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AN ANALYSIS OF TAGUCHI'S ON-LINE QUALITY MONITORING PROCEDURE FOR ATTRIBUTES WITH DIAGNOSIS ERRORS

by

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An Analysis of Taguchi's On-line Quality Monitoring Procedure for Attributes with Diagnosis Errors

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Abstract

The on-line quality monitoring procedure for attributes proposed by Taguchi, Elsayed and Hsiang for processes in which the fraction defective shifts from 0 at some point are examined under the assumption of diagnosis errors. The renewal reward (economic) approach of Nayebpour and Woodall is used to develop a mathematical expression for the long run expected cost per item produced and assessment of the optimum inspection interval is discussed.

Key Words: Taguchi's on-line monitoring procedure; diagnosis errors

1 Introduction

In Taguchi (1981, 1984, 1985) and Taguchi, Elsayed and Hsiang (1989), an economically designed on-line process monitoring strategy for attribute quality characteristics is proposed. The setting is that of a production process that produces items independently and whose fraction defective shifts at some point from the initial value of 0 to a value π, $0 < \pi < 1$. The procedure consists of inspecting a single item after every m produced and carrying out a process adjustment routine that restores fraction defective to its initial value as soon as a defective item is found. Assessment of the inspection interval m^* that minimizes the procedure's long run expected cost per item produced is examined in two cases: case 1, in which π=1, and case 2, in which $0 < \pi < 1$.

As noted by Nayebpour and Woodall (1993), the strategies proposed by Taguchi and his coauthors are closely related to the economic design of np charts as considered by many authors such as Gibra (1978), Montgomery, Heikes and Mance (1975), and Willians, Looney and Peters (1985). Nevertheless, no explicit assumption concerning the waiting time to process shift is made. Assuming that shift occurs after a geometrically distributed number of items is produced, Nayebpour and Woodall (1993) proceed to develop precise mathematical expressions to assess m^*, both in case 1 and case 2, and compare the results achieved with those of Taguchi et al.

In this paper, a general mathematical model is constructed to move a step ahead and examine the effect of diagnosis errors in the inspection procedure, an assumption not taken into account in previous studies. The basic assumptions and cost structure follow closely those of Nayebpour and Woodall (1993). Specifically we assume that:

(1.1) Items are individually and independently produced at each time unit;

(1.2) After a geometrically distributed number of time is produced, process fraction defective shifts from 0 to π, $0 < \pi \leq 1$;

(1.3) While production is on, a single item is inspected after every m, $(m > 1)$, produced;

(1.4) Inspection is instantaneous and imperfect in the sense that with probability α, $0 < \alpha < 1$, a non-defective item is considered defective and with probability β, $0 < \beta < 1$, a defective item is considered non-defective;

(1.5) As soon as an inspected item is considered defective, process adjustment begins. Process adjustment is a two-phase operation in which phase 1 (process stop) takes l, $l > 0$, time units to complete and phase 2 (process recovery) is instantaneous. During process stop inspection is suspended and after process adjustment fraction defective shifts back to 0 and the cycle just described is repeated;

(1.6) The cost of adjustment is C_1 if shift occurs during the cycle and C_2 otherwise;

(1.7) The cost of inspecting an item produced is C_3;

(1.8) The last unit inspected as well as the l units produced during process stop are scrapped and the unit costs of producing and scrapping defective and non-defective items are C_4 and C_5, respectively;

(1.9) The cost of producing and not scrapping a defective item is C_6;

There are of course some differences between the assumptions (1.1)-(1.9) and those in force in Nayebpour and Woodall (1993) that are worth commenting. The most important concerns (1.8), the scrapping of the last $(l+1)$ units produced in a cycle. Since inspection is allowed to be imperfect, retrospectively searching for defectives among the last $(m+l)$ items produced in a cycle, which is one of the options in Nayebpour and Woodall (1993), wouldn't be effective. We could however impose the scrapping of the last $(l+m)$ items instead of just the last $(l+1)$. Mathematically, this causes no extra difficulty and, in fact, one can readily obtain the appropriate results if it is interesting to consider scrapping the last $l+k$ units produced for some fixed value of k, $k \geq 1$. Imperfect inspection and scrapping also impose differences in cost structure. Since under imperfect inspection process adjustment can be incorrectly called up, a different cost of adjustment must be considered and this is what is done in (1.6). Scrapping on the other hand generates the extra loss of discarding non-defective item and this is taken care of in (1.8).

2 The mathematical setup

Since under the assumptions (1.1)-(1.9) the successive cycle costs and lengths, $\{(K_J, L_J) : j \geq 1\}$, can be thought of as a sequence of independent and identically distributed random vectors, the renewal theorem can be invoked to show that the monitoring procedure's cost up to the n-th item produced, $\Delta(n)$, satisfies

$$lim_{n\to\infty}\frac{1}{n}E[\Delta(n)] = \frac{E(K_1)}{E(L_1)} \tag{2.1}$$

The left hand side of (2.1) defines the objetive function of interest, namely, the procedure's long run expected cost per item produced C, for which we will now develop an expression in terms of basic process parameters. To do that we will describe a production cycle by the set

$$\{W, \xi, (D_1, S_1, T_1), \ \ldots, \ (D_\xi, S_\xi, T_\xi), R\} \tag{2.2}$$

of random elements, in which:

(2.2a) W denotes the (unobservable) time to process shift;

(2.2b) ξ denotes the (observable) total number of inspections;

(2.2c) $\{D_1, \ldots, D_\xi\}$ denote the (unobservable) numbers of defective items produced between the successive inspections;

(2.2d) $\{S_1, \ldots, S_\xi\}$ denote the (unobservable) states of the inspected itens,that is

$$\begin{aligned} S_j &= 1 \text{ if the j-th inspected item is defective} \\ &= 0 \text{ otherwise;} \end{aligned}$$

(2.2e) $\{T_1, \ldots, T_\xi\}$ denote the (observable) results of the successive inspections, that is

$$\begin{aligned} T_j &= 1 \text{ if the j-th inspected item is considered defective} \\ &= 0 \text{ otherwise;} \end{aligned}$$

(2.2f) R denotes the number of defective items produced during process stop.

