



# Theoretical and experimental constraints for the equation of state of dense and hot matter

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## Abstract

This review aims at providing an extensive discussion of modern constraints relevant for dense and hot strongly interacting matter. It includes theoretical first-principle results from lattice and perturbative QCD, as well as chiral effective field theory results. From the experimental side, it includes heavy-ion collision and low-energy nuclear physics results, as well as observations from neutron stars and their mergers. The validity of different constraints, concerning specific conditions and ranges of applicability, is also provided.

**Keywords** Multi-messenger physics · Neutron star · Dense matter · Heavy-ion collisions

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## 1 Introduction

Depending on conditions (thermodynamic variables), such as temperature and density, matter can appear in many forms (phases). Typical phases include solid, liquid, and gas; but many others can exist, such as plasmas, condensates, and superconducting phases (just to name a few). How matter transitions from one phase to another can also take many forms. A first-order phase transition is how water typically changes from solid to liquid or liquid to gas wherein the phase transition happens at a fixed temperature, free energy, and pressure, which leads to dramatic changes in certain thermodynamic properties (e.g., a jump in the density). At extremely large temperatures and pressures, for water, a crossover phase transition is reached between the liquid and gas phases: depending on what thermodynamic observable one looks at, the substance could look more like a liquid or a gas. In other words, the phase transition no longer takes place at a fixed temperature, free energy, and pressure, but rather across a range of them. Finally, bordering these two regimes, there exists a critical point that separates a crossover phase transition from a first-order one. To describe these different phases of matter, one requires an equation of state (EoS) that depends on the thermodynamic variables of the system. One should note, however, that the EoS is an equilibrium property, and, of course, out-of-equilibrium effects can also be quite relevant. For instance, imagine a body of water that is flowing and being cooled at the same time. In such a dynamical system, one also requires information about the transport coefficients in order to properly describe its behavior as it freezes.

In this work, we will concern ourselves with phases of matter that appear at high energy, relevant when studying the strong force. This is the force that binds together the nucleus, and leads to the generation of 99% of the visible matter in the universe. The theory that governs the strong force is quantum chromodynamics (QCD, Gross and Wilczek 1973; Politzer 1973). QCD describes the interactions of the smallest building blocks of matter (quarks and gluons). Quarks and gluons are normally not free (or “deconfined”) in Nature, but rather confined within hadrons. The latter comprise mesons (quark anti-quark pairs  $q\bar{q}$ ), baryons (three-quark states  $qqq$ ), or anti-baryons (three anti-quark states  $\bar{q}\bar{q}\bar{q}$ ).<sup>1</sup> The quark content and their corresponding quantum numbers (see Table 1) yield the quantum numbers of the hadron itself. One can calculate the thermodynamic properties of strongly interacting matter using either lattice QCD in the non-perturbative regime, or perturbative QCD (pQCD) where the coupling is small (high temperatures and/or extremely high densities).

Protons ( $uud$  quark state), neutrons ( $udd$  quark state), and, in rare cases, hyperons (baryons with strange quark content) form nuclei, the properties of which depend on the number of nucleons  $A$ , as well as the number of protons  $Z$  and the number of neutrons  $A - Z$  within the nucleus.<sup>2</sup> In principle, QCD also drives the properties of nuclei. However, in the vast majority of cases, it would not be convenient to calculate

<sup>1</sup> Several pentaquark ( $qqqq\bar{q}$ ) and tetraquark ( $qq\bar{q}\bar{q}$ ) states have also been observed in the past two decades but are not directly relevant to this work; we refer the reader to Chen et al. (2023) for an extended review of this topic.

<sup>2</sup> In the rare case of hypernuclei, one must also consider the number of hyperons  $H$  such that the total number of neutrons within the nucleus is  $A - Z - H$ .

**Table 1** Summary table of quarks including flavor, mass, and quantum numbers (Workman et al. 2022)

Flavor	Mass (MeV)	Charge (e)	Baryon number	Spin	Isospin (z-projection)	Strangeness
Up ( <i>u</i> )	$2.16^{+0.49}_{-0.26}$	$+\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2} (+\frac{1}{2})$	0
Down ( <i>d</i> )	$4.67^{+0.48}_{-0.17}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{2} (-\frac{1}{2})$	0
Strange ( <i>s</i> )	$93.4^{+8.6}_{-3.4}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	-1
Charm ( <i>c</i> )	$1270 \pm 20$	$+\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	0
Bottom ( <i>b</i> )	$4180^{+30}_{-20}$	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	0
Top ( <i>t</i> )	$1.73 \times 10^5 \pm 300$	$+\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	0	0

the properties of nuclei or nuclear matter (beyond densities and temperatures at which nuclei dissolve into a soup of hadrons) directly from the Lagrangian of QCD, both because of the numerical challenges but also because it would not be the most effective way (it would be akin to calculating the properties of a lake from the microscopic interactions of H<sub>2</sub>O molecules). The objective of this review article is to put together the constraints derived from fundamental theories (that are gauge invariant and renormalizable) and observations. For this reason, we do not discuss relativistic mean-field models. We also opted not to incorporate the different approaches used to describe nuclear matter, and refer instead to an excellent review on this subject (Oertel et al. 2017). Two different approaches are generally used to obtain the equation of state of nuclear matter from these constraints: ab-initio many-body methods using realistic interactions (these include Green Function methods, variational and Monte Carlo methods, (Dirac)–Brueckner–Hartree–Fock calculations and an example is the well known Akmal, Pandharipande, and Ravenhall EoS (Akmal et al. 1998)) or phenomenological approaches based in density functional theories applying effective interactions, including relativistic mean-field models with meson exchange forces and non-relativistic Skyrme and Gogny forces, see Oertel et al. (2017) for a review. Using these methods, one can calculate thermodynamic quantities at low temperatures (on the MeV scale) and around nuclear saturation density,  $n_{\text{sat}}$ , which represents the point on the saturation curve where the binding energy per nucleon in a nucleus is at its lowest, indicating a balance between the attractive and repulsive nuclear forces, therefore, maximal stability within the nuclear system (Haensel et al. 1981).

How can we solve QCD and study nuclear matter theoretically? How can we probe QCD and nuclear matter experimentally? What systems in Nature and in the laboratory are sensitive to quarks and gluons, hadrons, or nuclei? At large temperatures and vanishing net baryon densities  $n_B = 0$  (i.e., the same amount of baryons/quarks and anti-baryons/anti-quarks), the conditions are the same as those of the early universe and can be reproduced in the laboratory, at the Large Hadron Collider (LHC, Citron et al. 2019) and at the Relativistic Heavy Ion Collider (RHIC, STAR Collaboration 2014; Cebra et al. 2014) for top center-of-mass beam energies  $\sqrt{s_{NN}} = 200$  GeV. In equilibrium, lattice QCD can be used to calculate the EoS,

which can be extended to finite  $n_B$  using expansion schemes up to baryon chemical potentials (over temperature) of about  $\mu_B/T \sim 3.5$ . Medium to low energy RHIC collisions explored in the Beam Energy Scan (BES) phase I and II ( $\sqrt{s_{NN}} = 7.7 - 200$  GeV in collider mode), as well as existing and future fixed target experiments at RHIC (STAR Collaboration 2014; Cebra et al. 2014), SPS (Pianese et al. 2018), HADES (Galatyuk 2014, 2020), and FAIR (Fries 2006; Tahir et al. 2005; Lutz et al. 2009; Durante et al. 2019) can reach temperatures in the range  $T \sim 50 - 350$  MeV and baryon chemical potentials  $\mu_B \sim 20 - 800$  MeV, using a range of center of mass beam energies  $\sqrt{s_{NN}} \sim 2 - 11$  GeV. Therefore, these low-energy experiments provide a significant amount of information that can also be used to infer the EoS (Dexheimer et al. 2021c; Lovato et al. 2022b; Sorensen et al. 2024). However, these systems are probed dynamically and may be far from equilibrium, so one must not consider the EoS extracted from heavy-ions as data in the typical sense, but rather as a posterior model that is sensitive to priors and systematic uncertainties that may exist in that model. In the high temperature and/or chemical potential limit, systematic methods such as perturbative resummations can be used to calculate the EoS analytically directly from the QCD Lagrangian.

Low-energy nuclear experiments provide methods to extract key properties of nuclei. Most stable nuclei are composed of “isospin-symmetric nuclear matter”, i.e.  $Z = 0.5 A$ , such that the number of protons and neutrons are equal in the nucleus. For simplicity, one defines the charge fraction  $Y_Q = Z/A$ , which can also be related to the charge density  $n_Q$  (assuming a system of only hadrons, no leptons) over the baryon density  $n_B$  such that  $Y_Q = n_Q/n_B$  as well. Then, for symmetric nuclear matter  $Y_Q = 0.5$  and this is where most nuclear experiments provide information. However, heavy nuclei do become more neutron rich, such that  $Y_Q \sim 0.4$ . Note that, for the highest energies, heavy-ion experiments only probe  $Y_Q = 0.5$  as the nuclei basically pass through each other, and the fireball left behind cannot create net isospin ( $Y_Q \neq 0.5$ ) or strangeness ( $Y_S = S/A = n_s/n_B \neq 0$ ) during the very brief time of the collision (on the order of  $\sim 10$  fm/c or  $10^{-23}$  s).

All thermodynamic properties change as  $Y_Q$  varies. This can be measured experimentally in low-energy nuclear experiments around  $n_{\text{sat}}$  through the determination of the symmetry energy  $E_{\text{sym}}$ , which can be approximated as the difference between the energy per nucleon of  $Y_Q = 0$  (pure neutron matter  $E_{\text{PNM}}$ ) and  $Y_Q = 0.5$  matter (symmetric nuclear matter  $E_{\text{SNM}}$ )<sup>3</sup>

$$E_{\text{sym}} \equiv \frac{E_{\text{PNM}} - E_{\text{SNM}}}{N_B}. \quad (1)$$

The baryon number  $N_B$  is more comprehensive than  $A$ , as it also includes quarks, with  $N_B = 1/3$ . At saturation density, many other quantities can be determined such as the binding energy per nucleon, or the (in)compressibility of matter, in addition to  $n_{\text{sat}}$  itself. At small  $Y_Q$ , matter in neutron stars provides information about both

<sup>3</sup> The general definition of the symmetry energy is  $E_{\text{sym}} \equiv \frac{1}{2} \partial^2 (E/N_B) / \partial \beta^2$ , where  $E/N_B$  is the energy per baryon and  $\beta \equiv (n_n - n_p)/(n_n + n_p)$  in terms of neutron stars or  $\beta = 1 - 2 Y_Q$  (Bombaci and Lombardo 1991; Haensel 1977; Muller-Kirsten et al. 1999). Equation (1), commonly found in the literature, only shows terms up to second order in the expansion.

nuclear and QCD matter at low temperatures and medium-to-high densities. Matter in this case is necessarily charge-neutral, as  $Y_Q = Y_{lep}$ , the charge fraction of leptons (electrons and muons). On the other hand, weak(-force) equilibrium ensures  $\mu_Q = -\mu_e = -\mu_\mu$ , meaning that the charge chemical potential, the difference between the chemical potential of protons and neutrons (in the absence of hyperons), or up and down quarks, equals the ones of electrons and muons.

At saturation densities, a neutron star's internal composition is primarily made up of nucleons and leptons. However, as the density increases, other baryonic species may appear due to the rapid rise in baryon chemical potential associated with a higher density and reduce the ground state energy of the dense nuclear matter phase by opening new Fermi channels. Due to the long time-scales involved (when compared to weak interactions), matter in neutron stars can also include particles with net strangeness, hyperons. Here on Earth, hyperons can be produced but are unstable and quickly decay in  $\sim 10^{-8}$  seconds via weak interactions into protons and neutrons. In the high density regime in the core of neutron stars, hyperons cannot decay back to nucleons due to Pauli blocking, meaning that producing additional nucleons would increase the energy of the system (Joglekar et al. 2020; Blaschke et al. 2020). However, the appearance of hyperons softens the EoS of dense matter and lowers the maximum mass  $M_{\max}$  of neutron stars predicted by a given theory, which is incompatible with the observations of massive stars, see Sect. 8. This mismatch between experimental observations and theoretical calculations is referred to as the hyperon puzzle (Bednarek et al. 2012; Buballa et al. 2014). To make them compatible, additional repulsion is needed in the theory so that the EoS becomes stiffer. This additional effect can be introduced through the following known mechanisms, (i) hyperon-hyperon interaction via exchange of short-range vector mesons (Rijken and Schulze 2016), (ii) three body repulsive hyperonic force (Lonardoni et al. 2015; Gerstung et al. 2020; Logoteta et al. 2019), (iii) higher-order vector interactions (Bodmer 1991; Dexheimer et al. 2021a), (iv) excluded volume for hadrons (Hagedorn 1983; Dexheimer et al. 2013), and (iii) a phase transition to quark matter at a density less than or around the hyperon threshold (Vidana et al. 2005).

On the other hand, the generation of heavier non-strange baryons (resonances) in the core of neutron stars is still an open question (Weissenborn et al. 2012). Initially (Glendenning 1985), it was argued that resonances appear at much higher densities beyond the density of a neutron star core and, thus, they are not relevant for nuclear astrophysics. Nevertheless, an early appearance of  $\Delta$ -baryons at  $2\text{--}3\ n_{\text{sat}}$  was obtained in several works (Schürhoff et al. 2010; Drago et al. 2014; Li et al. 2018; Marquez et al. 2022). It was shown that, due to the isospin rearrangement that takes place when the  $\Delta$ 's appear, they do not produce an effect analogous to the hyperon puzzle and are able to replace baryons without clashing with  $M_{\max}$  constraints, producing smaller stars in better agreement with observations (Dexheimer et al. 2021b).

## 2 Executive summary

In this work, we discuss theoretical and experimental constraints for dense and hot matter, including astrophysical observations. For theoretical constraints, we restrict ourselves to those that are derived directly from first principles in particular regimes, where lattice QCD or pQCD calculations are possible, as well as from  $\chi$ EFT also in a particular regime, where it can be considered as the low-energy theory of QCD. For experimental constraints, we focus on measurements and, whenever possible, avoid mentioning quantities inferred from data. For example, by using yields of identified particles in heavy-ion collisions, it is possible to infer the temperature and baryon chemical potential at the point of chemical freeze-out.<sup>4</sup> However, the extracted  $\{T, \mu_B\}$  at fixed  $\sqrt{s_{NN}}$  and centrality are dependent on a number of assumptions such as the particle list, decay channels, decay widths, how interactions are described (if at all), etc. Thus, we only provide the hadron yields measured directly from experiments and not the thermodynamic quantities inferred from them, which are model dependent.

In the case of experimental low-energy nuclear results, the use of quantities inferred from data is unavoidable. Due to the importance of those results, we discuss them, while highlighting relevant dependencies. For astrophysical observations, posteriors are extracted from a combination of measured data and modeling where the systematic uncertainties are carefully taken into account. Nonetheless, there are certain caveats when one considers these posteriors that we would be remiss not to discuss. This context is important for theorists to understand before making comparisons between tidal deformabilities posteriors extracted from gravitational waves, mass-radius posteriors from NICER X-ray observations, and mass and/or radius extractions from other types of X-ray observations.

### 2.1 Theoretical constraints: lattice QCD

At vanishing  $n_B$  or, equivalently (at finite temperature),  $\mu_B = 0$ , lattice QCD calculations reliably provide the EoS for  $T \gtrsim 125$  MeV. They rely on solving QCD numerically on a very large grid of points in space and time. In this case, it has been determined that the change of phase between a hadron resonance gas (HRG) at low temperatures into a quark-gluon plasma at high temperatures is a smooth crossover. At finite  $\mu_B$ , the exponential of the QCD action becomes complex and cannot be used as a weight for the configurations generated in Monte Carlo simulations, which is known as the sign problem (Troyer and Wiese 2005; Dexheimer et al. 2021c). However, expansions around  $\mu_B = 0$  allow one to obtain the lattice QCD EoS up to a chemical potential dependent on temperature  $\mu_B \sim 3.5 T$  (Borsányi et al. 2021;

<sup>4</sup> Due to the rapid expansion and cooling of the quark-gluon plasma produced in heavy-ion collisions, at a point (chemical freeze-out) following the (pseudo)phase transition where quark and gluons have combined into hadrons, the particles become so far apart that chemical reactions are not longer possible. A second point (kinetic freeze-out) at even lower temperatures occurs (later in the reaction), where the particles become more dilute and kinetic reactions are no longer possible. It is generally believed that chemical freeze-out occurs near the quark deconfinement transition and can be used as a (close but not precise) proxy for the phase transition line.

Borsanyi et al. 2022). Furthermore, lattice QCD results can also constrain the hadronic spectrum through partial pressures (Alba et al. 2017) and provide insight into strangeness-baryon number interactions using cross-correlators (Bellwied et al. 2020). Despite these successes, the expanded lattice QCD EoS cannot reach temperatures and densities relevant to low-energy heavy-ion collisions and neutron stars.

## 2.2 Theoretical constraints: perturbative QCD

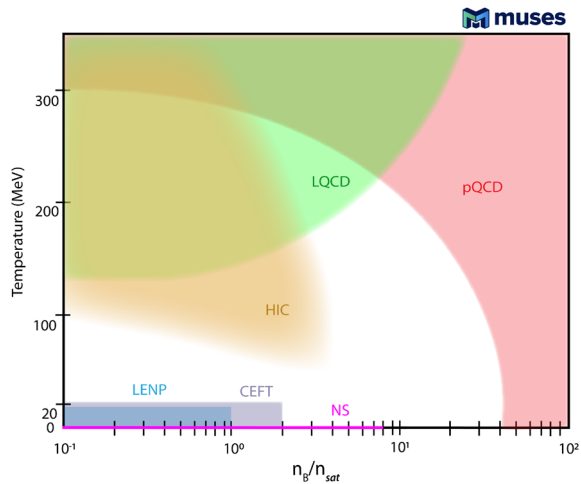
Although quarks are never truly free, due to asymptotic freedom, the coupling strength of the strong force ( $\alpha_s$ ) decreases logarithmically with energy and, more importantly, in a deconfined medium, Debye screening reduces the effective interaction between quarks and gluons. As a result, as the temperature and/or chemical potential(s) involved become large, one finds that perturbation theory becomes applicable and analytic calculations of the perturbative QCD (pQCD) EoS become reliable (Andersen et al. 2010a, b, 2011a, b; Mogliacci et al. 2013; Haque et al. 2014a, b; Haque and Strickland 2021; Ghiglieri et al. 2020). This occurs at  $T \gtrsim 300$  MeV at  $\mu_B = 0$  (Haque et al. 2014a, b; Haque and Strickland 2021) and at  $n_B \gtrsim 40 n_{\text{sat}}$  at  $T = 0$  (Andersen and Strickland 2002). In the latter case, note however that causality and stability bounds allow pQCD to be applied at lower densities (Komoltsev and Kurkela 2022). In practice, achieving agreement between perturbative QCD and lattice QCD requires resummations at all orders. The two main methods for accomplishing such resummations are effective field theory methods (Braaten and Nieto 1995, 1996a) and hard-thermal-loop perturbation theory (Andersen et al. 2002, 2004). Both resummation schemes have been extended to N2LO (next-to-next-to leading order) in their respective loop expansions at  $\mu_B = 0$ . At finite chemical potential, N2LO (Freedman and McLerran 1977a, b, c) and partial N3LO (next-to-next-to-next-to leading order) results are available (Gorda et al. 2018, 2021b, a).

