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BAYESIAN ANALYSIS OF A MULTIVARIATE NULL INTERCEPT ERRORS-IN-VARIABLES REGRESSION MODEL

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SUMMARY.

Longitudinal data are of great interest in analysis of clinical trials. In many practical situations the covariate can not be measured precisely and a natural alternative model is

the errors-in-variables regression models. In this paper we study a null intercept errors-in-variables regression model with a structure of dependency between the response variables within the same group. We apply the model to a real data presented in Hadgu and Koch (1999). In that study, volunteers with pre existing dental plaque were randomized to two experimental mouth rinses (A and B) or a control mouth rinse with double blinding. The dental plaque index was measured for each subject in the beginning of the study and two follow-up times, which leads to the presence of an interclass correlation. We propose the use of a Bayesian approach to model a multivariate null intercept errors-in-variables regression model to the longitudinal data. The proposed Bayesian approach accommodates the correlated measurements and incorporates the restriction that the slopes must lie in the [0,1] interval. A Gibbs sampler is used to perform the computations.

1 Introduction

Errors-in-variables regression models constitute an attractive alternative model in many practical experimental problems, as when the same response is observed on the same units under different experimental conditions. There are situations, where the use of the null intercept models are adequate. Aoki (2001) considered the use of the null intercept errors-in-variables model to reanalyse the data from a pretest/post-test study designed to compare two types of toothbrushes with respect to the efficacy in removing dental plaque. In that study, 26 preschoolers were evaluated under two different experimental conditions (toothbrushes), with respect to the dental plaque index before and after brushing with either a conventional or an experimental toothbrush. No intercepts were included in the proposed models, since null pretest dental plaque index imply null expected post-test values. As the covariate (plaque index) is measured imprecisely, an alternative way to analyze this data set is to consider the errors-in-variables models. In addition, as the same individuals were evaluated under two different experimental conditions the models accounted for the possible dependence of the within subjects measurements (see also, Aoki et al. (2001) and (2003)).

Considering the dental plaque index data just described, it was collected one measurement of the dental plaque index after the use of the experimental or conventional toothbrush by each subject. Motivated by the dental clinical trial from a recent paper by Hadgu and Koch (1999), we extended the univariate null intercept errors-in-variables regression model to a multivariate null intercept errors-in-variables regression model. In that study 109 adult male and female volunteers were randomized to two experimental mouth rinses (A or B) or a control mouth rinse and evaluated under these three experimental conditions (mouth rinses), with respect to the dental plaque index at baseline, after 3 months and after 6 months from the baseline with the use of the mouth rinse A, mouth rinse B or the control mouth rinse. As the covariate (plaque index) is measured imprecisely, an alternative way to analyze this data set is to consider the errors-in-variables models. In addition since the plaque index was collected at baseline and two follow up times, we need a model that takes into account the possible dependence on the outcome measurements. No intercepts are included in the proposed model, since null dental plaque index at baseline imply null expected post-test values, that is, the dental plaque index should not increase after the use of each mouth rinse. Also, the slope parameters must be restricted to the interval $[0, 1]$, as the slope parameters represent the percentage of dental plaque remaining after the use of each mouth rinse. One advantage of the Bayesian approach is that it allows incorporating this information (that the slopes must be restricted between 0 and 1) easily.

We develop the model considering the Bayesian approach and Gibbs Sampling algorithm to obtain the posterior summaries of interest. In Section 2, we describe the model.

In Section 3, we apply the model to the clinical trial and finally in Section 4 we discuss the results.

2 The Model

The proposed model to analyze the kind of data described earlier is given by

$$\begin{aligned} \mathbf{x}_i &= \xi_i + \delta_i, \\ \mathbf{y}_i &= \eta_i + \varepsilon_i, \\ \eta_i &= \mathbf{X}_i \beta_i, \quad i = 1, \ldots, p, \end{aligned} \tag{1}$$

