|x_m| > a$, as needed.

3 A remark about Birkhoff's theorem

As we noted earlier, the proof of Theorem 1 is non-constructive and we do not know of a constructive proof. As a possible first step towards such a proof we outline in the sequel a proof of Birkhoff's recurrence theorem, without using Zorn's lemma, valid for dynamical systems of our interest.

For s in $A^{\mathbb{N}}$ we define

$$\operatorname{seg} s = \{\, u \in A^* \mid s \in A^* u A^{\mathbb{N}} \,\},$$

and

$$\operatorname{syn} s = \{\, u \in A^* \mid \operatorname{seg} s - A^* u A^* \text{ is finite} \,\}.$$

Here, seg s and syn s stand for the set of segments of s and the set of segments of s that occur syndetically in s. It is well known that $s \in A^{\mathbb{N}}$ is uniformly recurrent if and only if seg s = syn s [4, p. 31]. Indeed, one can also prove that s is uniformly recurrent if and only if seg s is minimal in $\{\operatorname{seg} t \mid t \in A^{\mathbb{N}}\}$ (see [6, p.137]). A step in this proof is given by the lemma:

Lemma 5 *Let $s \in A^{\mathbb{N}}$ and $u \in \operatorname{seg} s - \operatorname{syn} s$. There exists $t \in A^{\mathbb{N}}$ such that* $\operatorname{syn} s \subseteq \operatorname{syn} t \subseteq \operatorname{seg} s - A^* u A^*$.

Proof. Since $u \notin \operatorname{syn} s$, the set $S = \operatorname{seg} s - A^* u A^*$ is infinite and it is clearly closed by taking segments of words in S. By König's lemma (or by compactness of $A^{\mathbb{N}}$) there exists $t \in A^{\mathbb{N}}$ such that $\operatorname{seg} t \subseteq S$. Since $\operatorname{seg} t \subseteq \operatorname{seg} s$, it follows that $\operatorname{syn} s \subseteq \operatorname{syn} t$ and the lemma is proved. ∎

Proposition 6 *Let (X, T) be a dynamical system, where $X \subseteq A^{\mathbb{N}}$, and T is the shift transformation. Then X contains a uniformly recurrent point.*

Proof. It is sufficient to prove that the orbit closure of every s in $A^{\mathbb{N}}$ contains a word t in $A^{\mathbb{N}}$ which is uniformly recurrent, i.e. such that $\operatorname{syn} s \subseteq \operatorname{syn} t = \operatorname{seg} t \subseteq \operatorname{seg} s$. To do this, we claim that there exists a sequence $s_0, s_1, \ldots$ of words in $A^{\mathbb{N}}$, such that $s_0 = s$,

$$\operatorname{seg} s_0 \supseteq \operatorname{seg} s_1 \supseteq \cdots \supseteq \operatorname{seg} s_n \supseteq \cdots$$

$$\operatorname{syn} s_0 \subseteq \operatorname{syn} s_1 \subseteq \cdots \subseteq \operatorname{syn} s_n \subseteq \cdots$$

and, for each $n \geq 1$, $\operatorname{syn} s_n \supseteq \operatorname{seg} s_n \cap A^n$. Indeed, s_{i+1} can be obtained from s_i by repeatedly applying Lemma 5, for words u of length $i + 1$, until

geting a word, s_{i+1}, for which every word of length at most $i+1$ in seg s_{i+1} belongs to syn s_{i+1}.

Consider now the set $S = \{ s_i \mid i \geq 0 \}$. If S is finite then, for some j, $s_j = s_{j+1} = \cdots$ and the theorem is proved for $t = s_j$, which is uniformly recurrent since seg $s_j = $ syn s_j. Otherwise, S is infinite and it has a limit point t, since $A^{\mathbb{N}}$ is a compact space. Now, the following inclusions can be proved:

$$\text{seg } t \subseteq \bigcap_{n \geq 0} \text{seg } s_n \subseteq \bigcup_{n \geq 0} \text{syn } s_n \subseteq \text{syn } t \subseteq \text{seg } t.$$

It follows that

$$\text{syn } s \subseteq \text{syn } t = \text{seg } t \subseteq \text{seg } s,$$

i.e. t is a uniformly recurrent word in the orbit closure of s. This completes the proof of the proposition. ∎

References

[1] J. Auslander, On the proximal relation in topological dynamics, *Proc. Amer. Math. Soc.* 11 (1960), 890–895.

