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An exponential property of lattices
over group rings

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1. Introduction

Let G be a finite group and let R be a complete rank one discrete valuation ring with prime ideal πR generated by π . Let p be the rational prime with $p \in \pi R$. The exponent of an RG -lattice M is $\exp(M) = \pi^a$ if $\pi^a R$ is the annihilator of the Tate Cohomology $\widehat{\text{Ext}}_{RG}^*(M, M)$. That is π^a is the least power of π such that multiplication by π^a factors through a projective RG -module. We say that an indecomposable lattice M possesses the exponential property (property E) if multiplication by π^{a-1} is in the socle of the ring $\widehat{\text{Ext}}_{RG}^0(M, M)$. Every absolutely irreducible lattice and every lattice whose rank is prime to p has this property.

Most of the results of this paper are based on the following interpretation of the exponential property. Suppose that L and M are RG -lattices such that M is indecomposable, has property E and $\exp(M) = \pi^a$. If γ is a homomorphism from L to M or from M to L and if $\pi^{a-1} \cdot \gamma$ does not factor through a projective then γ is split! Consequently, for a lattice M with property E, there is an easy method for measuring the multiplicity of M as a direct summand in a lattice L . This

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fact combined with Shapiro's Lemma or Frobenius reciprocity yields a simple proof that, for absolutely indecomposable lattices, property E is preserved under the Green correspondence. A more difficult argument shows that it is also preserved under the taking of sources. The proofs rely heavily on the theory of almost split sequences, and in some sense the results can be viewed as extensions of those in [2].

Conditions that are preserved under the Green correspondence and the taking of sources are extremely rare. Reinhard Knörr has also found such a condition which he calls "virtually irreducible" [5]. Picaronny and Puig have shown that Knörr's results follow from a more general theorem [7]. The results of this paper were announced at a recent Oberwolfach meeting at which several informal discussions centered on the question of whether the two conditions were equivalent. Indeed they are. Jacques Thévenaz, who was not at the meeting, had independently discovered property E, proved its equivalence with Knörr's condition and found another verification of the Picaronny-Puig theorem [9]. Knörr has also found a proof of the equivalence [6]. In section 4 of this paper we offer yet another demonstration of the equivalence of the conditions. Our proof is different in spirit, though not in essence, from those of Thévenaz and Knörr, in that it emphasizes the role of the almost split sequences as used throughout the paper. In particular we establish the relationship between the exponent of a lattice M and the image of the trace map on $\text{Hom}_{RG}(M, M)$.

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2. Definitions and Preliminaries

Let R be a complete discrete valuation ring of characteristic zero with prime ideal $\text{Rad } R = \pi R$ generated by a single element π . Suppose that $R/\pi R$ has characteristic $p > 0$. Recall that an RG-lattice is a finitely generated RG-module that is free as an R -module. If L and M are RG-lattices then $\underline{\text{Hom}}(L, M)$ is the torsion R -module

$$\underline{\text{Hom}}(L, M) = \underline{\text{Hom}}_{RG}(L, M) = \text{Hom}_{RG}(L, M)/A$$

where A is the submodule consisting of all $\alpha : L \rightarrow M$ such that α factors through a projective RG-module. Alternatively $\underline{\text{Hom}}(L, M)$ is the zeroth Tate cohomology group $\widehat{\text{Ext}}_{RG}^0(L, M) = \hat{H}^0(G, \text{Hom}_R(L, M))$. Note that for $\gamma \in \text{Hom}_{RG}(L, M)$, $|G| \cdot \gamma$ factors as the composition

$$L \xrightarrow{\sigma} RG \otimes_R L \xrightarrow{\tau} M$$

where $\sigma(\lambda) = \sum_{g \in G} g \otimes \lambda$ and $\tau(g \otimes \lambda) = g\alpha(g^{-1}\lambda) = \alpha(\lambda)$ for $\lambda \in L$ and $g \in G$. Since $RG \otimes_R L$ is a projective RG-module, we have that $|G| \cdot \underline{\text{Hom}}(L, M) = \{0\}$.

The exponent of a torsion R -module T is π^a if π^a is a generator for the annihilator in R of T . If $\alpha \in \text{Hom}_{RG}(L, M)$ then $\exp(\alpha) = \pi^a$ if $\pi^a \cdot \alpha : L \rightarrow M$ factors through a projective RG-module, but $\pi^{a-1} \cdot \alpha$ does not. That is, $\exp(\alpha)$ is a generator for the annihilator of the class of α in $\underline{\text{Hom}}(L, M)$. For an RG-lattice M , $\exp(M) = \exp(\text{Id}_M)$ where Id_M is the identity homomorphism.

For convenience of notation we shall write $\pi^a < \pi^b$ if $a < b$. We begin with a few easy statements. An explanation of the new notation is provided in the proof.

Lemma 2.1. Let L, M and N be RG -lattices.

i) If $\alpha : L \rightarrow M$, $\beta : M \rightarrow N$ are RG -homomorphisms then

$$\exp(\beta\alpha) \leq \exp(\alpha) \quad \text{and} \quad \exp(\beta\alpha) \leq \exp(\beta).$$

ii) $\exp(\text{Hom}(L, M)) \leq \exp(L)$ and $\exp(M)$.

iii) $\exp(L \otimes M) \leq \exp(L)$.

iv) $\exp(M) = \exp(M^*) = \exp(M \otimes M^*) = \exp(\text{Hom}_R(M, M))$
 $= \exp(\Omega^n(M))$ for any n .