where $\mathbf{x}_i^T = (x_{i1}, \ldots, x_{ini})$, $\mathbf{y}_i^T = \left(\mathbf{y}_{1i}^T,\ \mathbf{y}_{2i}^T\right) = (y_{1i1},\ \ldots,\ y_{1ini},\ y_{2i1}, \ldots, y_{2ini})$, $\boldsymbol{\beta}_i^T = (\beta_{1i},\ \beta_{2i})$, $\boldsymbol{\xi}_i^T = (\xi_{i1}, \ldots,\ \xi_{ini})$, $\boldsymbol{\delta}_i^T = (\delta_{i1}, \ldots, \delta_{ini})$, $\boldsymbol{\varepsilon}_i^T = \left(\boldsymbol{\varepsilon}_{1i}^T,\ \boldsymbol{\varepsilon}_{2i}^T\right) = (\varepsilon_{1i1}, \ldots,\ \varepsilon_{1ini},\ \varepsilon_{2i1}, \ldots,\ \varepsilon_{2ini})$, $\mathbf{X}_i = \begin{bmatrix} \boldsymbol{\xi}_i & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\xi}_i \end{bmatrix}$, with $\delta_{ij} \sim ind.\ N(0, \sigma^2)$, $\varepsilon_{kij} \sim ind.\ N(0, \sigma_\varepsilon^2)$, δ_{ij} and ε_{kij} not correlated and independent of $\xi_{ij} \sim ind\ N(\mu, \sigma_x^2)$, $i = 1, \ldots, p$, $j = 1, \ldots, ni$, $k = 1, 2$.

Considering the clinical trial described earlier, we have i=1,2,3 (mouth rinse type: Control, A and B, respectively), k=1,2 (1 represents the plaque index after 3 months and 2 represents the plaque index after 6 months from the baseline), x_{ij} represents the observed plaque index at baseline, while y_{1ij} and y_{2ij} represents, respectively, the observed plaque index after 3 months and after 6 months from the beginning of the study, with the use of the mouth rinse i for the subject j and ξ_{ij} the unobserved real plaque index at baseline.

Let us define by $\mathbf{Z}_{ij}$ the observed vector $[x_{ij}, y_{1ij}, y_{2ij}]^T$; then

$$\boldsymbol{Z}_{ij} \sim N_3 \left(\begin{bmatrix} \mu \\ \beta_{1i}\mu \\ \beta_{2i}\mu \end{bmatrix}, \begin{bmatrix} \sigma_x^2 + \sigma^2 & \beta_{1i}\sigma_x^2 & \beta_{2i}\sigma_x^2 \\ \beta_{1i}\sigma_x^2 & \beta_{1i}^2\sigma_x^2 + \sigma_{e_i}^2 & \beta_{1i}\beta_{2i}\sigma_x^2 \\ \beta_{2i}\sigma_x^2 & \beta_{1i}\beta_{2i}\sigma_x^2 & \beta_{2i}^2\sigma_x^2 + \sigma_{e_i}^2 \end{bmatrix} \right). \tag{2}$$

By using general properties of the multivariate normal distribution, it follows that the likelihood function for $\boldsymbol{\theta} = (\mu, \boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_p, \sigma_x^2, \sigma^2, \sigma_{e_1}^2, \ldots, \sigma_{e_p}^2)^T$ can be written as

$$L(\boldsymbol{\theta}) = (2\pi)^{-\frac{3n}{2}} \prod_{i=1}^{p} (\sigma_{ei}^{-n_i} \nu_i^{-\frac{n_i}{2}}) exp \left[-\frac{1}{2} \sum_{i=1}^{p} \frac{(\beta_{1i}^2 + \beta_{2i}^2)\sigma_x^2 + \sigma_{e_i}^2}{\nu_i} \sum_{j=1}^{n_i} (x_{ij} - \mu)^2 - \right.$$

$$\frac{1}{2} \sum_{i=1}^{p} \frac{1}{\nu_i} \left(\frac{\beta_{2i}^2 \sigma_x^2 \sigma^2}{\sigma_{e_i}^2} + \sigma_x^2 + \sigma^2 \right) \sum_{j=1}^{n_i} (y_{1ij} - \beta_{1i}\mu)^2 + \sum_{i=1}^{p} \frac{\beta_{1i}\sigma_x^2}{\nu_i} \sum_{j=1}^{n_i} (x_{ij} - \mu)(y_{1ij} - \beta_{1i}\mu) -$$