Notice that in case 1 ($\pi = 1$), $\{(D_1, S_1), \ldots, (D_\xi, S_\xi), R\}$ is determined by W and ξ, so that production cycle is completely described by $\{W, \xi, T_1, \ldots, T_\xi\}$. To describe the joint distribution of (2.2) we let W be geometrically distributed with parameter p, $0 < p < 1$, and $\{(D_j, S_j, T_j) : j \geq 1\}$ be conditionally independent given W such that for $d = 0, 1, \ldots, m_j$, $s = 0, 1$ and $t = 0, 1$,

$$
\begin{aligned}
P\{D_j &= d, S_j = s, T_j = t \mid W = w\} \\
&= 1_{\{0\}}(d) 1_{\{0\}}(s) \alpha^t (1-\alpha)^{(1-t)} \quad \text{if} \quad j \leq int(\tfrac{w}{m}) \\
&= B(m_j, \pi, d, s)[\pi \beta^{1-t}(1-\beta)^t]^s [(1-\pi)\alpha^t (1-\alpha)^{1-t}]^{1-s} \quad \text{if} \quad j > int(\tfrac{w}{m})
\end{aligned}
\tag{2.3}
$$

where

$$m_j = mj - 1 - w \vee m(j-1)$$

and

$$
\begin{aligned}
B(n, \pi, k, s) &= 1_{\{n\}}(k) 1_{\{1\}}(s) \quad \text{if} \quad \pi = 1 \\
&= C_n^k \pi^k (1-\pi)^{n-k} \quad \text{if} \quad 0 < \pi < 1 .
\end{aligned}
$$

Obviously,

$$\xi = inf\{j : T_j = 1\} \tag{2.4}$$

and we let R be conditionally independent of $\{(D_j, S_j, T_j) : j \geq 1\}$ given W and ξ, and such that for $r = 0, 1, \ldots, l$.

$$
\begin{aligned}
P\{R = r \mid \xi = k, W = w\} &= 1_{\{0\}}(r) \quad \text{if} \quad w \geq mk + l \\
&= B[(mk + l - w) \wedge l, \pi, r, 1] \quad \text{if} \quad w < mk + l
\end{aligned}
\tag{2.5}
$$

Since L_1 and K_1 in (2.1) have the same distribution as $m\xi + l$ and $\Gamma_1 1\{m\xi + l \le W\} + \Gamma_2 1\{m\xi \le W < m\xi + l\} + \Gamma_3 1\{W < m\xi\}$, respectively, where

$$\Gamma_1 = C_2 + \xi C_3 + (l+1)C_5,$$

$$\Gamma_2 = C_1 + \xi C_3 + RC_4 + (l-R+1)C_5$$

$$\Gamma_3 = C_1 + \xi C_3 + (S_\xi + R)C_4 + (l - S_\xi - R + 1)C_5 + Z_\xi C_6,$$

with $Z_1 = D_1$ and $Z_j = D_1 + S_1 + \ldots + D_{j-1} + S_{j-1} + D_j$ for $j > 1$, it follows from (2.3)-(2.5) that

$$E[L_1] = \frac{m(1-\gamma q^m)}{(1-(1-\alpha)q^m)(1-\gamma)} + l \tag{2.6}$$

and

$$E[K_1] = C_1 + (C_2 - C_1)\frac{\alpha q^{m+l}}{(1-(1-\alpha)q^m)} + C_3\frac{1-\gamma q^m}{(1-\gamma)(1-(1-\alpha)q^m)} +$$
$$(C_4 - C_5)\left[\frac{\pi\alpha q^m}{(1-(1-\alpha)q^m)}\left(l - \frac{q(1-q^l)}{1-q}\right) + \frac{\pi(1-q^m)}{(1-(1-\alpha)q^m)}\left(l + \frac{1-\beta}{1-\gamma}\right)\right] +$$
$$C_5(l+1) + C_6\left\{\frac{m\pi}{(1-(1-\alpha)q^m)} - \frac{\pi(1-q^m)}{(1-(1-\alpha)q^m)}\left[\frac{1}{1-q} - \left(m - \frac{\beta}{\gamma} - 1\right)\frac{\gamma}{1-\gamma}\right]\right\} \tag{2.7}$$

where $\gamma = \pi\beta + (1-\pi)(1-\alpha)$.

A sketch of the calculation leading to (2.6) and (2.7) can be found in the Appendix.

In order to compare (2.6) and (2.7) with the results of Nayebpour and Woodall (1993) when $\alpha = \beta = 0$, we must set $C_5 = 0$ and $C_4 = C_6$. Doing this, the expected cycle cost and length reduce to

$$E(K_1) = C_1 + C_3\frac{1-(1-\pi)q^m}{(1-q^m)\pi} + C_4\left[\left(\frac{m}{1-q^m} - \frac{q}{1-q}\right)\pi + m(1-\pi) + l\pi\right] \tag{2.8}$$

and

$$E(L_1) = m\frac{1-(1-\pi)q^m}{1-q^m}\pi + l. \tag{2.9}$$

Since $\frac{1-1(-\pi)q^m}{(1-q^m)\pi} = \frac{1}{1-q^m} + \frac{1-\pi}{\pi}$, (2.8) and (2.9) reduce to (4.4) and (4.3) of Nayebpour and Woodall(1993), respectively. Note that the difference between the coefficients of C_3 in (2.8) and C_i in (4.4) of Nayepour and Woodall(1993) is due to the fact that in the present setting, inspection is suspended during process stop. This causes the elimination of $int(\frac{l}{m})$ from the coefficient of C_3 in (2.7) and (2.8). There is also a match between the coefficients of C_4 in (2.9) and C_D in (4.5a) of Nayebpour and Woodall (1993), when no retrospective inspection is in force, and this completes the comparison.

