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JACOBI MATRICES AND TRANSVERSALITY

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0. INTRODUCTION

In the theory of Dynamical Systems, transversality of stable and unstable manifolds of critical elements plays a central role in connection with generic theory and structural stability (see, for instance, [7] for the finite dimensional case and [3] ,[4] for infinite dimensions). In spite of this fact there is no general available method in order to check if transversality holds for a given system, although the property is true, generically, by the Kupka-Smale theorem.

Recently, D. Henry [5] has proved that transversality holds for the dynamical system in the Sobolev space H_0^1 $(0,\pi)$. Generated by the scalar parabolic equation

$$(1) \qquad u_t = u_{xx} + \lambda f(u)$$

provided that f is a smooth function such that $f(0) = 0$, $f'(0) = 1$, $tf''(t) < 0$ if $t \neq 0$ and $\lambda > 0$ with the property that $\lambda^{\frac{1}{2}}$ is not a positive integer.

His nice proof is based on some results on the asymptotic behavior of solutions of linear non autonomous equations, which have a fearly general validity, and on a property which is specific of linear scalar parabolic equations, that is, the number of zeros of the solutions does not increases with time.

We may also consider the number of zeros as a functional $N_0:M\to Z$ defined on some subset M of the space H_0^1; the properties of N_0 which are essential for proving transversality are:

(α) the eigenvectors of the linearization of (1) around equilibria are in M and $N_0(w_k) = k - 1$ if w_k is an eigenvector corresponding to the k-th eigenvalue;

(β) N_0 is not increasing along trajectories of the linear variational equation of (1) around a solution of (1);

(γ) N_0 is continuous with respect to some topology stronger than the topology of H_0^1.

Once the number of zeros is considered as a functional with the three above properties, it seems natural to conjecture that the existence of a functional like N_0 should not be a very special property of Sturm-Liouville operators but that other class of self-adjoint operators should be connected with some other functional. Here we talk about class of operators rather than about single operators because in connection with the above functional N_0 we find the whole class of Sturm-Liouville operators and this is important in proving (β) and for defining the kind of allowed non linearities, which must have the property that the linear variational equation is always of Sturm-Liouville type. This, for instance, is not true if in (1) we replace f(u) by a non linearity like $\int_0^1 K(x,y)f(u(y))dy$.

In connection with the above conjecture it is natural to ask the following questions:

(a) What abstract properties should a functional N have in order that we can associate to N a family $\mathcal{A}$ of self adjoint operators which have, with respect to N, the same properties that the Sturm-Liouville ones have with respect to N_0.

(b) Given a generic self adjoint operator $\mathcal{A}$ with a simple spectrum containing only eigenvalues, can we always embedd it in a class corresponding to some functional N? If this is possible, in how many different ways it can be done.

(c) What are the implications of (a) and (b) for non linear problems, i.e., what is the class of non linear problems which is naturally associated to a given functional N in the sense that transversality holds.

The aim of the present paper is to examine these questions in the finite dimensional setting. In section I is introduced a functional N related with the class $\mathcal{S}$ of all symmetric operators of an Euclidean space E and Theorem 1 gives a necessary and sufficient condition in order that N is not increasing along the solutions of a non autonomous system $\dot{x} = A(t)x$, $A(t) \in \mathcal{S}$, beside other important properties of N. The condition states that, with respect to an orthonormal basis of E, A(t) has a positive Jacobi matrix, that is, a matrix $a_{ij} = a_{ij}(t)$ such that $a_{ij} = 0$, $j > i+1$ and $a_{i,i+1} > 0$. In section II a class of smooth autonomous system $\dot{x} = f(x)$ is considered in E, with the property that the derivative f'(x) has a matrix representation of positive Jacobi type for all $x \in E$, and Theorem 2 proves that if e^-

and e^+ are hyperbolic equilibria of $\dot{x} = f(x)$ and there exists a solution $\phi(t)$ connecting e^- and e^+, that is $\lim \phi(t) = e^{\pm}$ as $t \to \pm\infty$ then the unstable manifold of e^- is transversal to the stable manifold of e^+; under the same hypothesis, Theorem 3 shows that the non wandering set is the set of all equilibria. The section II finishes then with the construction of a class of Morse-Smale systems. Section III shows the relationships between symmetric operators with simple eigenvalues and the functional N. It is proved that such an operator has always a matrix representation of positive Jacobi type and one can see that, in finite dimension, it is given an answer for the above question (b), that is, to any symmetric operator with simple eigenvalues, one can associated a class of non linear perturbations for which transversality holds (Theorem 5).

I. JACOBI MATRICES

As we said above, the Jacobi matrices will be one of the most important tools in the present paper. A Jacobi matrix (a_{ij}) is a $(n \times n)$ symmetric, tridiagonal matrix such that $b_i = a_{i,i+1} \neq 0$, $i = 1,2,\dots n-1$. (Positive Jacobi if in addition we have $b_i > 0$).

Following [2] pag 105, let us consider the set $\mathcal{N}_0 \subset R^n$ such that $u = (u_1, u_2, \dots, u_n) \in \mathcal{N}_0$ if and only if $u_1 \neq 0$, $u_n \neq 0$, and $u_i = 0$ $(1<i<n)$ implies $u_{i-1} \cdot u_{i+1} < 0$; consider also the function $u \in \mathcal{N}_0 \mapsto S(u)$ in which $S(u)$ is the number of sign changes in the components u_i of u.

<u>Proposition 1</u>. <u>Let J be a positive Jacobi matrix</u>. <u>Then</u>

1. <u>J has simple eigenvalues</u> $\lambda_1 > \lambda_2 > \ldots > \lambda_n$

2. <u>for</u> η_i <u>eigenvector corresponding to</u> λ_i <u>one has</u> $\eta_i \in \mathcal{N}_0$ <u>and</u> $S(\eta_i) = i-1$, $i=1,2,\ldots,n$.

3. <u>if</u> $c_h, c_{h+1}, \ldots, c_k$ <u>are real numbers which are not all equal to zero and</u> $\eta = \sum_{i=h}^{n} c_i \eta_i$ <u>then: either</u> $\eta \in \mathcal{N}_0$ <u>and</u> $h-1 \leq S(\eta) \leq k-1$, or $\eta \notin \mathcal{N}_0$ <u>and there is a neighborhood U of</u> η <u>such that</u> $h-1 \leq S(\eta') \leq k-1$ <u>for all</u> $\eta' \in U \cap \mathcal{N}_0$.