$$\frac{1}{2} \sum_{i=1}^{p} \frac{1}{\nu_i} \left(\frac{\beta_{1i}^2 \sigma_x^2 \sigma^2}{\sigma_{e_i}^2} + \sigma_x^2 + \sigma^2 \right) \sum_{j=1}^{n_i} (y_{2ij} - \beta_{2i}\mu)^2 + \sum_{i=1}^{p} \frac{\beta_{2i}\sigma_x^2}{\nu_i} \sum_{j=1}^{n_i} (x_{ij} - \mu)(y_{2ij} - \beta_{2i}\mu) +$$

$$\left. \sum_{i=1}^{p} \frac{\beta_{1i}\beta_{2i}\sigma_x^2\sigma^2}{\nu_i \sigma_{e_i}^2} \sum_{j=1}^{n_i} (y_{1ij} - \beta_{1i}\mu)(y_{2ij} - \beta_{2i}\mu) \right], \quad (3)$$

with $\nu_i = (\beta_{1i}^2 + \beta_{2i}^2)\sigma^2\sigma_x^2 + (\sigma^2 + \sigma_x^2)\sigma_{e_i}^2$ and $i = 1, \ldots, p; j = 1, \ldots, n_i$.

As the likelihood function (3) is complex to obtain the full conditional distributions of the parameters, we introduce the latent variables (Tanner and Wong (1987)) ξ_{ij}, $i = 1, \ldots, p$; $j = 1, \ldots, n_i$ to simplify the implementation of the algorithm. Let us define $\mathbf{w}_{ij} = [\xi_{ij}, x_{ij}, y_{1ij}, y_{2ij}]^T$; in this case the likelihood function is given by

$$L(\boldsymbol{\theta}) = (2\pi)^{-2n} \sigma^{-n} \sigma_x^{-n} \prod_{i=1}^{p} (\sigma_{ei}^2)^{-n_i} exp \left[-\frac{1}{2} \sum_{i=1}^{p} \sum_{j=1}^{n_i} \frac{(\xi_{ij} - \mu)^2}{\sigma_x^2} - \frac{1}{2} \sum_{i=1}^{p} \sum_{j=1}^{n_i} \frac{(x_{ij} - \xi_{ij})^2}{\sigma^2} - \right.$$

$$\left. \frac{1}{2} \sum_{i=1}^{p} \sum_{j=1}^{n_i} \frac{(y_{1ij} - \beta_{1i}\xi_{ij})^2}{\sigma_{e_i}^2} - \frac{1}{2} \sum_{i=1}^{p} \sum_{j=1}^{n_i} \frac{(y_{2ij} - \beta_{2i}\xi_{ij})^2}{\sigma_{e_i}^2} \right], \quad (4)$$

$i = 1, \ldots, p; j = 1, \ldots, n_i; n = n_1 + \ldots + n_p$.

Note that the latent variables correspond to the true value of the unobserved explanatory variables and with the use of this latent variables the resulting likelihood function which is given in (4) is much simpler to work with and the full conditional distributions of the parameters can be easily obtained. Under a Bayesian framework, all unobservable quantities are considered as random variables. To each of them a prior distribution expressing the degree of belief about their values is assigned. In the proposed model, the unknown quantities are expressed in the vectors $\boldsymbol{\theta} = (\mu, \boldsymbol{\beta_1}, \ldots, \boldsymbol{\beta_p}, \sigma_x^2, \sigma^2, \sigma_{e_1}^2, \ldots, \sigma_{e_p}^2)^T$ and $\boldsymbol{\xi}_i = (\xi_{i1}, \ldots, \xi_{ini})^T$. The prior density is taken to be of the form,

$$\pi(\boldsymbol{\theta}) = \pi(\mu)\,\pi(\beta_{11})\ldots\pi(\beta_{1p})\,\pi(\beta_{21})\ldots\pi(\beta_{2p})\,\pi\left(\sigma_x^2\right)\pi\left(\sigma^2\right)\pi\left(\sigma_{e_1}^2\right)\ldots\pi\left(\sigma_{e_p}^2\right),$$