[2] M. Coudrain and M. P. Schützenberger, Une condition de finitude des monoids finiment engendrés, *C. R. Acad. Sci. Paris, Ser. A* 262 (1966), 1149–1151.

[3] R. Ellis, A semigroup associated with a transformation group, *Trans. Amer. Math. Soc.* 94 (1960), 272–281.

[4] H. Furstenberg, *Recurrence in Ergodic Theory and Combinatorial Number Theory*, Princeton University Press, Princeton, 1981.

[5] H. Furstenberg and B. Weiss, Topological dynamics and combinatorial number theory, *J. d'Analyse Math.* 34 (1978), 61–85.

[6] R. L. Graham, B. L. Rothschild and J. H. Spencer, *Ramsey Theory*, John Wiley & Sons, New York, 1980.

[7] N. Hindman, Finite sums from sequences within cells of a partition of $\mathbb{N}$, *J. Comb. Th. A* 17 (1974), 1–11.

RT-MAP-7701 - Ivan de Queiroz Barros
On equivalence and reducibility of Generating Matrices
of RK-Procedures - Agosto 1977

RT-MAP-7702 - V.W. Setzer
A Note on a Recursive Top-Down Analizer of N.Wirth - Dezembro 1977

RT-MAP-7703 - Ivan de Queiroz Barros
Introdução a Aproximação Ótima - Dezembro 1977

RT-MAP-7704 - V.W. Setzer, M.M. Sanches
A linguagem "LEAL" para Ensino básico de Computação - Dezembro 1977

RT-MAP-7801 - Ivan de Queiroz Barros
Proof of two Lemmas of interest in connection with discretization
of Ordinary Differential Equations - Janeiro 1978

RT-MAP-7802 - Silvio Ursic, Cyro Patarra
Exact solution of Systems of Linear Equations with Iterative Methods
Fevereiro 1978

RT-MAP-7803 - Martin Grötschel, Yoshiko Wakabayashi
Hypohamiltonian Digraphs - Março 1978

RT-MAP-7804 - Martin Grötschel, Yoshiko Wakabayashi
Hypotraceable Digraphs - Maio 1978

RT-MAP-7805 - W. Hesse, V.W. Setzer
The Line-Justifier: an example of program development by transformations
Junho 1978

RT-MAP-7806 - Ivan de Queiroz Barros
Discretização
Capítulo I - Tópicos Introdutórios
Capítulo II - Discretização
Julho 1978

RT-MAP-7807 - Ivan de Queiroz Barros
(Γ', Γ) - Estabilidade e Métodos Preditores-Corretores - Setembro 1978

RT-MAP-7808 - Ivan de Queiroz Barros
Discretização
Capítulo III - Métodos de passo progressivo para Eq. Dif. Ord. com
condições iniciais - Setembro 1978

RT-MAP-7809 - V.W. Setzer
Program development by transformations applied to relational Data-Base
queries - Novembro 1978

RT-MAP-7810 - Nguiffo B. Boyom, Paulo Boulos

RT-MAP-7811 - D.T. Fernandes e C. Patarra
Sistemas Lineares Esparsos, um Método Exato de Solução - Novembro 1978

RT-MAP-7812 - V.W. Setzer e G. Bressan
Desenvolvimento de Programas por Transformações: uma Comparação entre dois Métodos - Novembro 1978

RT-MAP-7813 - Ivan de Queiroz Barros
Variação do Passo na Discretização de Eq. Dif. Ord. com Condições Iniciais - Novembro 1978

RT-MAP-7814 - Martin Grötschel e Yoshiko Wakabayashi
On the Complexity of the Monotone Asymetric Travelling Salesman Polytope I: HIPOHAMILTONIAN FACETS - Dezembro 1978