Proof: If $\exp(\alpha) = \pi^a$ then $\pi^a(\beta\alpha) = \beta \circ (\pi^a\alpha)$ factors through a projective since $\pi^a\alpha$ does. So $\exp(\beta\alpha) \leq \exp(\alpha)$. The other inequality follows similarly. Statement (ii) follows from (i) and the fact that if $\alpha : L \rightarrow M$ then $\alpha = \text{Id}_M \circ \alpha \circ \text{Id}_L$.

The tensor product $L \otimes M$ is the tensor $L \otimes_R M$ over R with diagonal G -action. So $\text{Id}_{L \otimes M} = \text{Id}_L \otimes \text{Id}_M$. If $\pi^a \text{Id}_L$ factors through a projective P then $\pi^a \text{Id}_{L \otimes M} = (\pi^a \text{Id}_L) \otimes \text{Id}_M$ factors through $P \otimes_R M$ which is projective.

In (iv), M^* is the dual lattice $M^* \cong \text{Hom}_R(M, R)$, where R is the trivial RG -lattice. In general the action of G on $\gamma \in \text{Hom}_R(L, M)$ is defined by $(g\gamma)(\lambda) = g \cdot \gamma(g^{-1}\lambda)$ for all $\lambda \in L$. It can be shown that $\text{Hom}_R(L, M) \cong L^* \otimes M$. Suppose that m_1, \dots, m_t is an R -basis for M and that $\lambda_1, \dots, \lambda_t$ is the dual basis in M^* . The RG -homomorphism $\sigma : M \rightarrow M \otimes M^* \otimes M$, given by $\sigma(m) = \sum_1^t m \otimes \lambda_i \otimes m_i$, is split by the homomorphism $\tau : M \otimes M^* \otimes M \rightarrow M$ where $\tau(m \otimes \lambda \otimes m') = \lambda(m) \cdot m'$. So M is a direct summand of $M \otimes M^* \otimes M$ and by (iii)

$$\exp(M) \leq \exp(M \otimes M^* \otimes M) \leq \exp(M \otimes M^*) \leq \exp(M).$$

The last part of statement (iv) follows by dimension shifting in Tate cohomology. That is, $\Omega^n(M)$, for $n > 0$, is defined to be the image of ∂_n where

$$\dots \longrightarrow P_n \xrightarrow{\partial_n} P_{n-1} \longrightarrow \dots \longrightarrow P_0 \longrightarrow M \longrightarrow 0$$

is a minimal projective resolution of M . For $n < 0$, $\Omega^n(M) \cong (\Omega^{-n}(M^*))^*$. That is, $\Omega^n(M)$ is a lattice with no projective direct summands such that $\Omega^{-n}(\Omega^n(M)) \cong M$. These modules are defined and unique up to isomorphism.

Using the long exact sequence in Tate cohomology it can be seen that

$$\widehat{\text{Ext}}_{\text{RG}}^n(L, M) \cong \widehat{\text{Ext}}_{\text{RG}}^{n-1}(\Omega(L), M) \cong \widehat{\text{Ext}}_{\text{RG}}^{n+1}(L, \Omega(M)) \text{ for all } n.$$

Hence

$$\underline{\text{Hom}}(M, M) \cong \widehat{\text{Ext}}_{\text{RG}}^0(M, M) \cong \widehat{\text{Ext}}_{\text{RG}}^0(\Omega^n(M), \Omega^n(M))$$

and $\exp(M) = \exp(\Omega(M))$ for all n .

What follows relies heavily on the fact that when M is indecomposable then $\underline{\text{Hom}}(M, M)$ is a local ring with maximal ideal $\text{Rad}(\underline{\text{Hom}}(M, M))$. As a module over itself it has a simple socle. If $\exp(M) = \pi^a$ then $\text{Soc}(\underline{\text{Hom}}(M, M)) \subseteq \pi^{a-1} \cdot \underline{\text{Hom}}(M, M)$.

Definition. Let M be an RG-lattice with $\exp(M) = \pi^a \neq 1$. We say that M has property E if M is indecomposable and if

$$\text{Soc}(\underline{\text{Hom}}(M, M)) = \pi^{a-1} \cdot \underline{\text{Hom}}(M, M).$$

Lemma 2.2. If M is an absolutely irreducible RG-lattice then M has property E.

Proof. Let K be the field of quotients of R . Then $K \otimes_R M = V$ is an absolutely irreducible KG -module, and $M \cong 1 \otimes M \subseteq V$ is an RG -submodule of V . Clearly any RG -homomorphism $\theta : M \rightarrow M$ extends to an element of $\text{Hom}_{KG}(V, V) \cong K$. Consequently $\text{Hom}_{RG}(M, M) \subseteq \text{Hom}_{KG}(V, V)$ is an R submodule of K . We conclude that $\text{Hom}_{RG}(M, M) \cong R$ and that $\underline{\text{Hom}}(M, M) \cong R/\text{exp}(M) \cdot R$.

Lemma 2.3. If M has property E then so also does M^* and $\Omega^n(M)$ for all n .

Proof. This follows easily from the facts that

$\underline{\text{Hom}}(M, M) \cong \underline{\text{Hom}}(\Omega^n(M), \Omega^n(M))$ and this ring is anti-isomorphic to $\underline{\text{Hom}}(M^*, M^*)$.