where $\mu \sim N(m, v)$, $\beta_{ki} \sim Beta(a_{ki}, c_{ki})$, $k = 1, 2$, $i = 1, \ldots, p$, $\sigma_{e_i}^2 \sim IG(e_i, h_i)$, $i = 1, \ldots, p$, $\sigma_x^2 \sim IG(e_{p+1}, h_{p+1})$, $\sigma^2 \sim IG(e_{p+2}, h_{p+2})$, with $Beta(a, b)$ denoting the Beta distribution with parameters $a > 0$, $b > 0$, $N(a, b^2)$ denoting the Normal distribution with location parameter a and scale parameter $b > 0$, $IG(a, b)$ denoting the Inverse Gamma distribution with shape parameter $a > 0$ and scale parameter $b > 0$.

We introduce an additional hierarchical level by allowing h_i to follow a Gamma distribution, that is, $h_i \sim G(g_i, r_i)$, $i = 1, \ldots, p$, with $G(a, b)$ denoting the Gamma distribution with shape parameter $a > 0$ and scale parameter $b > 0$, a_{ki}, c_{ki}, $k = 1, 2$, $i = 1, \ldots, p$, e_l, $l = 1, \ldots, p+2$, g_m, r_m, $m = 1, \ldots, p$, represent specified prior parameters. The slope parameters must lie in the interval $[0, 1]$, as it represents the reduction rate of the dental plaque index after the use of the corresponding mouth rinses at 3 months and at 6 months from the beginning of the study. This restriction is easily accomplished by using a Beta distribution as the prior.

For the computations of the posterior distributions, we consider the Gibbs sampler algorithm (Smith and Roberts, 1993). In this direction, we first obtain the full conditional distributions of each of the unknowns. If this conditional distribution follows a simple form, we sample directly; otherwise, we use the Metropolis algorithm, as can be seen next.

Denoting $\boldsymbol{\theta}_{\{-\theta_i\}} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_{3p+3})$, with $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_{3p+3}) = (\mu, \beta_{11}, \ldots, \beta_{1p}, \beta_{21}, \ldots, \beta_{2p}, \sigma_x^2, \sigma^2, \sigma_{e_1}^2, \ldots, \sigma_{e_p}^2)^T$; $\boldsymbol{\xi}_{\{-ij\}} = (\xi_{i1}, \ldots, \xi_{ij-1}, \xi_{ij+1}, \ldots, \xi_{in_i})$ and $\boldsymbol{\xi} =$

$(\boldsymbol{\xi}_1, \ldots, \boldsymbol{\xi}_p)$, we obtain the full conditional distributions of the unknowns as follows:

1)-$\pi(\mu/\boldsymbol{\theta}_{\{-\mu\}}, \boldsymbol{\xi}) \propto N(m_1, v_1)$, with

$$m_1 = \frac{m\sigma_x^2 + v\sum_{i=1}^{p}\sum_{j=1}^{n_i}\xi_{ij}}{nv + \sigma_x^2} \quad \text{and} \quad v_1 = \frac{v\sigma_x^2}{nv + \sigma_x^2},$$

2)-$\pi(\sigma_{e_i}^2/\boldsymbol{\theta}_{\{-\sigma_{e_i}^2\}}, \boldsymbol{\xi}) \propto IG(e_{ii}, h_{ii})$, $i = 1, \ldots, p$, with

$$e_{ii} = n_i + e_i \quad \text{and} \quad h_{ii} = \sum_{j=1}^{n_i}\frac{(y_{1ij} - \beta_{1i}\xi_{ij})^2}{2} + \sum_{j=1}^{n_i}\frac{(y_{2ij} - \beta_{2i}\xi_{ij})^2}{2} + h_i,$$

3)-$\pi(\sigma_x^2/\boldsymbol{\theta}_{\{-\sigma_x^2\}}, \boldsymbol{\xi}) \propto IG(e_{(p+1,p+1)}, h_{(p+1,p+1)})$, with

$$e_{(p+1,p+1)} = \frac{n}{2} + e_{p+1} \quad \text{and} \quad h_{(p+1,p+1)} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\frac{(\xi_{ij} - \mu)^2}{2} + h_{p+1},$$