<u>Proof</u>. For $\rho > 0$ sufficiently large and I the identity matrix, all the entries of $J + \rho . I$ are positive together with all its principal minors. Since $J + \rho I$ and J have the same eigenvectors, the result follows from theorem 13, pag 105 of [2]. □

Let E be an Euclidean space, dim E = n, with inner product (,). A functional N can be introduced in the following way:

<u>Definition 1</u> - <u>Let</u> $\{e_i\}$ <u>be an orthonormal basis in E and</u> $x = \sum_{i=1}^{n} x_i e_i$ <u>be the representation of</u> $x \in E$ <u>in that basis</u>. <u>Let</u> $\mathcal{N} \subset E$ <u>be the set of vectors such that either</u> $x_i \neq 0$, $i=1,2,\ldots,n$, or $x_{i-1}x_{i+1} < 0$ <u>whenever</u> $x_i = 0$ (<u>we define</u> $x_0 = x_{n+1} = 0$). <u>For</u> $x \in \mathcal{N}$, $N(x)-1$ <u>is the number of times</u> x_i <u>changes sign when i goes from 1 to n</u>, <u>that is</u>, $N(x)-1 = S(x_1,\ldots,x_n)$.

We note that $\mathcal{N}$ is an open set and that $N : \mathcal{N} \to \{1,\ldots,n\}$ is a step function which is constant on each one of the connected components of $\mathcal{N}$. Therefore N is continuous in $\mathcal{N}$.

The next theorem says that the class of functionals satisfying the previous definition is in fact a good class from the point of view we have outlined in the introduction.

Let $\mathcal{S}$ be the set of all symmetric operators in E.

Theorem 1. Given a smooth function $t \in (a,b) \to A(t) \in \mathcal{S}$, a necessary and sufficient condition in order that:

(i) the flow $\phi(x,s,t)$ of the differential equation $\dot{x} = A(t)x$ satisfies the condition

(2) $N(\phi(x,s,t_2)) \leq N(\phi(x,s,t_1))$

whenever $t_2 \geq t_1$ and both sides of (2) are defined;

(ii) the eigenvectors of $A(t)$ belong to $\mathcal{N}$;

is that: for each t, the matrix representation in $\{e_i\}$ of the operator $A(t)$ be a positive Jacobi matrix, that is, if $a_{ij} = (e_i, A(t)e_j)$, then $a_{ij} = 0$, $j > i+1$ and $a_{i,i+1} > 0$. Moreover, if this condition is satisfied and $x \neq 0$, then the instants t such that $\phi(x,s,t) \notin \mathcal{N}$ are isolated.

Proof. Let $\bar{x} \in E$ be a vector such that $x_{i-1} = x_i = x_{i+1} = 0$ if $1<i<n$ or such that $x_1 = x_2 = 0$ if $i = 1$, $x_{n-1} = x_n = 0$ if $i = n$; assume all other x_j are non zero. Let $\tilde{x}$ be $\tilde{x}=\bar{x}+\alpha e_{i-1} + \beta e_{i+1}$ if $1<i<n$, $\tilde{x} = \bar{x} + \beta e_2$ if $i = 1$ and $\tilde{x} = \bar{x}+\alpha e_{n-1}$ if $i = n$. The definition of N implies that if α, β are positive, then $\varepsilon > 0$ sufficiently small and ξ such that $(\xi, e_i) = 0$ imply

$$N(\tilde{x} + \varepsilon(e_i+\xi)) \quad N(\tilde{x} - \varepsilon(e_i+\xi)).$$

It follows that a necessary condition in order that N is not increasing along the trajectory of $\dot{x} = A(t)x$ which goes through $\tilde{x}$ at time t is that the vector field $A(t)\tilde{x}$ satisfies the condition $(A(t)\tilde{x}, e_i) \geq 0$. By using the definition of $\tilde{x}$ we have the condition

$$(3) \quad (A(t)\bar{x}, e_i) + \alpha(A(t)e_{i-1}, e_i) + \beta(A(t)e_{i+1}, e_i) \geq 0,$$

$$\alpha,\ \beta > 0,\ \forall\ \bar{x}.$$

This inequality implies $(A(t)\bar{x}, e_i) = 0\ \forall\ \bar{x}$ because, otherwise, by replacing $\bar{x}$ with $\gamma\bar{x}$, $\gamma \neq 0$, in (3) we see that there is a γ such that (3) is violated. Now one obtains

$$(4) \quad \alpha(A(t)e_{i-1}, e_i) + \beta(A(t)e_{i+1}, e_i) \geq 0 \qquad \forall\ \alpha,\ \beta > 0.$$

which clearly implies $(A(t)e_{i-1}, e_i) \geq 0$, $(A(t)e_{i+1}, e_i) \geq 0$, that is, $a_{i,i+1} \geq 0$. On the other hand, we have seen that

$$(A(t)\bar{x}, e_i) = \sum_{j \neq i, i+1, i-1} x_j (A(t)e_j, e_i) = 0 \qquad \forall\ x_j$$

that is, $a_{ij} = (A(t)e_j, e_i) = 0$ for $j > i+1$. In particular this shows that the matrix of $A(t)$ is tridiagonal. When one of the elements $a_{i,i+1}$ is zero, the eigenvalue problem for (a_{ij}) splits in the eigenvalue problems for two submatrices and therefore there are eigenvectors of (a_{ij}) which have two zero consecutive components and it is obvious that these eigenvectors are not in $\mathcal{N}$, then $a_{i,i+1} > 0$. This concludes necessity.