Observe also that if, in the present setting, scrapping the last $(l+1)$ units produced in the cycle is replaced by 100% inspecting the last $(l+m)$ units, and C_4, C_5 and C_6 are substituted by C_d and C_D as in Nayebpour and Woodall (1993), the coefficients of these two new cost factors in the corresponding expression for (2.7) would be:

$$E(Z_\xi + S_\xi + R) - E(U_\xi) \tag{2.10}$$

and

$$E(U_\xi) \tag{2.11}$$

where $U_1 = 0$ and $U_k = D_1 + s_1 + \ldots + D_{k-1} + S_{k_1}$, $k > 1$. Carrying through the required calculation [see Appendix for details], we get:

$$E(Z_\xi + S_\xi + R) =$$

$$= \frac{m\pi}{1-(1-\alpha)q^m} - \frac{1-q^m}{1-(1-\alpha)q^m}\left\{\frac{1}{1-q} + \left[\frac{\beta}{\gamma} - (m-1)\right]\frac{\gamma}{1-\gamma}\right\}\pi$$

$$+ \frac{\pi(1-\beta)(1-q^m)}{(1-\gamma)(1-(1-\alpha)q^m)} + \frac{\alpha q^m}{1-(1-\alpha)q^m}\left[l - (1-q^l)\frac{q}{1-q}\right]\pi$$

$$+\quad \frac{1-q^m)}{1-(1-\alpha)q^m} l\pi$$

and

$$E(U_\xi) = \frac{m\gamma\pi}{1-(1-\alpha)q^m} + \frac{1-q^m}{1-(1-\alpha)q^m}\left[(m-1)\frac{\gamma^2}{1-\gamma} + \frac{\beta}{1-\gamma} - \frac{\gamma}{1-q}\right]\pi.$$

Thus, when $\alpha = \beta = 0$, (2.10) and (2.11) reduce to

$$E(Z_\xi + S_\xi + R) - E(U_\xi) = \left[\frac{m}{1-q^m} - \frac{q}{1-q}\right]\pi^2 + m\pi(1-\pi) + l\pi + (1-\pi) \tag{2.12}$$

and

$$E(U_\xi) = \left[\frac{m}{1-q^m} - \frac{q}{1-q}\right]\pi(1-\pi) + m(1-\pi)^2 - (1-\pi). \tag{2.13}$$

Comparing (2.12) and (2.13) with the coefficients of C_d and C_D in (4.2) of Nayebpour and Woodall (1993), respectively we notice that a missing factor of $(1-\pi)$ and $(\pi-1)$ in the later two. The two missing terms cancel out when $C_d = C_D$ so that when $\alpha = \beta = 0$, $E(Z_\xi + S_\xi + R)$ reduces to the coefficient of C_D in (4.5a) of Nayebpour and Woodall (1993). The difference originates since in the calculation of (4.1) in Nayebpour and Woodall (1993), the state of the inspected component is left random when in fact this is not the case. The state of the inspected components are in fact determined by ξ.

3 The optimium inspection interval

As in Nayebpour and Woodall (1993), the problem of finding an inspection interval that minimizes the procedures long run expected cost per item produced, that is, a positive integer m^* for which

$$f(m^*) = inf\{f(m) : m, 1, 2, \ldots\},$$

where $f(m)$ is defined by the ratio, $\frac{E(L_1)}{E(K_1)}$, of (2.6) to (2.7), is hard to solve analytically. The main reason is that the objective function $f(m)$ is of the form

$$\frac{k_0 + k_1 m + k_2 q^m + k_3 m q^m}{e_0 + e_1 m + e_2 q^m + e_3 m q^m},$$

with coefficients k_i and e_i, i=0,1,2, and 3, being well defined (but not very charming) fuctions of the system parameters, α, β, π and q, and cost factors, C_i, i=1,2,...,6.

The alternative route is, therefore, computer assisted simple direct search. Plots of $f(m)$ versus m as well as values of m^*, for specific choices of the system parameters and cost factors, can easily be obtained with the aid of a software package such as Mathematica or Maple. Figure 3.1, constructed with Mathematica, for example, exhibits the convex-like behavior of $f(m)$ versus m for

$$\alpha = 0.01, \beta = 0.01, \pi = 1, q = 0.999661, l = 1000 \quad (3.1)$$

and

$$C_1 = C_2 = 4.000, C_3 = 150, C_4 = C_6 = 5 \text{ and } C_5 = 5.5. \quad (3.2)$$

The values in (3.1) and (3.2) match, to some extent, the system parameters and cost factors reported by Berry(1974) and used in example 1.1 of Nayebpour and Woodall (1993) for comparison purposes. In several test plots for different sets of values, the behavior of $f(m)$ versus m exhibit the same overall pattern of Figure 3.1. The difference between them lies mainly in the extention of the flat portion of the graph around m^*. Figure 3.2 illustrates this fact when the value of π used in 3.1 shifts from 1 to 0.6. Figure 3.3 provides a sharper image of difference in flatness between the two previous plots.

Tables 3.1-3.3 exhibit the values of m^* and $f(m^*)$ for different choices of the system parameters α, β, π and q. The cost factors were kept fixed as in (3.2). The reported m^* values are the integer parts of the optimum values produced by Mathematica's Find-Minimum procedure. These values reveal that diagnosis errors can greatly influence the optimum choise of the inspection interval and some of the patterns are worth commenting. It is intuitive that an increase in β (with the other system parameters and cost factors remaining fixed) will slow down process shift detection, requiring an increase in inpection effort. This behavior is revealed in Tables 3.1-3.3, except in Table 3.3 when $q = 0.999661$. While in the former an increase only in β causes m^* to decrease and $f(m^*)$ to increase, in the later it causes both m^* and $f(m^*)$ to increase. It is also intuitive that an increase in α (with the other system parameters and cost factors remaining fixed) will speed up false alarm, requiring a reduced inspection effort. This behavior is revealed in Tables 3.1-3.3, except in Table 3.3 when $q = 0.999661$. While in the former an increase only in α causes both m^* and $f(m^*)$ to increase, in the later it causes m^* to increase but $f(m^*)$ may increase or decrease. It is also worth noticing that a decrease in q causes m^* initially to decrease and then increase.