4)-$\pi(\sigma^2/\boldsymbol{\theta}_{\{-\sigma^2\}}, \boldsymbol{\xi}) \propto IG(e_{(p+2,p+2)}, h_{(p+2,p+2)})$, with

$$e_{(p+2,p+2)} = \frac{n}{2} + e_{p+2} \quad \text{and} \quad h_{(p+2,p+2)} = \sum_{i=1}^{p}\sum_{j=1}^{n_i}\frac{(x_{ij} - \xi_{ij})^2}{2} + h_{p+2},$$

5)-$\pi(h_i/\boldsymbol{\theta}_{\{-h_i\}}, \boldsymbol{\xi}) \propto G(g_{ii}, r_{ii})$, $i = 1, \ldots, n$, with

$$g_{ii} = g_i + e_i \quad \text{and} \quad r_{ii} = r_i + \frac{1}{\sigma_{e_i}^2},$$

6)-$\pi(\xi_{ij}/\boldsymbol{\theta}, \boldsymbol{\xi}_{\{-ij\}}) \propto N(m_{q_{ij}}, v_{q_i})$, $i = 1, \ldots, p$, $j = 1, \ldots, n_i$, with

$$m_{q_{ij}} = \frac{(y_{1ij}\beta_{1i} + y_{2ij}\beta_{2i})\sigma_x^2\sigma^2 + (\mu\sigma^2 + \sigma_x^2 x_{ij})\sigma_{e_i}^2}{(\beta_{1i}^2 + \beta_{2i}^2)\sigma_x^2\sigma^2 + (\sigma_x^2 + \sigma^2)\sigma_{e_i}^2} \text{ and } v_{q_i} = \frac{\sigma_x^2\sigma^2\sigma_{e_i}^2}{(\beta_{1i}^2 + \beta_{2i}^2)\sigma_x^2\sigma^2 + (\sigma_x^2 + \sigma^2)\sigma_{e_i}^2}.$$

For the parameters β_{ki}, $k = 1, 2$ and $i = 1, \ldots, p$, we obtained the following full conditional distributions which led us to the use of the Metropolis algorithm:

$$\pi\left(\beta_{ki}/\boldsymbol{\theta}_{\{-\beta_{ki}\}}\right) \quad \propto \quad h(\beta_{ki})\Psi_{ki}\left(\beta_{ki},\sigma^2_{e_i},\xi_{ij}\right) \quad \text{with} \quad h(\beta_{ki}) = \beta_{ki}^{a_{ki}-1}\left(1-\beta_{ki}\right)^{b_{ki}-1},$$

$$\Psi_{ki}\left(\beta_{ki},\sigma^2_{e_i},\xi_{ij}\right) = exp\left(-\frac{1}{2}\sum_{j=1}^{n_i}\frac{(y_{kij}-\beta_{ki}\xi_{ij})^2}{\sigma^2_{e_i}}\right). \qquad (5)$$

As $h(\beta_{ki}) \sim Beta(a_{ki}, b_{ki})$, $k = 1, 2$ and $i = 1, \ldots, p$, we sample candidates from $h(\beta_{ki})$, and use the function (5) to obtain the probability of move given by

$$min\{\frac{\Psi_{ki}\left(\beta_{ki}^{(s)},\sigma_{e_i}^{2(s)},\xi_{ij}^{(s)}\right)}{\Psi_{ki}\left(\beta_{ki}^{(s-1)},\sigma_{e_i}^{2(s)},\xi_{ij}^{(s)}\right)},1\}, j = 1,\ldots,n_i,$$

where (s) indicates the iteration s, as can be seen in Chib and Greenberg (1995).

3 An application

In this section, we apply the multivariate null intercept errors-in-variables model developed in Section 2 to the longitudinal data presented in Hadgu and Koch (1999). The main objective of the analysis is to estimate β_{ki}, $k = 1, 2$, $i = 1, 2, 3$ and to test the hypothesis $\beta_{k1} = \beta_{k2}$, $k = 1, 2$ and $\beta_{k1} = \beta_{k3}$, $k = 1, 2$, which essentially corresponds, respectively, to a comparison of the experimental mouth rinse A (i=2) and B (i=3) with the Control mouth rinse (i=1) with respect to the efficacy in the prevention of the development of dental plaque after 3 (6) months from the beginning of the study for k=1 (k=2). Another hypothesis of interest is $\beta_{1i} = \beta_{2i}$, $i = 1, 2, 3$ which corresponds to analyzing if the mouth rinse i is long lasting, that is, if it continues to reduce the dental plaque over the entire time of the clinical trial.