To prove sufficiency let $Z(t) = \phi(x,s,t)$ and assume there exists $\bar{t} \in [t_1, t_2]$ such that $\bar{Z} = Z(\bar{t}) \notin \mathcal{N}$; then $\bar{Z} \neq 0$ because otherwise $Z(t) = 0\ \forall\ t \in [t_1, t_2]$ in contradiction with the hypothesis $Z(t_1) \in \mathcal{N}$. Since $\bar{Z} \neq 0$, and if $\bar{Z}_i = 0$, then one of the three following situations holds:

$$\bar{z}_1=\bar{z}_2=\ldots=\bar{z}_i=\ldots=\bar{z}_k=0,\ \bar{z}_{k+1}\neq 0 \quad \text{for some } k<n;$$

$$\bar{z}_{h-1}\neq 0,\ \bar{z}_h=\ldots=\bar{z}_i=\ldots=\bar{z}_n=0 \quad \text{for some } h>1;$$

$$\bar{z}_{h-1}\neq 0,\ \bar{z}_h=\ldots=\bar{z}_i=\ldots=\bar{z}_k=0,\ \bar{z}_{k+1}=0 \quad \text{for some } 1<h\leq i\leq k<n.$$

We will discuss only the last case under the assumption $\bar{z}_{h-1}<0$, $\bar{z}_{k+1}>0$. The other cases can be discussed in a similar way. Then, if we set $b_i = a_{i,i+1}$, $a_i = a_{ii}$ we have for $t=\bar{t}$

$$\dot{z}_h = b_{h-1}z_{h-1} + a_h z_h + b_h z_{h+1} = b_{h-1}\bar{z}_{h-1} < 0;$$

$$(5) \qquad \dot{z}_i = b_{i-1}z_{i-1} + a_i z_i + b_i z_{i+1} = 0,\ h<i<k;$$

$$\dot{z}_k = b_{k-1}z_{k-1} + a_k z_k + b_k z_{k+1} = b_k\bar{z}_{k+1} > 0.$$

From (5), if $h<i-1$ and $k>i+1$, it follows for $t=\bar{t}$:

$$\ddot{z}_{h+1} = b_h\dot{z}_h + a_{h+1}\dot{z}_{h+1} + b_{h+1}\dot{z}_{h+2} + \dot{b}_h z_h + \dot{a}_{h+1}z_{h+1} + \dot{b}_{h+1}z_{h+2} = b_h b_{h-1}\bar{z}_{h-1} < 0$$

$$\ddot{z}_{h-1} = b_{k-2}\dot{z}_{k-2} + a_{k-1}\dot{z}_{k-1} + b_{k-1}\dot{z}_k + \dot{b}_{k-2}z_{k-2} + \dot{a}_{k-1}z_{k-1} + \dot{b}_{k-1}z_k = b_{k-1}b_k\bar{z}_{k+1} > 0.$$

If $h<i-2$ and $k>i+2$ we obtain in a similar fashion that for $t=\bar{t}$, $\dddot{z}_{h+2}<0$ and $\dddot{z}_{k-2}>0$.

If $k-h+1$ is even this process ends with the computation of the derivatives of order $1/2\,(k-h+1)$ and we have $z_{h+i-1}(\bar{t}+\varepsilon) = \frac{1}{i!}\,z^{(i)}(\bar{t})\,\varepsilon^i + o(\varepsilon^i) < 0$ for $\varepsilon>0$ small and $i=1,\ldots,\frac{k-h+1}{2}$ and also $z_{k-i+1}(\bar{t}+\varepsilon) = \frac{1}{i!}\,z^{(i)}(\bar{t})\,\varepsilon^i + o(\varepsilon^i) > 0$ for $\varepsilon>0$ small and $i=1,\ldots,\frac{k-h+1}{2}$. Therefore, the number of changing of sign between h and k is well defined for $\varepsilon\neq 0$ and is exactly 1 for $\varepsilon>0$ while is clearly larger than 1 for $\varepsilon<0$. The discussion of the case in which $k-h+1$ is odd is similar and yields to the same conclusion. This proves that if $(a_{ij}) = (A(t)e_i, e_j)$ is a positive Jacobi matrix then we have (i). The fact that if (a_{ij}) is positive Jacobi implies (ii) follows from Proposition 1. The proof of Theorem 1 is complete. □

Remark that in proving Theorem 1 we have also proved the following result that we need for later reference:

Proposition 2: Assume there is an orthornomal basis $\{e_i\}$ in E such that $a_{ij}(t) = (A(t)e_i, e_j)$ is a positive Jacobi matrix for each $t \in (a,b)$ and let N be as in Definition 1 with respect to $\{e_i\}$, then

(i) If $x \neq 0$ the instants $\bar{t}$ such that $\phi(x,s,\bar{t}) \notin \mathcal{N}$ are isolated.

(ii) If $\phi(x,s,\bar{t}) \notin \mathcal{N}$ and N_m, N_M, are the minimum and the maximum of N in a small neighborhood of $\phi(x,s,\bar{t})$, then for $\varepsilon > 0$ small $N_m = N(\phi(x,s,\bar{t}+\varepsilon)) < N(\phi(x,s,\bar{t}-\varepsilon)) = N_M$.

An obvious extension of Theorem 1 is the following:

Proposition 3: If $f: \Omega \subset E \times R \to E$, Ω open, is a function such that there exists a smooth function $A(.): \Omega \to \mathcal{S}$ such that $f(x,t) = A(x,t)x$, $(x,t) \in \Omega$, and the matrix representation of $A(x,t)$ with respect to some fixed orthornomal basis $\{e_i\}$ is positive Jacobi for each $(x,t) \in \Omega$, then if N is the functional associated to $\{e_i\}$ and $\phi(x,s,t)$ is the solution operator of $\dot{x} = f(x,t)$ we have that $\phi(x,s,t) \in \mathcal{N}$ except possibly for t in a discrete set and

$$N(\phi(x,s,t_2)) \leq N(\phi(x,s,t_1))$$

whenever $t_2 \geq t_1$ and both sides are defined.

II. A CLASS OF MORSE-SMALE SYSTEMS

The results in the previous section allow us to formulate a sufficient condition in order that for a special class of systems the stable and unstable manifolds of hyperbolic equilibria intersect transversally. In fact:

Theorem 2. If $f:\Omega \subset E \to E$, Ω open, is a smooth function such that for all $x \in \Omega$ the matrix representation of the derivative $f'(x)$ with respect to some fixed orthonormal basis $\{e_i\}$ in E if of positive Jacobi type, then if e^-, $e^+ \in \Omega$ are hyperbolic equilibria of

(5) $\dot{x} = f(x)$

and $\phi(t)$ is a solution of (5) such that $\lim_{t\to\pm\infty} \phi(t) = e^{\pm}$, then $W^u(e^-) \pitchfork W^s(e^+)$.

