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J. L. Arraut
C. A. Maquera

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STRUCTURAL STABILITY OF SINGULAR ACTIONS OF $\mathbb{R}^n$ HAVING A FIRST INTEGRAL

J. L. ARRAUT AND C. A. MAQUERA

RESUMO. Começamos caracterizando a família de n-variedades analíticas reais fechadas e orientáveis que admitem uma ação analítica real de $\mathbb{R}^n$ com pelo menos uma órbita difeomorfa a $T^{n-1} \times \mathbb{R}$. Depois, para cada n-variedade fechada e orientável N definimos um subconjunto $\mathscr{C}_n$ de ações singulares em $A^\omega(\mathbb{R}^n, N)$ tal que cada ação $\varphi \in \mathscr{C}_n$ tem uma integral primeira não constante assim como algumas propriedades genéricas e logo provamos que φ é C^1 estruturalmente estável. Uma ação de $\mathbb{R}^n$ é chamada singular se cada órbita tem dimensão menor do que n.

ABSTRACT. We begin by characterizing the family of real analytic closed orientable n-manifolds, $n \geq 2$, that admit an analytic action of $\mathbb{R}^n$ with at least one orbit diffeomorphic to $T^{n-1} \times \mathbb{R}$. Next, for each closed orientable n-manifold N we define a subset $\mathscr{C}_n$ of singular actions in $A^\omega(\mathbb{R}^n, N)$ such that each action $\varphi \in \mathscr{C}_n$ has a non-constant first integral as well as some generic properties and then we prove that φ is C^1 structurally stable. An action of $\mathbb{R}^n$ is called singular if every orbit has dimension less than n.

1. INTRODUCTION

Let M be a closed connected real analytic m-manifold and $A^r(\mathbb{R}^n, M)$ the set of C^r, $1 \leq r \leq \omega$, actions of $\mathbb{R}^n$ on M whose infinitesimal generators are also of class C^r. We consider in $A^r(\mathbb{R}^n, M)$ the C^1-topology induced by the C^1-distance between infinitesimal generators. An action of $\mathbb{R}^n$ on a manifold is called singular if every orbit has dimension less than n.

In this paper we characterize the family of analytic closed orientable n-manifolds that admit an analytic action of $\mathbb{R}^n$ with at least one orbit diffeomorphic to $T^{n-1} \times \mathbb{R}$, see Theorem 2.11. This family consists of T^n and $\mathcal{H}_n$, the family of manifolds obtained by glueing two copies of $T^{n-2} \times D^2$ by an orientation preserving diffeomorphism of $T^{n-1} = \partial(T^{n-2} \times D^2)$. An action $\varphi \in A^r(\mathbb{R}^n, M)$ is said to be of *type* j, $0 \leq j \leq n$, if the union of the j-dimensional orbits is an open dense subset of M. Denote by $A^r_j(\mathbb{R}^n, M)$ the set of actions of type j. For each analytic closed connected orientable n-manifold N we define a subset $\mathscr{C}_n \subset A^\omega_{n-1}(\mathbb{R}^n, N)$ such that each action $\varphi \in \mathscr{C}_n$ has a non-constant first integral. Our main result is to prove that each action in $\mathscr{C}_n$ is C^1 structurally stable, see Theorem 2.15. It is proved in Lemma 2.8 that if $\varphi \in \mathscr{C}_n$, then there exists $\psi_\varphi \in A^\omega(\mathbb{R}^{n-1}, N)$ with the same orbits than φ. C. Perelló proved in [8] that Morse-Smale C^ω vector fields on an analytic orientable closed surface are C^1-structurally stable. It follows from his theorem that if $\varphi \in \mathscr{C}_2$, then ψ_φ is not C^1-structurally stable.

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The fact that there are C^1 structurally stable singular actions is not new. In fact, Kato and Morimoto proved in [5] the following theorem: Let $X \in \mathfrak{X}^r(N)$, $r \geq 1$ and $n \geq 3$, be an Anosov vector field. Then, $Y \in \mathfrak{X}^r(N)$ satisfies $[X, Y] = 0$ if and only if $Y = cX$, with $c \in \mathbb{R}$. As a consequence of this result, we can define an open set $\mathscr{B} \subset A^r(\mathbb{R}^n, N)$ of singular actions such that each element in $\mathscr{B}$ is C^1 structurally stable.

The fact that there are C^1 structurally stable actions with non-constant first integrals is also not new. Write $T^3 = S^1 \times T^2$ and let $\mathcal{F}$ be the foliation of T^3 with leaves $\{\theta\} \times T^2$, $\theta \in S^1$. Saldanha in [9], defined a subset of locally free actions $\mathscr{C} \subset A^\infty(\mathbb{R}^2, T^3)$ such that each $\varphi \in \mathscr{C}$ has $\mathcal{F}$ as underlying foliation and is C^1 structurally stable.

What is new in our result is that each $\varphi \in \mathscr{C}_n$, besides being C^1 structurally stable, is a singular action and at the same time has a non-constant first integral. Finally, we show that this phenomenum is typical of analytic actions. In [1], to be published, we consider actions $\varphi \in A_n^r(\mathbb{R}^n, N)$. We defined the concept of transversally hyperbolic compact orbit and proved for $n = 2$ and $n = 3$ that if $\varphi \in A_n^r(\mathbb{R}^n, N)$, $r \geq 1$, and every compact orbit is transversally hyperbolic, then φ is structurally stable. The problem of characterizing the structurally stable actions in $A^\omega(\mathbb{R}^n, N)$ is still an open and interesting problem.

2. Preliminaries and statement of the main results

M will denote a closed connected and orientable real analytic m-manifold. A C^r-action of Lie group G on M is a C^r-map $\varphi : G \times M \to M$, $1 \leq r \leq \omega$, such that $\varphi(e, p) = p$ and $\varphi(gh, p) = \varphi(g, \varphi(h, p))$, for each $g, h \in G$ and $p \in M$, where e is the identity in G. $\mathcal{O}_p = \{\varphi(g, p);\ g \in G\}$ is called the *φ-orbit* of p. $G_p = \{g \in G;\ \varphi(g, p) = p\}$ is called the *isotropy group* of p. For each $p \in M$ the map $g \mapsto \varphi(g, p)$ induces an injective immersion of the homogeneous space G/G_p in M with image $\mathcal{O}_p$. When $G = \mathbb{R}^n$, the possible φ-orbits are injective immersions of $T^k \times \mathbb{R}^\ell$, $0 \leq k + \ell \leq n$, where $T^k = S^1 \times \cdots \times S^1$, k times.

For each $0 \leq i \leq n-1$ let $\mathrm{Sing}_i(\varphi) = \{p \in M; \dim \mathcal{O}_p = i\}$ and $\mathrm{Sing}(\varphi) = \cup_{i=0}^{n-1} \mathrm{Sing}_i(\varphi)$. If $p \in \mathrm{Sing}(\varphi)$, $\mathcal{O}_p$ is called a *singular orbit* and when $p \in \mathrm{Sing}_0(\varphi)$, $\mathcal{O}_p$ is also called a *point orbit* and p a *fixed point* by φ. We also write $p \in \mathrm{Sing}_i^c(\varphi)$, $i = 1, \ldots, n-1$, when $\mathcal{O}_p$ is a T^i-orbit. If $\mathrm{Sing}(\varphi) = M$, we call φ a *singular action.*

For each $w \in \mathbb{R}^n \setminus \{0\}$ φ induces a C^r-flow $(\varphi_w^t)_{t \in \mathbb{R}}$ given by $\varphi_w^t(p) = \varphi(tw, p)$ and its corresponding C^{r-1}-vector field X_w defined by $X_w(p) = D_1\varphi(0, p) \cdot w$. If $\{w_1, \ldots, w_n\}$ is a base of $\mathbb{R}^n$ the associated vector fields $X_{w_1}, \ldots, X_{w_n}$ determine completely the action φ and are called a set of *infinitesimal generators* of φ. Note that $[X_{w_i}, X_{w_j}] = 0$ for any two of them.

Denote by $A^r(\mathbb{R}^n, M)$ the set of C^r-actions, $r \geq 1$, of $\mathbb{R}^n$ on M such that their canonical infinitesimal generators are also C^r vector fields. Given two actions $\{\varphi; X_1, \ldots, X_n\}$ and $\{\psi; Y_1, \ldots, Y_n\}$ define $d_k(\varphi, \psi) = \max_{1 \leq i \leq n} \|X_i - Y_i\|_k$. With this distance $A^r(\mathbb{R}^n, M)$ is a metric space and the corresponding topology is the C^k-topology. We say that φ is an action of *type j* and write $\varphi \in A_j^r(\mathbb{R}^n, M)$ if the union of the j-dimensional orbits is an open dense subset of M. Note that for analytic actions $A^\omega(\mathbb{R}^n, M) = A_0^\omega(\mathbb{R}^n, M) \cup A_1^\omega(\mathbb{R}^n, M) \cup \cdots \cup A_n^\omega(\mathbb{R}^n, M)$, see Lemma 2.6. This is not the case for non-analytic actions since it is possible to have $\varphi \in A^\infty(\mathbb{R}^n, M)$ which does not belong to any $A_j^\infty(\mathbb{R}^n, M)$, see [4].

Let $\varphi \in A^r(\mathbb{R}^k, M)$ and $X_1, \ldots, X_k$ a set of infinitesimal generators of φ. We shall denote by $\mathscr{G}(\varphi)$ the commutative Lie subalgebra of $\mathfrak{X}^r(M)$ generated by those vector fields.

Definition 2.1. An action $\psi \in A^r(\mathbb{R}^k, M)$ is said to *immerse (immerse properly)* in $A^r(\mathbb{R}^n, M)$, $k \leq n$, if there exist $\varphi \in A^r(\mathbb{R}^n, M)$ such that $\mathscr{G}(\psi)$ is a subalgebra (proper subalgebra) of $\mathscr{G}(\varphi)$. We shall write $\psi \mapsto \varphi$ to indicate that φ realizes the immersion of ψ. If $\psi \in A^r_k(\mathbb{R}^k, M)$ immerses properly in $A^r_n(\mathbb{R}^n, M)$ one shall say that ψ *embedds* in $A^r_n(\mathbb{R}^n, M)$.

Let $\psi \in A^r(\mathbb{R}^k, M)$ and $X_1, \ldots, X_k$ a set of infinitesimal generators. ψ can allways be immersed in $A^r(\mathbb{R}^n, M)$. In fact, put $X_{k+i} = \sum_{j=1}^k a_{ji} X_j$, $1 \leq i \leq n-k$, where each $a_{ji} : M \to \mathbb{R}$ is a first integral, perhaps constant, of each X_j, $j = 1, \ldots, k$. The action φ generated by $X_1, \ldots, X_k, X_{k+1}, \ldots, X_n$ realizes the immersion of ψ. Note that the immersion is proper if at least one a_{ji} is non-constant. On the other hand $\psi \in A^r_k(\mathbb{R}^k, M)$, in general, does not embedds in $A^r_n(\mathbb{R}^n, M)$.

The notions of topological equivalence and C^k structural stability that we use here for actions are the standard one's. The following two lemmas extend to actions of $\mathbb{R}^n$ classical lemmas in the theory of flows, see [1]. Let $D^m_\varepsilon = \{(x_1, \ldots, x_m) \in \mathbb{R}^m;\ |x_i| < \varepsilon\}$, $\varepsilon > 0$, and $\frac{\partial}{\partial x_i} = (0, \ldots, 0, 1, 0, \ldots, 0)$ the constant vector field.