At this point we need to recall a few facts about almost split sequences. The basic references to this material are [1] and [8]. For any indecomposable lattice M there is an exact sequence of RG -lattices

$$D : 0 \longrightarrow \Omega(M) \xrightarrow{\alpha} B \xrightarrow{\beta} M \longrightarrow 0$$

which is called the almost split sequence for M . The sequence has the property that if $\theta : L \rightarrow M$ is a homomorphism which is not a splittable epimorphism then there exist $\sigma : L \rightarrow B$ such that $\beta\sigma = \theta$. Likewise if $\varphi : \Omega(M) \rightarrow L$ is not a splittable monomorphism then there exists a homomorphism $\tau : B \rightarrow L$ such that $\tau\alpha = \varphi$. The middle term B is unique up to isomorphism. An exact sequence of the form as D is an almost split sequence if and only if the class of D in $\text{Ext}_{RG}^1(M, \Omega(M)) \cong \underline{\text{Hom}}(M, M)$ is in the $\underline{\text{Hom}}(M, M)$ -socle.

Theorem 2.4. Suppose that M is an indecomposable RG -lattice with $\text{exp}(M) = \tau^a$. Let

$$0 \longrightarrow \Omega(M) \xrightarrow{\alpha} B \xrightarrow{\beta} M \longrightarrow 0$$

be the almost split sequence of M . The following are equivalent.

- i) M has property E .
- ii) $B \cong \Omega(M/\pi^{a-1}M)$.
- iii) $\exp(B) < \exp(M)$.
- iv) $\exp(\beta) < \exp(M)$.
- v) If $\gamma : L \rightarrow M$ is a homomorphism that is not a splittable epimorphism then $\exp(\gamma) < \exp(M)$.
- vi) If $\gamma : L \rightarrow M$ is a monomorphism that is not an isomorphism then $\exp(\gamma) < \exp(M)$.
- vii) $\exp(\text{Rad Hom}(M, M)) < \exp(M)$.

Proof. Assume (i). Let P be a projective cover of M . Since M has property E , $\pi^{a-1} \cdot \text{Id}_M$ is in the socle of $\text{Hom}(M, M)$. Consequently in the commutative diagram

$$\begin{array}{ccccccc}
 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \\
 0 & \longrightarrow & \Omega(M) & \xrightarrow{\alpha} & B & \xrightarrow{\beta} & M \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \pi^{a-1} \\
 0 & \longrightarrow & \Omega(M) & \longrightarrow & P & \longrightarrow & M \longrightarrow 0 \\
 & & \downarrow & & \downarrow & & \downarrow \\
 & & & & M/\pi^{a-1}M & \xlongequal{\quad} & M/\pi^{a-1}M \\
 & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & &
 \end{array}$$

the top row is the almost split sequence. Here P is a projective cover for M and the top row is the pull-back. Hence (ii) is immediate.

If $B \cong \Omega(M/\pi^{a-1}M)$ then we have an exact sequence

$$0 \longrightarrow B \xrightarrow{i} P \xrightarrow{\theta} M/\pi^{a-1}M \longrightarrow 0.$$

where P is projective. Now $\pi^{a-1}P \subseteq i(B)$ and in turn $\pi^{a-1}B \subseteq i^{-1}(\pi^{a-1}P) \cong P$. It follows that $\pi^{a-1} \cdot \text{Id}_B$ factor through $i^{-1}(\pi^{a-1}P)$ which is projective. Therefore (ii) implies (iii).

That (iii) implies (iv) is a direct consequence of Lemma 2.1.ii.

Furthermore (iv) implies (v) because, by the nature of almost split sequences, any such γ as in (iv) must factor through B . It is clear that (vi) follows from (v). If $\theta : M \rightarrow M$ is an endomorphism that is not an isomorphism, then $\theta(M) \neq M$. Hence θ factors through the inclusion of $\theta(M)$ into M . By (vi) and (2.1.i) $\exp(\theta) < \exp(M)$. So (vii) is a consequence of (vi).

Finally assume (vii). Then multiplication by π^{a-1} is an endomorphism of $\underline{\text{Hom}}(M, M)$ whose kernel is $\text{Rad } \underline{\text{Hom}}(M, M)$ and whose image, $\pi^{a-1} \underline{\text{Hom}}(M, M)$, is a simple $\underline{\text{Hom}}(M, M)$ -module. Therefore M has property E.

Theorem 2.5. With the same hypothesis as Theorem 2.4, the following are equivalent.

- i) M has property E.
- viii) $\exp(\alpha) < \exp(M)$.
- ix) If $\gamma : M \rightarrow L$ is a homomorphism that is not a splittable monomorphism then $\exp(\gamma) < \exp(M)$.

Proof. If M has property E, then (2.4.iii) implies that $\exp(\alpha) < \exp(M)$. Suppose that (viii) holds. If $\mu : \Omega(M) \rightarrow \Omega(M)$ is in $\text{Rad Hom}(\Omega(M), \Omega(M))$, then by the nature of almost split sequences, μ factors through α and $\exp(\mu) \leq \exp(\alpha) < \exp(\Omega(M))$. It follows by theorem 2.4(vii) that $\Omega(M)$ has property E. By Lemma 2.3, $\Omega^{-1}(M)$ has property E, and in the almost split sequence

$$0 \longrightarrow M \xrightarrow{\alpha'} B' \longrightarrow \Omega^{-1}(M) \longrightarrow 0$$

$\exp(\alpha') < \exp(M)$. If $\gamma : M \rightarrow L$ is not a splittable monomorphism then γ factors through α' and $\exp(\gamma) \leq \exp(\alpha') < \exp(M)$. Hence (viii) implies (ix).