For the proof of this theorem we need some notation and two lemmas. Given an integer $0 \le h \le n$ we let K_h be the set of $x \in E$ such that one of the following is true:

a) $x = 0$

b) $x \in \mathcal{N}$ and $N(x) \le h$

c) $x \notin \mathcal{N}$ and there is a neighborhood U of x such that $N(x') \le h$ for $x' \in U \cap \mathcal{N}$.

Similarly we define K^h to be the set of $x \in E$ such that a) or b') or c') holds where b') and c') are like b) and c) with $\le h$ replaced by $> h$. The sets K_h, K^h so defined are cones. Moreover $K_h \setminus \{0\}$, $K^h \setminus \{0\}$ are open sets and $K_h \cap K^h = \{0\}$, $\overline{K_h \cup K^h} = E$.

Lemma 1. Let $a < 0 < b$ and $(a,b) \ni t \to A(t) \in \mathcal{S}$ be a smooth function such that for each $t \in (a,b)$ the matrix representation of A(t) with respect to some fixed orthonormal basis $\{e_i\}$ of

E is of positive Jacobi type and let N, K_h, K^h be the corresponding function and cones. Then:

(i) if $\Sigma_0 \subset K_h$ is a linear subspace and Σ_t is the image of Σ_0 at the time t under the equation

$$\dot{y} = A(t)y \quad , \tag{6}$$

then $\dim \Sigma_t = \dim \Sigma_0$ and for $t \geq 0$ it results $\Sigma_t \dot{\subset} K_h$

(ii) if $\Sigma^0 \subset K^h$ is a linear subspace and Σ^t is the image of Σ^0 under (6), then $\dim \Sigma^t = \dim \Sigma^0$ and for $t \leq 0$ it results $\Sigma^t \subset K^h$.

Proof. We only prove (i); the proof for (ii) is similar. The linearity of (6) implies $\dim \Sigma_t = \dim \Sigma_0$. To prove that $t \geq 0$ implies $\Sigma_t \subset K_h$, we note that from the assumptions and Theorem 1 it follows that, if $y \in \Sigma_t$ and $t>0$, then either $y \notin \mathcal{N}$ or $N(y) \leq h$. In the later case $y \in K_h$. If $y \notin \mathcal{N}$, then Proposition 2 yields $N(\Psi(y,t,t-\varepsilon)) = N_M$ where we have let $\Psi(y,s,t)$ be the solution map of (6). Define $y_0 = \Psi(y,t,0)$ if $\Psi(y,t,0) \in \mathcal{N}$ or $y_0 = \Psi(y,t,\sigma)$ for some small $\sigma \neq 0$ if $\Psi(y,t,0) \notin \mathcal{N}$ and let $t' = t-\varepsilon$ in the first case, $t' = t-\sigma-\varepsilon$ in the second case. By definition of K_h and Proposition 2 we have $y_0 \in \mathcal{N}$ and $N(y_0) \leq h$. Therefore, by Theorem 1 it follows

$$N_M = N(\Psi(y,t,t-\varepsilon)) = N(\Psi(y_0,0,t')) \leq h,$$

that shows $y \in K_h$. □

Lemma 2. Let $(-\infty, +\infty) \ni t \mapsto A(t) \in \mathcal{S}$ be a smooth function as in Lemma 1 and moreover assume there exist A^-, A^+ such that $\lim A(t) = A^{\pm}$ as $t \to \pm\infty$. Then if $y \neq 0$ there exists $\lim \Psi(y,s,t)/\|\Psi(y,s,t)\|$ as $t \to \pm\infty$ and it is equal to one of the eigenvectors of $A^{\pm}$.

Proof. Let $\rho:(-\infty,+\infty)\to(-1,+1)$ be a smooth function such that $\frac{d\rho}{dt} > 0$, $t \in (-\infty,+\infty)$, $\lim \rho(t) = \pm 1$ as $t\to\pm\infty$. Then equation (6) is equivalent to the system

$$(7)\begin{cases} \dot{s} = \sigma(s) \\ \dot{y} = B(s)y \end{cases} ,$$

where $\sigma(s) = \frac{d\rho}{dt}(\rho^{-1}(s))$, $B(s) = A(\rho^{-1}(s))$. By setting $\sigma(\pm1) = 0$, $B(\pm1) = A^{\pm}$, we can assume the right hand sides of (7) are defined also for $s = \pm1$. This fact and the linearity of $(7)_2$ imply that (7) induces a smooth vector field X on the fibration $F = [-1\ +1] \times (S^{n-1}/r)$, S^{n-1} being the (n-1)-dimensional unit sphere in E and (S^{n-1}/r) the corresponding projective space. Since $\sigma(\pm1) = 0$ the fibers $\{\pm1\} \times (S^{n-1}/r)$ are invariant under X and no other fiber of F is invariant because $\sigma(s) \neq 0$ for $s \in (-1,+1)$. The flow on the fibers $\{\pm1\}\times(S^{n-1}/r)$ is easy to describe. We consider the case $\{+1\}\times(S^{n-1}/r)$; the properties of the flow on $\{-1\}\times(S^{n-1}/r)$ is analogous. There are exactly n equilibria defined by $(1, W_i)$, $1\leq i\leq n$, where the W_i is a unitary eigenvector of A^+ which corresponds to the i^{th} eigenvalue, and all orbits in $\{1\} \times (S^{n-1}/r)$ connect two of these equilibria. It follows that any compact invariant set in $\{1\} \times (S^{n-1}/r)$ which does not reduce to a single equilibrium contains at least two equilibria. Now let γ be an orbit of X which is not contained in $\{1\} \times (S^{n-1}/r)$ and let $(s(t),u(t))$, $s(t) \in (0,1)$, $u(t) \in S^{n-1}$ be a representation of γ. Since F is compact, the w-limit set $w(\gamma)$ of γ is non empty and by the above discussion is contained in $\{1\} \times (S^{n-1}/r)$. If $w(\gamma)$ contains two equilibria defined by $(1, w_h)$, $(1,w_k)$ with $h \neq k$, then one can construct a sequence $\{t_j\}$, $t_j \to \infty$ as $j \to \infty$, such that $\lim u(t_{2j}) = w_h$ and $\lim u(t_{2j+1}) = w_k$ as $j \to \infty$. This and the continuity of N imply that for j sufficiently large it results $h = N(u(t_{2j})) \neq N(u(t_{2j+1})) = k$ in contradiction with the fact that N is not increasing along solutions of (6). Then $w(\gamma)$ reduces to a single equilibrium. From this and an analogous argument concerning the α-limit set of γ, the lemma follows. □