Lemma 2.2 (k-flow box). *Let $\varphi \in A^r(\mathbb{R}^k, M)$ with infinitesimal generators $X_1, \ldots, X_k$, and $\mathcal{O}_p$ a k-dimensional orbit. There exists a C^r- diffeomorphism $h : V_p \to D^m_\varepsilon$, where V_p is a neighborhood of p, such that $h_* X_i = \frac{\partial}{\partial x_i}$ in D^m_ε, for each $i = 1, \ldots, k$.*

Remark 2.3. *Note that the diffeomorphism $h = h(\varphi) : V_p \to D^m_\varepsilon$ depends continuously on φ in the following sense: given $\eta > 0$, there exists $\delta > 0$ such that if $\widetilde{\varphi} \in A^r(\mathbb{R}^k, M)$ is δ C^1-close to φ, then $h(\widetilde{\varphi}) : \widetilde{V}_p \to D^m_\varepsilon$ is η C^1-close to $h(\varphi)$ in $V_p \cap \widetilde{V}_p$.*

A pair (V_p, h) as in Lemma 2.2 will be called a *k-flow box* at p. By using Lemma 2.2 one can also prove:

Lemma 2.4 (Long k-flow box). *Let $\varphi \in A^r(\mathbb{R}^k, M)$, $\mathcal{O}_p$ a k-dimensional orbit of φ and $\gamma \subset \mathcal{O}_p$ homeomorphic to $[0,1]$. Then, there exists k-flow box (V_γ, h), where V_γ is a neighborhood of γ.*

Infinitesimal generators adapted to a singular orbit. Let $\mathcal{O}_p$ be a singular k-dimensional orbit of $\varphi \in A^r(\mathbb{R}^n, M)$ and G_p its isotropy group. $\mathcal{O}_p$ is a $T^\ell \times \mathbb{R}^{k-\ell}$-orbit with $0 \leq \ell \leq k < n$. Call G^0_p the connected component of G_p that contains the origin and let H be a k-dimensional subspace of $\mathbb{R}^n$ such that $\mathbb{R}^n = H \oplus G^0_p$. Let $\{w_1, \ldots, w_n\}$ be a base of $\mathbb{R}^n$ such that $\{w_1, \ldots, w_k\}$ is a base of H, $\{w_{k+1}, \ldots, w_n\}$ is a base of G^0_p and $\{w_1, \ldots, w_\ell\}$ generate the subgroup $G_p \cap H$. $\{X_i = X_{w_i}; i = 1, \ldots, n\}$ is a set of infinitesimal generators of φ. Note that $X_{k+1}(q) = \cdots = X_n(q) = 0$ for every $q \in \mathcal{O}_p$. We shall say that $X_1, \ldots, X_n$ is a set of *infinitesimal generators adapted to* $\mathcal{O}_p$.

Applying Lemma 2.2 to the action φ restricted to H, we obtain a chart $h : V_p \to D^m_\varepsilon$ of M such that if $(\theta, x) \in D^m_\varepsilon = D^k_\varepsilon \times D^{m-k}_\varepsilon$, then the vector fields X_i in this chart can be writen

$$(2.1)\qquad \begin{aligned} X_i(\theta, x) &= \frac{\partial}{\partial \theta_i}, \quad i = 1, \ldots, k \\ X_{k+i}(\theta, x) &= \sum_{j=1}^{k} a_{ji}(x) \frac{\partial}{\partial \theta_j} + \sum_{j=k+1}^{m} a_{ji}(x) \frac{\partial}{\partial x_j}, \quad i = 1, \ldots, n-k \end{aligned}$$

A chart like the one above is called *adapted to $\mathcal{O}_p$ at p*. The vector fields

$$\widehat{X}_i = \sum_{j=k+1}^{m} a_{ji}(x)\frac{\partial}{\partial x_j}, \quad i = 1, \dots, n-k,$$

define a local action φ_T of $\mathbb{R}^{n-k}$ on D_ε^{m-k} having $0 \in D_\varepsilon^{m-k}$ as a fixed point.

When p is a fixed point of φ, then a chart adapted to $\mathcal{O}_p$ at p will be any chart of M which contains p. In this case $\widehat{X}_i = X_i$, $i = 1, \dots, n$.

Remark 2.5. *Note that $\{X_1, \dots X_k, \widehat{X}_1, \dots, \widehat{X}_{n-k}\}$ define a local $\mathbb{R}^n$-action $\widehat{\varphi}$ on D_ε^m and that $\mathcal{O}_{(\theta,x)}(\widehat{\varphi}) = \mathcal{O}_{(\theta,x)}(h \circ \varphi \circ h^{-1})$ for each $(\theta, x) \in D_\varepsilon^m$.*

Lemma 2.6. *In a closed connected real analytic manifold M the following decomposition holds:*

$$A^\omega(\mathbb{R}^n, M) = A_0^\omega(\mathbb{R}^n, M) \cup A_1^\omega(\mathbb{R}^n, M) \cup \dots \cup A_n^\omega(\mathbb{R}^n, M).$$

Moreover, if $\varphi \in A_n^\omega(\mathbb{R}^n, N)$, then there is only a finite number of n-dimensional orbits, all of them homeomorphic.

Proof. Let $X_1, \dots, X_n$ be a set of infinitesimal generators of φ and $\mathcal{O}_p$ an orbit of maximal dimension k, $0 < k \leq n$. Fix a finite number of charts (U_i, x_i), $0 \leq i \leq \ell$, of M such that $M = \cup_{i=1}^{\ell} U_i$ and assume that $X_1, \dots, X_k$ are linearly independent in U_1. This last property propagates along the charts to an open and dense subset of M using the fact that a real analytic function defined on an open set $U \subset \mathbb{R}^m$ is zero either on U or on the complement of an open dense subset of U, see [7]. Let $\mathcal{O}_1, \mathcal{O}_2$ be two different n-dimensional orbits of $\varphi \in A^\omega(\mathbb{R}^n, N)$ and G_1, G_2 their respective isotropy groups. If $u \in G_p$, then $X_u^1|_{\mathcal{O}_1} = id$. Since X_u^1 is an analytic diffeomorphism, it follows that $X_u^1 = id$ on N. Thus, $u \in G_2$ i.e., $G_1 \subset G_2$. By the same argument $G_2 \subset G_1$ and therefore $G_1 = G_2$. Finally, using charts adapted to the singular orbits one shows that every $\varphi \in A_n^\omega(\mathbb{R}^n, N)$ has a finite number of n-dimensional orbits. □

Let $\mathcal{O}_p$ be a T^{n-2}-orbit of $\psi \in A^r(\mathbb{R}^{n-1}, N)$. It can be verified that $\widehat{X}_{n-1}$ has the following two properties:

(1) Although $\widehat{X}_{n-1}$ depends on the chart (V_p, h) which in turn depends on H, the fact that $0 \in D_\varepsilon^2$ be a center (saddle, node, focus) of $D\widehat{X}_{n-1}(0)$ does not depend on the chart.

(2) If $q \in \mathcal{O}_p$ and $q \neq p$, there exists a chart (V_p, h) adapted to $\mathcal{O}_p$ such that $q \in V_p$.

It follows from the two properties above that the following concept is well defined.

Definition 2.7. Let $\mathcal{O}_p$ be a T^{n-2}-orbit of $\psi \in A^r(\mathbb{R}^{n-1}, N)$. $\mathcal{O}_p$ is said to be *transversally simple* if there exists a chart adapted to $\mathcal{O}_p$ at p such that $0 \in D_\varepsilon^2$ is a simple singularity of $\widehat{X}_{n-1}$. When 0 is a center (saddle, node, focus) of $D\widehat{X}_{n-1}(0)$ we will say that $\mathcal{O}_p$ is *transversally a center (saddle, node, focus).*

Separatrices of a T^{n-2}-orbit that is transversally a saddle. Let $\mathcal{O}_p$ be a T^{n-2}-orbit of $\psi \in A^r(\mathbb{R}^{n-1}, N)$ that is transversally a saddle, $X_1, \dots, X_{n-2}, X_{n-1}$ infinitesimal generators adapted to $\mathcal{O}_p$ and $h : V_p \longrightarrow D_\varepsilon^{n-2} \times D_\varepsilon^2$ a chart adapted to $\mathcal{O}_p$ at p. Let ξ^s (ξ^u) be the stable (unstable) submanifold of $\widehat{X}_{n-1}$ at $0 \in D_\varepsilon^2$, $\Sigma = h^{-1}(D_\varepsilon^2)$ and $\eta^s = h^{-1}(\xi^s)$ ($\eta^s = h^{-1}(\xi^s)$). We know that $X_i^1(p) = p$, $i = 1, \dots, n-2$. Let $\pi_i : V_p \to \Sigma$ be the projection along the orbits of the action generated by $X_1, \dots, X_{i-1}, X_{i+1} \dots, X_{n-2}$ and $\omega_i : U \subset \Sigma \to \Sigma$ be the local diffeomorphism defined by $\omega_i = \pi_i \circ X_i^1$. Since $[X_i, \widehat{X}_{n-1}] = 0$, $i = 1, \dots, n-2$, it follows that ω_i takes orbits of $\widehat{X}_{n-1}$ in U into orbits of $\widehat{X}_{n-1}$ in Σ and in particular $\omega_i((\eta^s \cup \eta^u) \cap U) \subset \eta^s \cup \eta^u$. Therefore, the four connected components of $\eta^s \cup \eta^u \setminus \{p\}$ give rise to at most four φ-orbits and each of them is a $T^{n-2} \times \mathbb{R}$-orbit. This orbits will be called *separatrices* of $\mathcal{O}_p$.

Lemma 2.8. *Let $\mathcal{O}_p$ be a transversally simple T^{n-2}-orbit of $\varphi \in A^r_{n-1}(\mathbb{R}^n, N)$. There exists a neighborhood V of $\mathcal{O}_p$ and $\psi_\varphi \in A^r(\mathbb{R}^{n-1}, N)$ such that $\mathcal{O}_q(\psi_\varphi) = \mathcal{O}_q(\varphi)$ for each $q \in V$. In particular, if $r = \omega$, then $\mathcal{O}_q(\psi_\varphi) = \mathcal{O}_q(\varphi)$ for each $q \in N$.*

Proof. Let $X_1, \ldots, X_{n-2}, X_{n-1}, X_n$ infinitesimal generators adapted to $\mathcal{O}_p$. We know that $X_1, \ldots, X_{n-2}$ are linearly independent in a neighborhood V_1 of $\mathcal{O}_p$. Let $h : V_p \longrightarrow D^m_\varepsilon$ be a chart adapted to $\mathcal{O}_p$ at p. Since $\mathcal{O}_p$ is transversally simple, we can assume that $0 \in D^2_\varepsilon$ is a simple singularity of $\widehat{X}_{n-1}$. Thus, there is a neighborhood U of p such that if $q \in U \setminus \mathcal{O}_p$, then $X_{n-1}(q) \neq 0$. It is clear that X_{n-1} has no singularities on the saturated set $sat(U \cap V_1)$ of $U \cap V_1$ under the action defined by $\{X_1, \ldots, X_{n-2}\}$ and also that $sat(U \cap V_1)$ contains a neighborhood V of $\mathcal{O}_p$. Call ψ_φ the action of $\mathbb{R}^{n-1}$ on N whose infinitesimal generators are $\{X_1, \ldots, X_{n-2}, X_{n-1}\}$. Since $\varphi \in A^r_{n-1}(\mathbb{R}^n, N)$, it follows that $X_n = \sum_{i=1}^{n-1} f_i X_i$ in V, where $X_i f_j = 0$ for $i, j = 1, \ldots, n-1$. Note that when $r = \omega$, $V = N$. □

Definition 2.9. A T^{n-2}-orbit $\mathcal{O}_p$ of $\varphi \in A^r_{n-1}(\mathbb{R}^n, N)$ is *transversally simple (center, saddle, node, focus)* if and only if it has the same attribute as an orbit of $\psi_\varphi \in A^r(\mathbb{R}^{n-1}, N)$, where ψ_φ is given by Lemma 2.8.

Proposition 2.10. *If $\varphi \in A^r(\mathbb{R}^n, N)$ has an orbit $\mathcal{O}$ homeomorphic to $T^{n-1} \times \mathbb{R}$, then* Front$(\mathcal{O})$ *is the union of at most two T^k-orbits with $k \in \{n-2, n-1\}$.*

For each $n \geq 2$ let $\mathcal{H}_n$ be the family of all analytic closed connected and orientable manifolds that can be obtained by glueing two copies of $T^{n-2} \times D^2$. Note that $\mathcal{H}_2$ has only one element, which is S^2. Note also that $\mathcal{H}_3$ consists of 3-manifolds that admit a Heegaard splitting of genus one.