## 2.3 Theoretical constraints: chiral effective field theory

Chiral effective field theory ( $\chi$ EFT) offers a systematic, model-independent framework for investigating the characteristics of hadronic systems at the low energies relevant for nuclear physics where the particle momentum is similar to the pion mass ( $p \sim m_\pi$ ) with quantified uncertainties (Weinberg 1979; Epelbaum et al. 2009; Machleidt and Entem 2011; Drischler et al. 2021c).  $\chi$ EFT starts from the most general Lagrangian that is consistent with the symmetries, in particular the spontaneously broken chiral symmetry, of low-energy QCD with nucleons and pions as degrees of freedom. The theory offers an order-by-order expansion for two-nucleon and multi-nucleon interactions whose long-range features are governed by pion-exchange contributions constrained by chiral symmetry and whose short-distance details are encoded in a set of contact interactions with strengths fitted to two- and few-body scattering and bound-state data. Theoretical uncertainties can be estimated by examining the order-by-order convergence of the  $\chi$ EFT expansion, a feature that provides a crucial benefit over phenomenological approaches (Drischler



**Fig. 1** Regions of the QCD phase diagram where constraints from heavy-ion collisions (HIC), lattice QCD (LQCD), perturbative QCD (pQCD), low-energy heavy-ion collisions (LENP), chiral effective field theory ( $\chi$ EFT), and astrophysics (neutron stars, NS) are available



et al. 2021c). Significant advances in the application of Bayesian statistical methods have led to robust uncertainty quantification in calculations of the EoS up to fourth order in the chiral expansion that are applicable in the range of  $n_B \lesssim 2 n_{\text{sat}}$  (Hebeler et al. 2010; Sammarruca et al. 2015; Tews et al. 2018; Drischler et al. 2020).

## 2.4 QCD phase diagram

From a combination of lattice QCD results (at  $T \geq 135$  MeV and  $\mu_B/T \leq 3.5$ ), pQCD calculations (limits include  $T \gtrsim 300$  MeV at  $\mu_B/T \lesssim 1$  and at  $n_B \gtrsim 40 n_{\text{sat}}$  at  $T = 0$ ), and  $\chi$ EFT bands ( $T \lesssim 20$  MeV and  $n_B \lesssim 2 n_{\text{sat}}$ ) we now have three theoretical points of reference (or rather regimes) in the QCD phase diagram, see Fig. 1. Effective models—e.g., chiral models (Nambu and Jona-Lasinio 1961a, b; Hatsuda and Kunihiro 1994; Dexheimer and Schramm 2010; Motornenko et al. 2020) and holography (Rougemon et al. 2017; Critelli et al. 2017; Grefa et al. 2021, 2022; Demircik et al. 2022; Kovensky et al. 2023)—some describing the microscopic degrees of freedom and their interactions, are used to connect these regimes in the phase diagram and even propose entirely new phases of dense and hot matter. These models are fixed to be in agreement with theoretical and experimental (low-energy nuclear physics, heavy-ion collisions, and astrophysics) results in the relevant regimes.

## 2.5 Experimental constraints: heavy-ion collisions

In the laboratory, heavy-ion collisions probe finite temperatures in the range of  $T \sim 50$ – $650$  MeV, depending on the center of mass beam energy  $\sqrt{s_{NN}}$ , such that higher  $\sqrt{s_{NN}}$  probe high temperatures and lower  $\sqrt{s_{NN}}$  probe lower temperatures. The temperature and density of the system vary in space and time throughout the evolution, which is the hottest at early times. Depending on the choice of the experimental observables, one can obtain information at different temperatures and densities within the collisions. The final distribution of hadrons reflects the

temperature and chemical potentials at chemical freeze-out (although certain momentum dependent observables are also sensitive to the kinetic freeze-out, see e.g. Adamczyk et al. 2017 for a comparison between chemical and kinetic freeze-out).

When temperatures are high enough (i.e., high  $\sqrt{s_{NN}}$ ) for a quark-gluon plasma to be produced, such that hydrodynamics is a good dynamical description, lowering  $\sqrt{s_{NN}}$  corresponds to a lower initial temperature, a lower freeze-out temperature, and a larger  $n_B$ . However, for very low beam energies, below  $\sqrt{s_{NN}} \lesssim 4\text{--}7$  GeV, the hadron gas phase dominates, such that hadron transport models may be used. This then means that higher  $\sqrt{s_{NN}}$  reaches larger  $n_B$  whereas lower  $\sqrt{s_{NN}}$  reach a smaller range of  $n_B$ . The exact switching point from a quark-gluon plasma dominated- to hadronic-dominated dynamical description is unknown and still hotly debated within the community. The initial collision temperature  $T_0$  is model-dependent, so we do not include estimates for it in this work. The freeze-out temperature, however, can be more directly extracted from experimental data (with certain caveats that we will explain here) using particle yields and assuming thermal equilibrium at freeze-out. Additionally, the emission of photons and lepton pairs (dileptons), which are immune to strong interactions and can traverse the QGP, can be used to extract average temperatures at different points in the heavy-ion collision evolution, which can be used to pin down the temperature evolution (Strickland 1994; Schenke and Strickland 2007; Martinez and Strickland 2008; Dion et al. 2011; Shen et al. 2014; Gale et al. 2015; Bhattacharya et al. 2016; Ryblewski and Strickland 2015; Paquet et al. 2016; Kasmaei and Strickland 2019, 2020). On the other hand, the extraction of  $n_B$  is more model dependent. If a QCD critical point exists, then susceptibilities of the pressure will diverge exactly at the critical point and may have non-trivial behavior in the surrounding critical region (Stephanov 2009; Parotto et al. 2020; Mroczek et al. 2021). In equilibrium, these would determine the cumulants of the distribution of protons, such as the kurtosis. Measurements of the kurtosis (Adam et al. 2021b; Abdallah et al. 2023b; Adamczewski-Musch et al. 2020c; Acharya et al. 2020a), 6th-order cumulants (Abdallah et al. 2021c), and fluctuations of light nuclei (Abdulhamid et al. 2023) exist from BES-I across a variety of beam energies with large statistical error bars. BES-II (Tlusty 2018) will significantly improve the measurement precision. However, the data has not yet been released.

Looking to the future, the Compressed Baryonic Matter (CBM) Experiment at FAIR (GSI, Germany) will be an experimental facility that will be dedicated to explore low beam energies in fixed target mode with high luminosity, i.e., with a high collision rate (Spies 2022). CBM (Lutz et al. 2009; Durante et al. 2019) will allow us to constrain the EoS at high  $\mu_B$  and moderate temperatures. Eventually, from the wealth of experimental data in heavy-ion collisions, it will be possible to extract an EoS using model-to-data comparisons. However, that will require more sophisticated dynamical models that do not yet exist (Bluhm et al. 2020). It has already been identified that the azimuthal anisotropies of the momentum distribution of particles in collisions, otherwise known as flow harmonics, are sensitive to the EoS at these low beam energies (Danielewicz et al. 2002; Spieles and Bleicher 2020). However, there are still significant questions remaining about the correct dynamical model and other free parameters such as transport

coefficients. Depending on the model assumptions, one can obtain radically different posteriors of the EoS, or find different EoSs consistent with the data at these beam energies (a few examples include comparing the different results and conclusions from Danielewicz et al. 2002; Spieles and Bleicher 2020; Schäfer et al. 2022; Shen and Schenke 2022; Oliinychenko et al. 2023). Thus, in this work, we will only include a discussion on some of the key experimental measurements but cannot yet clarify the precise implications of the data.

## 2.6 Experimental constraints: low-energy nuclear physics

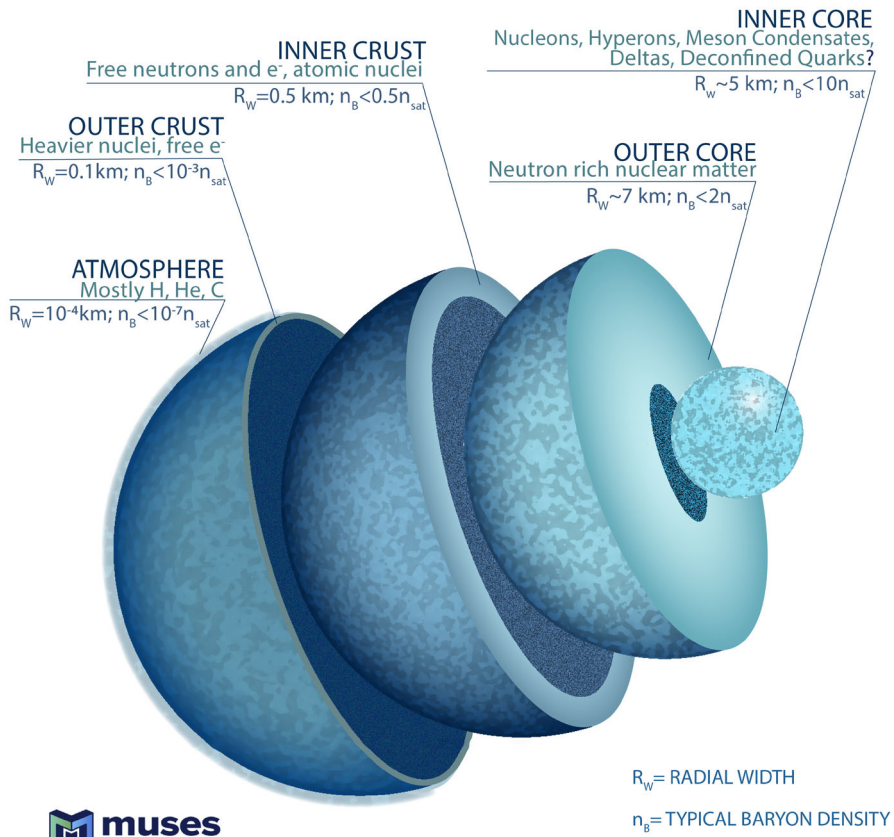
At significantly lower beam energies (approaching the  $T \rightarrow 0$  limit) there are a number of experiments that probe dense matter. These experiments study the properties of nuclei at (or near) saturation density. While most nuclei contain symmetric nuclear matter such that  $Y_Q \sim 0.5$ , heavy nuclei become more neutron-rich and may reach  $Y_Q \sim 0.4$ , while unstable nuclei close to the neutron drip line have much smaller values of  $Y_Q$ . However, neutron stars are composed of primarily asymmetric nuclear matter, with  $Y_Q \sim 0.001 - 0.2$ . Thus, one can use a Taylor series to expand between symmetric and asymmetric matter, known as the symmetry energy expansion. In this case, a few of its coefficients can be inferred from experimental measurements of, e.g., the neutron skin. Symmetric-matter properties include binding energy per nucleon, compressibility, and saturation density, and can also be inferred, together with the EoS. However, in this case there is model dependence which can be investigated by using different kinds of models.

In addition to the compressibility and the binding energy per nucleon, the effective mass of nucleons at saturation was shown to be important to study the nuclear EoS of hot stars (Raduta et al. 2021), the EoS of neutron stars with exotic particles at finite temperature (Raduta 2022), thermal effects in supernovae (Constantinou et al. 2015), and neutron-star mergers (see Raithel et al. 2019 and references therein). Nevertheless, the experimental determination of this quantity still includes large uncertainties and, therefore, will not be discussed in this review.

Beyond nucleons, properties of hyperons and  $\Delta$ -baryons can also be determined for symmetric matter. The most useful observable to constrain effective models is optical potentials at  $n_{\text{sat}}$ , which provides the result of the balance between attractive and repulsive strong interactions. At finite temperature, there is also data concerning the critical point for the liquid–gas phase transition (Elliott et al. 2012), where nuclei turn into bulk hadronic matter.

## 2.7 Experimental constraints: astrophysics

The high baryon density inside neutron stars makes them a natural laboratory to understand strong interaction physics under conditions that are impossible to achieve in a laboratory setting. Neutron stars are the end-life of massive stars, which run out of fuel for fusion and collapse gravitationally, violently exploding as supernovae. As a result, the cores of the remnant neutron stars possess densities of the order of several times  $n_{\text{sat}}$ . Neutron stars are stratified according to density, with different



**Fig. 2** Composition of a typical neutron star

types of co-existing phases categorized according to the radial coordinate (see Fig. 2). The outermost layer is the atmosphere, with a thickness of a few centimeters, which contains mostly hydrogen, helium, and carbon atoms. A little bit deeper, starts the outer crust with a layer of  $^{56}\text{Fe}$ . Electrons disassociate from specific nuclei and, moving towards the center of the star, nuclei become neutron richer and more massive, such that at the transition to the inner crust the nucleus  $^{118}\text{Kr}$  was determined as the most stable within several theoretical models (Ruester et al. 2006). There are, however, some models that may predict slightly larger nuclei at the outer-inner crust transition. In the inner crust, neutrons start to “drip out” of nuclei and, as a result, matter becomes a mixture of unbound electrons, unbound neutrons, and nuclei. At even higher densities in the outer core, above  $\sim n_{\text{sat}}$ , matter turns into a neutron-rich “soup” with no isolated nuclei. Going deeper into higher densities for the inner core, at about  $2 n_{\text{sat}}$ , hyperons,  $\Delta$ ’s, and meson condensates may appear and, eventually, deconfined quark matter may form. Regardless of the phase, fully evolved neutron stars fulfill chemical equilibrium and charge neutrality, either locally or globally, with mixtures of phases occurring in the latter case.

The EoS is related to microscopic equilibrium properties (pressure, energy density, etc.). Therefore, the nuclear EoS is not directly comparable to astrophysical observations, but it serves as an important input in calibrated models to calculate experimental observables, such as mass-radius relationships of compact stars. This is achieved by solving the Tolman–Oppenheimer–Volkoff (TOV) equations (Tolman 1939; Oppenheimer and Volkoff 1939), which are valid as long as rotational frequency ( $\nu$ ) and magnetic field ( $B$ ) effects are not significant. These results can be compared with astrophysical observations from neutron-star electromagnetic emissions, usually radio and X-ray, and most recently, gravitational wave emission from neutron-star mergers. In particular, observations from the National Radio Astronomy Observatory’s Green Bank Telescope (GBT, Demorest et al. 2010), NASA’s Neutron Star Interior Composition Explorer (NICER, Gendreau et al. 2016; Baubock et al. 2015; Miller 2016; Özel et al. 2016), and NSF’s Laser Interferometer Gravitational-wave Observatory (LIGO) together with Virgo (Abbott et al. 2017a, b; Gendreau et al. 2016) put strong constraints on the EoS (Gendreau et al. 2016; Annala et al. 2018). Many EoS models have been updated since these observations were made, to be in agreement with observations (Baym et al. 2018).

The most accurate neutron-star mass estimates come from the timing of radio pulsars in orbital systems with relativistic dynamical effects (Antoniadis et al. 2013; Cromartie et al. 2019; Fonseca et al. 2021b); they inform the EoS insofar as they set a lower bound on the maximum mass it must be able to support against gravitational collapse. The most reliable radius measurements stem from X-ray pulse profile modeling of rotating neutron stars (Miller et al. 2019; Riley et al. 2019a; Miller et al. 2021b; Riley et al. 2021a), and constrain the mass-radius relation predicted by the EoS. Meanwhile, gravitational-wave observations of merging neutron stars constrain the EoS via their tidal deformability (Abbott et al. 2017a, 2018, 2019, 2020a). If the gravitational waves are accompanied by a kilonova counterpart, as was the case for the binary neutron-star merger GW170817 (Abbott et al. 2017b), the lightcurve and spectrum of the electromagnetic emission, as well as its implications for the fate of the merger remnant, also inform the EoS (Bauswein et al. 2017; Margalit and Metzger 2017; Radice et al. 2018; Rezzolla et al. 2018; Ruiz et al. 2018; Shibata et al. 2017). Eventually, upgraded gravitational wave detectors will also be able to detect the post-merger signal (Carson et al. 2019) (the post-merger starts at the point where the two neutron stars touch). This signal is also sensitive to finite temperature effects that may even reach temperatures and densities similar to heavy-ion collisions (Adamczewski-Musch et al. 2019b), potential out-of-equilibrium effects due to the long-time scales associated with weak interactions (Alford et al. 2018, 2019; Alford and Harris 2019; Alford et al. 2021; Gavassino et al. 2021; Celora et al. 2022; Most et al. 2022) as well as deconfinement to quark matter (Bauswein et al. 2019; Most et al. 2019; Weih et al. 2020; Blacker et al. 2020; Tootle et al. 2022; Constantinou et al. 2021).

## 2.8 Organization of the paper

This review paper aims at compiling up-to-date constraints from high-energy physics, nuclear physics, and astrophysics that relate to the EoS and are, therefore,

fundamental to the understanding of current and future data from heavy-ion collisions to gravitational waves, making them relevant to a very broad community. Additionally, precise knowledge of the dense and hot matter EoS can help physicists to look beyond the standard model either for dark matter, which may accumulate in or around neutron stars, or for modified theories of gravity.

The paper is organized as follows: we first discuss the theoretical constraints of lattice (Sect. 3) and perturbative QCD (Sect. 4), followed by  $\chi$ EFT (Sect. 5). Then, we discuss experimental constraints from heavy-ion collisions (Sect. 6), (isospin symmetric and asymmetric) low-energy nuclear physics (Sect. 7), and astrophysical observations (Sect. 8). We provide a future outlook in Sect. 9, since a significant amount of new data is anticipated over the next decade.

### 3 Theoretical constraints: lattice QCD

Lattice QCD is the most suitable method to study strong interactions around and above the deconfinement phase transition region in the QCD phase diagram, due to its non-perturbative nature (Drischler et al. 2021b). As discussed in the introduction, due to the sign problem, first-principles lattice QCD results for the EoS at finite  $\mu_B$  are currently restricted. Since direct lattice simulations at  $\mu_B = 0$  and imaginary  $\mu_B$  are feasible, observables can be extrapolated using techniques involving zero or imaginary chemical potential simulations, i.e., analytical continuation, Taylor series and other alternative expansions. In this section, we present various constraints on the EoS, BSQ (baryon number, strangeness, and electric charge) susceptibilities, and partial pressures evaluated using lattice QCD.

#### 3.1 Equation of state

In Borsanyi et al. (2014a) and Bazavov et al. (2014), the EoS was obtained in lattice QCD simulations at  $\mu_B = 0$ . It was found that the rigorous continuum extrapolation results for 2+1 quark flavors are perfectly compatible with previous continuum estimates based on coarser lattices (Aoki et al. 2006; Borsanyi et al. 2010). The obtained pressure, entropy density, and interaction measure are displayed in the left panel of Fig. 3 alongside the predictions of the hadron resonance gas (HRG) model (Venugopalan and Prakash 1992) at low temperatures and the Stefan–Boltzmann (or conformal) limit of a non-interacting massless quarks gas at high  $T$ . They show full agreement with HRG results in the hadronic phase, and reach about 75% of the Stefan–Boltzmann limit at  $T \simeq 400$  MeV.

Furthermore, a Taylor series can be utilized to expand many observables to finite  $\mu_B/T$

$$\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k \quad (2)$$

where the susceptibilities  $\chi_{ijk}^{BQS}$  are defined as follows

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}. \quad (3)$$

They were obtained from lattice QCD calculations up to  $\mathcal{O}(\mu_B/T)^4$  for the full series of coefficients and up to  $\mathcal{O}(\mu_B/T)^8$  for some of the coefficients. The range of applicability of the Taylor expansion has recently been extended from  $\mu_B/T \leq 2$  (Guenther et al. 2017; Bazavov et al. 2017) to  $\mu_B/T \leq 2.5$  (Bollweg et al. 2022). Isentropic trajectories in the  $T - \mu_B$  plane have been extracted in Guenther et al. (2017), for which the starting points are the freeze-out parameters at different collision energies at RHIC (Alba et al. 2014). Strangeness neutrality and electric charge conservation were enforced by tuning the strange and electric charge chemical potentials,  $\mu_S(\mu_B, T)$  and  $\mu_Q(\mu_B, T)$ , to reproduce the conditions  $Y_S = 0$  and  $Y_Q = 0.4$  (Guenther et al. 2017).