Proof of Theorem 2. From lemma 2 it follows that $\dot\phi(t)/||\dot\phi(t)||$ approaches an eigenvector w_h^- of $f'(e^-)$ as $t \to -\infty$ and an eigenvector w_k^+ of $f'(e^+)$ as $t \to +\infty$. Clearly w_h^- corresponds to a positive eigenvalue of $f'(e^-)$ and w_k^+ to a negative eigenvalue of $f'(e^+)$. This and the fact that $k=N(w_k^+)\leq N(w_h^-)=h$ imply that if $m^- \geq h$ is the dimension of $W^u(e^-)$, then the dimension m^+ of $W^u(e^+)$ satisfies $m^+ \leq m^+-1$. It follows that $n-m^-+1$ eigenvectors $w^+_{m^-},\dots,w^+_n$ of $f'(e^+)$ are in $T_{e^+} W^s(e^+)$ and Proposition 1 (3.) implies that $\Sigma = \text{span}\,\{w^+_{m^-+1},\dots,w^+_n\}$ is contained in K^{m^-}. Since $K^{m^-}\setminus\{0\}$ is open and $W^s(e^+)$ is a smooth manifold, for any fixed $t \in (-\infty, +\infty)$, there is a $t^0 \geq t$ such that $T_{\phi(t_0)}W^s(e^+)$ contains an $n-m^-$ dimensional linear subspace Σ^0 which is contained in K^{m^-}. Let Σ^t be the image of Σ^0 under the linear equation

$$(8) \qquad \dot y = f'(\phi(t))y$$

Then $\Sigma^t \subset T_{\phi(t)}W^s(e^+)$ and Lemma 1 implies $\dim \Sigma^t = n-m^-$ and $\Sigma^t \subset K^{m^-}$ for $t \leq t^0$. A completely similar argument shows that $T_{\phi(t)}W^u(e^-)$ contains an m^--dimensional linear subspace Σ_t which is contained in K_{m^-}. Since $\dim \Sigma_t + \dim \Sigma^t = n$ and

$K_{m^-} \cap K^{m^-} = \{0\}$, we have $E = \Sigma_t \oplus \Sigma^t$ and therefore the theorem is is proved. □

Proposition 5. Let f satisfy the condition in Theorem 2 and let $e \in \Omega$ be a hyperbolic equilibrium of (5). Then the stable and unstable manifolds $W^u(e)$ and $W^s(e)$ of e are imbedded submanifolds of E. In fact, if $P: E \to T_e W^u(e)$ is the orthogonal projection onto the tangent space to $W^u(e)$ at e, then the restriction $P|W^u(e)$ is a diffeomorphism of $W^u(e)$ onto an open subset V of $T_e W^u(e)$, and a similar statement holds for $W^s(e)$.

Proof. By proposition 1 (3.), the tangent space $T_e W^u(e)$ is contained in K_{m^-}. Take an $x \in W^u(e)$; by definition of $W^u(e)$ we have $\phi(x,t) \to e$ as $t \to -\infty$. This and the smoothness of $W^u(e)$, together with the openess of $K_{m^-} \setminus \{0\}$, imply that there is a $t_0 < 0$ such that $T_{\phi(x,t_0)} W^u(e) \subset K_{m^-}$. By Lemma 1 it follows that $T_x W(e) \subset K_{m^-}$. Since x is generic we see that if Σ is an $(n - m^-)$-dimensional linear subspace in K^{m^-}, then $\Sigma \cap T_x W^u(e) = \{0\}$, $\forall x \in W^u(e)$. This equation implies that the restriction to $T_x W^u(e)$ of the orthogonal projection $P: E \to \Sigma^\perp$, $\Sigma^\perp$ the orthogonal complement of Σ, is one to one. Therefore from the implicit function theorem it follows that the restriction $P|W^u(e)$ is a diffeomorphism of $W^u(e)$ onto an open subset V of $\Sigma^\perp$. This proves that $W^u(e)$ is a graph over V and therefore an imbedded submanifold of E. In particular we may assume $\Sigma = T_e W^s(e)$ and therefore $\Sigma^\perp = T_e W^u(e)$. □

The condition imposed by Theorem 2 on the derivative $f'(x)$ implies that if $f(x) = \sum_{i=1}^{n} f_i(x_1, x_2, \ldots, x_n) e_i$, where $x = \sum_{i=1}^{n} x_i e_i$, then f_i depends at most on x_{i-1}, x_i, x_{i+1}.

A very simple example of a function f such that $f'(x)$ has a matrix representation of positive Jacoby type in some $\{e_i\}$ is

$$(9)\quad \begin{cases} f_1(x_1,x_2) = a_1(x_1)+b_1x_2 \\ f_i(x_{i-1},x_i,x_{i+1}) = a_i(x_i)+b_{i-1}x_{i-1}+b_ix_{i+1}, \quad 1<i<n. \\ f_n(x_{n-1},x_n) = a_n(x_n)+b_{n-1}x_{n-1} \end{cases}$$

where the $a_i(x_i)$ are arbitrary smooth functions of one real variable and the b_i are positive constants.

Suppose that each function $a_i(x_i)$ satisfies the condition $(a_i(x_i)/x_i)\to-\infty$ as $x_i\to\pm\infty$. Since

$$(x,f(x)) = x_1.a_1(x_1)+x_2.a_2(x_2)+\ldots+x_na_n(x_n)+$$
$$+2b_1x_1x_2+2b_2x_2x_3+\ldots+2b_{n-1}x_{n-1}x_n$$

we see that in the sphere $|x|=\rho>0$ for ρ big enough, one has $(x,f(x))<0$. This means that on the sphere $|x|=\rho$, ρ sufficiently big the vector field on R^n defined by (9) points inward. Using the Theorem 110 of [6] we conclude that the time one map ϕ associated to that vector field has a compact attractor $A(\phi)$. It is clear that if the functions $a_i(x_i)$ are polynomials of odd (≥ 3) degree and negative coeficients in the highest degree term we also obtain a compact attractor.