RT-MAP-7815 - Ana F. Humes e E.I. Jury
Stability of Multidimensional Discrete Systems: State-Space Representation Approach - Dezembro 1978

RT-MAP-7901 - Martin Grötschel, Yoshiko Wakabayashi
On the complexity of the Monotone Asymetric Travelling Salesman Polytope II: HYPOTRACEABLE FACETS - Fevereiro 1979

RT-MAP-7902 - M.M. Sanches e V.W. Setzer
A portabilidade do Compilador para a Linguagem LEAL - Junho 1979

RT-MAP-7903 - Martin Grötschel, Carsten Thomassen, Yoshiko Wakabayashi
Hypotraceable Digraphs - Julho 1979

RT-MAP-7904 - N'Guiffo B. Boyom
Translations non triviales dans les groupes (transitifs) des transformations affines - Novembro 1979

RT-MAP-8001 - Ângelo Barone Netto
Extremos detectáveis por jatos - Junho 1980

RT-MAP-8002 - Ivan de Queiroz Barros
Medida e Integração
Cap. I - Medida e Integração Abstrata - Julho 1980

RT-MAP-8003 - Routo Terada
Fast Algorithms for NP-Hard Problems which are Optimal or Near-Optimal with Probability one - Setembro 1980

RT-MAP-8004 - V.W. Setzer e R. Lapyda
Uma Metodologia de Projeto de Bancos de Dados para o Sistema ADABAS
Setembro 1980

RT-MAP-8005 - Imre Simon
On Brzozowski's Problem: $(1uA)^m = A*$ - Outubro 1980

RT-MAP-8006 - Ivan de Queiroz Barros
Medida e Integração

RT-MAP-8101 - Luzia Kazuko Yoshida e Gabriel Richard Bitran
Um algoritmo para Problemas de Programação Vetorial com Variáveis
Zero-Um - Fevereiro 1981

RT-MAP-8102 - Ivan de Queiroz Barros
Medida e Integração
Cap. III - Medidas em Espaços Topológicos - Março 1981

RT-MAP-8103 - V.W. Setzer, R. Lapyda
Design of Data Models for the ADABAS System using the Entity-Relationship
Approach - Abril 1981

RT-MAP-8104 - Ivan de Queiroz Barros
Medida e Integração
Cap. IV - Medida e Integração Vetoriais - Abril 1981

RT-MAP-8105 - U.S.R. Murty
Projective Geometries and Their Truncations - Maio 1981

RT-MAP-8106 - V.W. Setzer, R. Lapyda
Projeto de Bancos de Dados, Usando Modelos Conceituais
Este relatório Técnico complementa o RT-MAP-8103. Ambos substituem o
RT-MAP-8004 ampliando os conceitos ali expostos. - Junho 1981

RT-MAP-8107 - Maria Angela Gurgel, Yoshiko Wakabayashi
Embedding of Trees - August 1981

RT-MAP-8108 - Ivan de Queiroz Barros
Mecânica Analítica Clássica - Outubro 1981

RT-MAP-8109 - Ivan de Queiroz Barros
Equações Integrais de Fredholm no Espaço das Funções A-Uniformemente Contínuas
- Novembro 1981

RT-MAP-8110 - Ivan de Queiroz Barros
Dois Teoremas sobre Equações Integrais de Fredholm - Novembro 1981

RT-MAP-8201 - Siang Wun Song
On a High-Performance VLSI Solution to Database Problems - Janeiro 1982

RT-MAP-8202 - Maria Angela Gurgel, Yoshiko Wakabayashi
A Result on Hamilton-Connected Graphs - Junho 1982

RT-MAP-8203 - Jörg Blatter, Larry Schumaker
The Set of Continuous Selections of a Metric Projection in C(X)
- Outubro 1981

RT-MAP-8204 - Jörg Blatter, Larry Schumaker
Continuous Selections and Maximal Alternators for Spline Approximation
- Dezembro 1981

RT-MAP-8205 - Arnaldo Mandel
Topology of Oriented Matroids - Junho 1982

RT-MAP-8206 - Erich J. Neuhold
Database Management Systems; A General Introduction - Novembro 1982