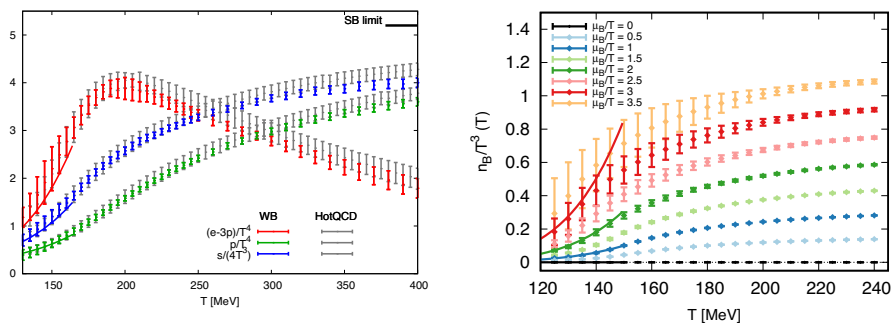
A new expansion scheme for extending the EoS of QCD to unprecedentedly large baryonic chemical potential up to  $\mu_B/T < 3.5$  has been proposed recently (Borsányi et al. 2021). The drawbacks of the conventional Taylor expansion approach, such as the challenges involved in carrying out such an expansion with a constrained number of coefficients and the low signal-to-noise ratio for the coefficients themselves, are significantly reduced in this new scheme (Borsányi et al. 2021). In the hadronic phase, a good agreement is found for the thermodynamic variables with HRG model results. This scheme is based on the following identity

$$\frac{\chi_1^B(T, \mu_B)}{\mu_B/T} = \chi_2^B(T', 0) \quad (4)$$

with

$$T'(T, \mu_B) = T(1 + \kappa_2^{BB}(T)\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^{BB}(T)\left(\frac{\mu_B}{T}\right)^4 + \mathcal{O}((\mu_B/T)^6). \quad (5)$$

The baryonic density as a function of the temperature for different values of  $\mu_B/T$



**Fig. 3** Left: comparison between the lattice QCD EoS at  $\mu_B = 0$  from the WB collaboration Borsanyi et al. (2014a) (colored points) and the HotQCD one (Bazavov et al. 2014) (gray points). Right: Baryonic density as a function of the temperature, for different values of  $\mu_B/T$ . At low temperature, full lines show results from the HRG model. Images reproduced with permission from [left] Ratti (2018), copyright by IOP, and [right] from Borsányi et al. (2021), copyright by the author(s)



from Borsányi et al. (2021) is shown in the right panel of Fig. 3. This extrapolation method was then generalized to include the strangeness neutrality condition (Borsanyi et al. 2022), which requires  $\mu_S \neq 0$ . The extrapolation approach is devoid of the unphysical oscillations that afflict fixed order Taylor expansions at higher  $\mu_B$ , even in the strangeness neutral situation. Effects beyond strangeness neutrality are estimated by computing the baryon-strangeness correlator to strangeness susceptibility ratio  $\frac{\chi_{11}^{BS}}{\chi_2^S}$  (discussed in the following subsection) at finite real  $\mu_B$  on the strangeness neutral line. This permits a leading order extrapolation in the ratio  $R = \chi_1^S/\chi_1^B$  (Borsanyi et al. 2022).

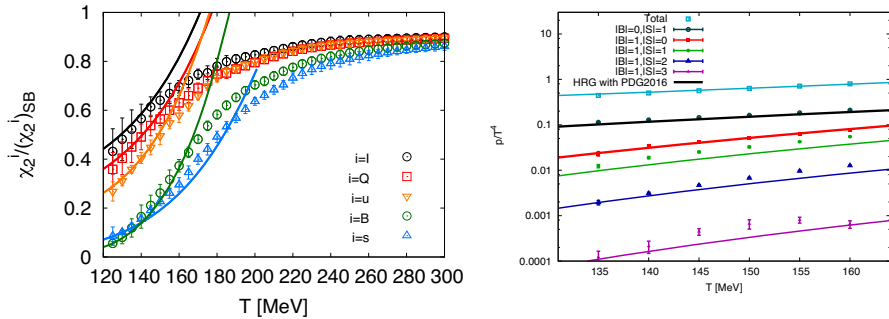
### 3.2 BSQ susceptibilities

Fluctuations of different conserved charges have been postulated as a signal of the deconfinement transition because they are sensitive probes of quantum numbers and related degrees of freedom. In heavy-ion collisions, one needs to relate fluctuations of net baryon number, strangeness, and electric charge with the event-by-event fluctuations of particle species. Non-diagonal correlators of conserved charges, like fluctuations, are useful for studying the chemical freeze-out in heavy-ion collisions. In thermal equilibrium, these correlators may be estimated using lattice simulations, and linked to moments of event-by-event distributions of multiplicity (i.e., number of particles of a given species in some kinematic region, typically these are all charged particles) distributions. They are defined as derivatives of the pressure with respect to the chemical potentials according to Eq. (3). The quark number chemical potentials appear as parameters in the Grand Canonical partition function. The derivative of this function with respect to these chemical potentials yields the susceptibilities and the non-diagonal correlators of the quark flavors. Quark flavor chemical potentials can be related to the conserved charge ones through the following relationships:  $\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q$ ,  $\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q$ , and  $\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S$ .

For  $T = 125 - 400$  MeV and at  $\mu_B = \mu_S = \mu_Q = 0$ , the Wuppertal-Budapest lattice QCD collaboration computed the non-diagonal (us) and diagonal (B,s,Q,I,u) susceptibilities for a system of 2+1 staggered quark flavors (Borsanyi et al. 2012), where I stands for isospin. Selected susceptibilities are shown in the left panel of Fig. 4. A Symanzik-improved gauge and a stout-link improved staggered fermion action were used in this analysis. The ratios of fluctuations were found, whose behavior may be recreated using hadronic observables, i.e. proxies, to compare either to lattice QCD findings or experimental observations (Bellwied et al. 2020).

Continuum extrapolated lattice QCD findings for  $\chi_{2,2}^{u,s}$ ,  $\chi_{2,2}^{u,d}$ ,  $\chi_{1,1}^{u,d}$ ,  $\chi_4^u$ ,  $\chi_4^B$  were presented in Bellwied et al. (2015). Second and fourth-order cumulants of conserved charges were constructed in a temperature range spanning from the QCD transition area to the region of resummed perturbation theory. It was found that, in the hadronic phase ( $T \sim 130$  MeV), the HRG model predictions accurately reflect the lattice data, whereas in the deconfined region ( $T \gtrsim 250$  MeV), a good agreement was found with three loop hard-thermal-loop (HTL) outcomes (Bellwied et al. 2015). Different diagonal and non-diagonal fluctuations of conserved charges are estimated up to





**Fig. 4** Left: Baryon number, strange quark, electric charge, isospin number, and up quark susceptibilities as functions of the temperature at  $\mu_B = 0$ . Right: compilation of partial pressures for different hadron families as functions of the temperature. Images reproduced with permission from [left] Borsanyi et al. (2012), copyright by SISSA, and [right] from Alba et al. (2017), copyright by APS

sixth-order on a lattice size of  $48^3 \times 12$  (Borsanyi et al. 2018). Higher-order fluctuations at zero baryon/charge/strangeness chemical potential are calculated. The ratios of baryon-number cumulants as functions of  $T$  and  $\mu_B$  are derived from these correlations and fluctuations, which fulfill the experimental criteria of proton/baryon ratio and strangeness neutrality and in turn, describe the observed cumulants as functions of collision energy from the STAR collaboration (Borsanyi et al. 2018). Ratios of fourth-to-second order susceptibilities for light and strange quarks were presented in Bellwied et al. (2013).

### 3.3 Partial pressures

Under the assumption that the hadronic phase can be treated as an ideal gas of resonances, and using lattice simulations, the partial pressures of hadrons were determined with various strangeness and baryon number contents. To explain the difference between the results of the HRG model and lattice QCD for some of them, the existence of missing strange resonances was proposed (Bazavov et al. 2013b; Alba et al. 2017). Note that partial pressures are only possible within the hadron resonance gas phase because (i) they require hadronic degrees-of-freedom and (ii) they are applicable under the assumption that the pressure can be written as separable components by the quantum number of the hadrons, i.e.,

$$P(\hat{\mu}_B, \hat{\mu}_S) = P_{00}^{BS} + P_{10}^{BS} \cosh(\hat{\mu}_B) + P_{01}^{BS} \cosh(\hat{\mu}_S) + P_{11}^{BS} \cosh(\hat{\mu}_B - \hat{\mu}_S) + P_{12}^{BS} \cosh(\hat{\mu}_B - 2\hat{\mu}_S) + P_{13}^{BS} \cosh(\hat{\mu}_B - 3\hat{\mu}_S) \quad (6)$$

where the coefficients  $P_{ij}^{BS}$  indicate the baryon number  $i$  and strangeness  $j$  of the family of hadrons for which the partial pressure is being isolated, and the dimensionless chemical potentials are written as  $\hat{\mu} = \mu/T$ . The calculations were made feasible by taking imaginary values of strange chemical potential in the simulations. For strange mesons, more interaction channels should be incorporated into the HRG

model, in order to explain the lattice data (Alba et al. 2017). The right panel of Fig. 4 shows a compilation of these partial pressures.

### 3.4 Pseudo-phase transition line

In a crossover, there is no sudden jump in the first derivatives of the pressure. Nevertheless, a (chiral) pseudo-phase transition line can be calculated based on where the order parameters change more rapidly. The exact location of the QCD transition line is a hot topic of research in the field of strong interactions. The most recent results are contained in Borsanyi et al. (2020). The transition temperature, obtained from the chiral condensate and its susceptibility, as a function of the chemical potential can be parametrized as

$$\frac{T_c(\mu_B)}{T_c(\mu_B=0)} = 1 - \kappa_2 \left( \frac{\mu_B}{T_c(\mu_B)} \right)^2 - \kappa_4 \left( \frac{\mu_B}{T_c(\mu_B)} \right)^4 + \dots \quad (7)$$

The crossover or pseudo-critical temperature  $T_c$  has been determined with extreme accuracy and extrapolated from imaginary up to real  $\mu_B \approx 300$  MeV. Additionally, the width of the chiral transition and the peak value of the chiral susceptibility were calculated along the crossover line. Both of them are constant functions of  $\mu_B$ . This means that, up to  $\mu_B = 300$  MeV, no sign of criticality has been observed in lattice results. In fact, at the critical point the height of the peak of the chiral susceptibility would diverge and its width would shrink. The small error reflects the most precise determination of the  $T - \mu_B$  phase transition line using lattice techniques. Besides  $T_c = 158 \pm 0.6$  MeV, the study provides updated results for the coefficients  $\kappa_2 = 0.0153 \pm 0.0018$  and  $\kappa_4 = 0.00032 \pm 0.00067$  (Borsanyi et al. 2020). Similar coefficients for the extrapolation of the transition temperature to finite strangeness, electric charge, and isospin chemical potentials were obtained in Bazavov et al. (2019), and are displayed in Table 2.

### 3.5 Limits on the critical point location

As mentioned in the previous subsection, in Borsanyi et al. (2020), by extrapolating the proxy for the transition width as well as the height of the chiral susceptibility peak from imaginary to real  $\mu_B$ , the strength of the phase transition was evaluated and no indication of criticality was found up to  $\mu_B \approx 300$  MeV. On the other hand, a phase transition temperature at  $\mu_B = 0$  of  $T_c = 132_{-6}^{+3}$  MeV was found in the chiral limit by the HotQCD collaboration using lattice QCD calculations with “rooted”

**Table 2** Continuum-extrapolated values of  $\kappa_2^X$  and  $\kappa_4^X$  (with  $\mu_Q = \mu_S = 0$  for  $X = B$ ,  $\mu_B = \mu_Q = 0$  for  $X = S$  and  $\mu_B = \mu_S = 0$  for  $X = Q, I$ ) from Bazavov et al. (2019)

$\kappa_2^B$	$\kappa_4^B$	$\kappa_2^S$	$\kappa_4^S$	$\kappa_2^Q$	$\kappa_4^Q$	$\kappa_2^I$	$\kappa_4^I$
0.016(6)	0.001(7)	0.017(5)	0.004(6)	0.029(6)	0.008(1)	0.026(4)	0.005(7)

staggered fermions (Ding et al. 2019). This transition temperature is computed with two massless light quarks and a physical strange quark based on two unique estimators. Since the curvature of the phase diagram is negative, a critical point in the chiral limit would sit at a temperature smaller than this one. The expectation is that the temperature of the critical point at physical quark masses has to be smaller than the one of the critical point in the chiral limit, and therefore definitely smaller than  $T_c = 132^{+3}_{-6}$  MeV.

## 4 Theoretical constraints: perturbative QCD

It is possible to compute analytically the QCD EoS directly from the QCD Lagrangian using finite temperature/density perturbation theory. However, in thermal and chemical equilibrium, when  $T \gg \mu_i$  (with quark chemical potentials  $\mu_i$ ), one finds that the naive loop expansion of physical quantities is ill-defined and diverges beyond a given loop order, which depends on the quantity under consideration. In the calculation of QCD thermodynamics, this stems from uncanceled infrared (IR) divergences that enter the expansion of the partition function at three-loop order. These IR divergences are due to long-distance interactions mediated by static gluon fields and result in contributions that are non-analytic in the strong coupling constant  $\alpha_s = g^2/4\pi$ , e.g.,  $\alpha_s^{3/2}$  and  $\log(\alpha_s)$ , unlike vacuum perturbation expansions which involve only powers of  $\alpha_s$ .

It is possible to understand at which perturbative order terms that are non-analytic in  $\alpha_s$  appear by considering the contribution of non-interacting static gluons to a given quantity. For simplicity, we now discuss the case of  $\mu_B = 0$  for this argument, but the same holds true at finite chemical potential. For the pressure of a gas of gluons one has  $P_{\text{gluons}} \sim \int d^3p p f_B(E_p)$ , where  $f_B$  denotes a Bose–Einstein distribution function and  $E_p$  is the energy of the in-medium gluons. The contributions from the momentum scales  $\pi T$ ,  $gT$  and  $g^2T$  can be expressed as

$$P_{\text{gluons}}^{p \sim \pi T} \sim T^4 f_B(\pi T) \sim T^4 + \mathcal{O}(g^2), \quad (8)$$

$$P_{\text{gluons}}^{p \sim gT} \sim (gT)^4 f_B(gT) \sim g^3 T^4 + \mathcal{O}(g^4), \quad (9)$$

$$P_{\text{gluons}}^{p \sim g^2T} \sim (g^2T)^4 f_B(g^2T) \sim g^6 T^4 \quad (10)$$

where we have used the fact that  $f_B(E) \sim T/E$  if  $E \ll T$ . This fact is of fundamental importance, since it implies that when the energy/momentum are *soft*, corresponding to electrostatic contributions  $p_{\text{soft}} \sim gT$ , one receives an enhancement of  $1/g$  compared to contributions from *hard* momenta,  $p_{\text{hard}} \sim T$ , due to the bosonic nature of the gluon. For *ultrasoft* (magnetostatic) momenta,  $p_{\text{ultrasoft}} \sim g^2T$ , the contributions are enhanced by  $1/g^2$  compared to the naive perturbative order. As Eqs. (8)–(10) demonstrate, it is possible to generate contributions of the order  $g^3 \sim \alpha_s^{3/2}$  from soft

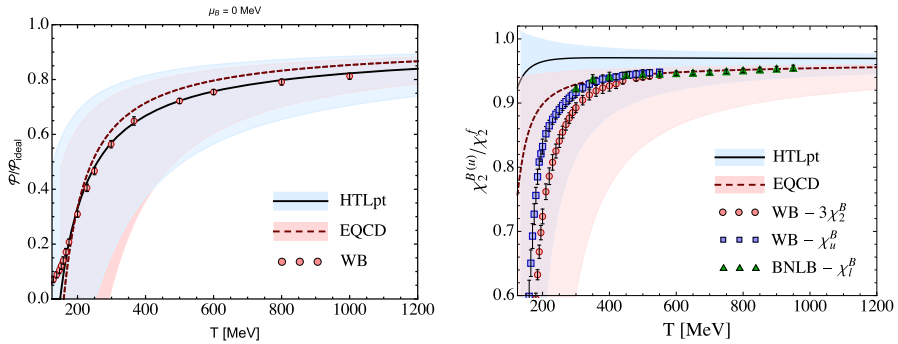
momenta and, in the case of the pressure, although perturbatively enhanced, ultrasoft momenta only start to play a role at order  $g^6 \sim \alpha_s^3$ .

Due to the infrared enhancement of electrostatic contributions, there is a class of diagrams called hard-thermal-loop (HTL) graphs that have soft external momenta and hard internal momenta that need to be resummed to all orders in the strong coupling (Braaten and Pisarski 1990a, b, c). There are now several schemes for carrying out such soft resummations (Arnold and Zhai 1994, 1995; Zhai and Kastening 1995; Braaten and Nieto 1995, 1996a; Kajantie et al. 1997; Andersen et al. 1999, 2000a, b; Blaizot et al. 1999a, b, 2001a, b; Andersen et al. 2002, 2004, 2010b, 2011c; Haque et al. 2014a, b). We note however, that even with such resummations, if one casts the result as a strict power series in the strong coupling constant the convergence of the perturbative series for the QCD free energy is quite poor. To address this issue, one must treat the soft sector non-perturbatively and re-sum contributions to all orders in the strong coupling constant. This can be done using effective field theory methods (Ghiglieri et al. 2020), approximately self-consistent two-particle irreducible methods (Blaizot et al. 1999a, b, 2001a, b), or the hard-thermal-loop perturbation theory reorganization of thermal field theory (Andersen et al. 1999, 2000b, 2002, 2004, 2010b, 2011c; Haque et al. 2014a, b).

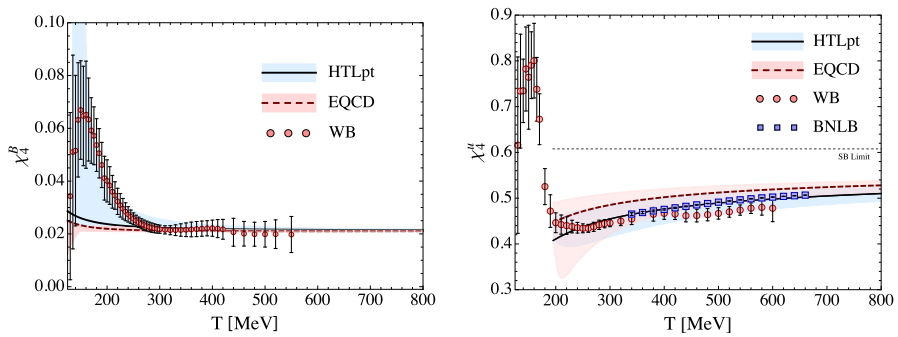
Thus, the calculation of the QCD EoS requires all-orders resummation, which can be accomplished in a variety of manners. Despite the fact that different methods exist, they all rely fundamentally on the use of so-called hard-thermal- or hard-dense-loops, which self-consistently include the main physical effect of the generation of in-medium gluon and quark masses at the one-loop level. By reorganizing the perturbative calculation of the QCD EoS around the high-temperature hard-loop limit of quantum field theory, the convergence of the perturbative series can be extended to phenomenologically relevant temperatures and densities. Below we summarize the results that have been obtained and compared to lattice QCD calculations where available.