Theorem 2.11. *Let N be a real analytic closed connected and orientable n-manifold, $n \geq 2$. Assume that $\varphi \in A^\omega(\mathbb{R}^n, N)$ has at least one $T^{n-1} \times \mathbb{R}$-orbit, then every n-dimensional orbit is a $T^{n-1} \times \mathbb{R}$-orbit and $\varphi \in A^\omega_n(\mathbb{R}^n, N)$. Furthermore,*

(1) *if* $\mathrm{Sing}^c_{n-2}(\varphi) = \emptyset$, *then N is homeomorphic to T^n,*

(2) *if* $\mathrm{Sing}^c_{n-2}(\varphi) \neq \emptyset$, *then* $\mathrm{Sing}^c_{n-2}(\varphi)$ *is the union of two T^{n-2}-orbits and $N \in \mathcal{H}_n$.*

Corollary 2.12. *Let N be a real analytic closed connected and orientable n-manifold and $\psi \in A^\omega_{n-1}(\mathbb{R}^{n-1}, N)$ that has a T^{n-1}-orbit $\mathcal{O}(\psi)$. If ψ embedds in $A^\omega_n(\mathbb{R}^n, N)$, then N is either homeomorphic to T^n or $N \in \mathcal{H}_n$. Moreover, in both cases $\mathcal{O}(\psi)$ has a neighborhood V that is a union of T^{n-1}-orbits.*

Definition 2.13. Let $\mathscr{C}^0_n$ be the set of actions $\varphi \in A^\omega_{n-1}(\mathbb{R}^n, N)$ such that there is at least one T^{n-2}-orbit $\mathcal{O}_p$ which is transversally a center and one T^{n-2}-orbit which is transversally a saddle. Let $\psi_\varphi \in A^\omega(\mathbb{R}^{n-1}, N)$ be the action constructed in Lemma 2.8 from φ and $\mathcal{O}_p$. Now define the subset $\mathscr{C}_n = \mathscr{C}_n(N) \subset \mathscr{C}^0_n$ by saying that $\varphi \in \mathscr{C}_n$ if and only if

1) $\mathrm{Sing}_i(\varphi) = \emptyset$, $i = 0, \ldots, n-3$, and $\mathrm{Sing}_{n-2}(\varphi) = \mathrm{Sing}^c_{n-2}(\varphi)$,
2) every T^{n-2}-orbit is transversally simple,
3) $\psi_\varphi \mapsto \varphi$ is a proper immersion,

4) If $\mathcal{O}_p$ is a T^{n-2}-orbit of φ that is transversally a saddle, then its separatrices are not separatrices of any other T^{n-2}-orbit that is also transversally a saddle.

Note that if $\mathcal{O}$ is a T^{n-2}-orbit of $\varphi \in \mathscr{C}_n$, then from condition 3) one obtains that $\mathcal{O}$ is transversally a center or a saddle. The following statement is a corollary of Proposition 2.10.

Corollary 2.14. *Let $\varphi \in \mathscr{C}_n^0(N)$ and ψ_φ as in Definition 2.13. Then ψ_φ can not be embedded in $A_n^\omega(\mathbb{R}^n, N)$.*

Theorem 2.15. *Let N be a real analytic closed orientable n-manifold and $\varphi \in \mathscr{C}_n \subset A_{n-1}^\omega(\mathbb{R}^n, N)$. Then φ is structurally stable and consequently $\mathscr{C}_n$ is an open set in $A^\omega(\mathbb{R}^n, N)$.*

3. Properties of actions in $\mathscr{C}_n$

In this section we prove some properties of actions $\varphi \in \mathscr{C}_n$ that are needed for the proof of Theorem 2.15. For the sake of clarity some of them are given first for $n = 2$ and then for $n > 2$.

Lemma 3.1 (Persistency of transversally simple T^{n-2}-orbits). *Let $\mathcal{O}_0$ be a transversally simple T^{n-2}-orbit of $\psi \in A^r(\mathbb{R}^{n-1}, N)$. Given a neighborhood V of $\mathcal{O}_0$ there exist a neighborhood $\mathcal{V}_\psi$ of ψ in $A^r(\mathbb{R}^{n-1}, N)$ such that each $\xi \in \mathcal{V}_\varphi$ has only one singular orbit $\mathcal{O}_0(\xi) \subset V$ and $\mathcal{O}_0(\xi)$ is also a transversally simple T^{n-2}-orbit.*

Proof. The proof is obtained using Remark 2.3. □

Remark 3.2. *Lemma 3.1 is also valid for actions $\varphi \in A_{n-1}^r(\mathbb{R}^n, N)$.*

For each $\varphi \in \mathscr{C}_n$ let K_φ be the set of points $p \in N$ such that $\mathcal{O}_p(\varphi)$ is not a T^{n-1}-orbit and $c_1, \ldots, c_k$ $(s_1, \ldots, s_\ell)$ be the T^{n-2}-orbits of φ that are transversally a center (saddle).

Proposition 3.3. $\mathscr{C}_2 \subset A_1^\omega(\mathbb{R}^2, N)$ *is an open set in* $A^\omega(\mathbb{R}^2, N)$.

Proof. Let $\varphi \in \mathscr{C}_2$. Denote by $c_1, \ldots, c_k$ the centers and by $s_1, \ldots, s_\ell$ the saddles of φ. Since $\varphi \in \mathscr{C}_2$, it follow that $k \geq 1$ and $\ell \geq 1$. Let X_1, X_2 be infinitesimal generators of φ such that the flow ψ_φ of X_1 has the same orbits as φ and $X_2 = fX_1$, with f non-constant. Given a neighborhood of each singular point of X_1, by Lemma 3.5, there exists a neighborhood $\mathcal{U} = \mathcal{U}(X_1)$ such that any $\widetilde{X}_1 \in \mathcal{U}$ has only one singularity in each of these neighborhoods and of the same type -center or saddle- as those of X_1. Assume now that $\mathcal{V}$ is a neighborhood of φ such that if $\widetilde{\varphi} \in \mathcal{V}$, then $\widetilde{X}_1 \in \mathcal{U}$. $\widetilde{\varphi}$ can not belong to $A_2^\omega(\mathbb{R}^2, N)$. If this were the case, then $\widetilde{\varphi}$ would have an $S^1 \times \mathbb{R}$-orbit intersecting any neighborhood of $\widetilde{c}_1$ and also an $\mathbb{R}$-orbit intersecting any neighborhood of $\widetilde{s}_1$, but this contradicts Proposition 2.10. Thus $\widetilde{\varphi} \in A_1^\omega(\mathbb{R}^2, N)$ and $\widetilde{X}_2 = \widetilde{f}\widetilde{X}_1$. Moreover, if $\mathcal{V}$ is sufficiently small, then $\widetilde{f}$ can not be constant. □

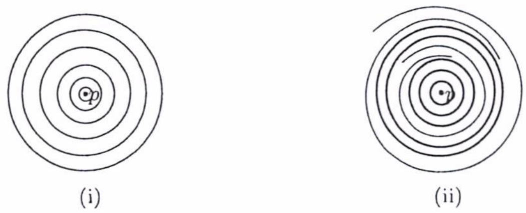
Theorem 3.4. *If $\varphi \in \mathscr{C}_2$, then there exists a neighborhood $\mathcal{V}$ of φ such that for each $\widetilde{\varphi} \in \mathcal{V}$*

$$K_{\widetilde{\varphi}} = \{\widetilde{c}_1, \ldots, \widetilde{c}_k\} \cup \{\widetilde{s}_1, \ldots, \widetilde{s}_\ell\} \cup \{\cup_{i=1}^{\ell} \widetilde{S}_{i1} \cup \widetilde{S}_{i2}\}$$

where, for each $i = 1, \ldots, \ell$, $\widetilde{S}_{i1}$ and $\widetilde{S}_{i2}$ are $\mathbb{R}$-orbits satisfying $\mathrm{Front}(\widetilde{S}_{i1}) = \widetilde{s}_i = \mathrm{Front}(\widetilde{S}_{i2})$.

Proof. By Proposition 3.3, it is enough to prove the theorem for a fixed $\varphi \in \mathscr{C}_2$. If $\mathcal{O}_q$ is a $\mathbb{R}$-orbit of ψ_φ, then $\alpha(q) = \omega(q) = s_i$ for some $i \in \{1, \ldots, k\}$. In fact, assume that $p \in \omega(q)$ is a regular point of ψ_φ. Take a transversal section Σ to the flow ψ_φ by p. There is a sequence $q_n \in \Sigma$, with $q_n = \psi_\varphi(t_n, q)$, converging to p. Let f be the real analytic first integral of ψ_φ given by Definition 2.13 condition 3). $f|_\Sigma$ is constant on $\{q_n\}$. This gives a contradiction and proves that $\omega(q) = s_i$ for some $i \in \{1, \ldots, k\}$. It follows from condition 4) that $\alpha(q) = s_i$, too. Since each saddle has two self-connections, the theorem is proved. □

To each fixed point p of $\varphi \in A^r(\mathbb{R}^n, M)$, $1 \leq r \leq \omega$, there is a linear action $\varrho : \mathbb{R}^n \to \mathrm{Aut}(T_pM)$ associated, given by $\varrho(v) = D\varphi_v(p)$ where $\varphi_v(\,\cdot\,) = \varphi(v, \,\cdot\,)$. Recall that if $\varphi \in A_1^r(\mathbb{R}^2, N^2)$, $1 \leq r \leq \omega$, has a fixed point $p \in N^2$ that is a center, then the phase portrait of the induced linear action ϱ is like Figure 1, (i).



(i) (ii)

FIGURE 1.

The following lemma gives a sufficient condition for the topological equivalence between φ and ϱ in the neighborhood of p.

Lemma 3.5. *Assume that $\varphi \in A_1^r(\mathbb{R}^2, N^2)$ has a first integral that is not constant on any open set. If $p \in \mathrm{Fix}(\varphi)$ is a center, then φ is topologically equivalent to ϱ in a neighborhood of p.*

Proof. Let X_1, X_2 be infinitesimal generators of φ such that p is a center of X_1 and $\psi_\varphi \in A^r(\mathbb{R}, N^2)$ is the flow of X_1. Let $h : V_p \to U_0$ be a chart of N such that $\mathrm{Sing}(X_1) \cap V_p = \{p\}$, $h(p) = 0$ and $D(h_* X_1)(0) = \begin{pmatrix} 0 & -\beta \\ \beta & 0 \end{pmatrix}$, $\beta \neq 0$. There exists a neighborhood $0 \in U \subset U_0$ such that if $x \in U$, then $h_* X_1(x)$ is transversal to the ray $0x$. It follows that any orbit of X_1 by $q \in h^{-1}(U)$ is either closed or a spiral that accumulates in p or in a closed orbit contained in $h^{-1}(U)$, see Figure 1 (ii). Let $C = \{q \in h^{-1}(U); \mathcal{O}_q(X_1) \text{ is closed}\}$. If $h^{-1}(U) \setminus C$ has non-empty interior, then the first integral would be constant in some open set. Thus $C = h^{-1}(U)$ and this proves the lemma. □

Proposition 3.6. *$\mathscr{C}_n \subset A_{n-1}^\omega(\mathbb{R}^n, N)$ is an open set in $A^\omega(\mathbb{R}^n, N)$.*

Proof. Denote by $c_1, \ldots, c_k$ and by $s_1, \ldots, s_\ell$ the T^{n-2}-orbits of φ that are transversally a center and a saddle, respectively. Since $\varphi \in \mathscr{C}_n$, $k \geq 1$ and $\ell \geq 1$. Let $X_1, \ldots, X_n$ be infinitesimal generators of φ such that the action ψ_φ generated by $X_1, \ldots, X_{n-1}$ has the same orbits than φ and $X_n = f_1 X_1 + \cdots + f_{n-1} X_{n-1}$, with f_i non-constant for at least one $i = 1, \ldots, n-1$. By Lemma 3.1, given a neighborhood of each T^{n-2}-orbit of ψ_φ there exists a neighborhood $\mathcal{U}_i = \mathcal{U}_i(X_i)$ of X_i, $i = 1, \ldots, n-1$, such that any action generated by $Y_1, \ldots, Y_{n-1}$, $Y_i \in \mathcal{U}_i$ has only one T^{n-2}-orbit of the same type than ψ_φ -transversally a center or saddle- in each of these neighborhoods. Now, assume that $\mathcal{V}$ is a neighborhood of φ such that if $\phi \in \mathcal{V}$, then $Y_i \in \mathcal{U}_i$, $i = 1, \ldots, n-1$. ϕ can not belong to $A_n^\omega(\mathbb{R}^n, N)$. If this were the case, then ϕ would have a $T^{n-1} \times \mathbb{R}$-orbit intersecting any neighborhood of $\widetilde{c}_1$ and also a $T^{n-2} \times \mathbb{R}$-orbit intersecting any neighborhood of $\widetilde{s}_1$, which contradicts Proposition 2.10. Thus, $\phi \in A_{n-1}^\omega(\mathbb{R}^n, N)$ and $Y_n = \widetilde{f}_1 Y_1 + \cdots + \widetilde{f}_{n-1} Y_{n-1}$. Finally if $\mathcal{V}$ is sufficiently small, then $\widetilde{f}_i$ can not be constant for every $i = 1, \dots, n-1$. □

Theorem 3.7. *If $\varphi \in \mathscr{C}_n$, then there exists a neighborhood $\mathcal{V} \subset \mathscr{C}_n$ of φ such that $K_{\widetilde{\varphi}} = \{\widetilde{c}_1, \dots, \widetilde{c}_k\} \cup \{\widetilde{s}_1, \dots, \widetilde{s}_\ell\} \cup (\cup_{i=1}^\ell \widetilde{S}_i)$ for each $\widetilde{\varphi} \in \mathcal{V}$ and for each $i = 1, \dots, \ell$, $\widetilde{S}_i$ satisfies one of the following statements:*

(1) *$\widetilde{S}_i$ is the union of two $T^{n-2} \times \mathbb{R}$-orbits $\widetilde{S}_{i1}, \widetilde{S}_{i2}$ such that $\widetilde{S}_{ij} \cup \widetilde{s}_i$ is homeomorphic to T^{n-1}, $j = 1, 2$.*

(2) *$\widetilde{S}_i$ is a $T^{n-2} \times \mathbb{R}$-orbit such that* $\mathrm{Front}(\widetilde{S}_i) = \widetilde{s}_i$.