It is also possible to construct examples where the b_i do not reduce to constants; for instance the condition on $f'(x)$ is still satisfied when we set in (9)

$$b_i = \sum_0^{\infty} \beta_j^i \; x_i^{2j} \; x_{i+1}^{2j} \;,$$

that is,

$$(10)\begin{cases} f_1 = a_1(x_1) + \sum_{j=0}^{\infty} \beta_j^1\, x_1^{2j}\, x_2^{2j+1} \\ f_i = a_i(x_i) + \sum_{j=0}^{\infty} x_i^{2j}(\beta_j^{i-1} x_{i-1}^{2j+1} + \beta_j^i x_{i+1}^{2j+1}), \\ f_n = a_n(x_n) + \sum_{j=0}^{\infty} \beta_j^n\, x_{n-1}^{2j+1} x_n^{2j} \end{cases}$$

with $\beta_0^i > 0$, $\beta_j^i \geq 0$ and the series converge.

The condition that the matrix representation of $f'(x)$ in $\{e_i\}$ is positive Jacobi not only put severe restriction on the functions f_i but also implies that the non wandering set is very simple. In fact:

Theorem 3. If $f: \Omega \subset E \to E$, $\Omega = \overset{o}{\Omega}$, is a smooth function such that for all $x \in \Omega$ the matrix representation of $f'(x)$ with respect to some orthonormal basis $\{e_i\}$ is of positive Jacobi type then the non wandering set of the dynamical system defined by the equation $\dot{x} = f(x)$ coincides with the set of equilibria.

Proof: Let $\bar{x}$ be a non wandering point which is not an equilibrium and let $\phi(x,t) = \sum_{i=1}^{n} \phi_i(x,t)e_i$ be the solution map. We can assume $\dot{\phi}(\bar{x},0) \in \mathcal{N}$ because by Proposition 2 if $\dot{\phi}(\bar{x},0) \notin \mathcal{N}$ we have $\dot{\phi}(\bar{x},\varepsilon) \in \mathcal{N}$ for $\varepsilon > 0$ small and therefore since the nonwandering set is invariant we replace $\bar{x}$ with $\bar{y} = \phi(\bar{x},\varepsilon)$. Since N is a continuous step function there is a flow box B containing $\bar{x}$ such that $x \in B$ implies $N(\dot{\phi}(x,0)) = N(\dot{\phi}(\bar{x},0))$. It is easy to see that $\dot{\phi}(\bar{x},0) \in \mathcal{N}$ implies $\dot{\phi}_1(\bar{x},0) \neq 0$ because N is not defined on vectors with the first or the last components equal to zero. We can choose B so that $\dot{\phi}_1(x,0) \neq 0$ for $x \in B$. Let π_1 be the hyperplane $\pi_1 = \{x \mid (x,e_1) = \bar{x}_1\}$ and let $K = \pi_1 \cap B$. Since $\dot{\phi}_1(\bar{x},0) = 0$, $\dot{\phi}_1(\bar{x},0)$ is not parallel to π_1, therefore by reducing the cross section of B if necessary we can ensure that the orbit through any point $x \in B$ crosses K transversally. Let To be an upper bound for the time that solutions spend in B and choose $T > To$, then $\bar{x}$ nonwandering implies there is an $x \in B$ such that $\phi(x,T) \in B$ and $T > To$ implies that $\phi(x,t)$ must leave the box B through one of its basis at some time and enter again in B from the other basis before T. It follows that we can assume that x and $\phi(x,T)$ belong to K and therefore that $x_1 = \phi_1(x,T) = \bar{x}_1$. This and the fact that $\dot{\phi}_1(x,0) . \dot{\phi}_1(x,T) > 0$ because $\dot{\phi}_1(y,0) \neq 0$ for $y \in B$ imply $\phi_1(x,T)$ has at least two extreme points in $(0\ T)$; therefore there is $\bar{t} \in (0,T)$ such that $\dot{\phi}_1(x,\bar{t}) = 0$. It follows that $\dot{\phi}(x,\bar{t}) \notin \mathcal{N}$ but then Proposition 2 implies $N(\dot{\phi}(x,\bar{t}+\varepsilon)) < N(\dot{\phi}(x,t-\varepsilon))$ for $\varepsilon > 0$ small. From this and the fact that N is not increasing along solutions of the linear variational equation $\dot{x} = f'(\phi(x,t))x$ it follows $N(\dot{\phi}(x,t)) < N(\dot{\phi}(x,0))$ which is a contradiction.□

Remark: In view of Theorems 2 and 3 above, if for all $x \in \Omega$ the matrix representation of $f'(x)$ with respect to some basis $\{e_i\}$ is of positive Jacobi type, the dynamical system defined by $\dot{x} = f(x)$ is a Morse-Smale system if and only if there is only a finite number of hyperbolic equilibria (this is the case for the systems with a compact attractor and hyperbolic equilibria).

As a special case of system (7) one consider the polynomial system in R^3:

$$(11)\begin{cases} \dot{x}_1 = b_1(x_2-x_1) + x_1(1-x_1^2) \\ \dot{x}_2 = b_2(x_3-x_2) + b_1(x_1-x_2) + x_2(1-x_2^2) \\ \dot{x}_3 = b_2(x_2-x_3) + x_3(1-x_3^2) \end{cases}$$

where

$$f'(x) = \begin{bmatrix} -b_1+1-3x_1^2 & b_1 & 0 \\ b_1 & -b_2-b_1+1-3x_2^2 & b_2 \\ 0 & b_2 & -b_2+1-3x_3^2 \end{bmatrix}$$

If we try to compute the equilibria we easily obtain a polynomial in x_1 of degree 27, that means, we always have at least one equilibrium and at most 27. If we make $b_1 = b_2 = 0$ one obtains exactly 27 hyperbolic equilibria; this implies that for $b_1, b_2 > 0$ and sufficiently small we are able to apply Theorems 2,3 and conclude that for $b_1, b_2 > 0$ small enough, system (11) is Morse-Smale with 27 hyperbolic equilibria and has a compact attractor.