#### 4.1 The resummed perturbative QCD EoS

The QCD EoS of deconfined quark matter at high chemical potential can be evaluated in terms of perturbative series in the running coupling constant  $\alpha_s$ . As a result, the neutron-star EoS can be studied using the weak coupling expansion (Kurkela et al. 2014; Annala et al. 2018; Shuryak 1978; Zhai and Kastening 1995; Braaten and Nieto 1996a, b; Arnold and Zhai 1995, 1994; Toimela 1985; Kapusta 1979; Annala et al. 2020; Kurkela et al. 2014, 2010; Freedman and McLerran 1977b, c; Ecker and Rezzolla 2022a; Altiparmak et al. 2022; Ecker and Rezzolla 2022b). The EoS and trace anomaly of deconfined quark matter have been calculated to three-loop order using HTL perturbation theory framework at small  $\mu_B$  and arbitrary  $T$ . Renormalization of the vacuum energy, the HTL mass parameters, and  $\alpha_s$  eliminate all UV divergences. The three-loop results for the thermodynamic functions are observed to be in agreement with lattice QCD data for  $T \gtrsim 2 - 3T_c$  after choosing a suitable mass parameter prescription (Andersen et al. 2011c). Furthermore, the QCD thermodynamic potential at nonzero temperature and



**Fig. 5** Left: the resummed QCD pressure for  $\mu_B = 0$  obtained using the three-loop EQCD and HTLpt perturbation theory results with lattice data from the Wuppertal-Budapest (WB) collaboration (Borsanyi et al. 2010). Right: the second-order light quark (and baryon) number susceptibilities. Lattice data are from the WB (Borsanyi 2013; Borsanyi et al. 2013) and BNLB collaborations (Bazavov et al. 2013a)

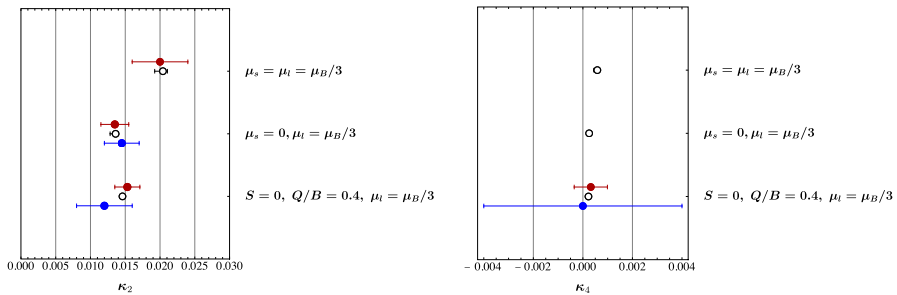


**Fig. 6** Left: the 4th baryon number susceptibility. Right: the 4th light quark number susceptibility. Lattice data sources are the same as in Fig. 5

chemical potential(s) has been calculated using N2LO at three-loop HTL perturbation theory which was used further to calculate the pressure, entropy density, trace anomaly, energy density, and speed of sound,  $c_s$ , of the QGP (Haque et al. 2014b). These findings were found to be in very good agreement with the data obtained from lattice QCD using the central values of the renormalization scales. This is illustrated in Figs. 5 and 6, which present comparisons of the resummed perturbative results with lattice data for the pressure and fourth-order baryonic and light-quark susceptibilities. In these figures, HTLpt corresponds to the N2LO hard thermal loop perturbation theory calculation of the EoS and EQCD corresponds to a resummed N2LO electric QCD effective field theory calculation of the same. The shaded bands indicate the size of the uncertainty due to the choice of renormalization scale.

## 4.2 The curvature of the QCD phase transition line

In another study, for the second- and fourth-order curvatures of the QCD phase transition line, the N2LO HTL perturbation theory predictions were shown. In all



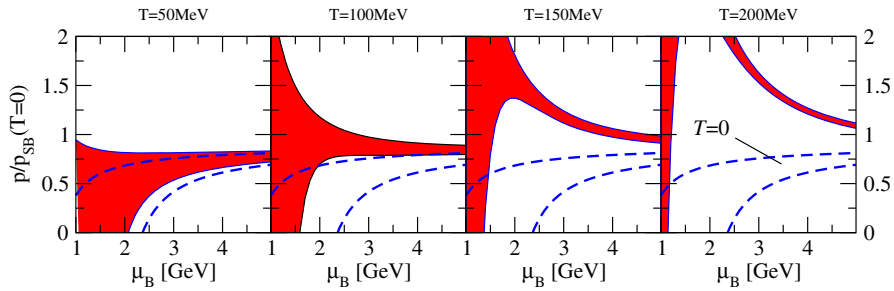
**Fig. 7** Left: filled circles are lattice calculations of the quadratic curvature coefficient  $\kappa_2$  (Cea et al. 2016; Bonati et al. 2015, 2018; Borsanyi et al. 2020; Bazavov et al. 2019), from top to bottom, respectively. Red-filled circles are results obtained using the imaginary chemical potential method and blue-filled circles are results obtained using Taylor expansions around  $\mu_B = 0$ . Black open circles are the N2LO HTL perturbation theory predictions. Right: filled circles are lattice calculations of quartic coefficient  $\kappa_4$  from Borsanyi et al. (2020), Bazavov et al. (2019), from top to bottom, respectively. The error bars associated with the HTL perturbation theory predictions result from variations of the assumed renormalization scale

three situations, (i)  $\mu_s = \mu_l = \mu_B/3$ , (ii)  $\mu_s = 0, \mu_l = \mu_B/3$ , and (iii)  $S = 0, Q/B = 0.4, \mu_l = \mu_B/3$ , it was shown that N2LO HTL perturbation theory is compatible with the already available lattice computations of  $\kappa_2$  and  $\kappa_4$  as defined in Eq. (7) (Haque and Strickland 2021). This is illustrated in Fig. 7, which presents comparisons of the resummed perturbative results with lattice data for the coefficients  $\kappa_2$  and  $\kappa_4$ .

### 4.3 Application at high density

Gorda et al. (2021b), at  $T = 0$ , calculated the N3LO contribution emerging from non-Abelian interactions among long-wavelength, dynamically screened gluonic fields using the weak-coupling expansion of the dense QCD EoS. In particular, they used the HTL effective theory to execute a comprehensive two-loop computation that is valid for long-wavelength, or soft, modes. In the plot of the EoS, unlike at high temperatures, the soft sector behaves well within cold quark matter, and the novel contribution reduces the renormalization-scale dependence of the EoS at high density (Gorda et al. 2021b). Working at exactly zero temperature is often a good approximation for fully evolved neutron stars but for the early stages of neutron-star evolution and neutron-star mergers, it is essential to incorporate temperature effects (Shen et al. 1998). However, the inclusion of finite temperature in high- $\mu_B$  quark matter gives rise to a technical difficulty for weak coupling expansions. It is no longer sufficient under this regime to handle simply the static sector of the theory nonperturbatively, but the  $T = 0$  limit's accompanying technical simplifications are also unavailable. In the EoS plots (see Fig. 8), the breakdown of the weak coupling expansion is observed by a rapid increase in the uncertainty of the result with an increase in temperature for tiny values of  $\mu_B$  (Kurkela and Vuorinen 2016).

The most up-to-date pQCD results at  $T = 0$  and finite densities can be found in Gorda et al. (2021a). The EoS derived in these calculations was applicable starting at  $n_B \sim 40 n_{\text{sat}}$  and above. However, there is an overall renormalization scale parameter,



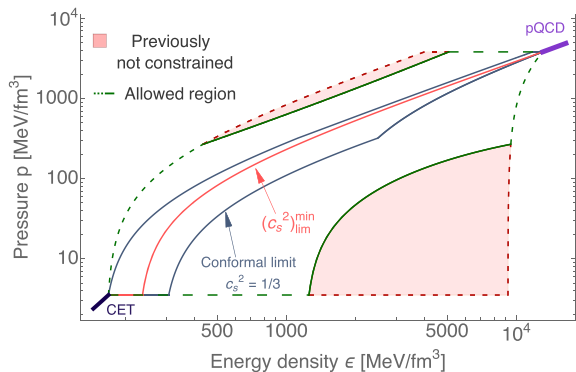
**Fig. 8** EoS of deconfined quark matter as a function of  $\mu_B$  at four different temperatures. The new result is shown by the red bands, with the widths resulting from a change in the renormalization scale  $\tilde{\Lambda}$  (Kurkela and Vuorinen 2016). The corresponding  $\mathcal{O}(g^4)$  result at absolute zero is shown by the dashed blue lines (Freedman and McLerran 1977c; Baluni 1978; Vuorinen 2003). Image reproduced with permission from Kurkela and Vuorinen (2016), copyright by the author(s)

$X$ , that is unknown. One can extrapolate down to lower densities assuming that the speed of sound squared should be bounded by causality and stability, i.e.,  $0 \leq c_s^2 \leq 1$ . The results were shown in Komoltsev and Kurkela (2022) where they varied  $X$  in the range  $1 \leq X \leq 4$ . The results of the constrained regime can be seen in Fig. 9. Various groups have then used these constraints in their neutron star EoS analyses (Marczenko et al. 2023; Somasundaram et al. 2023).

#### 4.4 Transport coefficients at finite $T$ and $\mu_B$

The quark-gluon plasma probed in heavy-ion collisions is not in equilibrium and viscous effects from shear and bulk viscosities are important for the evolution of the system. At the moment it is not yet possible to reliably compute the shear and bulk viscosities using first principle calculations (Meyer 2011). However, it is possible to perform calculations of these coefficients in the weak-coupling limit of QCD. The shear viscosity  $\eta$  and relaxation time  $\tau_\pi$  (the timescale within which the system

**Fig. 9** The extracted EoS constraints from pQCD and  $\chi$ EFT(CET in the figure) as a function of energy density are shown (see Sect. 5 for details). The excluded regions are in pink. Image reproduced with permission from Komoltsev and Kurkela (2022), copyright by APS



relaxes towards its Navier–Stokes regime, Denicol and Rischke 2021) are usually related through

$$\tau_\pi = C \frac{\eta}{\varepsilon + p} \quad (11)$$

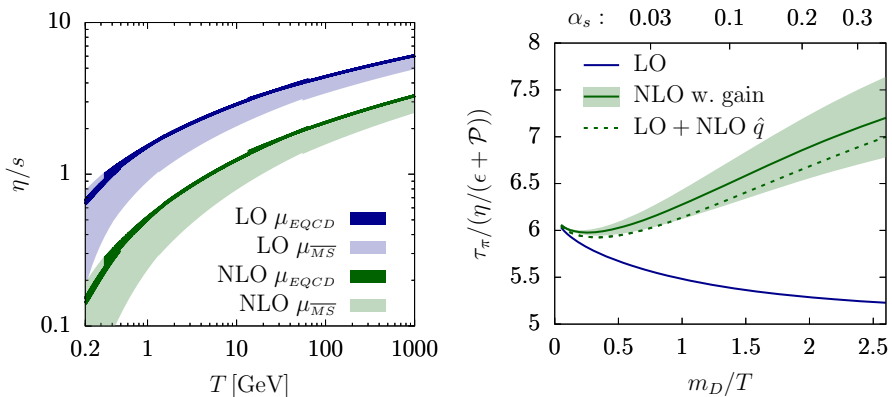
where  $C$  is a constant determined by the theory. Calculations of  $\eta/s$  in QCD have been completed up to NLO (next-to leading order, Ghiglieri et al. 2018a) and the constant  $C$  of the relaxation time at NLO (Ghiglieri et al. 2018b) for  $\mu_B = 0$ , as shown in Fig. 10.

Recently, the first calculations of shear viscosity at leading-log at finite  $\mu_B$  were performed in QCD in Danhoni and Moore (2023), as shown in Fig. 11.

Note, however, that at finite  $\mu_B$  the most natural dimensionless quantity involves the enthalpy ( $w = \varepsilon + p$ ), such that  $\eta T/w$  is the relevant quantity (the factor of  $T$  is to ensure that it remains dimensionless) to be used (Liao and Koch 2010). In the limit of vanishing baryon chemical potential, then

$$\lim_{\mu_B \rightarrow 0} \frac{\eta T}{\varepsilon + p} = \frac{\eta}{s} \quad (12)$$

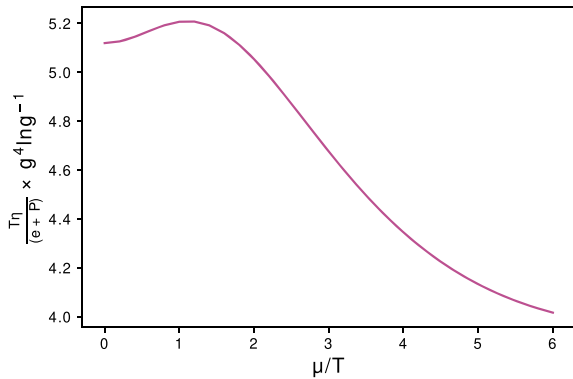
such that these results should be smoothly connected regardless of  $\mu_B$ . The relaxation time has not yet been calculated in QCD at finite  $\mu_B$ . Finally we note that, in typical relativistic viscous hydrodynamics simulations performed in heavy-ion collisions, a number of other transport coefficients are also needed. For example, using perturbative QCD, the bulk viscosity has been computed in Arnold et al. (2006), conductivity and diffusion in Arnold et al. (2000), and some second-order transport coefficients can be found in York and Moore (2009). In practice, these perturbatively-



**Fig. 10** Left: shear viscosity to entropy density ratio versus temperature derived from pQCD at LO (leading order) and NLO (next-to leading order) for two different choices of the running coupling for 3 flavors. Right: coefficient of the relaxation time for shear viscosity at leading order and next-to-leading order as a function of the Debye mass over temperature for QCD with 3 light flavors. Images reproduced with permission from [left] Ghiglieri et al. (2018a) and from [right] Ghiglieri et al. (2018b), copyright by the author(s)



**Fig. 11** Shear viscosity times temperature divided by the enthalpy versus chemical potential over temperature in 3 flavor QCD. Image reproduced with permission from Danhoni and Moore (2023), copyright by the author(s)



determined expressions are not the ones used in simulations, which often rely on simple formulas involving the transport coefficients determined from, for instance, kinetic theory models (Denicol et al. 2012, 2014) or holography (Kovtun et al. 2005; Finazzo et al. 2015; Rougemont et al. 2017; Grefa et al. 2022), see Everett et al. (2021).

## 5 Theoretical constraints: chiral effective field theory

In the opposite regime of low density and temperature, Chiral Effective Field Theory ( $\chi$ EFT) is used to calculate the EoS relevant around  $n_B \sim n_{\text{sat}}$  of neutron stars. For ab initio  $\chi$ EFT calculations, it is possible to study the EoS at arbitrary isospin asymmetry at both zero and nonzero temperature within a many-body perturbation theory (Drischler et al. 2014; Wellenhofer et al. 2016; Wen and Holt 2021; Somasundaram et al. 2021), or a many-body Brueckner–Hartree–Fock approach (Logoteta et al. 2016b, a). Recently, several benchmark calculations have been performed, considering the first and second generation of  $\chi$ EFT Norfolk NN and 3N interactions, to assess the possible error which is associated with the chosen method when solving the many-body Schrödinger equation (Piarulli et al. 2020; Lovato et al. 2022a). The authors have obtained a good agreement among the many-body techniques tested up to approximately the nuclear saturation density.

However, in practice it is convenient to first compute the EoS for symmetric nuclear matter ( $Y_Q = 0.5$ ) and pure neutron matter ( $Y_Q = 0$ ) and then interpolate between the two using the quadratic approximation for the isospin-asymmetry dependence of the EoS. From the density-dependent symmetry energy, one can extract the coefficients  $E_{\text{sym}}$ ,  $L$ ,  $K_{\text{sym}}$  from  $\chi$ EFT calculations

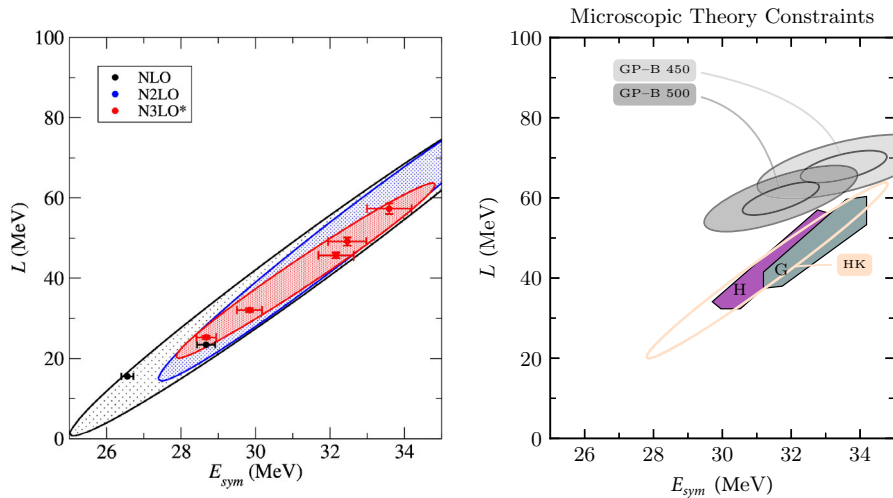
$$E_{\text{NS}} = E_{Y_Q=0.5} + N_B \left[ E_{\text{sym,sat}} + \frac{L_{\text{sat}}}{3} \left( \frac{n_B}{n_{\text{sat}}} - 1 \right) + \frac{K_{\text{sym,sat}}}{18} \left( \frac{n_B}{n_{\text{sat}}} - 1 \right)^2 \right] (1 - 2Y_Q)^2, \quad (13)$$

where  $E_{\text{NS}}$  is the ground-state energy at a given density and isospin asymmetry,

$E_{Y_Q=0.5}$  is the total energy for isospin-symmetric nuclear matter,  $E_{\text{sym,sat}} = \left( \frac{E_{Y_Q=0} - E_{Y_Q=0.5}}{N_B} \right)_{n_{\text{sat}}}$  is the symmetry energy at saturation,  $L_{\text{sat}} = 3n_{\text{sat}} \left( \frac{dE_{\text{sym}}}{dn_B} \right)_{n_{\text{sat}}}$  is the slope of the symmetry energy at saturation, and  $K_{\text{sym,sat}} = 9n_{\text{sat}}^2 \left( \frac{d^2 E_{\text{sym}}}{dn_B^2} \right)_{n_{\text{sat}}}$  is the symmetry energy curvature at saturation. Using this expansion scheme, it is possible to obtain the neutron star outer core EoS with quantified uncertainties from  $\chi$ EFT. The properties of the low-density crust (Lim and Holt 2017; Grams et al. 2022) and the high-density inner core (Hebeler et al. 2010; Tews et al. 2018; Lim et al. 2021; Drischler et al. 2021a; Brandes et al. 2023) require additional modeling assumptions.