The structure of $\psi \in A^\omega(\mathbb{R}^{n-1}, N)$ in the neighborhood of a T^{n-2}-orbit that is transversally a center is well determined when ψ has a non-constant first integral. More precisely:

Proposition 3.8. *Assume that $\psi \in A^r(\mathbb{R}^{n-1}, N)$ ($\varphi \in A_{n-1}^r(\mathbb{R}^n, N)$) has a T^{n-2}-orbit $\mathcal{O}_0$ that is transversally a center. If ψ (φ) has a first integral that is non-constant on any open subset of N, then there exists a ψ-invariant (φ-invariant) neighborhood V_0 of $\mathcal{O}_0$ such that $V_0 \setminus \mathcal{O}_0$ is a union of T^{n-1}-orbits.*

Proof. Let V be a neighborhood of $\mathcal{O}_0$ in N such that $V \cap \mathrm{Sing}(\varphi) = \mathcal{O}_0$. Take a chart $h : V_p \to D_\varepsilon^n$ adapted to $\mathcal{O}_0$ in p with $h(p) = 0$ and $V_p \subset V$. The infinitesimal generators of ψ in these coordinates are like in (2.1). Since $0 \in D_\varepsilon^2$ is a center of $D\widehat{X}_{n-1}(0)$ and ψ has a first integral that is not constant in V, so does $\widehat{X}_{n-1}$. By Lemma 3.5 there exists $\delta \in (0, \varepsilon)$ such that all orbits of $\widehat{X}_{n-1}$ within D_δ^2 are closed. Let ψ_0 be the action of $\mathbb{R}^{n-2}$ on N given by $X_1, \dots, X_{n-2}$. $\mathcal{O}_0$ is also an orbit of ψ_0 and $\Sigma = h^{-1}(D_\delta^2)$ is transversal at p to the orbits of ψ_0. If $q \in \Sigma \setminus \{p\}$, then $C_q = h^{-1}(\mathcal{O}_{h(q)}(\widehat{X}_{n-1}))$ is homeomorphic to S^1 and is contained in $\mathcal{O}_q$, see Figure 2. Let's consider the holonomy of $\mathcal{O}_0$ as a leaf of the foliation defined by ψ_0

$$\mathrm{Hol} : \pi_1(\mathcal{O}_0) \cong \mathbb{Z}^k \to \mathrm{Diff}^r(\Sigma, p). \tag{3.1}$$

The orbit α_i of X_i by p, $i = 1, \dots, n-2$, is closed and $\{[\alpha_1], \dots, [\alpha_{n-2}]\}$ is a set of generators of $\pi_1(\mathcal{O}_0)$. Let $\pi_i : V_p \to \Sigma$ be the projection along the orbits of the action generated by $X_1, \dots, X_{i-1}, X_{i+1} \dots, X_{n-2}$. Then, $\omega_i = \mathrm{Hol}(\alpha_i) = \pi_i \circ X_i^1$. Note that $\omega_i(C_q) = C_{\omega_i(q)}$ for each $q \in \Sigma$. We state that $\omega_i(C_q) = C_q$ for each $q \in \Sigma \setminus \{p\}$ and each $i \in \{1, \dots, n-2\}$, which in turn implies that $\mathcal{O}_q$ is a T^{n-1}-orbit. In fact, if for some $q \in \Sigma \setminus \{p\}$ there exists $i \in \{1, \dots, n-2\}$ such that $\omega_i(q) \notin C_q$, then either $\omega_i(C_q)$ or $\omega_i^{-1}(C_q)$ would be a circle $C_{q'}$ in the interior of C_q. This would imply that all orbits of ψ by points of the ring $R \subset \Gamma$ whose boundary is $C_q \cup C_{q'}$ would have a common orbit in its closure. Therefore, any first integral of ψ would be constant on the saturated of R, which is an open set. Finally, when $\varphi \in A_{n-1}^r(\mathbb{R}^n, N)$ it is enough to consider ψ_φ. □

With the same notation used in Proposition 3.8 and similar arguments we obtain:

Lemma 3.9. *Assume that $\psi \in A^r(\mathbb{R}^{n-1}, N)$ has a T^{n-2}-orbit $\mathcal{O}_0$ that is transversally a saddle. If ψ has a first integral that is not constant on any open set, then for each $i = 1, \dots, n-2$, either $\omega_i = id$ or $\omega_i^2 = id$ in the space of orbits of $(h^* \widehat{X}_{n-1})|_\Sigma$.*

Lemma 3.10. *If $\mathcal{O}$ is a $T^{n-2} \times \mathbb{R}$-orbit of $\varphi \in \mathscr{C}_n$, then $\mathcal{O}$ is a separatrix of some T^{n-2}-orbit $\mathcal{O}_0$ that is transversally a saddle.*

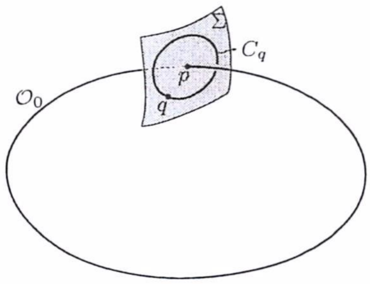


FIGURE 2.

Proof. Let C be a connected component of $\mathrm{cl}(\mathcal{O}) \setminus \mathcal{O}$, where $\mathrm{cl}(\mathcal{O})$ denote the closure of $\mathcal{O}$. C is a φ-invariant compact set. If $\mathrm{Sing}^c_{n-2} \cap C = \emptyset$, then C is a T^{n-1}-orbit and consequently every first integral of φ would be constant. If $\mathrm{Sing}^c_{n-2} \cap C \neq \emptyset$, call $\mathcal{O}_0$ the T^{n-2}-orbit contained in C. Since $\varphi \in \mathscr{C}_n$ any T^{n-2}-orbit is either transversally a center or a saddle. However, by Proposition 3.8 only the second case is possible. □

Proof of Theorem 3.7. By Proposition 3.6 it is enough to prove the theorem for a fixed $\varphi \in \mathscr{C}_n$. Let $\mathcal{U}$ be the family of φ-invariant open sets $U \subset N \setminus K_\varphi$ such that every orbit in U is a T^{n-1}-orbit. By Proposition 3.8, $\mathcal{U}$ is non-empty. The inclusion relation defines a parcial order in $\mathcal{U}$ and by Zorn's Lemma there exists a maximal element U_M in $\mathcal{U}$. Clearly $N \setminus U_M = K_\varphi$. We are going to show that

$$N \setminus U_M = \{c_1, \ldots, c_k\} \cup \{s_1, \ldots, s_\ell\} \cup (\cup_{i=1}^{\ell} S_i), \tag{3.2}$$

where S_i is the union of the separatrices of s_i, $i = 1, \ldots, \ell$. Let F be a connected component of $\mathrm{Front}(U_M)$. Assume that $F \cap \mathrm{Sing}^c_2(\varphi) = \emptyset$, then F is a T^{n-1}-orbit. Since φ is analytic F has trivial holonomy and there is a neighborhood V_F such that each orbit inside V_F is a T^{n-1}-orbit. The open set $U_M \cup V_F \in \mathcal{U}$ and contains U_M properly albeit this is a contradiction. Assume now that $F \cap \mathrm{Sing}^c_{n-2}(\varphi) \neq \emptyset$, then there exists a T^{n-2}-orbit $\mathcal{O} \subset F$. If $\mathcal{O} = c_j$, then by Proposition 3.8 $c_j = F$. If $\mathcal{O} = s_i$, by Lemmas 3.9 and 3.10 there exists a neighborhood V of s_i such that every orbit in $V \setminus (S_i \cup s_i)$ is a T^{n-1}-orbit. Thus the relation (3.2) is proved. It remains to proved that each S_i satisfies (1) or (2). With the notations introduced in the proof of the Proposição 3.8 put $\mathcal{O}_0 = s_i$. $\widehat{\omega}_j = h \circ \omega_j \circ h^{-1} : (D^2_\varepsilon, 0) \to (D^2_\varepsilon, 0)$, $j = 1, \ldots, n-2$, is a local diffeomorphism that preserves orbits of $\widehat{X}_{n-1}$, then

$$D\widehat{\omega}_j(0) D\widehat{X}_{n-1}(0) = D\widehat{X}_{n-1}(0) D\widehat{\omega}_j(0).$$

This implies that there exists a neighborhood Σ_0 of p in Σ such that $\omega_j(\Sigma_0 \cap \eta^k) \subset \eta^k$, $k = s, u$. Moreover, it follows from condition 4) of Definition 2.13 that S_i satisfies (1) or (2). □

Proposition 3.11. *If $\varphi \in A^\omega_{n-1}(\mathbb{R}^n, N)$ with $N \notin \mathcal{H}_n$ and φ satisfies conditions 1), 2) and 3) of Definition 2.13, then the existence of a T^{n-2}-orbit that is transversally a center implies the existence of another T^{n-2}-orbit that is transversally a saddle. In particular $\varphi \in \mathscr{C}^0_n(N)$.*

Proof. Let $\mathcal{O}$ be a T^{n-2}-orbit of φ that is transversally a center and $\mathcal{U}$ be the family of φ-invariant open sets $U \supset \mathcal{O}$ and homeomorphic to $T^{n-2} \times D^2$ such that every orbit in $U \setminus \mathcal{O}$ is a T^{n-1}-orbit. By condition 3) and Proposition 3.8, $\mathcal{U}$ is non-empty. The inclusion relation defines a parcial order in $\mathcal{U}$ and by Zorn's Lemma there exists a maximal element U_M in $\mathcal{U}$. As in the proof of Theorem

3.7, Front(U_M) can not be a T^{n-1}-orbit and by condition 1) Front(U_M) must contain a T^{n-2}-orbit $\mathcal{O}_1$. Now, if $\mathcal{O}_1$ is not transversally a saddle it has to be transversally a center and N would be contained in $\mathcal{H}_n$. □

It follows from Corollary 2.14 that an action φ, as in Proposition 3.11, does not embedded in $A^{\omega}_n(\mathbb{R}^n, N)$.

Let $\mathcal{O}_p$ be a $T^k \times \mathbb{R}^{n-k-1}$-orbit, $1 \le k \le n-1$, of $\psi \in A^r(\mathbb{R}^{n-1}, N)$ and $\{w^0_1, \dots, w^0_{n-1}\}$ a base of $\mathbb{R}^{n-1}$, where $\{w^0_1, \dots, w^0_k\}$ is a set of generators of the isotropy group G_p. Let $X_i = X_{w^0_i}$, $i = 1, \dots, n-1$. Note that for each $q \in \mathcal{O}_p$, $\mathcal{O}_q(X_i)$, $i = 1, \dots, k$ is periodic of period one.