For the general case of system (9) one can give, explicitily, conditions on the smooth functions $a_i(x_i)$ensuring that all equilibria are hyperbolic. In fact, we have

Proposition 6. Let $\bar{x} = (\bar{x}_1, \bar{x}_2, \ldots, \bar{x}_n)$ be an equilibrium point of system (9). Then, if: either $a_1'(\bar{x}_1) > 0$, $a_2'(\bar{x}_2) \leq 0$, $a_3'(\bar{x}_3) \geq 0$, ..., etc., or $a_1'(\bar{x}_1) < 0$, $a_2'(\bar{x}_2) \geq 0$, $a_3'(\bar{x}_3) \leq 0$, ..., etc., the point $\bar{x}$ is a hyperbolic equilibrium.

Proof. The linear part of (9) at $\bar{x}$ is the positive Jacobi matrix with determinant Δ_n:

$$\Delta_n = \begin{bmatrix} a_1'(\bar{x}_1) & b_1 & 0 & \ldots\ldots & 0 \\ b_1 & a_2'(\bar{x}_2) & b_2 & \ldots\ldots & 0 \\ 0 & b_2 & a_3'(\bar{x}_3) & \ldots\ldots & 0 \\ 0 & \ldots\ldots & b_{n-2} & a_{n-1}'(\bar{x}_{n-1}) & b_{n-1} \\ 0 & \ldots\ldots & 0 & b_{n-1} & a_n'(\bar{x}_n) \end{bmatrix}$$

From this we obtain:

$$\Delta_2 = a_1'(\bar{x}_1)a_2'(\bar{x}_2) - b_1^2$$
$$\Delta_3 = a_3'(\bar{x}_3)\Delta_2 - b_2^2 a_1'(\bar{x}_1)$$
$$\Delta_4 = a_4'(\bar{x}_4)\Delta_3 - b_3^2\,\Delta_2$$

and, in general, for $n \geq 5$ we have:

$$\Delta_n = a_n'(\bar{x}_n)\Delta_{n-1} - b_{n-1}^2\,\Delta_{n-2}.$$

A simple analysis shows, by induction, that under the conditions stated as hypothesis, the determinant Δ_n is never zero. □

Remark. When the system (9) is a polynomial system in which $a_i(x_i)$ has degree k_i, it is easy to see that, looking for equilibria, we have to find the real roots of a polynomial in x_1 of degree $k = k_1 x k_2 x \ldots x k_n$. The number of real roots is exactly the number of equilibria. If k is odd there is at least one equilibrium; the case $a_1(x_1) = 1$, $a_2(x_2) = a_3(x_3) = \ldots = a_n(x_n) = 0$ has no equilibrium points.

Theorem 4. Let F be the set of functions $f: E \to E$ which are bounded together with their first derivatives and have the property that for all $x \in E$ the matrix representation of $f'(x)$ with respect to a given basis $\{e_i\}$ is of positive Jacobi type and let F be endowed with the C^1 - topology. Then there is a residual set $G \subset F$ such that $f \in G$ has only hyperbolic equilibria.

Corollary. For $f \in G$ the dynamical system defined by the equation $\dot{x} = f(x)$ is a Kupka-Smale (Morse-Smale if there is only a finite number of equilibria) system.

Proof of Theorem 4

We quote from Eells [1] pag 781 the following:

Lemma 3. Let X,Y be C^r-manifolds ($r\geq 1$) modeled on Banach spaces. Consider a manifold $\mathcal{A} \subset C^r(X,Y)$ such that the evaluation map ev: $\mathcal{A} \times X \to Y$, $ev(\phi,x) = \phi(x)$, is C^r. Let K be a subset of X, B a C^r submanifold of Y and $\mathcal{A}(K,B) = \{\phi\in\mathcal{A}:\phi/K$ is transversal to B$\}$. Then if K is compact, $\mathcal{A}(K,B)$ is open in $\mathcal{A}$. Let dim X = n < ∞, codim (Y,B) = q < ∞, and $r>\max(n-q,0)$. If the evaluation map is transversal to B on K then $\mathcal{A}(K,B)$ is residual.

We will use now lemma 3; let $BC^1(R,R)$ be the Banach manifold of all C^1 real functions defined on R which are bounded with bounded derivative. Then the manifold $[BC^1(R,R)]^n$ is diffeomorphic to the manifold $\mathcal{A}\subset C^1(R^n,R^n\times R^n)$ defined in the following way: given a fixed f ε F then $\hat{g} \in \mathcal{A}$ if and only if $g = (g_1(x_1),\dots,g_n(x_n)) \in [BC^1(R,R)]^n$ and $\hat{g}(x_1\dots x_n) = ((x_1\dots x_n),(f_1(x_1\dots x_n) + g_1(x_1),\dots,f_n(x_1\dots x_n) + g_n(x_n)))$, $x = \sum_{i=1}^n x_ie_i$ and $f(x) = \sum_{i=1}^n f_i(x_1\dots x_n)e_i$. We make $X = R^n$, $Y = R^n \times R^n$, $B = R^n \times \{0\}$ (zero section) and apply lemma 3 for r = 1. The evaluation map

$$ev: \mathcal{A}\times R^n \to R^n \times R^n \quad \text{is differentiable and}$$

$$Dev(\hat{g},(x_1\dots x_n))\ (\hat{h},(y_1\dots y_n)) =$$
$$= ((y_1\dots y_n),(h_i(x_i)+g_i'(x_i)y_i+f_i'(x_1\dots x_n)(y_1\dots y_n)))$$

It is easy to see that Dev is surjective then transversal to the zero section $B = R^n \times \{0\}$ and by lemma 3 the set $\mathcal{A}(R^n,B) = \{\phi\in\mathcal{A}|\phi\pitchfork B\}$ is residual (then dense) in the manifold $\mathcal{A}$. But if B_n is the closed ball of E with radius n call $G_n = \{f \in F: \bar{f} \pitchfork_{B_n} E\times\{0\}\}$. G_n is open in F by continuity and compactness of B_n. Also $F \cap G_n$ is dense in F because $\mathcal{A}(R^n,B)$

is dense in F; finally, since $F \cap G_n$ is open and dense in F and

$$G = \bigcap_{n=1}^{\infty} F \cap G_n$$

it follows that G is residual in F and G has only hyperbolic equilibria. □

III. SYMMETRIC OPERATORS WITH SIMPLE SPECTRUM

Lemma 4. If $\mathscr{A}: E \to E$ is a symmetric operator with simple eigenvalues, then there exists an orthonormal basis $\{e_i\}$ in E such that the corresponding matrix representation is a positive Jacobi matrix.