The energy per baryon of symmetric nuclear matter,  $E_{Y_Q=0.5}/N_B$ , and pure neutron matter,  $E_{Y_Q=0}/N_B$ , has been computed from  $\chi$ EFT at different orders in the chiral expansion and different approximations in many-body perturbation theory. As a representative example, in Holt and Kaiser (2017) the EoS was computed up to third order in many-body perturbation theory, including self-consistent second-order single-particle energies. Chiral nucleon–nucleon interactions were included up to N3LO, while three-body forces were included up to N2LO in the chiral expansion. Using the above approach, the authors gave error bands on the EoS (including  $E_{\text{sym,sat}}$  and slope parameter  $L_{\text{sat}}$ ), taking into account uncertainties from the truncation of the chiral expansion and the choice of resolution scale in the nuclear interaction. The incorporation of third-order particle-hole ring diagrams (frequently overlooked in EoS computations) helped to reduce theoretical uncertainties in the neutron matter EoS at low densities, but beyond  $n_B \gtrsim 2n_{\text{sat}}$  the EoS error bars become large due to the breakdown in the chiral expansion. Recent advances in automated diagram and code generation have enabled studies at even higher orders in the many-body perturbation theory expansion (Drischler et al. 2021c, 2019). In the following we focus on selected results obtained in many-body perturbation theory and refer the reader to Lynn et al. (2019), Carlson et al. (2015), Gandolfi et al. (2020), Tews (2020), Rios (2020), Hagen et al. (2014) for comprehensive review articles on many-body calculations in the frameworks of quantum Monte Carlo, self-consistent Green's functions method, and coupled cluster theory.

The left panel of Fig. 12 shows the correlation between the symmetry energy  $E_{\text{sym}}$  and slope parameter  $L$  at saturation for different choices of the high-momentum regulating scale (shown as different data points) and order in the chiral expansion (denoted by different colors) all calculated at 3rd order in many-body perturbation theory including self-consistent (SC) nucleon self-energies at second order (Holt and Kaiser 2017). The ellipses show the 95% confidence level at orders NLO, N2LO, and the N3LO\* (where the star denotes that the three-body force is included only at N2LO) (Holt and Kaiser 2017). Interestingly, one finds that even EoS calculations performed at low order in the chiral expansion produce values of  $E_{\text{sym}}$  and  $L$  at saturation that tend to lie on a well-defined correlation line. The N3LO\* (red) ellipse illustrates the range of symmetry energy  $28 \text{ MeV} < E_{\text{sym,sat}} < 35 \text{ MeV}$  and slope parameter  $20 \text{ MeV} < L_{\text{sat}} < 65 \text{ MeV}$ , both of which are quite close to the findings of prior microscopic computations (Hebeler et al. 2010; Gandolfi et al. 2012) that also used neutron matter calculations plus the empirical saturation properties of



**Fig. 12** Adapted from Holt and Kaiser (2017), Drischler et al. (2021c). Left: correlation between the symmetry energy  $E_{\text{sym}}$  and its slope,  $L$  at saturation, from  $\chi$ EFT calculations at different orders in the chiral expansion (Holt and Kaiser 2017). Right: microscopic constraints on the  $E_{\text{sym}} - L$  correlation from Hebeler et al. (2010), Gandolfi et al. (2012), Holt and Kaiser (2017), Drischler et al. (2020)

symmetric nuclear matter to deduce  $E_{\text{sym}}$  and  $L$ . All three sets of results are shown in the right panel of Fig. 12 and labeled ‘H’ (Hebeler et al. 2010), ‘G’ (Gandolfi et al. 2012), and ‘HK’ (Holt and Kaiser 2017) respectively. In contrast, a recent work (Drischler et al. 2020) that analyzed correlated  $\chi$ EFT truncation errors in the EoS for neutron matter and symmetric nuclear matter using Bayesian statistical methods found  $E_{\text{sym}} = 31.7 \pm 1.1$  MeV and  $L = 59.8 \pm 4.1$  MeV at saturation, shown as ‘GP-B 500’ in the right panel of Fig. 12. The obtained value of  $E_{\text{sym}}$  was similar to those from Gandolfi et al. (2012), Hebeler et al. (2010), Holt and Kaiser (2017), but  $L$  was systematically larger. The results, however, are in good agreement with standard empirical constraints (Drischler et al. 2020; Li et al. 2019) discussed in Sect. 7.2.1.

Chiral effective field theory has also been employed to study the liquid–gas phase transition and thermodynamic EoS at low temperatures ( $T < 25$  MeV) in isospin-symmetric nuclear matter (Wellenhofer et al. 2014). The EoS has been computed using  $\chi$ EFT nuclear potentials at resolution scales of 414, 450, and 500 MeV. The results from

**Table 3** The N3LO contributions for binding energy per nucleon at saturation, saturation density, compressibility at saturation, and liquid–gas phase-transition critical values of temperature, density, and pressure at different resolution scales. Table adapted from Wellenhofer et al. (2014)

Resolution scale	$\frac{B}{A}$ (MeV)	$n_{\text{sat}}$ (fm $^{-3}$ )	$K$ (MeV)	$T_c$ (MeV)	$n_c$ (fm $^{-3}$ )	$P_c$ (MeV/fm $^3$ )
414 ( $M^*/M$ )	−15.79	0.171	223	17.4	0.066	0.33
450 ( $M^*/M$ )	−15.50	0.161	244	17.2	0.064	0.32
500 (no $M^*/M$ )	−16.51	0.174	250	19.1	0.072	0.42

this study are tabulated in Table 3. In particular, the values of the liquid–gas critical endpoint in temperature, pressure, and density agree well with the empirical multifragmentation and compound nuclear decay experiments discussed in Sect. 7.3. In addition, at low densities and moderate temperatures, the pure neutron matter EoS is well described within the virial expansion in terms of neutron–neutron scattering phase shifts. The results from chiral effective field theory have been shown (Wellenhofer et al. 2015) to be in very good agreement with the model-independent virial EoS. Since finite-temperature effects are difficult to reliably extract empirically, chiral effective field theory calculations have been used to constrain the temperature dependence of the dense-matter EoS in recent tabulations Du et al. (2019, 2022) for astrophysical simulations.

## 6 Experimental constraints: heavy-ion collisions

Given that hot and dense matter can be created experimentally in heavy-ion collisions, constraints on its EoS can be extracted via experimental measurements obtained from such collisions. As explained previously in Sect. 2.5, we will especially focus in this paper on the data themselves, and avoid citing quantities inferred from the data, as the latter come with an associated model dependence. We will mention particle production yields and their ratios, as well as fluctuation observables of particle multiplicities. Then, we will review experimental results on flow harmonics, to end with Hanbury–Brown–Twiss (HBT) interferometry measurements, also referred to in the field as femtoscopy.

We remind the reader that, as a general rule of thumb, high center of mass energy collisions,  $\sqrt{s_{NN}} \gtrsim 200$  GeV, are in the regime of the phase diagram where  $\mu_B \ll T$ , such that the numbers of particles and anti-particles are approximately equal (i.e.,  $n_B \sim 0$ ). As one lowers  $\sqrt{s_{NN}}$ , baryons are stopped within the collision such that higher  $n_B$  is reached. At sufficiently low beam energies,  $\sqrt{s_{NN}} \lesssim 4 - 7$  GeV,<sup>5</sup> matter is dominated by the hadron gas phase, such that lowering  $\sqrt{s_{NN}}$  leads to lower temperatures and lower  $n_B$ .

### 6.1 Particle yields

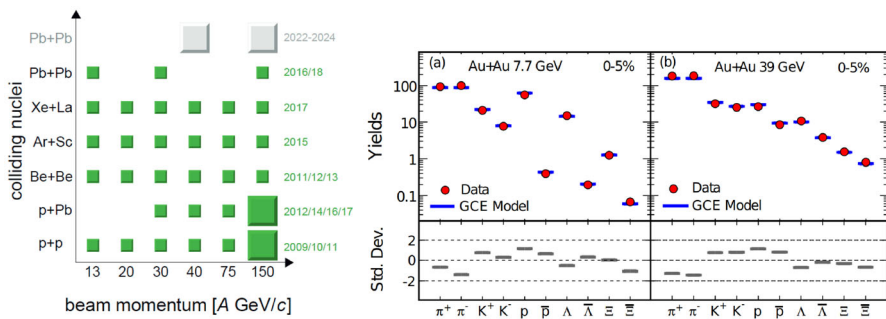
Particle production spectra are part of the simplest experimental observables used in heavy-ion collisions, to access thermodynamic properties and characteristics of the hot and dense matter. Starting with the integrated production yields of identified hadrons, they can be measured to help determine properties from the evolution of the system, in particular at chemical and kinetic freeze-out. These steps designate the ending of inelastic collisions between formed hadrons (what fixes the chemistry of the system) and in turn the ceasing of all elastic collisions (after which particles stream freely to the detectors) in the evolution of a heavy-ion collision. Statistical hadronization models (Hagedorn 1965; Dashen et al. 1969; Becattini and Passaleva 2002; Wheaton et al. 2011; Petran et al. 2014; Andronic et al. 2018, 2019; Vovchenko and Stoecker 2019) are fitted to these yields and ratios by varying over  $T$

<sup>5</sup> Note that the exact beam energy where this occurs is still under debate.

and  $\mu_B$ , in order to extract the respective chemical freeze-out values. Despite their ability to reproduce particle yields successfully, those models are limited in scope since they do not reproduce the dynamics of a collision and hinge on the assumption of thermal equilibrium, which is not necessarily achieved within the short time scales of heavy-ion collisions. Similar information can also be inferred for kinetic freeze-out, using a so-called blast-wave model (Schnedermann et al. 1993). The data from low transverse momentum particles (i.e.,  $p_T \lesssim 2 - 3$  GeV/c) measured at mid-rapidity (approximately transverse to the beam line direction) is used for statistical hadronization fits, because these particles spend the longest time within the medium (so they are more likely to be thermalized) and they have low enough momentum to avoid contributions from jet physics.

Measurement of yields for the most common light hadron species (namely  $\pi^\pm$ ,  $K^\pm$ ,  $p/\bar{p}$ ) and strange hadrons ( $\Lambda/\bar{\Lambda}$ ,  $\Xi^-/\bar{\Xi}^+$  and  $\Omega^-/\bar{\Omega}^+$ ) was achieved by STAR at RHIC and ALICE at the LHC. As part of the BES program, STAR has measured these hadron yields in Au+Au collisions at center-of-mass energies of  $\sqrt{s_{NN}} = 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4$  and 200 GeV (Abelev et al. 2009; Adamczyk et al. 2017; Adam et al. 2020b, d) and in U+U collisions at  $\sqrt{s_{NN}} = 193$  GeV (Abdallah et al. 2023d). Motivated by results from the past SPS-experiment NA49 obtained from Pb+Pb collisions at  $\sqrt{s_{NN}} < 20$  GeV (Alt et al. 2008), the NA61/SHINE experiment at CERN has conducted a scan in system-size and energy (in the same energy range as NA49). The diagram of all collided systems as a function of collision energy and nuclei is displayed in the left panel of Fig. 13. They published data for light hadrons in Ar+Sc (Acharya et al. 2021b) and Be+Be collisions (Acharya et al. 2021a) so far, while results from Xe+La and Pb+Pb collisions should become available in the next few years (Kowalski 2022). At LHC energies, the higher  $\sqrt{s_{NN}}$  produces significantly more particles, allowing more precise measurements of the species. For this reason, the ALICE experiment has measured yields not only for light and (multi)strange hadrons, but also for light nuclei and hyper-nuclei, in Pb+Pb collisions at 2.76 TeV/A in particular (Abelev et al. 2013a, c, 2014a; Adam et al. 2016f; Acharya et al. 2018e; Adam et al. 2016a), and more recently in Pb+Pb collisions at 5.02 TeV/A too (Acharya et al. 2020c, 2023c), as well as Xe+Xe collisions at 5.44 TeV/A (Acharya et al. 2021e).

The yields of particle production can also be used as an indicator of the onset of deconfinement, notably thanks to the strangeness enhancement: strange quark-antiquark pairs are expected to be produced at a much higher rate in a hot and dense medium than in a hadron gas. Hence, one should expect in particular an increase of multi-strange baryons compared to light-quark-compound hadrons in collision systems where the QGP has been formed, which has been observed experimentally in heavy-ion collisions at several energies (Antinori et al. 2006; Abelev et al. 2008b, 2014a). Moreover, the distinctive non-monotonic behavior of the  $K^+/\pi^+$  ratio as a function of the collision energy can also be considered as a sign of the onset of deconfinement, according to some authors (Gazdzicki and Gorenstein 1999; Poberezhnyuk et al. 2015). This so-called “horn” in the  $K^+/\pi^+$  ratio has been notably observed in Pb+Pb (Afanasiev et al. 2002; Alt et al. 2008) and Au+Au collisions (Akiba et al. 1996; Ahle et al. 2000; Abelev et al. 2009, 2010b; Adamczyk



**Fig. 13** Left: diagram of different collided systems as a function of collision energy. Right: yields of hadrons measured by STAR in central Au+Au collisions at 7.7 and 39 GeV/A, compared with results from a grand canonical statistical hadronization model. Images reproduced with permission from [left] Kowalski (2022), copyright by the author(s), and [right] Adamczyk et al. (2017), copyright by APS

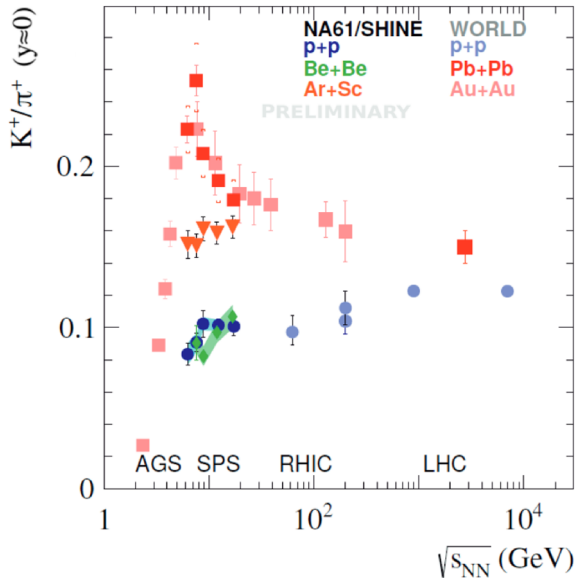
et al. 2017), but absent from p+p data (Aduszkiewicz et al. 2017; Abelev et al. 2010b; Aamodt et al. 2011c; Abelev et al. 2014c) and Be+Be collisions results (Acharya et al. 2021a). Recent results from Ar+Sc collisions (Kowalski 2022) have, however, stirred up doubts regarding the interpretation of this observable, as the value of this ratio from such collision system is closer to the one measured in big systems, while no horn structure is seen, similar to small systems. All these results for different systems can be seen in Fig. 14.

## 6.2 Fluctuation observables

In heavy-ion collisions, observables measuring fluctuations are among the most relevant for the investigation of the QCD phase diagram. Within the assumption of thermodynamic equilibrium, cumulants of net-particle multiplicity distributions become directly related to thermodynamic susceptibilities, and can be compared to results from lattice QCD (see Sect. 3.2) to extract information on the chemical freeze-out line (Alba et al. 2014, 2015, 2020). Moreover, large, relatively long-range fluctuations are expected in the neighborhood of the conjectured QCD critical point, making fluctuation observables very promising signatures of criticality (Stephanov et al. 1999; Stephanov 2009; Athanasiou et al. 2010). It has also been proposed that the finite-size scaling of critical fluctuations could be employed to constrain the location of the critical point (Palhares et al. 2011; Fraga et al. 2010; Lacey 2015; Lacey et al. 2016). In Fraga et al. (2010), finite-size scaling arguments were applied to mean transverse-momentum fluctuations measured by STAR (Adams et al. 2005c, 2007) to exclude a critical point below  $\mu_B \lesssim 450$  MeV.

Fluctuations of the conserved charges  $B$ ,  $S$ , and  $Q$  are of particular importance. As mentioned already in Sect. 3.2, these fluctuations can be used to probe the deconfinement transition, as well as the location of the critical endpoint. Calculated via the susceptibilities expressed in Eq. (3), which can be evaluated via lattice QCD

**Fig. 14** Preliminary results for the ratio of  $K^+/\pi^+$  yields at mid-rapidity as a function of the collision energy, compared for different collision systems. Image reproduced with permission from Kowalski (2022), copyright by the author (s)



simulations or HRG model calculations, they can also be related to the corresponding cumulants of conserved charges  $C_{lmn}^{BSQ}$ ,<sup>6</sup> following the relation

$$C_{lmn}^{BSQ} = VT^3 \times \chi_{lmn}^{BSQ} \quad (14)$$

with the volume  $V$  and temperature  $T$  of the system, and  $l, m, n \in \mathbb{N}$  (Luo and Xu 2017). The cumulants are also theoretically related to the correlation length of the system  $\xi$ , which is expected to diverge in the vicinity of the critical endpoint. In particular, the higher order cumulants are proportional to higher powers of  $\xi$ , making them more sensitive to critical fluctuations (Stephanov et al. 1999; Stephanov 2009; Athanasiou et al. 2010).

In heavy-ion collisions, however, it is impossible to measure the fluctuations of conserved charges directly, because one cannot detect all produced particles (e.g., neutral particles are not always possible to measure, so that baryon number fluctuations do not include neutrons). Nevertheless, it is common to measure the cumulants of identified particles' net-multiplicity distributions, using some hadronic species as proxies for conserved charges (Koch et al. 2005). Net-proton distributions are used as a proxy for net-baryons (Aggarwal et al. 2010; Adam et al. 2019b; Abdallah et al. 2021b), net-kaons (Adamczyk et al. 2018c; Ohlson 2018; Adam et al. 2019c) or net-lambdas (Adam et al. 2020a) are used as a proxy for net-strangeness, and net-pions+protons+kaons (Adam et al. 2019c) has been recently used as a proxy for net-electric charge, instead of the actual net-charged unidentified hadron distributions. Mixed correlations have also been measured (Adam et al. 2019c),

<sup>6</sup> Note that such cumulants are often referred to as  $\kappa_{lmn}^{BSQ}$  in the literature; we used a different symbol here to avoid confusion with the  $\kappa$  coefficients of the pseudo-transition temperature parametrisation from lattice QCD in Sect. 3.4.

although alternative ones have been suggested, that would provide more direct comparisons to lattice QCD susceptibilities (Bellwied et al. 2020).

These net-particle cumulants can be used to construct ratios, as they are connected with usual statistic quantities characterizing the net-hadron distributions  $N_\alpha = n_\alpha - n_{\bar{\alpha}}$  (with  $n_{\alpha/\bar{\alpha}}$  being respectively the number of hadrons or anti-hadrons of hadronic specie  $\alpha$ ). Hence, relations between such ratios and the mean  $\mu_\alpha$ , variance  $\sigma_\alpha$ , skewness  $S_\alpha$  or kurtosis  $\kappa_\alpha$  can be expressed as follows

$$\frac{\sigma_\alpha^2}{\mu_\alpha} = \frac{C_2^\alpha}{C_1^\alpha} = \frac{\langle (\delta N_\alpha)^2 \rangle}{\langle N_\alpha \rangle}, \quad (15)$$

$$S_\alpha \sigma_\alpha = \frac{C_3^\alpha}{C_2^\alpha} = \frac{\langle (\delta N_\alpha)^3 \rangle}{\langle (\delta N_\alpha)^2 \rangle}, \quad (16)$$

$$\kappa_\alpha \sigma_\alpha^2 = \frac{C_4^\alpha}{C_2^\alpha} = \frac{\langle (\delta N_\alpha)^4 \rangle}{\langle (\delta N_\alpha)^2 \rangle} - 3 \langle (\delta N_\alpha)^2 \rangle, \quad (17)$$

$$\frac{\kappa_\alpha \sigma_\alpha}{S_\alpha} = \frac{C_4^\alpha}{C_3^\alpha} = \frac{\langle (\delta N_\alpha)^4 \rangle - 3 \langle (\delta N_\alpha)^2 \rangle^2}{\langle (\delta N_\alpha)^3 \rangle}, \quad (18)$$

with  $\delta N_\alpha = N_\alpha - \langle N_\alpha \rangle$ , and  $\langle \rangle$  denoting an average over the number of events in a fixed centrality class at a specific beam energy (Luo and Xu 2017). These ratios allow for more direct comparisons to theoretical calculations of susceptibilities because the leading order dependence on volume and temperature cancels out (see Eq. (14)).