Lemma 3.12. *Assume that there exists a neighborhood V of p such that each orbit in $V \setminus \mathcal{O}_p$ is a T^{n-1}-orbit. Then, there exist a neighborhood V_0 of p with $V_0 \subset V$ and C^r functions $w_i : V_0 \to \mathbb{R}^{n-1}$, $i = 1, \dots, k$, such that $w_i(p) = w^0_i$ and for each $q \in V_0$, every orbit of $X_{w_i(q)}$, $i = 1, \dots, k$ is periodic of period one.*

Proof. Let $h : V_p \subset V \to D^n_\varepsilon$ with $h(p) = 0$, be a $(n-1)$-flow box at p. Let $D_i = D_i(\varepsilon) = \{(x_1, \dots, x_n) \in D^n_\varepsilon; x_i = 0\}$ and $\Sigma_i = \Sigma_i(\varepsilon) = h^{-1}(D_i)$. The functions $\tau_i : V_p \to (-\varepsilon, \varepsilon)$ given by $\tau_i(q) = -x_i(q)$, where $h(q) = (x_1(q), \dots, x_n(q))$, are such that $X_i^{\tau_i(q)}(q) \in \Sigma_i$, for $i = 1, \dots, n-1$. We know that $X^1_1(p) = p$, $i = 1, \dots, k$. Therefore, there exists $0 < \delta < \varepsilon$ such that $X^1_i(\Sigma_i(\delta)) \subset V_p$, $i = 1, \dots, k$. Let $\Sigma_p = \Sigma_p(\delta) = \cap_{i=1}^{n-1} \Sigma_i(\delta)$. Σ_p is a transversal section to $\mathcal{O}_p$ at p. For each $i = 1, \dots, k$, consider the function $w_i : \Sigma_p \to \mathbb{R}^{n-1}$ given by

$$w_i(q) = \sum_{j=1}^{i-1} \tau_j(X^1_i(q))w^0_j + (1 + \tau_i(X^1_i(q)))w^0_i + \sum_{j=i+1}^{n-1} \tau_j(X^1_i(q))w^0_j. \tag{3.3}$$

It can be verified that every orbit of $X_{w_i(q)}$ inside $\mathcal{O}_q$, $q \in U$, is periodic of period one and $w_i(p) = w^0_i$, $i = 1, \dots, k$. We can extend the functions w_i to the open set $V_0 = \cup_{q \in \Sigma_p}(\mathcal{O}_q \cap V)$ by defining $w_i(q) = w_i(\Sigma_p \cap \mathcal{O}_q)$ and this completes the proof. □

Lemma 3.13. *Assume that $\psi \in A^{\omega}(\mathbb{R}^{n-1}, N)$ immerses properly in $\mathscr{C}_n$. Let S_1, S_2 be the separatrices of a T^{n-2}-orbit $\mathcal{O}$ that is transversally a saddle and G_0, G_1, G_2 the isotropy subgroups of $\mathcal{O}$, S_1, S_2, respectively. Then, there exists a linear $(n-2)$-subspace H of $\mathbb{R}^{n-1}$ transversal to G_0 such that $G_1 = G_0 \cap H = G_2$.*

Proof. For each $i = 1, 2$, G_i is a subgroup of G_0 isomorphic to $\mathbb{Z}^{n-2}$. Let H_i be the linear subspace of $\mathbb{R}^{n-1}$ generated by G_i. Then $\dim H_i = n-2$ and H_i is transversal to G^0_0. We first show that $H_1 = H_2$. Assume that $H_1 \ne H_2$ and choose $u_1 \in G_1 \setminus G_2$, $u_2 \in G_2 \setminus G_1$ such that $w = u_1 - u_2 \in G^0_0$. Let $X_w, X_{u_1}, X_{u_2} \in \mathfrak{X}^r(N)$ be the associated vector fields. Then $X_w = X_{u_1} - X_{u_2}$ or equivalently $X^t_w = X^t_{u_1} \circ X^{-t}_{u_2}$. Take infinitesimal generators $X_1, \dots, X_{n-1}$ of ψ adapted to $\mathcal{O}$ so that $X_{n-1} = X_w$ and a chart (V, h) adapted to $\mathcal{O}$ at $p \in \mathcal{O}$. We will show now that $DX^1_{u_i}(p) = id$, $i = 1, 2$. Consider the map $h \circ X^1_{u_i} \circ h^{-1} : D^n_\varepsilon \to D^n_\varepsilon$. At $0 \in D^n_\varepsilon \subset \mathbb{R}^n$ take a base $\{\partial/\partial\theta_1, \dots, \partial/\partial\theta_{n-2}, v_s, v_u\}$ such that v_s (v_u) is tangent to ξ^s (ξ^u). Since $S_i \cap \eta^s \ne \emptyset$ and $S_i \cap \eta^u \ne \emptyset$ it follows that $D(h \circ X^1_{u_i} \circ h^{-1})(0)(v_s) = v_s$ and $D(h \circ X^1_{u_i} \circ h^{-1})(0)(v_u) = v_u$. Since $u_i \in G_0$, it follows that $D(h \circ X^1_{u_i} \circ h^{-1})(0)(\partial/\partial\theta_j) = \partial/\partial\theta_j$, $j = 1, \dots, n-2$. We conclude that $DX_{u_i}(p) = id$. Thus, $DX^1_w(p) = id$, which is equivalent to $DX_w(p) = 0$. However, this contradicts the fact that $\mathcal{O}$ is transversally a saddle and proves that $H_1 = H_2 = H$. Finally the fact that $G_1 = G_0 \cap H = G_2$ follows applying Lemma 3.12 to the action $\psi|_H$ on $S_1 \cup S_2$. □

4. Proof of the main results

4.1. Proof of Theorem 2.11. The following proposition is an extension of a result used by Lima in [6] for $n = 3$. Let $D = \{(x, y) \in \mathbb{R}^2; x^2 + y^2 \leq 1\}$.

Proposition 4.1. *Let $Y_1, \ldots, Y_{s-1}$, $s \geq 3$, be a set of commutative vector fields on $T^{s-2} \times D$ that are tangent to the boundary and also linearly independent there. Then, in every disk $\{\theta\} \times D \subset T^{s-2} \times D$ there exists an interior point where they are linearly dependent.*

Proof. Let $\varphi : \mathbb{R}^{s-1} \times (T^{s-2} \times D) \to T^{s-2} \times D$ be the action induced by $Y_1, \ldots, Y_{s-1}$. Since these vector fields are tangent to the boundary $\partial(T^{s-2} \times D) = T^{s-2} \times S^1$ and linearly independent there, we know that $\partial(T^{s-2} \times D)$ is an orbit of φ. Let H be the isotropy group of this orbit. Choose infinitesimal generators $X_1, \ldots, X_{s-1}$ of φ such that at each point $\theta = (\theta_1, \ldots, \theta_{s-1}) \in T^{s-1}$, $X_i(\theta) = \frac{\partial}{\partial\theta_i}$, $i = 1, \ldots, s-1$, and consider the map $\alpha : \partial D \to SO(s) \subset V_s(\mathbb{R}^s)$, where $V_s(\mathbb{R}^s)$ is the manifold of oriented bases of $\mathbb{R}^s$, given by $\alpha = (X_1, \ldots, X_s)$, where $X_s = X_1 \wedge \cdots \wedge X_{s-1}$. For each $s > 2$ $\pi_1(V_s(\mathbb{R}^s)) = \pi_1(SO(s)) = \mathbb{Z}_2$ and its generator is given by the inclusion of the circle $SO(2)$ into $SO(s)$. Since we can interpret α as been this inclusion, it follows that α, as a map into $V_s(\mathbb{R}^s)$, can not be extended to D and the proposition is proved. □

Corollary 4.2. *Let $\psi \in A^r(\mathbb{R}^{s-1}, M^s)$, $s \geq 3$, and $p \in \mathrm{Fix}(\psi)$ be an isolated singularity. Then, there exists a neighborhood V of p in M^s which does not contain any T^{s-1}-orbit.*

Proof. Let V be a neighborhood of p such that $V \cap \mathrm{Sing}(\psi) = \{p\}$ and assume that there is a T^{s-1}-orbit $\mathcal{O} \subset V$. Then, $\mathcal{O}$ embedds in $\mathbb{R}^s$ as an orbit and we can assume that $\mathcal{O} = \partial(T^{s-2} \times D)$ with $T^{s-2} \times D \subset V$. By Proposition 4.1, there are infinitely many singular points of ψ in the interior of $T^{s-2} \times D$ and this is a contradiction. □

Proof of Proposition 2.10. Front($\mathcal{O}$) has at most two connected components and each one of them contains at least a compact orbit. Let C be a connected component and $\mathcal{O}_0 \subset C$ a T^k-orbit with isotropy group G_0. We will show that $C = \mathcal{O}_0$. Assume, for a moment, that C contains another orbit $\mathcal{O}_1$ and take $p \in \mathcal{O}_0$ and $q \in \mathcal{O}_1$. Let $u_1, \ldots, u_{n-1}$ be a set of generators of G, the isotropy group of $\mathcal{O}$. Then, all X_{u_i}-orbits through $\mathcal{O}$ are periodic of period one. Even more, there are sequences $\{p_j \in \mathcal{O}; j \in \mathbb{N}\}$ and $\{t_{ij} \in [0, 1]; i = 1, 2 \ldots, n-1 \text{ and } j \in \mathbb{N}\}$ such that

$$\lim_{j\to\infty} p_j = p \quad \text{and} \quad \lim_{j\to\infty} \varphi\Big(\sum_{i=1}^{n-1} t_{ij}u_i, p_j\Big) = q.$$

For each $i = 1, \ldots, n-1$, we can assume, extracting a subsequence if necessary, that $t_{ij} \to t_i \in [0, 1]$. Then $\varphi(\sum_{i=1}^{n-1} t_i u_i, p) = q$, which contradicts the fact that $\sum_{i=1}^{n-1} t_i u_i \in G_0$.

Now, we will show that $k \in \{n-2, n-1\}$. Assume, for a moment, that $k < n-2$, then $s = n - k > 2$. Let $p \in \mathcal{O}_0$ and $h : V_p \to D_1^n$ a chart adapted to $\mathcal{O}_0$ at p and φ_T the induced local action of $\mathbb{R}^s$ on D^s. The image of $\mathcal{O} \cap V_p$ by h intersects D^s in a $T^{s-1} \times \mathbb{R}$-orbit $\widehat{\mathcal{O}}$ of φ_T such that $0 \in D^s$ is a connected component of Front($\widehat{\mathcal{O}}$) and, therefore, an isolated singular point of φ_T. The generators $u_1, \ldots, u_{s-1}$ of the isotropy group of $\widehat{\mathcal{O}}$ define a local action of $\mathbb{R}^{s-1}$ on D^s and this action has T^{s-1}-orbits arbitrarily close to 0. This contradicts Corollary 4.2 and proves that $k \in \{n-2, n-1\}$. □

Proof of Theorem 2.11. Let $\mathcal{O}$ be a $T^{n-1} \times \mathbb{R}$-orbit and $\mathcal{U}$ the family of all φ-invariant neighborhoods $U \supset \mathcal{O}$, homeomorphic to $T^{n-1} \times \mathbb{R}$, that do not contain a $T^s \times \mathbb{R}^{n-s}$-orbit with $s \neq n-1$. The inclusion relation defines a parcial order in $\mathcal{U}$ and by Zorn's Lemma there exists a maximal element U_M in $\mathcal{U}$. We are going to show that $\mathrm{cl}(U_M) = N$. In fact, assume that $\mathrm{cl}(U_M) \neq N$, and let F be a connected component of $\mathrm{cl}(U_M) \setminus U_M$. There are points of F in the closure of at least one $T^{n-1} \times \mathbb{R}$-orbit $\mathcal{O}_1 \subset U_M$ and since $\mathrm{Front}(\mathcal{O}_1)$ is φ-invariant, then $F \subset \mathrm{Front}(\mathcal{O}_1)$. By Proposition 2.10 and the fact that U_M is maximal, we can assume that F is a T^{n-1}-orbit. Since N is orientable there exists another $T^{n-1} \times \mathbb{R}$-orbit $\mathcal{O}_2 \subset N \setminus U_M$ such that $F \subset \mathrm{Front}(\mathcal{O}_2)$. The open set $U_M \cup F \cup \mathcal{O}_2 \in \mathcal{U}$ and contains properly U_M. This contradicts the fact that U_M is maximal and proves that every n-dimensional orbit is a $T^{n-1} \times \mathbb{R}$-orbit.

Let $\mathcal{O}_i$, $1 \leq i \leq \ell$, be the n-dimensional orbits of φ and G_i the isotropy group of $\mathcal{O}_i$. From the analyticity of φ one obtains that $G_1 = \cdots = G_\ell$, which are isomorphic to $\mathbb{Z}^{n-1}$. Let $\{u_1, \ldots, u_{n-1}, u_n\}$ be a set of generators of $\mathbb{R}^n$, as a vector space, such that $\{u_1, \ldots, u_{n-1}\}$ is a set of generators of G_1 and $X_i = X_{u_i}$, $1 \leq i \leq n$, the associated vector fields. $\{X_1, \ldots, X_{n-1}\}$ are the infinitesimal generators of an action φ_H of $\mathbb{R}^{n-1}$ on N that is the restriction of φ to the subspace H generated by $\{u_1, \ldots, u_{n-1}\}$. Note that if $p \in \mathcal{O}_1 \cup \cdots \cup \mathcal{O}_\ell$, then $\mathcal{O}_p(\varphi_H)$, the orbit of φ_H by p, is a T^{n-1}-orbit and if $p \in N \setminus \mathcal{O}_1 \cup \cdots \cup \mathcal{O}_\ell$, then it is, by Proposition 2.10, a T^{n-1}-orbit or a T^{n-2}-orbit. Even more, we can verify that the map $\overline{\varphi_H} : T^{n-1} \times N \to N$ given by $\overline{\varphi_H}(e^{2\pi i t_1}, \ldots, e^{2\pi i t_{n-1}}, p) = \varphi_H(t_1 u_1, \ldots, t_{n-1} u_{n-1}, p)$ is in fact an analytic action of T^{n-1} with the same underlying foliation than the one given by φ_H. Assume that $\mathrm{Sing}^c_{n-2}(\varphi) = \emptyset$, then by Lemma 3.12 the isotropy group of every orbit of $\overline{\varphi_H}$ is the same; or, in other words, $\overline{\varphi_H}$ is a free action. Thus, N is a principal T^{n-1} bundle over S^1 and consequently N is homeomorphic to T^n. Finally, assume $\mathrm{Sing}^c_{n-2}(\varphi) \neq \emptyset$. Since U_M is homeomorphic to $T^{n-1} \times \mathbb{R}$ and $\mathrm{cl}(U_M) = N$, it follows that $\mathrm{Sing}^c_{n-2}(\varphi)$ is the union of two T^{n-2}-orbits. Thus $N \in \mathcal{H}_n$. □

4.2. **Proof of Theorem 2.15,** $n = 2$. Before starting the proof of Theorem 2.15 we exhibit an example of an action $\varphi \in \mathscr{C}_2$.