An argument similar to that in the last theorem shows that (ix) implies (2.4. vii) and hence also (i).

Remark 2.6. It can be easily seen that if M has property E and $\exp(M) = \pi^a$, then in the notation of the theorems

$$\exp(B) = \exp(\alpha) = \exp(\beta) = \pi^{a-1}.$$

The reason is that if, for example, $\exp(\beta) < \pi^{a-1}$ then $\pi \cdot \text{Id}_M$ which has exponent π^{a-1} could not factor through β .

Corollary 2.7. Suppose that M is an indecomposable RG-lattice that is not isomorphic to a direct summand of the RG-lattice L . If M has property E then $\exp(\text{Hom}(L, M)) < \exp(M)$ and $\exp(\text{Hom}(M, L)) < \exp(M)$.

Proof. This is immediate from 2.4(v) and 2.5(ix).

We say that an RG-lattice is absolutely indecomposable provided $\text{Hom}_{\text{RG}}(M, M) / \text{Rad Hom}_{\text{RG}}(M, M) \cong R/\pi R$. An indecomposable lattice M has multiplicity t in lattice L if L is isomorphic to a direct sum of t copies of M and a lattice U , no direct summand of which is isomorphic to M .

Corollary 2.8. Suppose that M is an absolutely indecomposable lattice with property E, and that $\exp(M) = \pi^a$. For any RG-lattice L , the multiplicity of M in L is equal to

$$\text{Dim}_{R/\pi R} \pi^{a-1} \text{Hom}(L, M) = \text{Dim}_{R/\pi R} \pi^{a-1} \text{Hom}(M, L).$$

Proof. Write $L = V_1 \otimes \dots \otimes V_s$ where each V_i is an indecomposable RG-lattice. Then

$$\underline{\text{Hom}}(L, M) = \sum_{i=1}^s \underline{\text{Hom}}(V_i, M).$$

But $\pi^{a-1} \underline{\text{Hom}}(V_i, M) = \{0\}$ if $V_i \not\cong M$, while $\text{Dim}_{R/\pi} \pi^{a-1} \underline{\text{Hom}}(V_i, M) = 1$ if $V_i \cong M$, by the last corollary and the theorems.

Corollary 2.9. Let M be an indecomposable RG-lattice such that either the rank of M is prime to p or $\exp(M) = \pi^n$ where $|G| \cdot R = \pi^n R$. Then M has property E.

Proof. First note that if p does not divide $r = \text{Rank}_R(M)$ then the trace map

$$\text{Tr} : M \otimes M^* \cong \text{Hom}_R(M, M) \rightarrow R$$

is split by the map $\sigma : R \rightarrow \text{Hom}_R(M, M)$ where $\sigma(b) = b/r \cdot \text{Id}_M$ for $b \in R$. Therefore

$$\exp(M) = \exp(M \otimes M^*) \geq \exp(R) = \pi^n \geq \exp(M).$$

On the other hand if $\exp(M) = \pi^n$ then there exists an element $\alpha \in \text{Hom}_{RG}(R, \text{Hom}_R(M, M)) \cong \text{Hom}_{RG}(M, M)$ with $\exp(\alpha) = \pi^n$. Since R has property E by Lemma 2.2, R is a direct summand of $\text{Hom}_R(M, M)$ and M is a splitting trace lattice [2]. Then by Theorem 3.6 of [2], M has property E. The fact of note is that the middle term (called E in [2]) of the almost split sequence for R is $\Omega(R/\pi^{n-1}R)$ and hence the middle term of the almost split sequence for M is the nonprojective part of the product $\Omega(R/\pi^{n-1}R) \otimes M \cong \Omega(M/\pi^{n-1}M) \otimes (\text{proj.})$

Corollary 2.10. Let L and M be RG-lattices, and suppose that M has property E. Then $\exp(M) = \exp(L \otimes M)$ if and only if M is a direct summand of $L^* \otimes L \otimes M$.

Proof. This is an easy consequence of the fact that $\text{Hom}_{RG}(L \otimes M, L \otimes M) \cong \text{Hom}_{RG}(M, L^* \otimes L \otimes M)$.

3. Green Correspondents and Sources

If M and N are RG-lattices we write $M|N$ to signify that M is a direct summand of N . Let H be a subgroup of G . If L is an RH-lattice then the RG-lattice induced from L is $L^{\uparrow G} = RG \otimes_{RH} L$. An RG-lattice M is relatively RH-projective if $M|L^{\uparrow G}$ for some RH-lattice L .

Lemma 3.1. Let H be a subgroup of G , L an RH-lattice and M an RG-lattice.

- $\exp(L^{\uparrow G}) = \exp(L)$.
- $\exp(M_H) \leq \exp(M)$.
- If $L|M_H$ and M has property E then $\exp(L) = \exp(M)$ if and only if $M|L^{\uparrow G}$.
- If $M|L^{\uparrow G}$ and L has property E then $\exp(M) = \exp(L)$ if and only if $L|M_H$.