RT-MAP-8207 - Béla Bollobás
The Evolution of Random Graphs - Novembro 1982

RT-MAP-8208 - V.W. Setzer
Um Grafo Sintático para a Linguagem PL/M-80 - Novembro 1982

RT-MAP-8209 - Jayme Luiz Szwarcfiter
A Sufficient Condition for Hamilton Cycles - Novembro 1982

RT-MAP-8301 - W.M. Oliva
Stability of Morse-Smale Maps - Janeiro 1983

RT-MAP-8302 - Belá Bollobás, Istvan Simon
Repeated Random Insertion into a Priority Queue - Fevereiro 1983

RT-MAP-8303 - V.W. Setzer, P.C.D. Freitas e B.C.A. Cunha
Um Banco de Dados de Medicamentos - Julho 1983

RT-MAP-8304 - Ivan de Queiroz Barros
O Teorema de Stokes em Variedades Celuláveis - Julho 1983

RT-MAP-8305 - Arnaldo Mandel
The 1-Skeleton of Polytopes, oriented Matroids and some other lattices -
- Julho 1983

RT-MAP-8306 - Arnaldo Mandel
Alguns Problemas de Enumeração em Geometria - Agosto 1983

RT-MAP-8307 - Siang Wun Song
Complexidade de E/S e Projetos Optimais de Dispositivos para Ordenação -
- Agosto 1983

RT-MAP-8401-A - Dirceu Douglas Salvetti
Procedimentos para Cálculos com Splines
Parte A - Resumos Teóricos - Janeiro 1984
RT-MAP-8401-B
Parte B - Descrição de Procedimentos - Janeiro 1984
RT-MAP-8401-C
Parte C - Listagem de Testes - Janeiro 1984

RT-MAP-8402 - V.W. Setzer
Manifesto contra o uso de computadores no Ensino de 1º Grau - Abril 1984

RT-MAP-8403 - G. Fusco e W.M. Oliva
On Mechanical Systems with Non-Holonomic Constraints: Some Aspects of the
General Theory and Results for the Dissipative Case - Julho 1984

RT-MAP-8404 - Imre Simon
A Factorization of Infinite Words - Setembro 1984 - São Paulo - IME-USP
7 pg.

RT-MAP-8405 - Imre Simon
The Subword Structure of a Free Monoid - Setembro 1984 - São Paulo - IME-U
6 pg.

RT-MAP-8406 - Jairo Z. Gonçalves e Arnaldo Mandel
Are There Free Groups in Division Rings? - Setembro 1984 - São Paulo - IME-
25 pg.

RT-MAP-8407 - Paulo Feofiloff and D.H. Younger

RT-MAP-8605 - Angelo Barone Netto e Mauro de Oliveira Cesar
Nonconservative Positional Systems - Stability - Junho 1986
Sao Paulo - IME-USP - 14 pg.

RT-MAP-8606 - Júlio Michael Stern
Fatoração L - U e Aplicações - Agosto 1986
Sao Paulo - IME-USP - 105 pg.

RT-MAP-8607 - Afonso Galvão Ferreira
O Problema do Dobramento Optimal de PLAs - Agosto 1986
Sao Paulo - IME-USP - 73 pg.

RT-MAP-8608 - Gaetano Zampieri
Liapunov Stability for Some Central Forces - Novembro 1986
Sao Paulo - IME-USP - 17 pg.

RT-MAP-8701 - A. Bergamasco, P. Cordaro and J. Hounie
Global Properties of a Class of Vector Fields in the Plane - Fevereiro 1
Sao Paulo - IME-USP - 37 pg.

RT-MAP-8702 - P. Cordaro and J. Hounie
Local Solvability in C_c^∞ of Over-Determined Systems of Vector Fields -
Fevereiro - Sao Paulo - IME-USP - 32 pg.

RT-MAP-8703 - Imre Simon
The Nondeterministic Complexity of a Finite Automaton - Fevereiro 1987
Sao Paulo - IME-USP - 20 pg.

RT-MAP-8704 - Imre Simon
Infinite Words and a Theorem of Hindman - Abril 1987
Sao Paulo - IME-USP - 8 pg.