Proof. Let $\{f_i\}$ any orthonormal basis of E and A the corresponding symmetrix matrix of $\mathscr{A}$. We look for an orthogonal matrix H such that $H^T A H$ is of positive Jacobi type. In order to show this we let S be a generic symmetric matrix, D a diagonal matrix and consider the system:

$$(12) \quad \begin{cases} H^T A H = D + S \\ H^T H = I \end{cases}$$

in the unknowns H and D. If Λ is the diagonal matrix of the eigenvalues of A and V is the orthogonal matrix the i^{th} column of which are the components of the unitary eigenvector of A corresponding to λ_i we see that $H = V$, $D = \Lambda$ and $S = 0$ is a solution of (12). Therefore, if we show that the linearization of (12) around $(V, \Lambda, 0)$ can be solved uniquely, the implicit function theorem will imply that (12) has a solution $(H(S), D(S))$ for S near zero. That linearization is

$$(13)\quad \begin{cases} h^T A\, V + V^T A\, h = d + s \\ h^T V + V^T h = 0 \end{cases}$$

where (h,d,s) are the variations of (H,D,S). Let $\hat{h} = h^T V$ then the second equation (13) becomes $\hat{h} + \hat{h}^T = 0$, that is, $\hat{h}$ is a skew symmetric matrix. By using the fact that $AV = V\Lambda$, the first of the equations (13) becomes

$$(14)\quad \hat{h}\Lambda - \Lambda\hat{h} = d + s.$$

The generic element of the matrix on the left hand side of (14) is given by $(\lambda_j - \lambda_i)\hat{h}_{ij}$. Since $\lambda_i \neq \lambda_j$ (for $i \neq j$) one obtains $\hat{h}_{ij} = s_{ij}/(\lambda_j - \lambda_i)$ for $i \neq j$ and $d_i = -s_{ii}$. By the implicit function theorem we obtain the functions H(S) and D(S). Restricting these functions to the set of positive Jacobi matrices Lemma 4 is proved. □

From Lemma 4 and Theorem 1 it follows:

Theorem 5. Let $A: E \to E$ be a symmetric operator with simple eigenvalues $\lambda_1 > \lambda_2 > \ldots > \lambda_n$ and corresponding eigenvectors $v_1, v_2, \ldots, v_n$. There exists an orthonormal basis $\{e_i\}$ such that N is the functional associated to $\{e_i\}$ in the sense of definition 1, then:

(i) $v_1, v_2, \ldots, v_n \in \mathcal{N}$.

(ii) $N(v_j) = j$, $j = 1, 2, \ldots, n$.

(iii) if $x \neq 0$ then $e^{At}.x \in \mathcal{N}$ for $t \in R \setminus K$ where K is a discrete set and $N(e^{At_2}x) \leq N(e^{At_1}x)$ whenever $t_2 \geq t_1$, $t_1, t_2 \in R \setminus K$.

(iv) the basis $\{e_i\}$ can be chosen in infinitely many different ways.

This theorem and the results in II allow the identification of classes of non linear perturbations for the linear equation

(13) $\dot{x} = A x$

for which transversality automatically holds.

Theorem 6. Let $A:E\to E$ be a symmetric operator with simple eigenvalues and let $\{e_i\}$ be one of the basis in Theorem 5. Let $g_i:R^n\to R$ be smooth functions with the property that $\frac{\partial g_i}{\partial x_j}$ is a matrix of positive Jacobi type and $g:E\to E$ be the function $g = \sum_{i=1}^{n} g_i . e_i$. Then

(i) if $e^{\pm}$ are hyperbolic equilibria of

(16) $\dot{x} = A x + g(x)$

and $\phi(t)$ is a solution of (14) such that $\lim_{t\to\pm\infty}\phi(t) = e^{\pm}$ then $W^u(e^-) \pitchfork W^s(e^+)$.

(ii) the nonwandering set of the dynamical system defined by (16) is equal to the set of all equilibrium points.

In spite of its simplicity this theorem is quite surprising because it says that at least in finite dimension we can associate to any selfadjoint operator with simple spectrum a class of non-linear perturbations for which transversality holds. It is reasonable to believe that some extension of this result to infinite dimension is possible. This conjecture is partially confirmed by Theorem 7.13 in Stone [8] which states that if A is a self adjoint operator acting in a separable Hilbert Space H and A has a "simple" spectrum then there is a basis $\{e_i\}$ in H such that with respect to $\{e_i\}$, A is represented by an infinite Jacobi matrix.

REFERENCES

[1] J.EELLS JR., "A Setting for Global Analysis", Bull of AMS, Vol. 72, nº 5, Sept. 1966, pag 751-807.

[2] F.R.GANTMACHER, The Theory of Matrices. Chelsea Publ. Co., N.York, 1959, vol. 2.

[3] J.K.HALE, "Infinite Dimensional Dynamical Systems", LCDS Report 81-22 (Nov. 1981), Division of Applied Math., Brown Un.

[4] J.K.HALE, L.T.MAGALHÃES and W.M.OLIVA, An Introduction to Infinite Dynamical Systems - Geometric Theory, Springer Appl. Math. Sciences, 47(1984).

[5] D.B.HENRY, "Some Infinite-Dimensional Morse-Smale Systems defined by Parabolic Partial Differential Equations", J. Diff.Equations, Vol. 59, nº 2, Sept. 1985, pag 165-205.

[6] W.M.OLIVA, "Stability of Morse-Smale Maps", Rel.Tec.MAP - 8301 (Jan.1983), Inst. Mat. Est., Univ. of São Paulo.

[7] J.PALIS & W.DE MELLO, Geometric Theory of Dynamical Systems - An Introduction, Springer-Verlag (1982).

[8] M.H.STONE, Linear Transformations in Hilbert Space and Their Applications to Analysis, AMS Coll.Publ., Vol. XV, 1932.

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