Proof. All of these are applications of Frobenius reciprocity or Shapiro's Lemma which says that

$$\text{Hom}_{RG}(M, L^{\uparrow G}) \cong \text{Hom}_{RH}(M_H, L). \quad (3.2)$$

In (a) let $M = L^{\uparrow G}$ and note that $L | (L^{\uparrow G})_H$. So the exponent of the right side of (3.2) is $\exp(L)$. In (b) let $L = M_H$ and note that the left side of (3.2) has exponent at most equal to $\exp(M)$. Equation (3.2) and the theorems of the previous section imply (c) and (d) directly.

A vertex for an indecomposable RG-lattice M is a minimal p -subgroup Q of G such that $M | L^{\uparrow G}$ for some RQ -lattice L . Suppose that Q is a vertex for M and that $N_G(Q) \subseteq H \subseteq G$. The H -Green correspondent of M is an indecomposable U with vertex Q such that $M | U^{\uparrow G}$. Note that U is a unique up to isomorphism. Conversely if U is an RH -lattice with vertex Q then there is a unique RG -lattice M with vertex Q such that $U | M_H$. In this situation

$$M_H = U \oplus U' \quad \text{and} \quad U^{\uparrow G} = M \oplus M'$$

where $U | U'$ and $M | M'$. Details and proofs can be found in [4].

Theorem 3.3. Let M be an indecomposable RG -lattice with vertex Q . Suppose that $N_G(Q) \subseteq H \subseteq G$ and that L is an H -Green correspondent for M . Assume that L and M are absolutely indecomposable. Then M has property E if and only if L has property E .

Proof. Suppose that $\pi^a = \exp(M) = \exp(L)$. By (3.2) we have

$$\text{Dim}_{R/\pi R} \pi^{a-1} \underline{\text{Hom}}_{RG}(M, L^{\uparrow G}) = \text{Dim}_{R/\pi R} \pi^{a-1} \underline{\text{Hom}}_{RH}(M_H, L) \quad (3.4)$$

By (2.8), if M has property E , then the left side of (3.4) has value 1.

Since $\underline{\text{Hom}}_{RH}(L, L)$ is a direct summand of $\underline{\text{Hom}}_{RH}(M_H, L)$ and $\exp(L) = \pi^a$, we have that $\pi^{a-1} \underline{\text{Hom}}_{RH}(L, L) = R/\pi R$ if M has property E .

But this implies that L has property E . Similarly if L has property E then the right side of (3.4) is 1, and M must also have property E .

Remark 3.5. The proof of the theorem actually implies that if one of L or M is absolutely indecomposable and has property E then the other must be absolutely indecomposable in addition to having property E .

Let M be an RG -lattice with vertex Q . A Q -source for M is an RQ -lattice V such that $V \mid M_Q$ and $M \mid V \uparrow^G$. Any two Q -sources for M are G -conjugate. That is if V and V' are Q -sources for M then there exists $x \in N_G(Q)$ such that $V' \cong x \otimes V$.

Theorem 3.5. Suppose that M is an RG -lattice with vertex Q . Let L be an RQ -lattice which is a source for M . Assume that L and M are absolutely indecomposable. If M has property E then so also does L .

The proof proceeds by several steps which we number for the reader's convenience.

Step I. We may assume that $Q \triangleleft G$. If Q is not normal in G , then replace G by $H = N_G(Q)$ and replace M by its H -Green correspondent N . Since M has property E , so also does N . Moreover L is also a source for N .

Step II. Suppose that N is an RQ -lattice such that $\exp(\underline{\text{Hom}}(N, L)) = \exp(L)$ or $\exp(\underline{\text{Hom}}(L, N)) = \exp(L)$. Then $N \cong x \otimes L$ for some $x \in G = N_G(Q)$.

Assume that $\exp(\underline{\text{Hom}}_{RQ}(N, L)) = \exp(L)$. Note that by (3.1), $\exp(L) = \exp(M)$. So

$$\exp(\underline{\text{Hom}}_{\text{RG}}(N^{\uparrow G}, M)) = \exp(\underline{\text{Hom}}_{\text{RQ}}(N, M_Q)) = \exp(M) .$$

Because M has property E , $M|N^{\uparrow G}$ and N must also be a source of M .

But this says that $N \cong x \otimes L$ for some $x \in G$.

We will require the following general result.

Lemma 3.6. Suppose that $0 \rightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \rightarrow 0$ is an exact sequence of RG-lattices. If $B \cong A \otimes C$ then the sequence splits.

Proof. The long exact sequence on Tate cohomology implies that

$$\dots \rightarrow \underline{\text{Hom}}(C, A) \xrightarrow{\alpha_*} \underline{\text{Hom}}(C, B) \xrightarrow{\beta_*} \underline{\text{Hom}}(C, C) \rightarrow \dots$$

is an exact sequence of finitely generated torsion R-modules. Hence these modules have finite composition length. If $B \cong A \otimes C$ then

$$\underline{\text{Hom}}(C, B) \cong \underline{\text{Hom}}(C, A) \otimes \underline{\text{Hom}}(C, C) ,$$

and the composition length of the image of β_* is at least as large as that of $\underline{\text{Hom}}(C, C)$. Consequently β_* must be surjective. This implies the existence of a homomorphism $\theta: C \rightarrow B$ such that $\beta\theta - \text{Id}_C$ factors through a projective. An easy homological argument completes the proof of the lemma.