FIGURE 3.

Example: Let N be a closed orientable surface, $H : N \to \mathbb{R}$ an analytic Morse function with at least one singularity of index -1, $X_H \in \mathfrak{X}^\omega(N)$ the associated Hamiltonian. Assume that there are no connections between two different saddle points of X_H. Then, $\{X_H, HX_H\}$ are infinitesimal generators of an action $\varphi \in \mathscr{C}_2$ (see Figure 3).

Proof of Theorem 2.15, n=2. Without loss of generality we can assume that $\widetilde{c}_i = c_i$, $i = 1, \ldots, k$ and $\widetilde{s}_j = s_j$, $j = 1, \ldots, \ell$. Let $F_j : V_j \to \widetilde{V}_j$ be a topological equivalence between X_1 and $\widetilde{X}_1$ at s_j and $U_j \subset V_j$ a cross shaped neighborhood of s_j such that $\mathrm{Front}(U_j) = (\cup_{i=1}^4 B_{ji}) \cup (\cup_{i=1}^2 (A_{ji}^1 \cup A_{ji}^2))$, where B_{ji} is a piece of orbit of X_1 and A_{j1}^i, $i = 1, 2$ intersects S_{j1} and is transversal to X_1, see Figure 4. Let $\widetilde{U}_j = F_j(U_j)$, $\widetilde{B}_{ji} = F_j(B_{ji})$ and $\widetilde{A}_{ji}^l = F_j(A_{ji}^l)$. Reparametrizing the time t of the flows X_1^t and $\widetilde{X}_1^t$, we can assume that $X_1^1(A_{ji}^1) = A_{ji}^2$ and $\widetilde{X}_1^1(\widetilde{A}_{ji}^1) = \widetilde{A}_{ji}^2$ for $i = 1, 2$. Let W_j $(\widetilde{W}_j)$ be the saturated by X_1 $(\widetilde{X}_1)$ of closure of U_j $(\widetilde{U}_j)$. Then, $\mathrm{Front}(W_j)$ $(\mathrm{Front}(\widetilde{W}_j))$ is the union of three closed orbits of X_1 $(\widetilde{X}_1)$. If $p \in W_j \setminus U_j$, there exists $t \in (0,1)$ such that $X_1^t(p) \in A_{ji}^l$ for some $i, l \in \{1, 2\}$. Extend F_j to W_j by defining $F_j(p) = \widetilde{X}_1^{-t}(F_j(X_1^t(p)))$. It is clear that $F_j : W_j \to \widetilde{W}_j$ is a topological equivalence between X_1 and $\widetilde{X}_1$ that preserves orientation. Note that it is possible to reduce the size of W_j to guarantee that $W_{j_1} \cap W_{j_2} = \emptyset$ if $j_1 \neq j_2$.

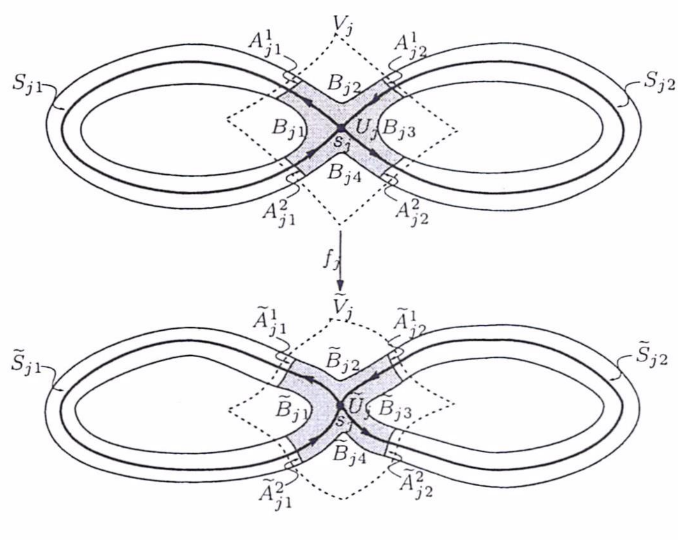


FIGURE 4.

Next, we are going to extend the equivalences F_j to a global topological equivalence F between X_1 and $\widetilde{X}_1$. Fix a Riemannian metric on N and let Z be the vector field obtained by rotating X_1 by a right angle. Clearly $\mathrm{Sing}(Z) = \mathrm{Sing}(X_1) = \mathrm{Sing}(\widetilde{X}_1)$ and taking $\mathcal{V}$, smaller, if necessary, Z will be transversal to both X_1 and $\widetilde{X}_1$ at points in $N \setminus \mathrm{Sing}(Z)$. Each connected component C of $N \setminus \cup_{j=1}^{\ell} W_j$ is homeomorphic either to an open disk or to $S^1 \times (0,1)$. In the first case, $C \cap \mathrm{Fix}(\varphi)$ is a center and in the second case, it is empty. To each C corresponds a unique connected component $\widetilde{C}$ of $N \setminus \cup_{j=1}^{\ell} \widetilde{W}_j$ with the same properties than C. We shall define $F : C \to \widetilde{C}$ for each C. There are two cases:

(i) *C is an open disk.* There exists a unique $j \in \{1, \ldots, \ell\}$, such that $\mathrm{Front}(C) \subset \mathrm{Front}(W_j)$, $\mathrm{Front}(\widetilde{C}) \subset \mathrm{Front}(\widetilde{W}_j)$ and $F_j(\mathrm{Front}(C)) = \mathrm{Front}(\widetilde{C})$. For each point $p \in \mathrm{Front}(C)$ let $L_p = \mathrm{cl}(\mathcal{O}_p(Z) \cap C)$ and $\widetilde{L}_{F_j(p)} = \mathrm{cl}(\mathcal{O}_{F_j(p)}(Z) \cap \widetilde{C})$. Fix a point $p_0 \in \mathrm{Front}(C)$ and extend $F_j : L_{p_0} \to \widetilde{L}_{F_j(p_0)}$ as a homeomorphism. If $q \in C$, there is a unique L_p that contains q. Now, let $q_1 = \mathcal{O}_q(X_1) \cap L_{p_0}$. Define $F(q) = \mathcal{O}_{F_j(q_1)}(\widetilde{X}_1) \cap \widetilde{L}_{F_j(p)}$.

(ii) *C is homeomorphic to $S^1 \times (0,1)$.* $\mathrm{Front}(C) = A_0 \cup A_1$, where A_0 and A_1 are S^1-orbits. There exists $i, j \in \{1, \ldots, \ell\}$, $i \neq j$ such that A_0 (A_1) is a connected component of $\mathrm{Front}(W_i)$ ($\mathrm{Front}(W_j)$). For the unique $\widetilde{C}$ associated to C we also have $\mathrm{Front}(\widetilde{C}) = \widetilde{A}_0 \cup \widetilde{A}_1$, where $\widetilde{A}_0$ ($\widetilde{A}_1$) is a connected component of $\mathrm{Front}(\widetilde{W}_i)$ ($\mathrm{Front}(\widetilde{W}_j)$). Let $G : \mathrm{cl}(C) \to S^1 \times [0,1]$ and $\widetilde{G} : \mathrm{cl}(\widetilde{C}) \to S^1 \times [0,1]$ homeomorphisms such that the orbits of $X_1|_{\mathrm{cl}(C)}$ and $\widetilde{X}_1|_{\mathrm{cl}(\widetilde{C})}$ are $G^{-1}(S^1 \times \{t\})$ and $\widetilde{G}^{-1}(S^1 \times \{t\})$, $t \in [0,1]$. The two homeomorphisms $h_0 : S^1 \times \{0\} \to S^1 \times \{0\}$ and $h_1 : S^1 \times \{1\} \to S^1 \times \{1\}$ given by $h_0 = \widetilde{G} \circ F_i \circ G^{-1}$ and $h_1 = \widetilde{G} \circ F_j \circ G^{-1}$ are both isotopic to the identity and therefore isotopic among themselves. Let h_t be the isotopy between h_0 and h_1 and define $H : S^1 \times [0,1] \to S^1 \times [0,1]$ by $H(\theta, t) = (h_t(\theta), t)$. H is a homeomorphism preserving the foliation $S^1 \times \{t\}$ and the restriction of the topological equivalence F to be closure of C will be the homeomorphism $\widetilde{G}^{-1} \circ H \circ G : \mathrm{cl}(C) \to \mathrm{cl}(\widetilde{C})$. □

4.3. **Proof of Theorem 2.15, general case.** Fix $i \in \{1, \dots, \ell\}$ and let $X_1, \dots, X_{n-1}$ be infinitesimal generators of ψ adapted to $\mathcal{O} = s_i$. Let S_i be as in relation (3.2) and G_i the isotropy group of S_i (well defined because of Theorem 3.7 and Lemma 3.13). Call H the $(n-2)$-linear subspace of $\mathbb{R}^{n-1}$ generated by G_i and $\psi_0 = \psi|_H$ is defined by $X_1, \dots, X_{n-2}$.

Lemma 4.3. *Assume that $\psi \in A^\omega(\mathbb{R}^{n-1}, N)$ immerses properly in $\mathscr{C}_n$. Let S be the union of the separatrices of a T^{n-2}-orbit $\mathcal{O}$ which is transversally a saddle and $p \in \mathcal{O}$. Then, there exists an eight shaped curve Γ with $p \in \Gamma$ and $\Gamma \subset \mathrm{cl}(S)$, which is transversal to the orbits of ψ_0 within $\mathrm{cl}(S)$, such that $\Gamma \setminus \{p\} = \Gamma_1 \cup \Gamma_2 \subset S$, $\Gamma_1 \cap \Gamma_2 = \emptyset$ and $\Gamma_i \cup \{p\}$ is homeomorphic to S^1, $i = 1, 2$.*

Proof. Let $h : V \longrightarrow D_\varepsilon^{n-2} \times D_\varepsilon^2$ be a chart adapted to $\mathcal{O}$ at p and η^s (η^u) be the stable (unstable) submanifold of $(h|_\Sigma)^* \widehat{X}_{n-1}$ at $p \in \Sigma = h^{-1}(D_\varepsilon^2)$. Put $\eta^i \setminus \{p\} = \eta_1^i \cup \eta_2^i$, $i = s, u$. Note that $\eta_j^s, \eta_j^u \subset S_j$, $j = 1, 2$. Let $q_j^i \in \eta_j^i$, $i = u, s$ be such that $\mathcal{O}_{q_j^s}(\psi_0) \neq \mathcal{O}_{q_j^u}(\psi_0)$ for $j = 1, 2$. Then, there exist $v_1, v_2 \in \mathbb{R}^{n-1} \setminus H$ such that $q_j^u \in \mathcal{O}_{q_j^s}(X_{v_j})$, $j = 1, 2$. Furthermore, since $v_1, v_2 \in \mathbb{R}^{n-1} \setminus H$ we know that $\mathcal{O}_{q_j^s}(X_{v_j})$, $j = 1, 2$ is transversal to the orbits of ψ_0. Consider the arc $\overline{q_1^i q_2^i} \subset \eta^i$ with extremes q_1^i and q_2^i, $i = s, u$, and the arc $\overline{q_j^s q_j^u} \subset \mathcal{O}_{q_j^s}(X_{v_j})$ with extremes q_j^s, q_j^u, $j = 1, 2$. The curve

$$\Gamma = \overline{q_1^s q_2^s} \cup \overline{q_1^u q_2^u} \cup \overline{q_1^s q_1^u} \cup \overline{q_2^s q_2^u}$$

is transversal to the ψ_0-orbits. We can assume, without lost of generality that the connected components of $\Gamma \setminus \{p\}$ are C^∞ curves. □

Remark 4.4. The curve Γ, although self-intersecting at p, has a C^∞ tangent vector field, which is linearly independent with $X_1, \dots, X_{n-2}$ at each point $q \in \Gamma$ and also $\Gamma \setminus \{p\} \subset S$. Let $U \subset \Sigma$ be a cross shaped neighborhood of p as in the proof of Theorem 2.15, $n = 2$. It is possible to extend U to a 2-dimensional submanifold $\mu \supset \Gamma$ which is transversal to the orbits of ψ_0 and therefore also to those of ψ. The intersection of the orbits of ψ with μ define a foliation, which is given by the orbits of a vector field X that satisfies the following properties:

(1) $X|_U = (h^* \widehat{X}_{n-1})|_U$,
(2) $\mathrm{Sing}(X) = \{p\}$ and p is a saddle singularity.
(3) The connected components of $\Gamma \setminus \{p\}$ are the separatrices of X at p.
(4) Every orbit of X by a points in $\mu \setminus \Gamma$ is closed.