While direct comparisons between theoretically calculated susceptibilities and multi-particle cumulants have been made, certain caveats exist. First of all, these comparisons are only valid if the chosen particle species are good proxies for their respective conserved charge (see e.g. Chatterjee et al. 2016; Bellwied et al. 2020). Additionally, there are fundamental conceptual differences between the assumed in-equilibrium and infinite volume lattice QCD calculations on one side, and the highly dynamical, far-from-equilibrium, short-lived, and finite-size system created in heavy-ion collisions on the other side. While building cumulant ratios cancels the trivial dependence on volume and temperature, it does not prevent volume fluctuations that can affect the signal, especially for higher order cumulants (Gorenstein and Gazdzicki 2011; Konchakovski et al. 2009; Skokov et al. 2013; Luo and Xu 2017). Calculating the cumulants as a function of the centrality class will generally increase the signal, as the volume of systems varies within a single centrality class (Luo et al. 2013). Also known as the centrality bin-width effect, this artificial modification of the measured fluctuations can be minimized by using small centrality classes, and some correction methods (Sahoo et al. 2013; Gorenstein 2015). A second consequence is the fact that the finite size of the system limits the growth of  $\xi$ . The correlation length must be smaller than the size of the system itself and  $\xi$  is even smaller when the system is inhomogeneous (Stephanov et al. 1999). Because the system only approaches the critical point for a finite period of time, the growth of  $\xi$  would be consequently limited, restraining even more the size of measured fluctuations



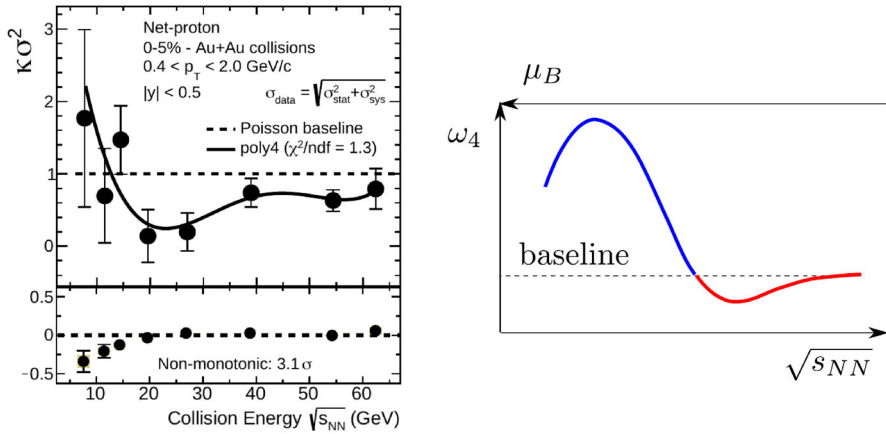
amplitude (Berdnikov and Rajagopal 2000; Hippert et al. 2016; Herold et al. 2016). Critical lensing effects may somewhat compensate for some of these effects by drawing more of the system towards the critical regime (Dore et al. 2022).

Finally, the width of the rapidity (angle with respect to the beam line) window in which particle cumulants are measured is important. The signal from critical fluctuations is expected to have a correlation range of  $\Delta y \sim 1$ , hence cumulants should be measured with particles in a rapidity of this order at least to be sensitive to criticality (Ling and Stephanov 2016). Another alternative is to use factorial cumulants  $\hat{C}_n$  (also referred to as correlation functions), which can be expressed as linear combinations of cumulants  $C_n$  and are better suited for acceptance dependence studies because of their linear scaling with the rapidity acceptance (Ling and Stephanov 2016; Bzdak et al. 2017). Moreover, acceptance cuts may also affect fluctuation observables and, together with detection efficiency effects, contribute with spurious binomial fluctuations (Pruneau et al. 2002; Bzdak and Koch 2012; Garg et al. 2013; Karsch et al. 2016; Hippert and Fraga 2017). Resonance decays can also lead to spurious contributions, as has been discussed in Begun et al. (2006), Sahoo et al. (2013), Garg et al. (2013), Nahrgang et al. (2015), Bluhm et al. (2017), Mishra et al. (2016), Hippert and Fraga (2017).

### 6.2.1 Net- $p$ fluctuations

Experimental collaborations commonly use the net-proton distribution as a proxy for the baryon number  $B$ , even though protons experience isospin randomization during the late stages of the collision (Kitazawa and Asakawa 2012b; Nahrgang et al. 2015). This process causes the original nucleon isospin distribution to be blurred and is due to the reactions  $p + \pi^{0/-} \leftrightarrow \Delta^{+/0} \leftrightarrow n + \pi^{+/0}$  that nucleons undergo several times during the hadronic cascade. These reactions do not affect net- $B$  fluctuations, but do affect net-proton fluctuations. Since protons are the only nucleons measured in the final state, isospin randomization has to be taken into account when comparing both quantities (Kitazawa and Asakawa 2012a).

The ALICE collaboration has published measurements of  $C_1^p$  and  $C_2^p$  net-proton cumulants and their ratios in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV (Acharya et al. 2020a) and at  $\sqrt{s_{NN}} = 5.02$  TeV  $C_3^p$  was also measured (Acharya et al. 2023b). As the system is created at almost vanishing baryonic chemical potential at such high energies, those cumulants can be compared to lattice QCD susceptibility results like the one discussed in Sect. 3.2, keeping in mind the subtleties of such comparison mentioned in the previous paragraph. However, no critical signal is expected in such collisions, they are mostly used to study correlation dynamics and the effect of global and local charge conservation (ALICE Collaboration 2022). Only higher-order cumulants, from  $C_6^p$  and beyond, are expected to exhibit  $O(4)$  criticality (Friman et al. 2011; Almasi et al. 2019). At RHIC energies, the STAR experiment has measured net-proton (factorial) cumulants as one of the main objectives of the BES program, in Au+Au collisions from  $\sqrt{s_{NN}} = 200$  GeV down to  $\sqrt{s_{NN}} = 7.7$  GeV for cumulants up to  $C_4^p$ , and even down to  $\sqrt{s_{NN}} = 3$  GeV for up to  $C_6^p$  (Abdallah et al. 2021b; Aboona et al. 2023b). One of the most interesting results is the energy dependence of



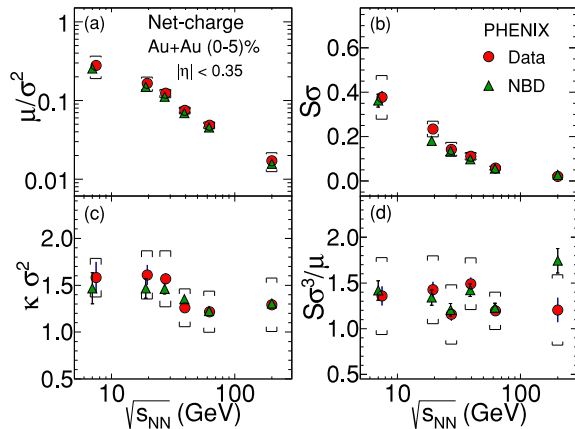
**Fig. 15** Left: energy dependence of the  $C_4/C_2(=\kappa\sigma^2)$  ratio from net- $p$  distribution for  $|y| < 0.5$  and  $0.4 < p_T < 2.0$  GeV/c, measured in 0–5% Au+Au collisions by the STAR collaboration. Image adapted from Abdallah et al. (2021b). Right: expected behavior of the  $C_4/C_1(=\omega_4)$  ratio for net- $p$ , in the case of a freeze-out line passing through the critical region near the critical endpoint. Image reproduced with permission from Bzdak et al. (2020), copyright by Elsevier

the  $C_4^p/C_2^p$  ratio, shown in the left panel of Fig. 15, which exhibits a non-monotonic behavior with a significance of  $3.1\sigma$  towards low collision energies (Abdallah et al. 2021b). The right panel of Fig. 15 shows a theoretical calculation of one possible critical point from Stephanov (2011) using a 3D Ising model. Such non-monotonic behavior of the net-proton kurtosis, and more specifically the peak arising after the dip when going to lower energy, had been predicted as an effective sign of the existence a critical region in the phase diagram in Stephanov (2011)—later work (Mroczek et al. 2021, 2023a) has found exceptions to this using the same framework but incorporating all higher order terms, demonstrating that the peak in kurtosis is the most important signal for the critical point but the bump is not always present. Further investigations are already planned to enlighten this special result, by collecting more data in low-energy collisions and especially exploring energies below  $\sqrt{s_{NN}} = 7.7$  GeV in the BES-II program (Tlusty 2018). They will complete the results of the HADES collaboration, which published a complete analysis for net- $p$  cumulants up to  $C_4^p$ , in Au+Au collisions at  $\sqrt{s_{NN}} = 2.4$  GeV (Adamczewski-Musch et al. 2020c).

## 6.2.2 Net-charged hadron fluctuations

Electric charge fluctuations are the easiest to measure experimentally, as charged particle distributions are accessible even without having to identify the detected particles. Both STAR (Adamczyk et al. 2014a) and PHENIX (Adare et al. 2016a) collaborations have published results of net- $Q$  cumulants up to  $C_4^Q$  in Au+Au collisions from 7.7 to 200 GeV/A, shown for PHENIX in Fig. 16, with no evidence of a peak that could hint at the presence of a critical endpoint. The same net- $Q$  cumulants have also been measured by the NA61/SHINE experiment in smaller

**Fig. 16** Energy dependence of  $\mu/\sigma^2 \sim C_1/C_2$ ,  $S\sigma \sim C_3/C_2$ ,  $\kappa\sigma^2 \sim C_4/C_2$  and  $S\sigma^3/\mu \sim C_3/C_1$  for net- $Q$  in central Au+Au collisions for particles with  $0.3 < p_T < 2.0$  GeV and within  $|\eta| < 0.35$ , from the PHENIX collaboration. Data are compared with negative binomial-distribution (NBD). Image reproduced with permission from Adare et al. (2016a), copyright by APS

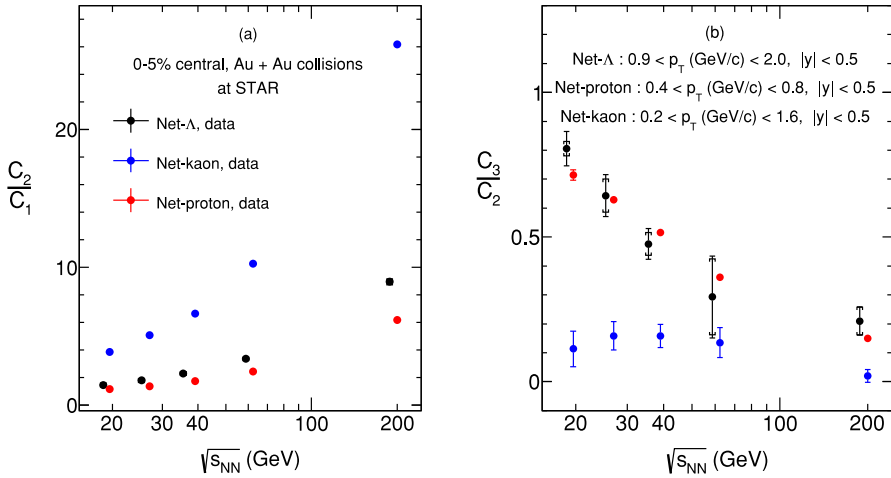


systems (Be+Be and Ar+Sc) for several collision energies within  $5.1 \leq \sqrt{s_{NN}} \leq 17.3$  GeV, without any sign of criticality (Marcinek 2023). Combining net-p and net-Q fluctuations can be used to extract the  $T, \mu_B$  at freeze-out for a specific  $\sqrt{s_{NN}}$  and centrality class (normally central collisions of 0–5%). This has been done within a hadron resonance gas model where acceptance cuts and isospin randomization can be taken into account (Alba et al. 2014, 2015, 2020) but consistent results have also been found from lattice QCD susceptibilities as well (Borsanyi et al. 2014b) that cannot take those effects into account.

### 6.2.3 Net-K, net- $\Lambda$ fluctuations

In the strangeness sector, net-kaon (specifically  $K^\pm$ ) distributions are used as a proxy, since they are abundantly produced and easily reconstructed in heavy-ion collisions. Because all other strange particles carry baryon number, the resulting cumulants separated by particle species can provide varying results (Zhou et al. 2017). These cumulants have been measured extensively by the STAR experiment, again in Au+Au collisions from 7.7 to 200 GeV/A up to  $C_4^K$  (Adamczyk et al. 2018c), see Fig. 17. The ALICE collaboration has also published some preliminary results of  $C_1$  and  $C_2$  for net- $K$  distributions, along with net- $\pi$  and net- $p$  results, in Pb+Pb collisions at 2.76 TeV/A (Ohlson 2018). Recently, net- $\Lambda$  cumulants up to  $C_3^\Lambda$  order and their ratios have also been measured by the STAR experiment, in Au+Au collisions at  $\sqrt{s_{NN}} = 19.6, 27, 39, 62.4$  and 200 GeV (Adam et al. 2020a). Note that  $\Lambda$  results inherently include contamination from  $\Sigma^0$  baryons, which decay with a branching ratios of 100% via the channel  $\Sigma^0 \rightarrow \Lambda + \gamma$  and cannot be discriminated from primary  $\Lambda$  production. Such results on event-by-event fluctuations of  $\Lambda$  baryons are important to investigate the interplay between baryon number and strangeness conservation at hadronization.

It was proposed in Bellwied et al. (2013) from lattice QCD that there may be a flavor hierarchy wherein strange particles freeze-out at a higher temperature than light particles. The idea relies on the change in the degrees of freedom in

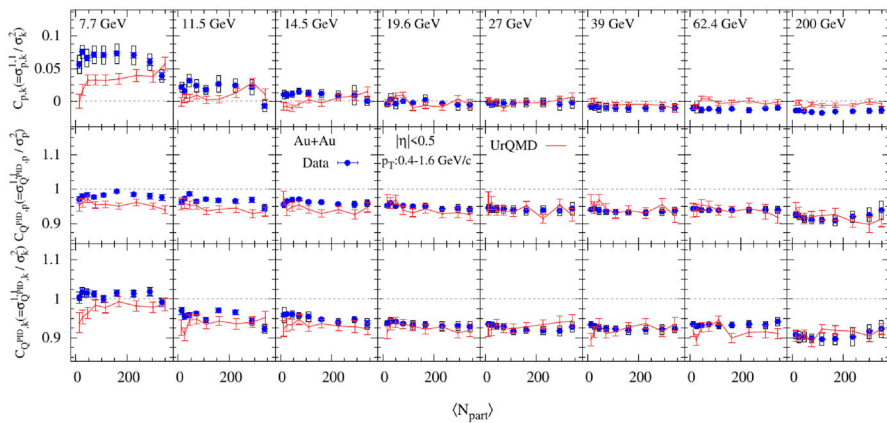


**Fig. 17** Energy dependence of  $C_2/C_1$  and  $C_3/C_2$  for net- $\Lambda$ , net- $K$  and net- $p$  within  $|y| < 0.5$  in central Au+Au collisions, from the STAR collaboration. Image reproduced with permission from Adam et al. (2020a), copyright by APS

comparisons between various lattice susceptibilities for light and strange quarks to the hadron resonance gas model. The original lattice QCD paper suggested strange particles hadronize at  $T \sim 10 - 15$  MeV higher temperatures than light particles. Using the net- $K$  and net- $\Lambda$  results, it has been shown from both a hadron resonance gas (Bellwied et al. 2019; Bluhm and Nahrgang 2019) and lattice QCD (Noronha-Hostler et al. 2016a) that a preference for a higher freeze-out temperature for strangeness is preferred (although the exact temperature is somewhat model dependent).

## 6.2.4 Mixed conserved charges

In addition to the so-called “diagonal” cumulants, i.e., cumulants of net-multiplicity distribution for hadronic species related to a single conserved charge, the STAR collaboration measured off-diagonal cumulants that represent correlations between different conserved charges (Koch et al. 2005; Majumder and Muller 2006). STAR extracted results for covariances, i.e.  $C_2$  mixed-cumulants of net- $Q$ , net- $p$ , and net- $K$  distributions (proxies for  $B$  and  $S$  respectively) and their ratios, from Au+Au collisions in the usual BES-I collision energy range  $7.7 \leq \sqrt{s_{NN}} \leq 200$  GeV (Adam et al. 2019c). The full suite of experimental observables was  $\sigma_{Q,p}^{1,1}$ ,  $\sigma_{Q,K}^{1,1}$ ,  $\sigma_{Q,K}^{1,1}$ ,  $\sigma_{Q,p}^{1,1}/\sigma_p^2$ ,  $\sigma_{Q,K}^{1,1}/\sigma_K^2$ ,  $\sigma_{p,K}^{1,1}/\sigma_K^2$ . While both  $\sigma_{Q,p}^{1,1}/\sigma_p^2$  and  $\sigma_{Q,K}^{1,1}/\sigma_K^2$  ratios show only a small collision energy dependence and no peculiar behavior, the  $\sigma_{p,K}^{1,1}/\sigma_K^2$  ratio exhibits a global sign change around  $\sqrt{s_{NN}} \sim 20$  GeV, as can be seen in Fig. 18. Even though not straightforward to interpret, this result might provide important insight into the onset of deconfinement. In Bellwied et al. (2020) it was argued that the current off-diagonal cumulants were not the best to reproduce lattice QCD due to



**Fig. 18** From Adam et al. (2019c). Centrality dependence of covariance to variance ratios for net- $Q$ , net- $p$ , and net- $K$  distributions in Au+Au collisions at different collision energies from the STAR collaboration, compared with results from ultra-relativistic quantum molecular dynamics (UrQMD) simulations

“missing” hadrons that could not be measured experimentally (e.g. neutrons). Thus, it was suggested to instead measure  $\sigma_\Lambda^2/(\sigma_K^2 + \sigma_\Lambda^2)$  and  $\sigma_K^2/2(\sigma_\Lambda^2 + \sigma_K^2)$  to assess, respectively, strange baryon correlations and strange electric charge correlations. These can be reconstructed using the data from Adam et al. (2020a, 2020d).

## 6.2.5 Summary of cumulant observables for conserved charge proxies

In terms of theoretical comparisons to these data points, each theoretical approach has its own caveats. In principle, lattice QCD only provides results for the infinite volume limit, it cannot account for decays, cannot account for limited particle species (i.e. effects like isospin randomization are missed), nor can it account for kinematic cuts. However, using partial pressures (Noronha-Hostler et al. 2016a) it is at least possible to capture fluctuations of certain hadronic species (i.e. kaons) more directly from lattice QCD. A hadron resonance gas approach does have the advantage of fitting lattice QCD very well at temperatures below  $T \sim 150\text{--}165$  MeV (the exact temperature depends on the observable), can take into account isospin randomization and kinematic cuts, and can calculate quantities for specific particle species. However, a hadron resonance gas model is dependent on the particle list considered (incomplete particle lists can lead to misleading results) and cannot take into account dynamical effects or out-of-equilibrium effects. A third option is also often used, which are hadron transport codes like UrQMD (Bass et al. 1998; Bleicher et al. 1999) or SMASH (Weil et al. 2016; Hammelmann and Elfner 2023) that can take into account all the dynamical and out-of-equilibrium effects. However, transport models have the caveats that they cannot take into account decays and interactions of more than 2 bodies (i.e.  $1 \rightarrow 3$  body decays are excluded, even though they are known to exist experimentally) and their connection to temperature is more tenuous.