Returning to the proof of Theorem 3.5, we assume that L does not have property E . Let $0 \rightarrow \Omega(L) \xrightarrow{\alpha} W \xrightarrow{\beta} L \rightarrow 0$ be the almost split sequence for L . Then by Theorem 2.4, $\exp(\beta) = \exp(L)$, and by Step II, $W \cong (x \otimes L) \otimes U$ for some $x \in G$ and some RG-lattice U . Also we have that $\exp(\alpha) = \exp(L)$ and $W \cong \Omega(y \otimes L) \otimes V \cong (y \otimes \Omega(L)) \otimes V$ for some $y \in G$ and some RQ-lattice V . We claim that $\Omega(L) \cong z \otimes L$ for some $z \in G$. Otherwise, by the Krull-Schmidt theorem and a rank argument, the almost split sequence for L has the form

$$0 \rightarrow \Omega(L) \xrightarrow{\alpha} (y \otimes \Omega(L)) \oplus (x \otimes L) \xrightarrow{\beta} L \rightarrow 0 \quad (3.7)$$

for some $x, y \in G$. Then inducing to G we get the sequence

$$0 \rightarrow \Omega(L)^{\uparrow G} \rightarrow \Omega(L)^{\uparrow G} \oplus L^{\uparrow G} \rightarrow L^{\uparrow G} \rightarrow 0 \quad (3.8)$$

since $(x \otimes L)^{\uparrow G} \cong L^{\uparrow G}$. This sequence must split by the previous lemma. However this is impossible because sequence (3.7) is a direct summand of the restriction to Q of (3.8). More generally the technique proves the following.

Step III. If L does not have property E , then the almost split sequence for L has the form

$$0 \rightarrow L_1 \xrightarrow{\alpha} L_2 \oplus U \xrightarrow{\beta} L \rightarrow 0 \quad (3.9)$$

where $L_1 \cong \Omega(L) \cong z \otimes L$, $L_2 \cong x \otimes L$ for some $x, z \in G$ and $U \cong g \otimes L$ for any $g \in G$.

The point here is that if U were isomorphic to $g \otimes L$ for some $g \in G$ then the sequence (3.9) when induced to G would split as before.

Step IV. If L does not have property E then in (3.9), $L_1 \cong L_2 \cong L$.

The assumption that L does not have property E implies that $\text{Dim}_{R/\pi R} \pi^{a-1} \text{Hom}(L, L) \geq 2$ where $\pi^a = \exp(L)$. That is, there exists an endomorphism $\gamma: L \rightarrow L$ such that γ is not an isomorphism but $\exp(\gamma) = \pi^a$. Then γ factors as $\gamma = \beta\theta$ for some $\theta: L \rightarrow L_2 \oplus U$. Let

$$\begin{aligned} \rho_1: L_2 \oplus U &\rightarrow U & ; & & i_1: U &\rightarrow L_2 \oplus U & ; \\ \rho_2: L_2 \oplus U &\rightarrow L_2 & ; & & i_2: L_2 &\rightarrow L_2 \oplus U & . \end{aligned}$$

be the projection and injection maps such that $\rho_1 i_1 = \text{Id}_U$, $\rho_2 i_2 = \text{Id}_{L_2}$. $i_1 \rho_1 + i_2 \rho_2 = \text{Id}_{(L_1 \oplus U)}$. Then $\gamma = \beta \theta = \beta i_1 \rho_1 \theta + \beta i_2 \rho_2 \theta$ and since $\exp(\rho_1 \theta) < \pi_a = \exp(\gamma)$ we must have that $\exp(\beta i_2) = \exp(\rho_2 \theta) = \pi_a$. Now suppose that $L_2 \neq L$. Then $\rho_2 \theta$ is not an isomorphism, and it is not an epimorphism. The almost split sequence for L_2 has the form

$$0 \rightarrow L_3 \xrightarrow{\alpha'} L_1 \oplus U' \xrightarrow{\beta'} L_2 \rightarrow 0$$

where it can be assumed that $\beta'|_{L_1} = \rho_2 \alpha$ and $U' \neq g \oplus L$ for any $g \in G$. This is because $\rho_2 \alpha$ is an irreducible map and so it must occur in this sequence [1]. Since $\rho_2 \theta$ is not an epimorphism, there exist $\sigma: L \rightarrow L_1 \oplus U'$ such that $\beta' \sigma = \rho_2 \theta$. That is, we have a commutative diagram

$$\begin{array}{ccccc}
 & & & L & \\
 & & & \swarrow & \searrow \\
 & & \phi & & \gamma \\
 & & \swarrow & \searrow & \\
 (L_1 \oplus U') \oplus U & \xrightarrow{(\beta', 1)} & L_2 \oplus U & \xrightarrow{\beta} & L
 \end{array}$$

where $\phi = (\sigma, \rho_1 \theta)$. Since $\exp(\gamma) = \pi^a = \exp(L)$ we must have that $\beta(\beta', 1)|_{L_1}: L_1 \rightarrow L$ has exponent π^a . But this map is the same as $\beta i_2 \rho_2 \alpha: L_1 \rightarrow L$, and we know that $\beta i_2 \rho_2 \alpha + \beta i_1 \rho_1 \alpha = \beta \alpha = 0$. This requires that $\beta i_1 \rho_1 \alpha$ also have exponent π^a . However $\exp(\beta i_1) < \pi^a$ and we have a contradiction. Therefore $L_2 = L$. The same analysis on the sequence (3.10) proves that $L_1 = L_2$.