Test cases with other values of π and q also revealed either extremely long flat portions of $f(m)$ around m^* or a monotone decreasing behavior of $f(m)$. The FindMinimum procedure of Mathematica failed to provide a solution to m^* in these cases. To illustrate this fact, note that setting $\pi = 0.4$ and keeping the other system parameters and cost factors as in (3.1) and (3.2), we get from Mathematica that

$$f(100000) = 2.42006$$
$$f(1108432) = 2.42001$$
$$f(m) = 2.4 \text{ for } m \geq 1108433$$

and the application of FindMinimum does not provide a solution to m^*. Also setting $q = 0.998$ and keeping the other system parameters and cost factors as in (3.1) and (3.2), we get from Mathematica that

$$f(100000) = 5.01624$$
$$f(327969009) = 5.00001$$
$$f(m) = 5 \text{ for } m \geq 327969010$$

and the application of FindMinimum also does not provide a solution to m^*.

Table 3.1.A - Values of m^* and $(f(m^*))$ for $\pi = 0.9999$ when $\beta = 0$, $\pi = 1$ when $\beta > 0$.

α	q	Values of β							
		0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.00	0.999990	2491	2467	2373	2259	2044	1841	1646	1456
		(0.209)	(0.210)	(0.215)	(0.221)	(0.235)	(0.252)	(0.271)	(0.295)
	0.999900	868	860	829	791	720	652	586	522
		(1.137)	(1.140)	(1.153)	(1.169)	(1.206)	(1.248)	(1.298)	(1.359)
	0.999661	595	590	571	547	501	458	416	374
		(2.669)	(2.673)	(2.687)	(2.707)	(2.750)	(2.799)	(2.858)	(2.930)
	0.999000	863	859	844	826	790	756	726	702
		(4.705)	(4.706)	(4.713)	(4.722)	(4.741)	(4.763)	(4.789)	(4.820)
0.01	0.999990	3175	3145	3025	2880	2606	2348	2101	1858
		(0.240)	(0.242)	(0.248)	(0.256)	(0.274)	(0.294)	(0.318)	(0.348)
	0.999900	1087	1077	1039	992	903	818	737	656
		(1.207)	(1.211)	(1.226)	(1.246)	(1.289)	(1.339)	(1.398)	(1.470)
	0.999661	720	714	691	663	609	557	507	456
		(2.731)	(2.735)	(2.752)	(2.774)	(2.822)	(2.878)	(2.943)	(3.023)
	0.999000	984	980	964	945	908	873	842	819
		(4.717)	(4.718)	(4.725)	(4.734)	(4.753)	(4.776)	(4.801)	(4.832)
0.05	0.999990	5060	5012	4822	4594	4161	3752	3359	2974
		(0.323)	(0.327)	(0.337)	(0.349)	(0.376)	(0.407)	(0.444)	(0.489)
	0.999900	1703	1688	1630	1559	1423	1293	1167	1042
		(1.391)	(1.396)	(1.417)	(1.445)	(1.506)	(1.575)	(1.656)	(1.753)
	0.999661	1085	1077	1045	1005	929	854	781	708
		(2.896)	(2.901)	(2.922)	(2.950)	(3.011)	(3.081)	(3.161)	(3.256)
	0.999000	1347	1342	1327	1307	1269	1235	1206	1188
		(4.747)	(4.749)	(4.756)	(4.765)	(4.785)	(4.807)	(4.832)	(4.861)
0.10	0.999990	6712	6648	6400	6100	5530	4991	4472	3962
		(0.397)	(0.400)	(0.412)	(0.427)	(0.462)	(0.502)	(0.549)	(0.606)
	0.999900	2248	2229	2155	2064	1890	1722	1558	1396
		(1.539)	(1.546)	(1.572)	(1.606)	(1.679)	(1.762)	(1.858)	(1.972)
	0.999661	1413	1404	1365	1317	1223	1131	1039	948
		(3.025)	(3.031)	(3.055)	(3.088)	(3.157)	(3.235)	(3.324)	(3.429)
	0.999000	1666	1661	1646	1628	1593	1562	1539	1529
		(4.769)	(4.771)	(4.778)	(4.787)	(4.807)	(4.828)	(4.852)	(4.879)

Table 3.1.B - Values of m^* and $(f(m^*))$ for $\pi = 0.9999$ when $\beta = 0$, $\pi = 1$ when $\beta > 0$