In Table 4 we summarize the latest results from ALICE, STAR, HADES, and NA61/SHINE on all the cumulants for net-proton, net-charge, and net-strangeness.

**Table 4** Summary of cumulant ratios of different particle species for which measurements have been published by experiments across several collision energies ( $\sqrt{s_{NN}}$ ) and systems

Ratio	System	Experiment	$\sqrt{s_{NN}}$
$C_2^p/C_1^p$	Au+ Au	HADES (Adamczewski-Musch et al. <a href="#">2020c</a> )	2.4 GeV
		STAR (Abdallah et al. <a href="#">2021b</a> )	7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, 200 GeV
$C_2^p/\langle p + \bar{p} \rangle$	Pb+Pb	ALICE (Acharya et al. <a href="#">2020a</a> , <a href="#">2023b</a> )	2.76, 5.02 TeV
$C_3^p/C_1^p$	Au+ Au	PHENIX (Adare et al. <a href="#">2016a</a> )	7.7, 19.6, 27, 39, 62.4, 200 GeV
$C_3^p/C_2^p$	Au+ Au	HADES (Adamczewski-Musch et al. <a href="#">2020c</a> )	2.4 GeV
		STAR (Abdallah et al. <a href="#">2021b</a> )	7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, 200 GeV
$C_4^p/C_2^p$	Au+ Au	HADES (Adamczewski-Musch et al. <a href="#">2020c</a> )	2.4 GeV
		STAR (Aboona et al. <a href="#">2023b</a> ; Abdallah et al. <a href="#">2021b</a> )	3, 7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, 200 GeV
$C_5^p/C_1^p$	Au+ Au	STAR (Aboona et al. <a href="#">2023b</a> )	3, 7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, 200 GeV
$C_6^p/C_2^p$	Au+ Au	STAR (Aboona et al. <a href="#">2023b</a> )	3, 7.7, 11.5, 14.5, 19.6, 27, 39, 54.4, 62.4, 200 GeV
$C_2^Q/C_1^Q$	Au+ Au	PHENIX (Adare et al. <a href="#">2016a</a> )	7.7, 19.6, 27, 39, 62.4, 200 GeV
	Au+ Au	STAR (Adamczyk et al. <a href="#">2014a</a> )	7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV
$C_3^Q/C_1^Q$	Au+ Au	PHENIX (Adare et al. <a href="#">2016a</a> )	7.7, 19.6, 27, 39, 62.4, 200 GeV
$C_3^Q/C_2^Q$	Au+ Au	PHENIX (Adare et al. <a href="#">2016a</a> )	7.7, 19.6, 27, 39, 62.4, 200 GeV
		STAR (Adamczyk et al. <a href="#">2014a</a> )	7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV
$C_4^Q/C_2^Q$	Au+ Au	PHENIX (Adare et al. <a href="#">2016a</a> )	7.7, 19.6, 27, 39, 62.4, 200 GeV
		STAR (Adamczyk et al. <a href="#">2014a</a> )	7.7, 11.5, 19.6, 27, 39, 62.4, 200 GeV
$C_2^K/C_1^K$		STAR (Adamczyk et al. <a href="#">2018c</a> )	7.7, 11.5, 14.5, 19.6, 27, 39, 62.4, 200 GeV
$C_3^K/C_2^K$		STAR (Adamczyk et al. <a href="#">2018c</a> )	7.7, 11.5, 14.5, 19.6, 27, 39, 62.4, 200 GeV
$C_4^K/C_2^K$		STAR (Adamczyk et al. <a href="#">2018c</a> )	7.7, 11.5, 14.5, 19.6, 27, 39, 62.4, 200 GeV
$C_2^\Lambda/C_1^\Lambda$		STAR (Adam et al. <a href="#">2020a</a> )	19.6, 27, 39, 62.4, 200 GeV
$C_3^\Lambda/C_2^\Lambda$		STAR (Adam et al. <a href="#">2020a</a> )	19.6, 27, 39, 62.4, 200 GeV

We note that new results are anticipated later this year from STAR's Beam Energy Scan II program that will significantly reduce the error bars from BES I and also provide new beam energies in the fixed target regime i.e. between  $\sqrt{s_{NN}} = 3 - 7.7$  GeV.

### 6.2.6 Other types of fluctuations

Other observables used to study two-particle correlations are the dynamical fluctuations,  $v_{\text{dyn}}$ , which can be used in the case of incomplete particle detection, even though  $v_{\text{dyn}}$  intrinsically depends on multiplicity (Gavin and Kapusta 2002; Pruneau et al. 2002). Two-species correlations between proton, kaon, and pion distributions have been investigated by the NA49, STAR, and ALICE collaborations, in very central Pb+Pb and Au+Au collisions for  $6.3 \leq \sqrt{s_{NN}} \leq 17.3$  GeV,  $7.7 \leq \sqrt{s_{NN}} \leq 200$  GeV and  $\sqrt{s_{NN}} = 2.76$  TeV, respectively (Anticic et al. 2014; Abdelwahab et al. 2015; Acharya et al. 2019d). The energy dependence of  $v_{\text{dyn}}[\pi, K]$  and  $v_{\text{dyn}}[p, K]$  from NA49 data, in particular, displays a strong variation below center-of-mass energy  $\sqrt{s_{NN}} \sim 10$  GeV, as can be seen in Fig. 19. This could indicate a change in the production mechanism of such particles, hinting at differences in the phases probed in these collisions (Acharya et al. 2019d). The ALICE collaboration recently presented preliminary results on the system-size dependence of  $v_{\text{dyn}}[+, -]$  normalized by charged particle density to remove its intrinsic multiplicity dependence. The data collected from p+p, p+Pb, and Pb+Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV, and Xe+Xe collisions at  $\sqrt{s_{NN}} = 5.44$  TeV display a decreasing trend with the size of collided systems that no model has been able to completely reproduce (Sputowska 2022).

Intensive and strongly intensive quantities can be used to investigate hadron number fluctuations, getting rid of the volume dependence and volume fluctuations, as proposed in Gorenstein and Gazdzicki (2011). Some of the results introduced earlier, like the scaled cumulants published by ALICE in Acharya et al. (2023b), STAR in Abdallah et al. (2021b) or NA61/SHINE in Marcinek (2023) are in fact intensive quantities by construction.

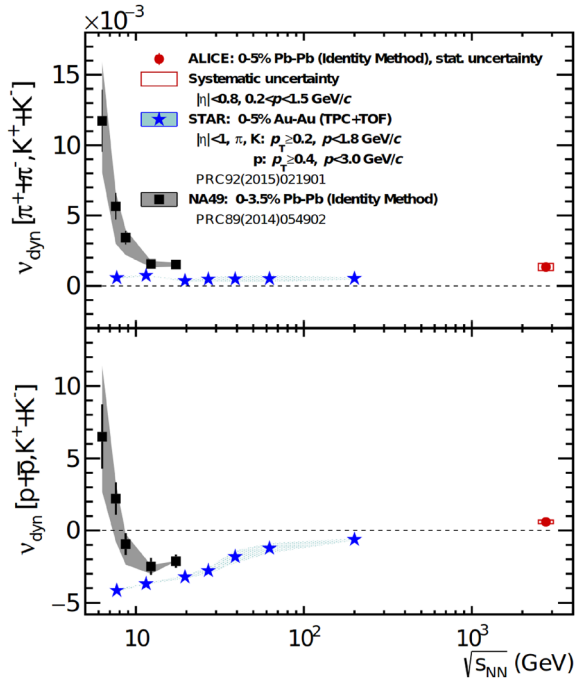
Scaled factorial moments  $F_r(M)$  of order  $r$  can be used to study the presence of the critical endpoint or phase transition, as they are defined to be sensitive to large multiplicity fluctuations caused by criticality (Bialas and Peschanski 1986; Satz 1989; Bialas and Hwa 1991). They are defined as

$$F_r(M) = \frac{\left\langle \frac{1}{M^D} \sum_{i=1}^{M^D} n_i(n_i - 1) \dots (n_i - r + 1) \right\rangle}{\left\langle \frac{1}{M^D} \sum_{i=1}^{M^D} n_i \right\rangle^r} \quad (19)$$

with  $M^D$  the number of bins in which the  $D$ -dimensional momentum space is partitioned,  $n_i$  the multiplicity of particles of interest in the  $i^{\text{th}}$  bin, and  $r$  the order of the moment of interest (Wu et al. 2020). The NA61/SHINE collaboration has measured  $F_2(M)$  for protons in central Pb+Pb and Ar+Sc collisions, and  $F_{2,3,4}(M)$  for negatively charged hadrons at several energies below  $\sqrt{s_{NN}} = 17$  GeV, showing no signal of any criticality in such systems (Adhikary 2022).

Fluctuations of the mean transverse momentum, as those of conserved quantities, would also diverge at the critical point in equilibrium and are expected to be enlarged in its vicinity (Stephanov et al. 1999). Mean transverse momentum correlations have

**Fig. 19** Combined results from STAR measurements in central Au+Au collisions, and ALICE and NA49 measurements in central Pb+Pb collisions on the energy dependence of  $v_{\text{dyn}}[\pi, K]$  and  $v_{\text{dyn}}[p, K]$  (Acharya et al. 2019d)



been measured by the STAR collaboration, for beam energies of  $\sqrt{s_{\text{NN}}} = 20, 62, 130$  and 200 GeV (Adams et al. 2005c, 2007).

### 6.3 Flow harmonics

Another important class of observables giving information about the dynamics of heavy-ion collisions and the EoS of nuclear matter are the flow harmonics  $v_n$ . They are the coefficients of the Fourier expansion of an  $N$ -particle triple-differential distribution

$$E \frac{d^3N}{d^3p} = \frac{1}{2\pi} \cdot \frac{d^2N}{p_T dp_T dy} \left( 1 + 2 \sum_{n=1}^{\infty} v_n \cdot \cos(n(\phi - \Psi)) \right) \quad (20)$$

where  $E$  is the particle energy,  $p_{(T)}$  its (transverse) momentum,  $y$  its rapidity,  $\phi$  its azimuthal angle and  $\Psi$  the event plane angle. The flow coefficients are defined as  $v_n = \langle \cos(n(\phi - \Psi)) \rangle$ ,  $\langle \dots \rangle$  denoting an average over many collision events (Voloshin and Zhang 1996; Poskanzer and Voloshin 1998). These flow coefficients measure the azimuthal anisotropies of particle distributions and are a signature of collective expansion, with  $v_2$  measurements playing an important role in the conclusion that QGP formation was observed in Au+Au collisions at RHIC (Adcox et al. 2005; Back et al. 2005c; Adams et al. 2005b).



### 6.3.1 Measuring collective flow across $\sqrt{s_{NN}}$ : event-plane versus multi-particle cumulant methods

While collective flow harmonics may initially appear deceptively simple as just a cos term, there are a number of subtle details that the reader must be aware of before making direct theory-to-experiment comparisons. There are two primary methods used to calculate  $v_n$  for low  $p_T$  particles (for the relevance to the EoS we will focus only on low  $p_T$  particles but at high  $p_T$  other technical details exist both on the theory and experimental side, some of which are discussed in Betz et al. (2017)). Before explaining the two methods, it is important to first understand that collective flow is not just a scalar quantity but rather it is a vector that contains both the magnitude of the flow  $v_n$  that is 0 for a circular event and 1 for the extreme of the corresponding shape (i.e. for elliptical flow,  $v_2$ , it would appear as a line) and the corresponding event-plane angle  $\Psi_n$  that is the direction of the flow vector. Then, the flow harmonic can be defined as the complex vector

$$V_n = v_n e^{in\Psi_n}. \quad (21)$$

To be clear,  $\Psi_n$  is the event plane angle reconstructed by the detector that includes all the usual caveats of having finite number of particles, acceptance cuts, and efficiencies. However, in principle, if all particles were measured to infinite precision one could rigorously define the underlying event plane  $\Phi_n$  that is the actual event plane of the given event. Due to the previously mentioned detector effects,  $\Psi_n \neq \Phi_n$  and the dispersion in this relationship can be defined as the resolution,  $R$ , for a specific flow harmonic

$$R(v_n) \equiv \langle e^{in(\Psi_n - \Phi_n)} \rangle \quad (22)$$

wherein the bracket  $\langle \dots \rangle$  indicates an averaging over a large ensemble of events.

At this point, we can discuss the two different methods for calculating flow harmonics. The first one is the “**event plane method**” (Poskanzer and Voloshin 1998), which was the first technique used to calculate flow harmonics. At that time it was assumed that dynamical fluctuations would have a negligible effect on the extraction of the event plane, such that  $v_n$  would essentially be the same for all events within a fixed centrality class. In this method,  $\Psi_n$  is determined from two or more subsets of particles (A and B, known as subevents) within a single event such that

$$\psi_n^{A,B} \equiv \frac{1}{n} \frac{\sum_{i \in A,B} w_i \sin(n\phi_i)}{\sum_{i \in A,B} w_i \cos(n\phi_i)} \quad (23)$$

where  $n$  is the number of particles considered and  $w_i$  is a relevant weight (such as energy or momentum). At this point, it is important to discuss the type of particles considered. The most standard flow measurements are all charged particles. However, it can be of interest to study the flow of *identified particles* such as protons, pions, or kaons. In that case, one “particle of interest”, subevent A, is taken (e.g. kaons) and one “reference particle”, subevent B, is taken from all charged particles. Because particles of interest tend to be rarer, in most cases only one particle of

interest is considered rather than 2 (although exceptions exist). Next, the flow harmonic is determined via

$$v_n\{EP\} \equiv \frac{\langle \cos(\phi_n - \Psi_n^A) \rangle_{poi}}{\sqrt{\langle \cos(\phi_n - \Psi_n^A) \rangle_{all}}} \quad (24)$$

where the average in the numerator is only of particle of interest and the average in the denominator is over all charged particles.

The second method is the “multi-particle cumulant” method (Borghini et al. 2001c; Bilandzic et al. 2011, 2014) that correlates  $m$  number of particles. In this method (here we follow the formalism used in Luzum and Petersen 2014) one can correlate  $m$  particles such that

$$\langle m \rangle_{n_1, n_2, \dots, n_m} \equiv \langle \langle \cos(n_1 \phi^{a_1} + n_2 \phi^{a_2} + \dots + n_m \phi^{a_m}) \rangle_m \rangle_{ev} \quad (25)$$

where averages over particles are indicated by the subscript  $m$  and averages over events are indicated by a subscript  $ev$ . While not shown here, often these averages also include weights (such as by multiplicity) when averaging over multiple events (we will revisit this concept later). At this point we should note that Eq. (25) leads to two distinct contributions that are flow  $v_n$  (single particle distribution) and non-flow  $\delta_{n,p}$  (genuine  $p$ -particle correlations that arise from things like a  $\rho \rightarrow \pi\pi$  decay). Because hydrodynamics leads to only flow, experimentalists use various methods to minimize non-flow in their data analysis. Returning to Eq. (25), a 2-particle correlation of all charged particles (i.e. with no particle of interest) leads to

$$v_n\{2\} \equiv \sqrt{\langle v_n^2 \rangle_{ev} + \langle \delta_{2,2} \rangle_{ev}} \quad (26)$$

where one gets a contribution both from genuine flow and 2-particle correlations. In order to minimize the non-flow contribution, rapidity gaps are taken within the experiments. They remove decay and jet effects that occur close to each other in rapidity. Thus, after these cuts it is reasonable to assume that

$$v_n\{2\} \approx \sqrt{\langle v_n^2 \rangle_{ev}} \quad (27)$$

such that in theoretical calculations one can directly calculate  $v_n$  for a single event and then take the root-mean-squared over many events to calculate  $v_n\{2\}$ . Notice that event-by-event flow fluctuations will also contribute to Eq. (27) and to multi-particle cumulants in general, and should be taken into account when comparing to hydrodynamic simulations.

Depending if one is in a high resolution limit (i.e.  $v_n \gg 1/\sqrt{N}$  and  $R \rightarrow 1$ ) or the low resolution limit (i.e.  $v_n \sqrt{N} \ll 1$  and  $R \rightarrow v_n \sqrt{N}$ ) the correct theoretical quantity to calculate varies (Luzum and Ollitrault 2013) such that

$$v_n\{EP\} \xrightarrow{\text{Highres}} \langle v_n \rangle \quad (28)$$

$$v_n\{EP\} \xrightarrow{\text{Lowres}} \sqrt{\langle v_n^2 \rangle}. \quad (29)$$

However, most calculations fall between the high and low resolution limits, leading to ambiguous comparisons to experimental data. Thus, only the multi-particle cumulant method that explicitly defines a two particle correlation as

$$v_n\{2\} \equiv \sqrt{\langle v_n^2 \rangle} \quad (30)$$

provides a method for unambiguous comparisons between theory and experiment.

Due to these uncertainties in the comparison between theory and experiment, only the multi-particle cumulant method ensures an apples-to-apples comparison (Luzum and Ollitrault 2013). However, currently multi-particle cumulants have not been adopted uniformly across  $\sqrt{s_{NN}}$  but rather, at low energies the event-plane method is still used and at high-energies multi-particle cumulants are the standard. As explained in Luzum and Ollitrault (2013), this may lead to just a difference of few percentage points in the results, but for precision calculations that can lead to ambiguities.

An additional caveat when comparing high and low beam energies is the choice of event plane angle that is used in the experimental analysis. At high  $\sqrt{s_{NN}}$ , in the rare occasions in which the event-plane angle is used, the latter is always consistent with the flow harmonic such that  $v_2$  is measured with  $\Psi_2$ ,  $v_3$  is measured with  $\Psi_3$  and so on. However, at low beam energies all collective flow harmonics are measured with the event-plane method, relative to spectator reaction plane,  $\Psi_1$  (Reisdorf et al. 2012). Thus, the interpretation of the flow harmonics is quite different than those measured at high  $\sqrt{s_{NN}}$ . At the time of writing, we are not aware of a systematic study within a theoretical model comparing the differences between these measurements across  $\sqrt{s_{NN}}$ . However, experiments have compared these methods and found differences between them (Bastid et al. 2005; Aamodt et al. 2011b).

Other caveats exist when comparing theory to experiment. The averaging over events,  $\langle \dots \rangle_{ev}$  normally is weighted by a certain factor  $W_i$  that is a function of the multiplicity,  $M$ , of a given event such that

$$\langle \dots \rangle_{ev} \equiv \frac{\sum_i^{ev} \langle \dots \rangle_i W_i}{\sum_i^{ev} W_i} \quad (31)$$

where for a two-particle correlation  $W = M(M-1)$ . The averaging in Eq. (31) biases experimental observables (within a fixed centrality class) to events with high multiplicities because events with large  $M$  have a large weight. Additionally, experiments often perform calculations in smaller centrality bins (e.g. 0.5%) which are then re-assembled into broader centrality bins such as ranges of 5% or 10% depending on the statistics. While these are minor effects, they do play a role at the few percent level, especially for correlations of 4+ particles (Gardim et al. 2017; Betz et al. 2017).

For the following sections, we will consider integrated flow harmonics, which include all particles in certain kinematic ranges in transverse momentum,  $p_T$ , and rapidity,  $y$ , or pseudorapidity,  $\eta$ . Later we will discuss differential flow, where one still integrates over (pseudo)rapidity but lets  $p_T$  vary. For the latter approach, the

particle of interest is at a fixed  $p_T$  and the reference particles are taken across a much wider  $p_T$  range. For these calculations, a scalar product must be used

$$v_n\{SP\}(p_T) \equiv \frac{\langle v_n v_n(p_T) \cos n(\psi_n - \psi_n(p_T)) \rangle}{\sqrt{\langle v_n^2 \rangle}} \quad (32)$$

where  $v_n(p_T)$  and  $\psi_n(p_T)$  indicate the magnitude and angle of the flow harmonic at a fixed  $p_T$ . One can write a very similar equation integrating over  $p_T$  and instead varying (pseudo)rapidity.