We can now complete the proof of Theorem 3.5. Wiedemann has shown that if Λ is an order such that some Λ -lattice has an almost split sequence as in Steps III and IV, then Λ is Morita equivalent to a Bass order in a separable skewfield. Hence Λ could not be a group ring. So L must have property Ξ .

Corollary 3.11. Let M be an RG -lattice with vertex Q and Q -source L . Suppose that M and L are absolutely indecomposable. Then the multiplicity of M in $L^{\uparrow G}$ is the same as the multiplicity of L in M_Q .

Proof. The corollary follows directly from the theorem, Corollary 2.8 and the isomorphism (3.4) with $H = Q$.

Remark 3.12. Actually this technique proves a somewhat stronger result. That is, if $H \subseteq G$, and if L and M are respectively RH - and RG -lattices that are absolutely indecomposable and have property E , then the multiplicity of M in $L^{\uparrow G}$ is the same as the multiplicity of L in M_H .

In general the converse of Theorem 3.5 is not true. That is, the source of a lattice may have property E even though the lattice itself does not. This principal is illustrated in the following.

Proposition 3.13. Let Q be a normal subgroup of a p -group G . Suppose that L is an RQ -lattice which is absolutely indecomposable and has property E . Then $L^{\uparrow G}$ has property E if and only if $x \otimes L \neq L$ for all $x \in G$, $x \notin Q$.

Proof. First note that $L^{\uparrow G}$ is absolutely indecomposable by Green's indecomposability theorem [4]. So $L^{\uparrow G}$ has property E if and only if

$$1 = \dim_{R/\pi R} \pi^{a-1} \text{Hom}_{RG}(L^{\uparrow G}, L^{\uparrow G}) = \dim_{R/\pi R} \pi^{a-1} \text{Hom}_{RQ}(L, L^{\uparrow G})$$

where $\pi^a = \exp(L) = \exp(L^{\uparrow G})$. But $L^{\uparrow G}|_Q \cong \sum x \otimes L$ where the sum is over a complete set of representatives of the left cosets of Q in G .

4. Property E and Virtually Irreducible Lattices

As mentioned in the introduction, in this section we give another verification that property E is equivalent to the virtually irreducible property of Knörr [5]. All such verifications ultimately depend on some form of a duality on the cohomology of lattices. The one which we choose here is the standard Tate duality (see [3]). We should point out that several of the results of this paper could have been proved directly from the duality. Indeed it is an essential ingredient in the proof of the existence of almost split sequences.

The duality asserts that there is a nondegenerate bilinear pairing

$$\hat{H}^m(G, L) \otimes_R \hat{H}^{-m}(G, L^*) \rightarrow R/|G| \cdot R$$

for any RG-lattice L and any m . The map is defined as the composition

$$\hat{H}^m(G, L) \otimes_R \hat{H}^{-m}(G, L^*) \rightarrow \hat{H}^0(G, L \otimes L^*) \rightarrow \hat{H}^0(G, R) \cong R/|G| \cdot R$$

of the cup product with the homomorphism arising from the RG-homomorphism

$$\theta: L \otimes L^* \rightarrow R \text{ that takes } \mu \otimes \lambda \text{ to } \lambda(\mu) \text{ for } \mu \in L \text{ and } \lambda \in L^* .$$

Note that under the isomorphism $L \otimes L^* \cong \text{Hom}_R(L, L)$, θ becomes the trace map. In particular if $m = 0$ and $L \cong \text{Hom}_R(M, M)$ for M an RG-lattice, then $\hat{H}^0(G, \text{Hom}_R(M, M)) = \underline{\text{Hom}}(M, M)$, and it can be checked that the duality

$$\underline{\text{Hom}}(M, M) \otimes \underline{\text{Hom}}(M, M) \rightarrow R/|G| \cdot R$$

is given by $\text{cls}(\alpha) \otimes \text{cls}(\beta) \rightarrow \text{Tr}(\alpha\beta) + |G|R$ for $\alpha, \beta \in \text{Hom}_{RG}(M, M)$.

Proposition 4.2. Let M be an RG-lattice. Then

$$\exp(M) \cdot \text{Tr}(\text{Hom}_{RG}(M, M)) = |G| \cdot R .$$

That is if $|G|R = \pi^n R$ and if $\text{Tr}(\text{Hom}_{RG}(M, M)) = \pi^b R$ then $\exp(M) = \pi^{n-b}$.

Proof. The trace map $\text{Tr}: \text{Hom}_R(M, M) \rightarrow R$ defines a homomorphism

$$\text{Hom}(M, M) \cong \hat{H}^0(G, \text{Hom}_R(M, M)) \xrightarrow{\text{Tr}_*} \hat{H}^0(G, R) \cong R/|G| \cdot R$$

where $\text{Tr}_*(\text{cls}(\alpha)) = \text{Tr}(\alpha) + |G| \cdot R$, for $\alpha \in \text{Hom}_{RG}(M, M)$. If $\exp(M) = \pi^a$ then $\text{Tr}_*(\pi^a \cdot \text{cls}(\alpha)) = \pi^a \text{Tr}(\alpha) + |G| \cdot R = 0 + |G| \cdot R$. Hence $\pi^a \geq \pi^{n-b}$.