Let $\mathcal{V}$ be a neighborhood of $\varphi \in \mathscr{C}_n$ as in Proposition 3.6. By this proposition and Theorem 3.7 for each $\phi \in \mathcal{V}$ we have

$$K_\phi = \{\widetilde{c}_1, \dots, \widetilde{c}_k\} \cup \{\widetilde{s}_1, \dots, \widetilde{s}_\ell\} \cup (\cup_{i=1}^\ell \widetilde{S}_i)$$

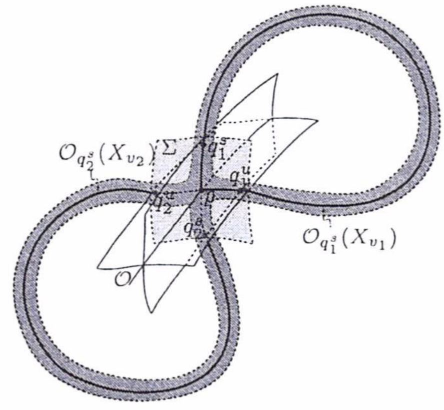


FIGURE 5.

where, for each $i = 1, \dots, \ell$, $\widetilde{S}_i$ has the same properties as S_i (Theorem 3.7, (1) or (2)).

Let $\{w_1^0, \dots, w_{n-1}^0\}$ be a base of $\mathbb{R}^{n-1}$, where $\{w_1^0, \dots, w_k^0\}$ is a set of generators of G_p, the isotropy group of a $T^k \times \mathbb{R}^{n-k-1}$-orbit $\mathcal{O}_p$, $n-2 \leq k \leq n-1$, of ψ_φ. Let $X_i = X_{w_i^0}$, $i = 1, \dots, n-1$, and $w_i : V_0 \to \mathbb{R}^{n-1}$, $i = 1, \dots, k$, the C^r functions given by Lemma 3.12. if $\phi \in \mathcal{V}$ and $\{u_1, \dots, u_{n-1}\}$ is a base of $\mathbb{R}^{n-1}$, we will denote by $\{Y_{u_1}, \dots, Y_{u_{n-1}}\}$ the corresponding infinitesimal generators of ψ_ϕ. Let $Y_i = Y_{w_i^0}$, $i = 1, \dots, n-1$.

Lemma 4.5. *Given $\varphi \in \mathscr{C}_n$ and $d > 0$ there exist a neighborhood $\mathcal{V}_0 \subset \mathcal{V}$ of $\varphi \in \mathscr{C}_n$ and a neighborhood $V_1 \subset V_0$ of p that satisfy: if $\phi \in \mathcal{V}_0$, then there exist C^ω functions $\widetilde{w}_i : V_1 \to \mathbb{R}^{n-1}$, $i = 1, \dots, k$, such that for each $q \in V_1$, $\|\widetilde{w}_i(q) - w_i(q)\| < d$ and every orbit of $Y_{\widetilde{w}_i(q)}$, $i = 1, \dots, k$ inside $\mathcal{O}_q(\psi_\phi)$ is periodic of period one.*

Proof. We shall use the same notations as in the proof of Lemma 3.12. Let $\mathcal{V}_1 \subset \mathcal{V}$ be a neighborhood of φ such that $Y_i^1(\Sigma_i(\delta)) \subset V_0$ for all $i = 1, \dots, k$. There exists $\alpha > 0$ such that

$$\widetilde{\Sigma}_i = \cup_{|t_i| < \alpha} Y_1^{t_1} \circ \cdots \circ Y_{i-1}^{t_{i-1}} \circ Y_{i+1}^{t_{i+1}} \circ \cdots \circ Y_{n-1}^{t_{n-1}}(\Sigma_p(\delta)),\ i = 1, \dots, n-1,$$

is C^r close to Σ_i and transversal to Y_i. Let $V_1 = \cup_{|t| < \alpha} Y_i^t(\widetilde{\Sigma}_i)$. For ϕ sufficiently C^1 close of φ the C^ω functions $\widetilde{\tau}_i : V_1 \to (-\varepsilon, \varepsilon)$ such that $Y_i^{\widetilde{\tau}_i(q)}(q) \in \widetilde{\Sigma}_i$, $i = 1, \dots, n-1$, are C^1 close of the corresponding $\tau_i|_{V_1}$ of ψ_φ. Note that $\Sigma_p = \Sigma_p(\delta) = \cap_{i=1}^{n-1} \widetilde{\Sigma}_i(\alpha)$. For each $i = 1, \dots, k$, the functions $\widetilde{w}_i : \Sigma_p \to \mathbb{R}^{n-1}$ given by

$$\widetilde{w}_i(q) = \sum_{j=1}^{i-1} \widetilde{\tau}_j(Y_i^1(q)) w_j^0 + (1 + \widetilde{\tau}_i(Y_i^1(q))) w_i^0 + \sum_{j=i+1}^{n-1} \widetilde{\tau}_j(Y_i^1(q)) w_j^0$$

are such that every orbit of $Y_{\widetilde{w}_i(q)}$, $i = 1, \dots, k$, inside $\mathcal{O}_q(\psi_\phi)$ is periodic of period one. Furthermore, there exist a neighborhood $\mathcal{V}_0 \subset \mathcal{V}_1$ of φ such that $\phi \in \mathcal{V}_0$ implies that $\widetilde{w}_i$ is $d - C^1$-close of w_i. □

Proof of Theorem 2.15, general case. Let $\psi_\varphi \in A^\omega(\mathbb{R}^{n-1}, N)$ be the action associated to φ given by Lemma 2.8. Put a Riemannian metric on N and let ξ be the normal bundle to $\mathscr{F}(\psi_\varphi)$, the codimension 1 underlying foliation of ψ_φ. ξ is orientable and integrable. Let $\mathcal{V}$ be a neighborhood of $\varphi \in \mathscr{C}_n$ as in Proposition 3.6. Without loss of generality, we can assume that if $\phi \in \mathcal{V}$, then there exists $p_j \in \widetilde{s}_j \cap s_j$, $j = 1, \dots, \ell$. Let $\Gamma_j \ni p_j$ be the eight shaped curve given by Lemma 4.3 for the action ψ_φ and (μ_j, X^j_φ) the 2-dimensional submanifold and the vector field, given by Remark 4.4. It is possible to construct each μ_j in such a way that ξ be tangent to μ_j at each $p \in \mu_j \setminus \{p_j\}$. If $\mathcal{V}$ is sufficiently small, then μ_j is also transversal to the ψ_ϕ-orbits for every $\phi \in \mathcal{V}$. The intersection of the ψ_ϕ-orbits with μ_j, $j = 1, \dots, \ell$, are the orbits of a vector field X^j_ϕ. By Theorem 3.7, $\widetilde{S}_j \cap \mu_j = \widetilde{\Gamma}_j$ is an eight shaped curve and the orbits of X^j_ϕ near of $\widetilde{\Gamma}_j$ are periodic. With similar arguments to those used in the case $n = 2$, we obtain a topological equivalence $F_j : \nu_j \to \widetilde{\nu}_j$ between X^j_φ and X^j_ϕ, where ν_j $(\widetilde{\nu}_j)$ is a X^j_φ-invariant (X^j_ϕ-invariant) closed neighborhood of Γ_j $(\widetilde{\Gamma}_j)$ and $\nu_j \cup \widetilde{\nu}_j \subset \mu_j$. Given $\delta > 0$, we can reduce the size of $\mathcal{V}$ and also of ν_j in such a way that for every $p \in \nu_j$, $d(p, F_j(p)) < \delta$.

Let W_j $(\widetilde{W}_j)$ be the saturated by ψ_φ (ψ_ϕ) of ν_j $(\widetilde{\nu}_j)$. Then $\mathrm{Front}(W_j)$ $(\mathrm{Front}(\widetilde{W}_j))$ is the union of three T^{n-1}-orbits if S_j $(\widetilde{S}_j)$ satisfies Theorem 3.7, (1) and of two T^{n-1}-orbits if S_j $(\widetilde{S}_j)$ satisfies Theorem 3.7, (2). Fix $j \in \{1, \dots, \ell\}$ and let $w^0_1, \dots, w^0_{n-2} \in \mathbb{R}^{n-1}$ be a generators of G_j, the isotropy group of S_j. Therefore, for each $p \in S_j$, $\mathcal{O}_p(X_{w^0_i})$ is periodic of period one $\forall i \in \{1, \dots, n-2\}$. Fix $q_j \in \Gamma_j$ and let $w_i : V_j \to \mathbb{R}^{n-1}$, $i = 1, \dots, n-2$, where V_j is a neighborhood of p_j in μ_j, be the functions given by Lemma 3.12. Recall that $w_i(q_j) = w^0_i$. Note that for each $p \in V_j$, $w_1(p), \dots, w_{n-2}(p)$ is a set of generators of the isotropy group of $\mathcal{O}_p((\psi_\varphi)_0)$. Thus, we can assume that each w_i is defined on ν_j and also in W_j, if we wish. Let $\widetilde{w}_i : \widetilde{W}_j \to \mathbb{R}^{n-1}$, $i = 1, \dots, n-2$, be the functions given by Lemma 4.5, for $k = n-2$. For each $p \in V^0_j$, $\widetilde{w}_1(p), \dots, \widetilde{w}_{n-2}(p)$ is a set of generators of the isotropy group of $\mathcal{O}_p((\psi_\phi)_0)$ and thus, we can assume that $\widetilde{w}_i$ is defined on $\widetilde{\nu}_j$ or even in $\widetilde{W}_j$. We are going to extend F_j to a map $W_j \to \widetilde{W}_j$. If $p \in W_j \setminus \nu_j$, there exists $t_1, \dots, t_{n-2} \in (0,1)$ such that $f(p) = X^{t_1}_{w_1(p)} \circ \cdots \circ X^{t_{n-2}}_{w_{n-2}(p)}(p) \in \nu_j$. Define

$$F_j(p) = Y^{t_1}_{\widetilde{w}_1(F_j(f(p)))} \circ \cdots \circ Y^{t_{n-2}}_{\widetilde{w}_{n-2}(F_j(f(p)))}(F_j(f(p))).$$

$F_j : W_j \to \widetilde{W}_j$ is a topological equivalence that preserves orientation between ψ_φ and ψ_ϕ and by reducing the size of $\mathcal{V}$ and ν_j we can assume that $d(p, F_j(p)) < \delta$ for each $p \in W_j$, where δ is given. Note that there is no problem in assuming that $W_j \cap W_k = \emptyset$ if $j \neq k$.

We are going to extend the equivalences F_j to a global topological equivalence F between ψ_φ and ψ_ϕ. Each connected component C of $N \setminus \cup^\ell_{j=1} W_j$ is homeomorphic either to $T^{n-2} \times D$ or to $T^{n-1} \times (0,1)$, where D is an open 2-disk. In the first case $C \cap \mathrm{Sing}^c_{n-2}(\psi_\varphi)$ is a T^{n-2}-orbit that is transversally a center and in the second case is empty. To each C corresponds a unique connected component $\widetilde{C}$ of $N \setminus \cup^\ell_{j=1} \widetilde{W}_j$ with the same properties than C. We shall define $F : C \to \widetilde{C}$ for each C. There are two cases:

(i) C is homeomorphic to $T^{n-2} \times D$. There exists a unique $j \in \{1, \dots, \ell\}$ such that $\mathrm{Front}(C) \subset \mathrm{Front}(W_j)$, $\mathrm{Front}(\widetilde{C}) \subset \mathrm{Front}(\widetilde{W}_j)$ and $F_j(\mathrm{Front}(C)) = \mathrm{Front}(\widetilde{C})$. For each point $p \in \mathrm{Front}(C)$ let $L_p = \mathrm{cl}(\mathcal{O}_p(\xi) \cap C)$, where $\mathcal{O}_p(\xi)$ is the integral curve of ξ by p and $\widetilde{L}_{F_j(p)} = \mathrm{cl}(\mathcal{O}_{F_j(p)}(\xi) \cap \widetilde{C})$. Fix a point $p_0 \in \mathrm{Front}(C)$, and extend $F_j : L_{p_0} \to \widetilde{L}_{F_j(p_0)}$ as an homeomorphism. If $q \in C$, then there is a unique $p \in \mathrm{Front}(C)$ such that $q = \mathcal{O}_q(\psi_\varphi) \cap L_p$. Let $q_1 = \mathcal{O}_q(\psi_\varphi) \cap L_{p_0}$ and define $F(q) = \mathcal{O}_{F_j(q_1)}(\psi_\phi) \cap \widetilde{L}_{F_j(p)}$.