Considering different initial values, we generated five parallel independent runs of the Gibbs sampler chain with size 20.000, disregarding the first 18.000. Since the successive realization of each chain are correlated, we considered a spacing of size 10, obtaining a sample of size 200 from each chain. To monitor the convergence of the chain we have used the between and within sequence information, following the approach developed in Gelman and Rubin (1992) to obtain the potential scale reduction, $\hat{R}$. In all cases, these values were less than 1.01 indicating the convergence of the chain. The choice for the values of the hyperparameters in the prior distribution for $\boldsymbol{\theta}$ was based on a careful preliminary data analysis including the values of the maximum likelihood estimators.

In Table 1, we present the posterior means and the corresponding standard deviations for the components of the vector of parameters $\boldsymbol{\theta}$; in Figure 1 we show the chain behavior of the posterior marginal densities of β_{13} and β_{23}, followed by the histogram and the autocorrelation plot (considering the remaining parameters of interest, namely β_{11},β_{12},β_{21} and β_{22}, the corresponding graphs are all similar).

Table 1: Posterior Mean and Standard Deviation

μ	β_{11}	β_{12}	β_{13}	β_{21}	β_{22}	β_{23}
2.534	0.707	0.525	0.510	0.689	0.501	0.412
(0.032)	(0.035)	(0.046)	(0.032)	(0.034)	(0.044)	(0.032)

σ_x^2	σ^2	$\sigma_{e_1}^2$	$\sigma_{e_2}^2$	σ_{e3}^2
0.103	0.010	0.280	0.443	0.232
(0.015)	(0.005)	(0.047)	(0.081)	(0.039)

The primordial interest of the experiment was to compare the efficiency of the two experimental mouth rinses A and B with the Control mouth rinse, namely, we are interested in comparing the slope parameters β_{k2} and β_{k3} with respect to β_{k1}, $k = 1, 2$. One way of testing this hypothesis (β_{k1}=β_{ki}, $k = 1, 2$,$i = 1, 2$) is to consider Monte Carlo estimates based on the generated gibbs samples of $\beta_{k1} - \beta_{k2}$ and $\beta_{k1} - \beta_{k3}$, $k = 1, 2$, and verify if the value zero belongs to the credibility region. Another question of interest is whether the mouth rinses continues to reduce the plaque index after 3 months, which means to compare the slope parameters β_{1i} and β_{2i}, $i = 1, 2, 3$. Table 2 displays the values of the mean and 95% credibility region for the questions of interest just described, as well as the comparison of the experimental mouth rinses A and B (β_{k2} and β_{h3}, $k = 1, 2$).

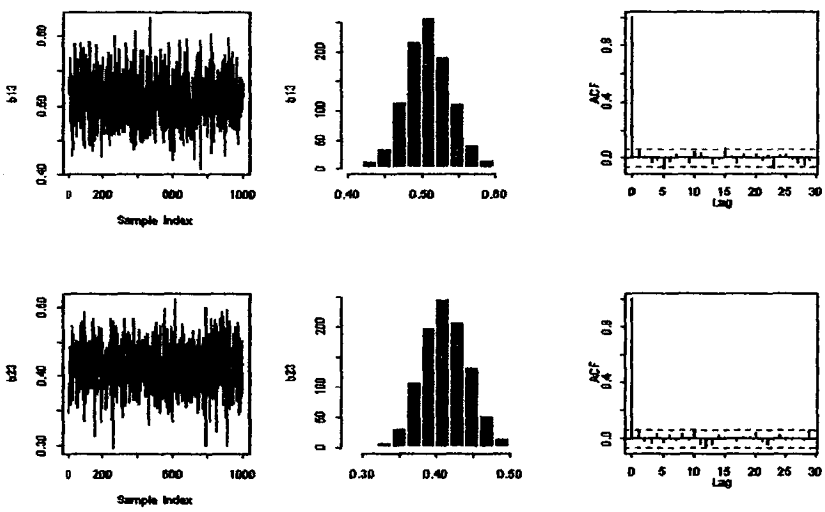

Figure 1: Sequence of the samples β_{13} and β_{23} (mouth rinse B), histogram and the autocorrelation plot.