α	q	Values of β							
		0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.20	0.999990	9167	9083	8749	8345	7576	6846	6142	5450
		(0.499)	(0.503)	(0.519)	(0.540)	(0.585)	(0.636)	(0.697)	(0.771)
	0.999900	3063	3038	2943	2825	2598	2377	2160	1944
		(1.740)	(1.748)	(1.779)	(1.820)	(1.909)	(2.008)	(2.122)	(2.256)
	0.999661	1905	1893	1847	1789	1674	1560	1446	1330
		(3.190)	(3.197)	(3.226)	(3.262)	(3.340)	(3.427)	(3.525)	(3.639)
	0.999000	2109	2105	2092	2076	2048	2024	2010	2014
		(4.794)	(4.796)	(4.803)	(4.812)	(4.830)	(4.851)	(4.873)	(4.898)
0.30	0.999990	11098	10996	105977	10114	9192	8315	7468	6634
		(0.577)	(0.581)	(0.600)	(0.624)	(0.677)	(0.737)	(0.807)	(0.893)
	0.999900	3704	3676	3565	3429	3163	2904	2648	2392
		(1.882)	(1.891)	(1.926)	(1.971)	(2.069)	(2.179)	(2.303)	(2.448)
	0.999661	2289	2276	2225	2162	2034	1907	1778	1647
		(3.302)	(3.309)	(3.339)	(3.378)	(3.461)	(3.552)	(3.654)	(3.771)
	0.999000	2424	2420	2409	2396	2372	2355	2348	2360
		(4.809)	(4.811)	(4.817)	(4.826)	(4.844)	(4.864)	(4.885)	(4.908)
0.40	0.999990	12743	12628	12174	11625	10575	9575	8607	7654
		(0.641)	(0.646)	(0.666)	(0.694)	(0.752)	(0.820)	(0.898)	(0.992)
	0.999900	4250	4219	4097	3946	3650	3360	3073	2785
		(1.99372)	(2.003)	(2.041)	(2.090)	(2.194)	(2.310)	(2.442)	(2.595)
	0.999661	2612	2599	2545	2478	2343	2206	2067	1925
		(3.385)	(3.393)	(3.424)	(3.465)	(3.550)	(3.644)	(3.748)	(3.866)
	0.999000	2671	2667	2657	2646	2626	2613	2611	2630
		(4.819)	(4.821)	(4.827)	(4.836)	(4.853)	(4.872)	(4.892)	(4.914)
0.50	0.999990	14202	14074	13574	12967	11806	10698	9625	8567
		(0.696)	(0.701)	(0.724)	(0.753)	(0.817)	(0.890)	(0.975)	(1.077)
	0.999900	4734	4700	4568	4406	4085	3770	3457	3141
		(2.086)	(2.096)	(2.136)	(2.187)	(2.297)	(2.418)	(2.555)	(2.712)
	0.999661	2896	2882	2826	2756	2615	2472	2326	2177
		(3.452)	(3.460)	(3.492)	(3.533)	(3.620)	(3.715)	(3.820)	(3.938)
	0.999000	2873	2870	2861	2851	2834	2824	2826	2850
		(4.826)	(4.828)	(4.835)	(4.843)	(4.860)	(4.878)	(4.898)	(4.918)

Table 3.2.A - Values of m^* and $(f(m^*))$ for $\pi = 0.75$

α	q	Values of β: 0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.00	0.999990	2254 (0.223)	2236 (0.224)	2166 (0.229)	2081 (0.234)	1913 (0.247)	1747 (0.262)	1583 (0.280)	1416 (0.303)
	0.999900	828 (1.169)	822 (1.172)	798 (1.183)	768 (1.197)	710 (1.228)	652 (1.265)	595 (1.310)	537 (1.365)
	0.999661	682 (2.667)	678 (2.670)	661 (2.680)	640 (2.695)	598 (2.726)	557 (2.763)	516 (2.806)	475 (2.860)
	0.999000								
0.01	0.999990	2885 (0.257)	2863 (0.259)	2774 (0.264)	2665 (0.271)	2451 (0.287)	2240 (0.306)	2030 (0.329)	1818 (0.356)
	0.999900	1045 (1.240)	1038 (1.243)	1008 (1.255)	971 (1.272)	898 (1.309)	827 (1.352)	755 (1.403)	682 (1.467)
	0.999661	838 (2.716)	833 (2.719)	813 (2.731)	788 (2.746)	738 (2.780)	689 (2.820)	640 (2.867)	592 (2.923)
	0.999000								
0.05	0.999990	4668 (0.347)	4633 (0.348)	4492 (0.357)	4318 (0.367)	3977 (0.391)	3642 (0.418)	3309 (0.451)	2972 (0.491)
	0.999900	1679 (1.417)	1667 (1.421)	1621 (1.438)	1565 (1.461)	1453 (1.510)	1343 (1.566)	1233 (1.632)	1121 (1.711)
	0.999661	1312 (2.836)	1305 (2.839)	1277 (2.853)	1243 (2.871)	1174 (2.910)	1105 (2.955)	1037 (3.006)	968 (3.066)
	0.999000								
0.10	0.999990	6297 (0.419)	6250 (0.421)	6064 (0.431)	5834 (0.445)	5383 (0.474)	4939 (0.508)	4499 (0.548)	4055 (0.597)
	0.999900	2271 (1.551)	2256 (1.556)	2197 (1.576)	2125 (1.602)	1981 (1.659)	1838 (1.723)	1695 (1.797)	1550 (1.885)
	0.999661	1764 (2.920)	1756 (2.923)	1723 (2.938)	1682 (2.958)	1600 (2.999)	1518 (3.045)	1436 (3.097)	1354 (3.157)
	0.999000								

Table 3.2.B - Values of m^* and $(f(m^*))$ for $\pi = 0.75$

α	q	Values of β							
		0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.20	0.999990	8870	8805	8550	8236	7621	7017	6419	5819
		(0.515)	(0.518)	(0.531)	(0.548)	(0.584)	(0.625)	(0.674)	(0.732)
	0.999900	3216	3196	3120	3024	2836	2648	2460	2271
		(1.717)	(1.723)	(1.745)	(1.775)	(1.839)	(1.911)	(1.992)	(2.086)
	0.999661	2474	2464	2427	2379	2284	2189	2094	1999
		(3.013)	(3.017)	(3.033)	(3.052)	(3.095)	(3.140)	(3.191)	(3.247)
	0.999000								
0.30	0.999990	11045	10967	10657	10277	9532	8802	8082	7361
		(0.582)	(0.585)	(0.600)	(0.618)	(0.659)	(0.705)	(0.757)	(0.820)
	0.999900	4012	3989	3900	3789	3569	3350	3130	2909
		(1.822)	(1.828)	(1.852)	(1.884)	(1.951)	(2.025)	(2.108)	(2.203)
	0.999661	3046	3037	2997	2947	2848	2748	2647	2547
		(3.066)	(3.070)	(3.085)	(3.105)	(3.146)	(3.191)	(3.239)	(3.292)
	0.999000								
0.40	0.999990	13025	12935	12579	12140	11284	10447	9622	8799
		(0.632)	(0.636)	(0.651)	(0.672)	(0.715)	(0.764)	(0.819)	(0.883)
	0.999900	4727	4702	4604	4481	4236	3993	3749	3503
		(1.896)	(1.902)	(1.927)	(1.960)	(2.028)	(2.104)	(2.187)	(2.280)
	0.999661	3536	3526	3486	3436	3335	3234	3132	3030
		(3.100)	(3.104)	(3.119)	(3.138)	(3.178)	(3.221)	(3.268)	(3.317)
	0.999000								
0.50	0.999990	14890	14789	14391	13901	12944	12011	11093	10181
		(0.671)	(0.675)	(0.691)	(0.712)	(0.758)	(0.808)	(0.866)	(0.932)
	0.999900	5389	5362	5256	5124	4860	4598	4335	4069
		(1.950)	(1.957)	(1.982)	(2.015)	(2.085)	(2.160)	(2.243)	(2.334)
	0.999661	3966	3956	3486	3867	3767	3666	3564	3463
		(3.123)	(3.127)	(3.119)	(3.160)	(3.200)	(3.241)	(3.285)	(3.337)
	0.999000								