On the other hand, the nondegeneracy of the pairing (4.1) requires that there exist some $\beta \in \text{Hom}_{RG}(M, M)$ such that

$$\text{Tr}_*(\pi^{a-1} \text{cls}(\beta)) = \pi^{a-1} \text{Tr}(\beta) + |G| \cdot R \neq |G| \cdot R.$$

Therefore $\pi^{a-1} \text{Tr}(\beta) \cdot R = \pi^{n-1} R$. Hence $\text{Tr}(\beta) \cdot R = \pi^{n-a} R$. This proves the proposition.

Proposition 4.3 ([9], [6]). Let M be an indecomposable lattice.

Then M has property E if and only if

$$\text{Tr}(\text{Rad Hom}_{RG}(M, M)) \neq \text{Tr}(\text{Hom}_{RG}(M, M)). \quad (4.4)$$

If this is the case and if M is absolutely irreducible then $\text{Rank}_R(M) \cdot R = \pi^{n-a} R$ where $\exp(M) = \pi^a$ and $\pi^n R = |G| \cdot R$. (A virtually irreducible lattice is defined to be one that is absolutely indecomposable and satisfies condition (4.4).)

Proof. Consider the diagram

$$\begin{array}{ccccccc} & & & & \text{Hom}_R(M, M) & & \\ & & & & \downarrow \hat{\gamma} & & \\ 0 & \longrightarrow & \Omega(R) & \longrightarrow & B & \xrightarrow{\beta} & R \longrightarrow 0 \\ & & \parallel & & \downarrow & & \downarrow \pi^{a-1} \\ 0 & \longrightarrow & \Omega(R) & \longrightarrow & P & \longrightarrow & R \longrightarrow 0 \end{array} \quad (4.5)$$

where P is a projective cover for R , the upper row is the pull back of the lower along π^{a-1} and $B \cong \Omega(R/\pi^{a-1}R)$. Following the argument in [7], we have a commutative diagram

$$\begin{array}{ccc}
 \text{Hom}_{RG}(\text{Hom}_R(M,M), B) & \xrightarrow{B_*} & \text{Hom}_{RG}(\text{Hom}_R(M,M), R) \\
 \downarrow \varphi_1 & & \downarrow \varphi_2 \\
 \text{Hom}_{RG}(M, M \otimes B) & \xrightarrow{(1 \otimes \beta)_*} & \text{Hom}_{RG}(M, M)
 \end{array} \quad (4.6)$$

where φ_1 and φ_2 are natural isomorphisms. Considering the tensor product of (4.5) with M , we have the exact sequence

$$0 \rightarrow M \otimes \Omega(R) \rightarrow M \otimes B \xrightarrow{1 \otimes \beta} M \rightarrow 0 \quad (4.7)$$

where $M \otimes \Omega(R) \cong \Omega(M) \otimes Q$ and $M \otimes B \cong (M/\pi^{a-1}M) \otimes Q'$ for some projective modules Q and Q' . Hence from Theorem 2.4 we get that M has property E if and only if (4.7) is the almost split sequence for M modulo the projectives Q and Q' . Hence M has property E if and only if

$$(1 \otimes \beta)_*(\text{Hom}_{RG}(M, M \otimes B)) = \text{Rad}(\text{Hom}_{RG}(M, M)).$$

For $\gamma \in \text{Hom}_{RG}(M, M)$, $\varphi_2^{-1}(\gamma) = \hat{\gamma}$ where $\hat{\gamma}(f) = \text{Tr}(\gamma \circ f)$ for $f \in \text{Hom}_R(M, M)$. Let $\sigma: R \rightarrow \text{Hom}_R(M, M)$ by $\sigma(r) = r \cdot \text{Id}_M$ for $r \in R$. Hence if γ is in the image of $(1 \otimes \beta)_*$, then $\pi^{a-1} \hat{\gamma} \sigma: R \rightarrow R$ factors through P (see (4.5)). But $\hat{\gamma} \sigma(1) = \text{Tr}(\gamma \circ \text{Id}_M) = \text{Tr}(\gamma)$. Therefore if γ is in the image of $(1 \otimes \beta)_*$ then $\pi^{a-1} \hat{\gamma} \sigma$ factors through a projective and its image must be contained in $|G| \cdot R$. It follows that M has property E if and only if $\pi^{a-1} \text{Tr}(\text{Hom}_{RG}(M, M))$ is in $|G| \cdot R$. So in this case

$$\exp(M) \text{Tr}(\text{Rad Hom}_{RG}(M, M)) \subseteq \pi |G| \cdot R. \quad (4.8)$$

The first statement of the proposition is proved by comparing the above to Proposition 4.2.

Now suppose that M has property E and is absolutely indecomposable.

If $\gamma \in \text{Hom}_{RG}(M, M)$, then $\gamma = k \cdot \text{Id}_M + \theta$ for some $k \in R$ and

$\theta \in \text{Rad Hom}_{RG}(M, M)$. Moreover γ is an isomorphism if and only if

$k \notin \text{Rad } R$. By Proposition 4.2 and (4.8) there exists some isomorphism γ such that

$$\begin{aligned} \exp(M) \cdot \text{Tr}(\gamma) \cdot R &= \exp(M) \cdot \text{Tr}(\text{Id}_M)R \\ &= \exp(M) \cdot \text{Rank}_R(M) \cdot R \subseteq \pi|G|R \end{aligned}$$

This completes the proof.

Remark 4.9. Corollary 2.9 follows easily from propositions 4.2, 4.3 and the simple fact that $\text{Tr}(\text{Rad Hom}_{RG}(M, M)) \subseteq \pi R$ for an indecomposable RG -lattice M .

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