Table 2: Posterior mean and 95% credibility region

$\beta_{11}-\beta_{12}$	$\beta_{11}-\beta_{13}$	$\beta_{12}-\beta_{13}$	$\beta_{21}-\beta_{22}$	$\beta_{21}-\beta_{23}$
0.182	0.197	0.015	0.188	0.277
(0.070,0.295)	(0.107,0.288)	(-0.096,0.124)	(0.079,0.301)	(0.185,0.372)

$\beta_{22}-\beta_{23}$	$\beta_{11}-\beta_{21}$	$\beta_{12}-\beta_{22}$	$\beta_{13}-\beta_{23}$
0.089	0.018	0.023	0.098
(-0.023,0.194)	(-0.072,0.117)	(-0.101,0.148)	(0.013,0.187)

Analyzing these results we conclude that both of the experimental mouth rinses were more efficient then the Control mouth rinse in reducing the plaque index after 3 months and after 6 months, while there are no difference between the two experimental mouth rinses. Another conclusion is that the experimental mouth rinse B continues to reduce plaque over the entire time of the clinical trial, whereas the mouth rinse A and Control mouth rinse are not long lasting. As expected these results are in accordance with those obtained in Hadgu and Koch (1999).

4 Discussion

In many problems the covariate can not be measured precisely, as is the case of the dental plaque discussed in this paper considering three kinds of mouth rinses, as well as, the experiment studied in Aoki et al. (2001) described in the Introduction, considering two types of toothbrushes. The main objective of that study was to compare the effectiveness of the experimental toothbrush to the conventional one, but in that case there was only one measurement after the use of each toothbrush. In that way, we extended the model to a multivariate null intercept errors-in-variables regression model which could be easily adjusted with the use of the Bayesian approach (see Aoki et al (2003) for the Bayesian approach of the univariate case). Also, the Bayesian approach enabled us to deal with the problem of restriction of the slope parameters in the interval $[0, 1]$ easily. On the other hand, the optimization problems with restriction on the parametric space are usually quite complicated, which would be the case if we consider the classical approach with the use of the maximum likelihood estimation. Even without considering the restriction of the parameters in the interval $[0, 1]$, if we consider the model proposed in Section 2, the maximum likelihood estimator can not be obtained explicitly which would lead us to relay on iterative numerical methods such as EM algorithm. The test of hypothesis of interest would be based on asymptotic results. Another point to consider is the dependence on the outcome measurements (dental plaque index after 3 months and after 6 months from the baseline with the use of the three mouth rinses), which was naturally incorporated with the use of the structural model.

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RESUMO

Dados longitudinais são de grande interesse em análises de ensaios clínicos. Em muitas situações práticas a covariável não pode ser observada diretamente e um modelo alternativo natural seria o modelo de regressão com erros nas variáveis. Neste artigo vamos estudar um modelo de regressão com erros nas variáveis com intercepto nulo, com uma estrutura de dependência entre as variáveis respostas dentro do mesmo grupo. O modelo é aplicado a um conjunto de dados reais apresentado em Hadgu and Koch (1999). Neste estudo, voluntários com placa dentária pré existente foram aleatorizados entre dois líquidos de bochecho experimentais (A e B) ou um líquido de bochecho controle. O índice de placa dentária foi medido em cada indivíduo no início do estudo e em dois tempos consecutivos, o que nos leva a presença de uma correlação entre as medições de placa dentária realizadas nestes dois períodos consecutivos. Nós propomos o uso de metodologias Bayesianas para analisar o modelo de regressão com erros nas variáveis multivariado com intercepto nulo utilizando os dados longitudinais. O modelo proposto acomoda as medidas correlacionadas e incorpora a restrição de que os coeficientes angulares devem estar no intervalo [0,1]. O amostrador de Gibbs é utilizado para fazer os cálculos computacionais necessários.

NOTAS DO ICMC

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