(ii) C is homeomorphic to $T^{n-1} \times (0,1)$. Then, $\text{Front}(C) = A_0 \cup A_1$, where A_0 and A_1 are T^{n-1}-orbits of ψ_φ. There exist $i, j \in \{1, \dots, \ell\}$, $i \neq j$, such that A_0 (A_1) is a connected component of $\text{Front}(W_i)$ ($\text{Front}(W_j)$). For the corresponding $\widetilde{C}$, $\text{Front}(\widetilde{C}) = \widetilde{A}_0 \cup \widetilde{A}_1$, where $\widetilde{A}_0$ ($\widetilde{A}_1$) is a connected component of $\text{Front}(\widetilde{W}_i)$ ($\text{Front}(\widetilde{W}_j)$). Let $\mathcal{O}(\xi)$ be an integral curve of ξ that cuts every T^{n-1}-orbit in $\text{cl}(C)$ and every T^{n-1}-orbit in $\text{cl}(\widetilde{C})$ and put $p_0 = \mathcal{O}(\xi) \cap A_0$ $\widetilde{p}_0 = \mathcal{O}(\xi) \cap \widetilde{A}_0$, $p_1 = \mathcal{O}(\xi) \cap A_1$, $\widetilde{p}_1 = \mathcal{O}(\xi) \cap \widetilde{A}_1$, $[p_0, p_1]$ ($[\widetilde{p}_0, \widetilde{p}_1]$) the arc within $\mathcal{O}(\xi)$ with extremes p_0 and p_1 ($\widetilde{p}_0$ and $\widetilde{p}_1$). Define $g : [0,1] \to [p_0, p_1]$ and $\widetilde{g} : [0,1] \to [\widetilde{p}_0, \widetilde{p}_1]$ as homeomorphisms. For each $p \in [p_0, p_1]$ ($p \in [\widetilde{p}_0, \widetilde{p}_1]$) let G_p ($\widetilde{G}_p$) the isotropy group of p under ψ_φ (ψ_ϕ). G_p ($\widetilde{G}_p$) is generated by $w_1(p), \dots, w_{n-2}(p), w_{n-1}(p)$ ($\widetilde{w}_1(p), \dots, \widetilde{w}_{n-2}(p), \widetilde{w}_{n-1}(p)$). Consider the map $\psi : \mathbb{R}^{n-1} \times [p_0, p_1] \to \text{cl}(C)$ ($\widetilde{\psi} : \mathbb{R}^{n-1} \times [\widetilde{p}_0, \widetilde{p}_1] \to \text{cl}(\widetilde{C})$) given by:

$$\begin{aligned} \psi(t_1, \dots, t_{n-1}, p) &= \psi_\varphi(t_1 w_1(p) + \dots + t_{n-1} w_{n-1}(p), p) \\ \widetilde{\psi}(t_1, \dots, t_{n-1}, p) &= \psi_\phi(t_1 \widetilde{w}_1(p) + \dots + t_{n-1} \widetilde{w}_{n-1}(p), p). \end{aligned}$$

Since $\psi(\mathbb{R}^{n-1} \times \{p\})$ ($\widetilde{\psi}(\mathbb{R}^{n-1} \times \{p\})$) is the $T^{n-1} - \psi_\varphi$-orbit ($T^{n-1} - \psi_\varphi$-orbit) by p, then ψ ($\widetilde{\psi}$) induces a map $\psi_1 : T^{n-1} \times [p_0, p_1] \to C$ ($\widetilde{\psi}_1 : T^{n-1} \times [\widetilde{p}_0, \widetilde{p}_1] \to \widetilde{C}$). Define $H : T^{n-1} \times [0,1] \to \text{cl}(C)$ and $\widetilde{H} : T^{n-1} \times [0,1] \to \text{cl}(\widetilde{C})$ by $H(x,s) = \psi_1(x, g(s))$ and $\widetilde{H}(x,s) = \widetilde{\psi}_1(x, \widetilde{g}(s))$. Again, by reducing the size of $\mathcal{V}$, of ν_i, ν_j and by the proximity of the functions w_i and $\widetilde{w}_i$ in the neighborhood of a T^{n-1}-orbit given by Lemma 4.5, for $k = n-1$, we can assume that for every $x \in T^{n-1}$

$$\begin{aligned} d((x,0), \widetilde{H}^{-1} \circ F_i \circ H(x,0)) < \delta \\ d((x,1), \widetilde{H}^{-1} \circ F_j \circ H(x,1)) < \delta \end{aligned},$$

where $\delta > 0$ is given. Therefore $h_0 = \widetilde{H}^{-1} \circ F_i \circ H(\ ,0) : T^{n-1} \to T^{n-1}$ and $h_1 = \widetilde{H}^{-1} \circ F_j \circ H(\ ,1) : T^{n-1} \to T^{n-1}$ are both isotopic to identity and thus, isotopic between themselves. Let h_s be the isotopy between h_0 and h_1 and define $F : \text{cl}(C) \to \text{cl}(\widetilde{C})$ by $F(q) = \widetilde{H}^{-1} \circ h \circ H^{-1}(q)$, where $h : T^{n-1} \times [0,1] \to T^{n-1} \times [0,1]$, $h(x,s) = (h_s(x), s)$. F is the desired topological equivalence. □

4.4. **The case C^r.** The following lemma is proved in [2, page 74] and will be useful in this section.

Lemma 4.6. *Let $\mathscr{F}$ be a codimension one transversally orientable C^r foliation, $r \geq 1$. Assume that $F \in \mathscr{F}$ is a compact leaf with trivial holonomy, then there exists an open neighborhood V of F saturated by $\mathscr{F}$ and a C^r-diffeomorphism $h : F \times (-1,1) \to V$ such that the leaves of $\mathscr{F}$ inside V are the submanifolds $h(F \times \{t\})$, $t \in (-1,1)$ and $F = h(F \times \{0\})$. In particular, $V \setminus F$ has two connected components.*

Let $\mathcal{O}_0$ be a T^{n-1}-orbit of $\varphi \in A^r(\mathbb{R}^n, N)$ and $\{w_1, \dots, w_{n-1}, w_n\}$ a base of $\mathbb{R}^n$ such that $\{w_1, \dots, w_{n-1}\}$ is a set of generators of its isotropy group G_0. Put $X_i = X_{w_i}$, $i = 1, \dots, n$.

Proposition 4.7. *Let $\varphi \in A^r(\mathbb{R}^n, N)$, $\mathcal{O}_0$ a T^{n-1}-orbit and assume that there exists a neighborhood V_0 of $\mathcal{O}_0$ such that every orbit in V_0 is a T^{n-1}-orbit. Then there exists a diffeomorphism $f : V \to T^{n-1} \times (-1,1)$, where $V \subset V_0$ is a neighborhood of $\mathcal{O}_0$, such that*

$$f_* X_i(\theta, x) = a_{1i}(x) \frac{\partial}{\partial \theta_1} + \dots + a_{(n-1)i}(x) \frac{\partial}{\partial \theta_{n-1}}, \quad i = 1, \dots, n, \tag{4.1}$$

where $\theta = (\theta_1, \dots, \theta_{n-1}) \in T^{n-1}$ and the functions $a_{ij} : T^{n-1} \times (-1,1) \to \mathbb{R}$ are of class C^r, $j = 1, \dots, n-1$.

Proof. By Lemma 4.6 we can assume that $\mathcal{O}_0 = T^{n-1} \times \{0\}$, $V = T^{n-1} \times (-1,1)$ and that $\mathcal{O}_x = T^{n-1} \times \{x\}$ is the orbit of φ by the point $(\theta, x) \in V$. Let H be the vector subspace of $\mathbb{R}^n$ generated by G_0. By Lemma 3.12, there exist C^r functions $w_i : (-1,1) \to H$, $i = 1, \dots, n-1$, such that G_x, the isotropy group of $\mathcal{O}_x$, is generated by $\{w_1(x), \dots, w_{n-1}(x)\}$. For each point $x \in (-1,1)$ there exists a diffeomorphism $f_x : \mathcal{O}_x \to \mathcal{O}_x$ such that $(f_x)_* X_{w_i(x)} = \frac{\partial}{\partial \theta_i}$, $i = 1, \dots, n-1$. Since X_j commutes with $X_{w_i(x)}$, for every $j = 1, \dots, n$ and $i = 1, \dots, n-1$, it follows that

$$(f_x)_* X_j(\theta, x) = \sum_{i=1}^{n-1} a_{ij}(x) \frac{\partial}{\partial \theta_i}, \quad j = 1, \dots, n.$$

Define $f : T^{n-1} \times (-1,1) \to T^{n-1} \times (-1,1)$ by $f(\theta, x) = f_x(\theta)$. Since w_i is a C^r function, we conclude that f is a C^r diffeomorphism such that

$$f_* X_j(\theta, x) = \sum_{i=1}^{n-1} a_{ij}(x) \frac{\partial}{\partial \theta_i}, \quad j = 1, \dots, n.$$

□

Proposition 4.8. *Let $\varphi \in \mathrm{Act}^r(\mathbb{R}^n, N)$. Assume that there exists a φ-invariant neighborhood V_0 of $\mathcal{O}_0$ such that every orbit inside V_0 is a T^{n-1}-orbit. Then, φ can not be locally structurally stable at $\mathcal{O}_0$.*

Proof. By Proposition 4.7 we can assume that $\mathcal{O}_0 = T^{n-1} \times \{0\}$, $V_0 = T^{n-1} \times (-1,1)$ and that the infinitesimal generators have the form

$$X_i(\theta, x) = a_{1i}(x) \frac{\partial}{\partial \theta_1} + \cdots + a_{(n-1)i}(x) \frac{\partial}{\partial \theta_{n-1}}, \quad i = 1, \dots, n.$$

Given $\varepsilon > 0$ there exist $\delta > 0$ and a function $\widetilde{a}_{nn} : (-1,1) \to \mathbb{R}$ such that $\widetilde{a}'_{nn}(0) \neq 0$, $\widetilde{a}_{nn}(x) = 0$ if $x \geq |2\delta|$ and $\|\widetilde{a}_{nn}\|_1 < \varepsilon$. Let $A_{nn}(x) = (a_{ij}(x))$, $i, j = 1, \dots, n-1$, and define for each $k = 1, \dots, n-1$,

$$\widetilde{a}_{kn}(\theta, x) = \widetilde{a}_{nn}(x) \langle [A_{nn}(x)]^{-1} \cdot \alpha'_k(x),\ \theta \rangle + a_{kn}(x),$$

where $\alpha_k(x)$ is the column vector $\left(a_{k1}(x), \dots, a_{k(n-1)}(x)\right)^t$. The vector fields $X_1, \dots, X_{n-1}$ together with

$$\widetilde{X}_n = \begin{cases} \sum_{k=1}^{n-1} \widetilde{a}_{kn}(\theta, x) \frac{\partial}{\partial \theta_k} + \widetilde{a}_{nn}(x) \frac{\partial}{\partial x}, & \text{in } T^{n-1} \times (-1,1) \\ X_n, & \text{otherwise} \end{cases}$$

define an action $\widetilde{\varphi} \in A^r(\mathbb{R}^n, N)$ that can be taken arbitrarily C^1-close of φ by choosing appropriaty ε. Note that $\widetilde{\varphi}$ has a $T^{n-1} \times \mathbb{R}$-orbit inside V_0 and thus can not be topologically equivalent to φ. □

In Proposition 4.8, the fact that $\varphi \in A^r(\mathbb{R}^n, N)$ is crucial. In fact Saldanha in [9] showed that there are structurally stable actions of $\mathbb{R}^2$ on T^3 with all orbits being T^2-orbits.

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J. L. ARRAUT AND C. A. MAQUERA, DEPARTAMENTO DE MATEMÁTICA, ICMC/USP - SÃO CARLOS, CAIXA POSTAL 668, 13560-970 SÃO CARLOS, SP,
E-mail address: arraut@icmc.usp.br
E-mail address: cmaquera@icmc.usp.br

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