Table 3.3.A - Values of m^* and $(f(m^*))$ for $\pi = 0.50$

α	q	Values of β							
		0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.00	0.999990	2098	2085	2031	1963	1827	1688	1546	1398
		(0.235)	(0.236)	(0.240)	(0.245)	(0.257)	(0.271)	(0.288)	(0.309)
	0.999900	864	859	839	814	764	712	660	606
		(1.176)	(1.178)	(1.187)	(1.199)	(1.224)	(1.255)	(1.292)	(1.337)
	0.999661	2993	3002	3047	3119	3344	3778	4779	9088
		(2.473)	(2.473)	(2.475)	(2.478)	(2.484)	(2.489)	(2.495)	(2.499)
	0.999000								
0.01	0.999990	2705	2688	2619	2533	2359	2183	2002	1814
		(0.271)	(0.272)	(0.277)	(0.283)	(0.297)	(0.314)	(0.334)	(0.360)
	0.999900	1108	1102	1077	1045	983	919	854	787
		(1.238)	(1.240)	(1.250)	(1.263)	(1.292)	(1.326)	(1.366)	(1.416)
	0.999661	3737	3749	3805	3894	4161	4648	5695	9357
		(2.476)	(2.476)	(2.478)	(2.481)	(2.485)	(2.490)	(2.495)	(2.499)
	0.999000								
0.05	0.999990	4481	4454	4344	4208	3933	3655	3371	3078
		(0.359)	(0.361)	(0.368)	(0.377)	(0.397)	(0.420)	(0.447)	(0.481)
	0.999900	1858	1848	1810	1763	1668	1572	1473	1373
		(1.383)	(1.385)	(1.397)	(1.413)	(1.448)	(1.487)	(1.532)	(1.586)
	0.999661	5719	5736	5809	5920	6229	6735	7662	9873
		(2.480)	(2.481)	(2.482)	(2.484)	(2.488)	(2.491)	(2.495)	(2.498)
	0.999000								
0.10	0.999990	6195	6159	6015	5835	5474	5109	4739	4357
		(0.426)	(0.428)	(0.436)	(0.447)	(0.470)	(0.497)	(0.528)	(0.565)
	0.999900	2609	2597	2549	2489	2368	2246	2123	1996
		(1.480)	(1.483)	(1.496)	(1.513)	(1.549)	(1.590)	(1.636)	(1.688)
	0.999661	7197	7214	7288	7395	7684	8124	8852	10272
		(2.482)	(2.482)	(2.483)	(2.485)	(2.488)	(2.491)	(2.494)	(2.497)
	0.999000								

Table 3.3.B - Values of m^* and $(f(m^*))$ for $\pi = 0.50$

α	q	Values of β							
		0	0.01	0.05	0.1	0.2	0.3	0.4	0.5
0.20	0.999990	9095	9046	8850	8606	8118	7629	7135	6632
		(0.507)	(0.510)	(0.519)	(0.531)	(0.557)	(0.586)	(0.620)	(0.659)
	0.999900	3893	3877	3816	3740	3586	3432	3276	3118
		(1.584)	(1.587)	(1.600)	(1.617)	(1.653)	(1.692)	(1.735)	(1.784)
	0.999661	8948	8962	9024	9113	9337	9655	10126	10889
		(2.482)	(2.482)	(2.483)	(2.485)	(2.487)	(2.490)	(2.493)	(2.495)
	0.999000								
0.30	0.999990	11732	11673	11436	11141	10555	9969	9381	8787
		(0.556)	(0.558)	(0.568)	(0.581)	(0.607)	(0.637)	(0.671)	(0.709)
	0.999900	5035	5018	4948	4862	4688	4513	4337	4160
		(1.637)	(1.640)	(1.653)	(1.670)	(1.704)	(1.741)	(1.782)	(1.826)
	0.999661	10042	10054	10104	10174	10349	10584	10912	11394
		(2.481)	(2.482)	(2.483)	(2.485)	(2.486)	(2.488)	(2.491)	(2.493)
	0.999000								
0.40	0.999990	14273	14205	13932	13593	12921	12252	11585	10915
		(0.588)	(0.590)	(0.600)	(0.613)	(0.640)	(0.669)	(0.702)	(0.738)
	0.999900	6093	6074	6000	5906	5718	5530	5341	5150
		(1.668)	(1.671)	(1.683)	(1.699)	(1.732)	(1.767)	(1.805)	(1.845)
	0.999661	10835	10845	10886	10942	11080	11260	11500	11833
		(2.480)	(2.481)	(2.481)	(2.483)	(2.485)	(2.487)	(2.489)	(2.491)
	0.999000								
0.50	0.999990	16781	16705	16399	16020	15269	14526	13786	13048
		(0.609)	(0.611)	(0.621)	(0.634)	(0.661)	(0.690)	(0.721)	(0.756)
	0.999900	7088	7069	6990	6892	6695	6497	6298	6099
		(1.685)	(1.688)	(1.700)	(1.715)	(1.747)	(1.781)	(1.816)	(1.853)
	0.999661	11458	11466	11500	11546	11657	11797	11980	12224
		(2.480)	(2.479)	(2.480)	(2.481)	(2.483)	(2.485)	(2.487)	(2.490)
	0.999000								

Appendix

For the sake of completeness we shall here present a sketch of the calculation that led to (2.6) and (2.7), from the model setup in (2.3)-(2.5).

From the remark following (2.5), we have:

$$E[L_1] = mE[\xi] + l \tag{A.1}$$

and

$$\begin{aligned} E[K_1] &= C_1 + (C_2 - C_1)P(m\xi + l \le W) + C_3E(\xi) + \\ &\quad (C_4 - C_5)[E(R; m\xi \le W < m\xi + l) + E(R + S_\xi; W < m\xi)] + \\ &\quad C_5(l+1) + C_6E(Z_\xi; W < m\xi). \end{aligned} \tag{A.2}$$

Now

$$\begin{aligned} E(\xi) &= \sum_{n=0}^{\infty} P(\xi > n) \\ &= 1 + \sum_{n=1}^{\infty} E[P(\xi > n \mid W)] \\ &= 1 + \sum_{n=1}^{\infty}\sum_{j=0}^{\infty}\sum_{w=mj}^{m(j+1)-1} pq^w (1-\alpha)^{j\wedge n}\gamma^{(n-j)\wedge n} \\ &= \frac{1 - \gamma q^m}{(1-(1-\alpha)q^m)(1-\gamma)}, \end{aligned} \tag{A.3}$$

where $\gamma = \pi\beta + (1-\alpha)(1-\pi)$, since from (2.3)

$$\begin{aligned} P(\xi > n \mid W = w) &= \prod_{j=1}^{n} P(T_j = 0 \mid W = w) \\ &= (1-\alpha)^{int(w/m)\wedge n}\gamma^{n - int(w/m)\wedge n}. \end{aligned}$$

Similarly,

$$\begin{aligned} P(m\xi + l \le W) &= \sum_{k=1}^{\infty}\sum_{w=mk+l}^{\infty} pq^w (1-\alpha)^{k-1}\alpha \\ &= \frac{\alpha q^{m+l}}{1-(1-\alpha)q^m} \end{aligned} \tag{A.4}$$

since from (2.3)

$$P(\xi = k \mid W = w) = (1-\alpha)^{k-1}\alpha \quad \text{if} \quad int(w/m) = l \geq k$$

$$= (1-\alpha)^l \gamma^{k-l-1}(1-\gamma) \qquad \text{otherwise.} \tag{A.5}$$

The expected value of R on the set $A = \{m\xi \leq W < m\xi + l\}$ is obtained by noting that on A, R is, conditionally on W and ξ, binomially distributed with mean $\pi(m\xi + l - w)$. Consequently, it follows from (A.5) that

$$\begin{aligned} E[R; m\xi \leq W < m\xi + l] &= E[E(R \mid W, \xi); m\xi \leq W < m\xi + l] \\ &= \pi E[m\xi + l - W; m\xi \leq W < m\xi + l] \\ &= \pi \sum_{k=1}^{\infty} \sum_{w=mk}^{mk+l-1} (mk + l - w) pq^w (1-\alpha)^{k-1}\alpha \\ &= \frac{\pi\alpha q^m}{1-(1-\alpha)q^m}[l - \frac{q(1-q^l)}{1-q}]. \end{aligned} \tag{A.6}$$

Similarly,

$$\begin{aligned} E[R; W < m\xi] &= \pi l P(W < m\xi) \\ &= \pi l \sum_{j=0}^{\infty} \sum_{w=mj}^{m(j+1)-1} \sum_{k=j+1}^{\infty} pq^w (1-\alpha)^j \gamma^{k-j-1} 1 - \gamma \\ &= \frac{\pi l(1-q^m)}{1-(1-\alpha)q^m} \end{aligned} \tag{A.7}$$

since on the set $B = \{W < m\xi\}$ R is, conditionally on W and ξ, binomialy distributed with mean πl. From (A.6) and (A.7), one gets

$$E[R] = \frac{\alpha q^m}{1-(1-\alpha)q^m}[l - (1-q^l)\frac{q}{1-q}] + \frac{1-q^m}{1-(1-\alpha)q^m} l\pi. \tag{A.8}$$

The calculation of the expected value of S_ξ on the set B is straightforward since

$$E[S_\xi \mid W = w, \xi = k] = P(S_\xi = 1 \mid \xi = k, W = w) = \frac{(1-\beta)}{(1-\gamma)}\pi \text{ if w } < \text{ mk}$$

$$= 0 \text{ otherwise .}$$

So,

$$
\begin{aligned}
E[S_\xi] = E[S_\xi; W < m\xi] &= E[E(S_\xi \mid W, \xi); W < m\xi] \\
&= \frac{1-\beta}{1-\alpha}\pi P(W < m\xi) \\
&= \frac{(1-q^m)(1-\beta)\pi}{(1-(1-\alpha)q^m)(1-\gamma)}.
\end{aligned}
\tag{A.9}
$$

It is also not difficult to see that

$$
\begin{aligned}
E[Z_\xi \mid W = w, \xi = k] &= [m(int(w/m)+1) - w - 1]\pi + \\
&\quad [k - (int(w/m)+1)][(m-1)\pi + \frac{\beta}{\gamma}\pi]
\end{aligned}
$$

on B. From this result it follows that

$$
\begin{aligned}
E[Z_\xi] = E[Z_\xi; W &< m\xi] = E[E(Z_\xi \mid W, \xi); W < m\xi] \\
&= \sum_{j=0}^{\infty} \sum_{w=mj}^{m(j+1)-1} \sum_{k=j+1}^{\infty} \{[m(j+1) - w - 1] + [k - (j+1)][(m-1) + \\
&\quad \frac{\beta}{\gamma}]\}\pi p q^w (1-\alpha)^j \gamma^{k-j-1}(1-\gamma) \\
&= \frac{m\pi}{1-(1-\alpha)q^m} - \frac{1-q^m}{1-(1-\alpha)q^m}\left\{\frac{1}{1-q} + \right. \\
&\quad \left. [\frac{\beta}{\gamma} - (m-1)]\frac{\gamma}{1-\gamma}\right\}\pi.
\end{aligned}
\tag{A.10}
$$

Substituting the expressions (A.3), (A.4), (A.6)-(A.10) in (A.1) and (A.2) lead to (2.6) and (2.7).

Finally, letting $U_1 = 0$ and $U_k = D_1 + S_1 + \ldots + D_{k-1} + S_{k-1}$ for $k > 1$, it is not difficult to see that

$$
E[U_\xi \mid W, \xi] = [m(int(\frac{W}{m}) + 1) - W - 1]\pi
$$

$$+([-int(\frac{w}{m})-2)(m-1]\pi$$

$$+[k-int(\frac{W}{m})-1)\frac{\beta}{\gamma}]\pi.$$

Consequently,

$$\begin{aligned}
E[U_\xi] = E[U_\xi; W &< m\xi] = E[E[U_\xi \mid W, \xi]; W < m\xi] \\
&= \sum_{j=0}^{\infty} \sum_{W=mj}^{m(j+1)-1} \sum_{k=j+2}^{\infty} \Big[(m(j+1)-w-1)\pi + (k-j-2)(m-1)\pi \\
&\quad + (k-j-1)\frac{\beta}{\gamma}\pi\Big] pq^w (1-\alpha)^j \gamma^{k-j-1}(1-\gamma) \\
&= \frac{m\gamma\pi}{1-(1-\alpha)q^m} + \frac{1-q^m}{1-(1-\alpha)q^m}[(m-1)\frac{\gamma^2}{1-\gamma} + \frac{\beta}{1-\beta} - \frac{\gamma}{1-q}]\pi.
\end{aligned}$$

References

[1] Taguchi, G. (1981), *"On-line Quality Control During Production"*, Japanese Standards Association, Tokyo.

[2] Taguchi, G. (1984), *"Quality Evaluation for Quality Assurance"*, American Supplier Institute, Dearborn, MI.

[3] Taguchi, G. (1985), *"Quality Engineering in Japan"*, Communications in Statistics - Theory ans Methods, **14**, 2785–2801.

[4] Taguchi, G., Elsayed, E. A. and Hsiang, T. (1989), *"Quality Engineering in Production Systems"*, McGraw-Hill, New York, NY.

[5] Nayebpour, M. R. and Woodall, W. H (1993), *"An Analysis of Taguchi's On-line Quality Monitoring Procedure for Attributes"*, Technometrics, **35**, 53–60.

[6] Gibra, I. N. (1978), *"Economically Optimal Determination of the Parameters of np-Control Charts"*, Journal of Quality Technology, **10**, 12–19.

[7] Montgomery, D. C., Heikes, R. G. and Mance, J. F. (1975), *"Economic Design of Fraction Defective Control Charts"*, Management Science, **21**, 1272–1284.

[8] Williams, W. W., Looney, S. W. and Peters, M. H. (1985), *"Use of Curtailed Sampling Plans in the Economic Design of np-Control Charts"*, Technometrics, **27**, 57–63.

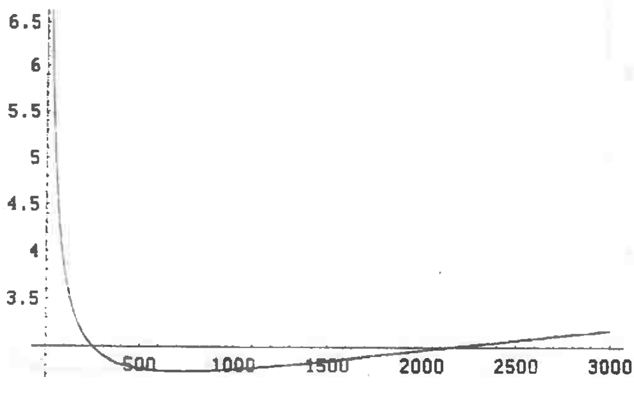

Figure 3.1: Values of f(m) versus m for (3.1) and (3.2).

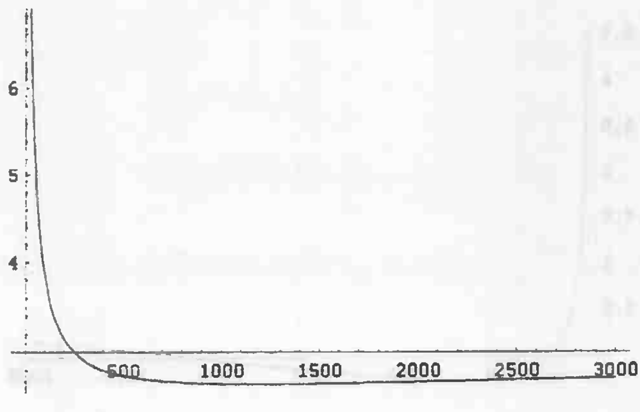

Figure 3.2: Values of f(m) versus m for $\pi = 0.6$ and other parameters remaining fixed

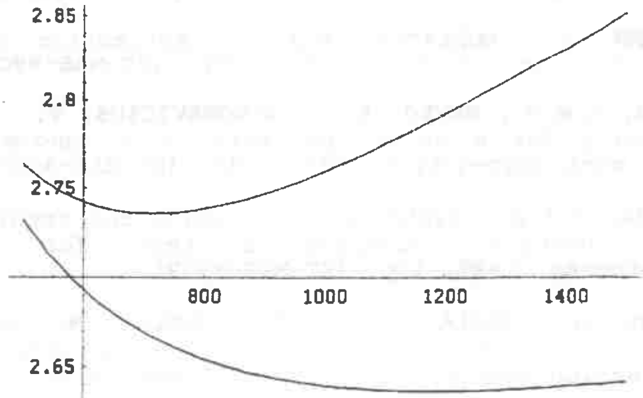

Figure 3.3: Values of f(m) versus m for $0.6 < \pi < 1$ and other parameters remaining fixed

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