### 6.3.2 Directed flow, $v_1$

Due to event-by-event fluctuations with rapidity, there are two contributions (Teaney and Yan 2011; Gardim et al. 2011) to directed flow ( $v_1$ )

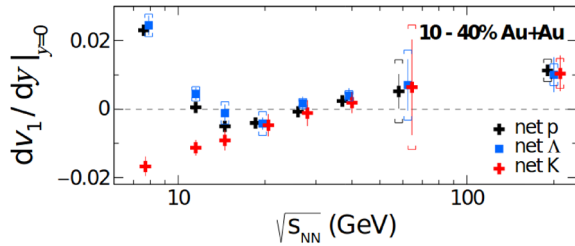
$$v_1(y)e^{i\Psi_1(y)} = v_{1,even}(y)e^{i\Psi_{1,even}(y)} + v_{1,odd}(y)e^{i\Psi_{1,odd}(y)} \quad (33)$$

where  $v_{1,even}(y) = v_{1,even}(-y)$ ,  $\Psi_{1,even}(y) = \Psi_{1,even}(-y)$ , and  $\Psi_{1,odd}(y) = \Psi_{1,odd}(-y)$  such that they are even across rapidity, whereas  $v_{1,odd}(-y) = -v_{1,odd}(y)$  such that it is odd in rapidity. Before the discovery of event-by-event fluctuations (i.e. if one assumes no initial state fluctuations) only  $v_{1,odd}(y)$  is relevant. Thus,  $v_{1,even}(y)$  is an entirely fluctuation-driven quantity.

Even though knowledge of  $v_{1,even}(y)$  has existed for over a decade, it has been studied very little because issues related to momentum conservation make it difficult to perform meaningful comparisons between theory and experiment. On the other hand,  $v_{1,odd}(y)$  has been used quite extensively at low  $\sqrt{s_{NN}}$  to study the EoS. In fact, it is more common to consider the slope of directed flow with respect to the rapidity i.e.  $dv_1/dy$ , which is a notoriously difficult quantity to reproduce in theoretical calculations. Thus, from this point forward we will only consider  $v_{1,odd}(y)$  and drop the “odd” sub-index for convenience.

Directed flow is expected to be sensitive to the EoS, in particular to the presence of a 1st order phase transition, or the compressibility of nuclear matter (Stoecker 2005; Reisdorf and Ritter 1997). For this reason,  $v_1$  of charged particles has been measured in Pb+Pb collisions by the NA49 experiment at  $\sqrt{s_{NN}} = 8.7$  GeV and  $\sqrt{s_{NN}} = 17.3$  GeV (Alt et al. 2003) and by the ALICE experiment at  $\sqrt{s_{NN}} = 2.76$  TeV (Abelev et al. 2013b). The same has been done in Au+Au collisions, from  $\sqrt{s_{NN}} = 2$  GeV to  $\sqrt{s_{NN}} = 8$  GeV by the E895 experiment (Liu et al. 2000), and at several collision energies within  $7.7 \leq \sqrt{s_{NN}} \leq 200$  GeV by the PHOBOS experiment (Back et al. 2006), and by the STAR experiment for identified particles (Adamczyk et al. 2014b, 2018b). An interesting observation from these results is the discontinuous slope of the mid-rapidity proton directed flow as a function of the energy, exhibiting a minimum in the  $\sqrt{s_{NN}} = 10 - 20$  GeV energy range, as displayed in Fig. 20. The interpretation of such results is still unclear though, despite the fact that it has been proposed as a sign for a 1st order phase transition, as no model can explain this behavior properly (Singha et al. 2016). At last,  $v_1$  measurements have been achieved in asymmetric Au+Cu collisions at  $\sqrt{s_{NN}} =$

**Fig. 20** Directed flow of net- $p$ ,  $\Lambda$  and  $K$  particle distributions at mid-rapidity as a function of the energy, for Au+Au collisions at intermediate centrality (10–40%). Image adapted from Adamczyk et al. (2018b)



200 GeV too, for charged and identified particles by both PHENIX (Adare et al. 2016c) and STAR collaborations (Adamczyk et al. 2018a).

### 6.3.3 Elliptic flow, $v_2$

Elliptic flow, another name for the second flow harmonic  $v_2$ , characterises the ellipticity of the final-state particle momentum distribution, and is proportional to the initial spatial eccentricity of the collision system (Snellings 2011). A nonzero value of  $v_2$  is naturally expected to arise due to the initial asymmetry of the colliding part of the system in non-central heavy-ion collisions, especially when going towards more peripheral events, where the almond shape of the overlapping region is accentuated. The pressure gradient, created by the very high initial energy density in the overlapping region, will hence convert the spatial anisotropies into momentum anisotropies while the system cools down (Snellings 2011). Using hydrodynamics or transport models, one can then extract the EoS (Pratt et al. 2015) and transport coefficients of the quark-gluon plasma like the shear viscosity over entropy ratio  $\eta/s$  (Bernhard et al. 2019), which are nevertheless highly model-dependent and thus out of the scope of this paper. Because the connection to the EoS is highly dependent on a number of other quantities such as the initial state amongst many others, we refer an interested reader to reviews, e.g. Heinz and Snellings (2013).

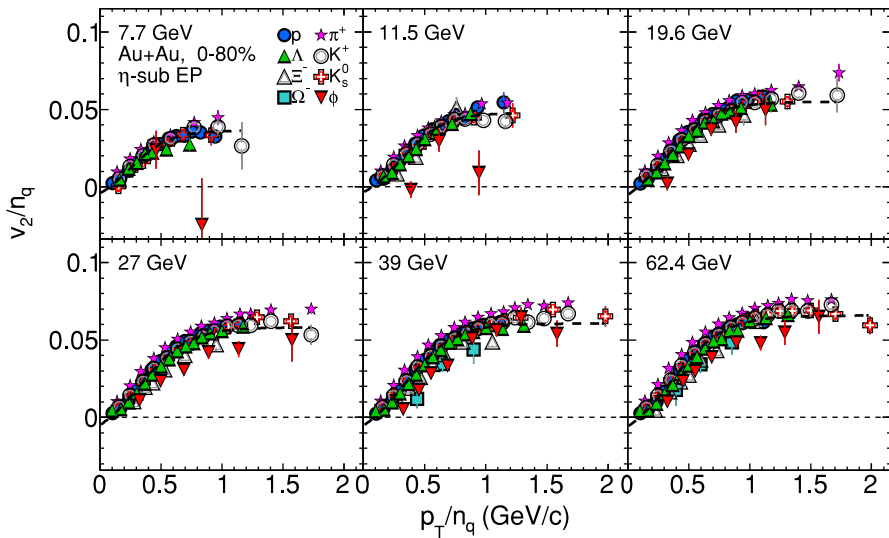
Elliptic flow has been measured by several collaborations at the LHC in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV and  $\sqrt{s_{NN}} = 5.02$  TeV for charged particles (Aad et al. 2012a; Chatrchyan et al. 2013, 2014b; Acharya et al. 2018d) and also for identified hadrons (Abelev et al. 2015; Acharya et al. 2018b), as well as in Xe+Xe collisions at  $\sqrt{s_{NN}} = 5.44$  TeV (Acharya et al. 2018a; Aad et al. 2020; Sirunyan et al. 2019a). At collision energies of the BESII program, between  $\sqrt{s_{NN}} = 7.7$  and 200 GeV,  $v_2$  has also been extensively studied in Au+Au collision systems for both charged particles (Adler et al. 2002; Back et al. 2005b, a; Adamczyk et al. 2012; Adare et al. 2015a) and identified particles (Adler et al. 2001a; Adams et al. 2005d; Adamczyk et al. 2016b; Abdallah et al. 2023a) by the STAR, PHENIX and PHOBOS collaborations. Elliptic flow of smaller systems, namely Cu+Cu collisions at  $\sqrt{s_{NN}} = 62.4$  and 200 GeV (Alver et al. 2007; Abelev et al. 2010a; Adare et al. 2015a), Ru+Ru and Zr+Zr collisions at  $\sqrt{s_{NN}} = 200$  GeV (Sinha 2023) have been measured as well. In addition,  $v_2$  data have also been extracted for the large, deformed system of U+U collisions at  $\sqrt{s_{NN}} = 193$  GeV (Abdallah et al. 2021a). All of these results show two strong indicators of the common origin of produced

particles from a moving fluid, namely the QGP. The first one is the mass ordering of the  $p_T$ -differential  $v_2$  from identified hadrons at low  $p_T$  ( $< 2$  GeV), with lighter species exhibiting a larger  $v_2$  than heavier ones (Abelev et al. 2010a; Adamczyk et al. 2013a). It can be understood by assuming that all particles originate from common cells of fluid with a given velocity, thus causing a shift of the  $v_2$  signal in momentum due to the mass differences (Huovinen et al. 2001; Huovinen and Ruuskanen 2006). The second one is the so-called Number of Constituent Quarks (NCQ) scaling (Molnar and Voloshin 2003) at intermediate  $p_T$  ( $> 1$  GeV), namely the fact that all  $v_2$  signals from different species match when plotted as a function of  $p_T$  or the reduced transverse mass, also called the transverse kinetic energy,  $KE_T = m_T - m_0$ , all of them divided by the number of valence quarks  $N_q$  ( $= 2$  for mesons and  $= 3$  for baryons) (Adare et al. 2007; Abelev et al. 2010a; Adamczyk et al. 2013a; Acharya et al. 2018b, 2021c). Assuming a common origin of the produced hadrons, this scaling shown in Fig. 21 illustrates that collectivity arises from quarks as degrees of freedom, before being translated into flow of the hadrons formed by quark coalescence (Fries et al. 2003; Molnar and Voloshin 2003). Such observations constitute, among other probes, a good indicator of the formation of a quark-gluon plasma. However,  $N_q$  scaling starts breaking down in minimum bias Au+Au collisions from  $\sqrt{s_{NN}} = 11.5$  GeV and below, as can be seen in Fig. 21 with the  $v_2/N_q$  of  $\phi$  mesons (although this interpretation is discussed in Hirano et al. (2008)).

At even lower beam energies of Au+Au collisions at  $\sqrt{s_{NN}} = 3$  GeV, the  $v_2$  measurements by the STAR experiment (Abdallah et al. 2022c) show no  $N_q$  scaling at all. This result tends to indicate the absence of efficient quark coalescence in the system created in such collisions, thus suggesting a dominance of the hadronic phase over deconfined partonic flow. Current hydrodynamic calculations can provide a reasonable match to experimental data down to  $\sqrt{s_{NN}} = 7.7$  GeV (Shen and Schenke 2022) and  $\sqrt{s_{NN}} = 4.3$  GeV (Schäfer et al. 2022). However, we point out that other effects may destroy  $N_q$  scaling that have not yet been explored at these energies in full (3+1)D relativistic viscous hydrodynamical simulations. For example, the existence of a critical point leads to critical fluctuations that may have dramatic effects on transport coefficients, which in turn affect collective flow (see Lovato et al. 2022b for further discussion on flow at these beam energies). We also caution that NCQ scaling is only approximate and expected to hold only at intermediate  $p_T$  (Molnar and Voloshin 2003). Precise NCQ scaling for  $v_2\{2\}(p_T/N_q)/N_q$  is not observed even at LHC energies (Abelev et al. 2015; Acharya et al. 2018b, 2021c), so one must be careful when drawing conclusions from its absence. Overall, a much clearer scaling can be seen when plotting against  $KE_T/N_q$  (Adare et al. 2007; Abelev et al. 2015).

### 6.3.4 Triangular flow $v_3$ and beyond

Let us first discuss triangular flow before moving onto higher-order flow harmonics. The existence of triangular flow was not understood in the field of heavy-ion collisions for many years. In part, this occurred because experimentalists measured triangular flow with the wrong event plane angle (e.g. using  $\Psi_2$  instead of  $\Psi_3$ ,

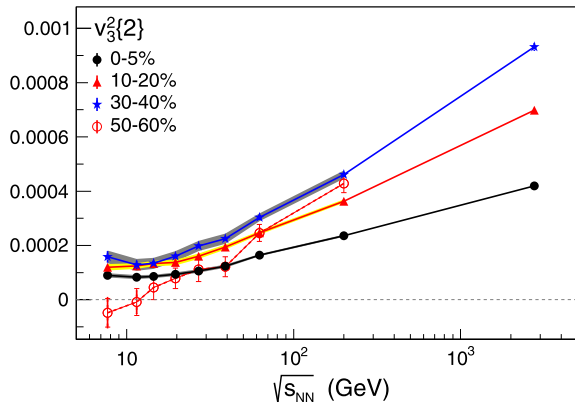


**Fig. 21** NCQ scaling of the elliptic flow of identified hadrons ( $\pi^+$ ,  $K^+$ ,  $K_s^0$ ,  $\phi$ ,  $p$ ,  $\Lambda$ ,  $\Xi^-$  and  $\Omega^-$ ) for 0–80% Au+Au collisions at  $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39$  and  $62.4$  GeV. Image reproduced with permission from Adamczyk et al. (2013a), copyright by APS

leading to a result of  $v_3(\Psi_2) = 0$ . However, the key issue was that the importance of event-by-event fluctuations and how significantly they could affect flow harmonics was not understood. The calculations from Takahashi et al. (2009) of 2 particle correlations, were performed in the first theoretical framework to use event-by-event fluctuating initial conditions and was the only model able to reproduce the experimental results at the time. This made it possible for experimentalists to understand the significance of triangular flow and discover the correct method for calculating it (Alver and Roland 2010). This discovery of nonzero  $v_3$  that can only arise from event-by-event fluctuating initial conditions led to a radical change in the field of heavy-ion collisions because many assumptions had to be changed, but it allowed for new ways to probe the QGP that did not exist previous to the discovery.

Higher-order flow harmonics, from  $v_3$  and up to  $v_6$ , have also been measured in order to get complementary information concerning what can be already determined from  $v_{1,2}$  results. As they require a rather high number of events to be measured with reasonable uncertainties, they have been measured almost exclusively at top energy of the BES program in Au+Au collisions, i.e.,  $\sqrt{s_{NN}} = 200$  GeV (Adare et al. 2016b; Adam et al. 2020c; Abdallah et al. 2022a), and also in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV (Aamodt et al. 2011b; Aad et al. 2012a; Adam et al. 2016d; Chatrchyan et al. 2014a, b) and  $\sqrt{s_{NN}} = 5.02$  TeV (Adam et al. 2016b; Aaboud et al. 2018; Acharya et al. 2020b), as well as in Xe+Xe collisions at  $\sqrt{s_{NN}} = 5.44$  TeV (Acharya et al. 2018a; Aad et al. 2020). STAR (AuAu  $\sqrt{s_{NN}} = 7.7 - 200$  GeV) has now measured the suppression of  $v_3\{2\}$  at low beam energies (Adamczyk et al. 2016a) where a minimum is reached (but  $v_3\{2\}$  does not precisely reach 0) whose results are still open for interpretation, see Fig. 22. Additionally, HADES has also

**Fig. 22** All charged particles  $v_3^2\{2\}$  from the STAR collaboration for various centralities in Au+Au collisions at  $\sqrt{s_{NN}} = 7.7, 11.5, 19.6, 27, 39$  and  $62.4$  GeV. Image reproduced with permission from Adamczyk et al. (2016a), copyright by APS



calculated higher-order flow harmonics at  $\sqrt{s_{NN}} = 2.4$  GeV (Adamczewski-Musch et al. 2020a), albeit with the event-plane method with respect to the  $\Psi_1$  event plane angle, see Fig. 23.

In the context of BESI, the STAR collaboration has performed a comprehensive analysis of 3-particle azimuthal anisotropy correlations  $C_{m,n,m+n}$ , between different harmonics  $m$ ,  $n$  and  $m+n$ , over a wide range of multiplicities, transverse momenta, and energies from  $\sqrt{s_{NN}} = 7.7$  GeV to 200 GeV (Adamczyk et al. 2018d). The dependence displayed by results on the pseudo-rapidity differences between particles suggests the breaking of longitudinal boost invariance or dominance of unconventional non-flow contributions.

### 6.3.5 4+ particle flow cumulant observables

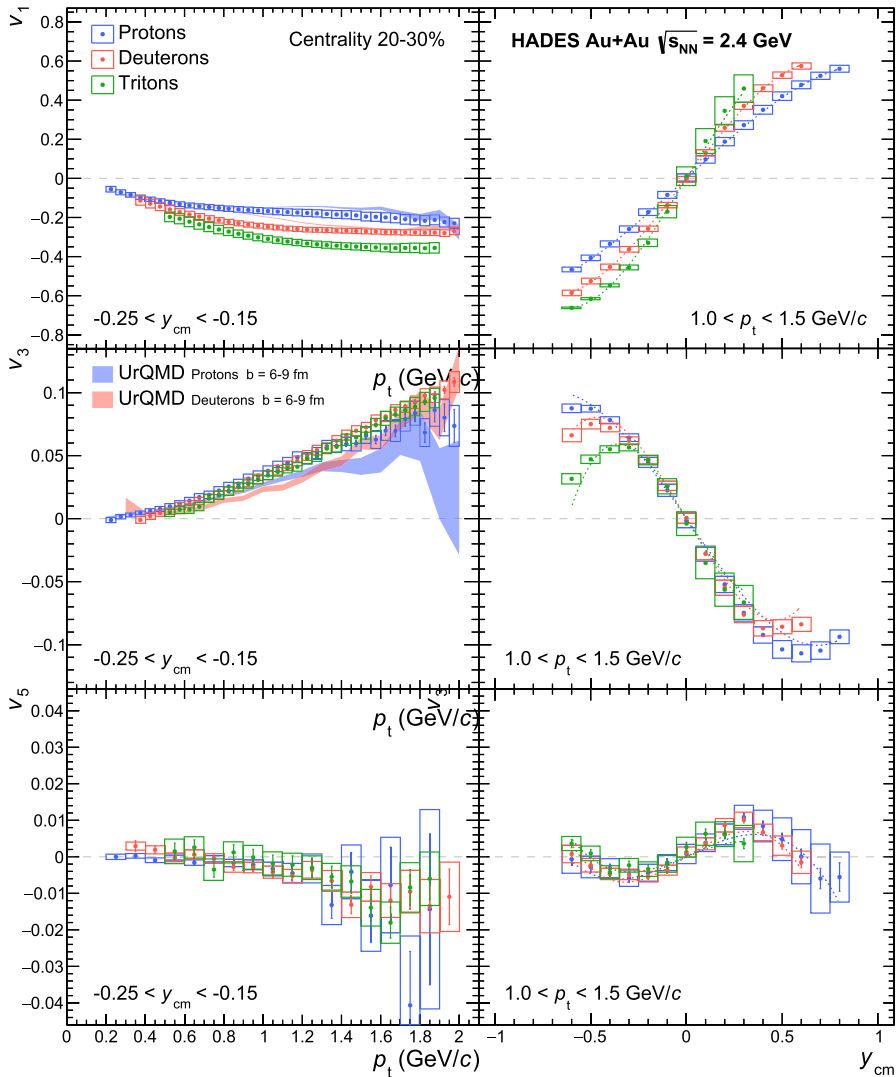
The multi-particle cumulant method used to extract  $v_n\{2\}$  in Eq. (30) can be extended to cumulants of higher order, denoted  $v_n\{2k\}$  (Borghini et al. 2001a, b, c; Bilandzic et al. 2011, 2014; Di Francesco et al. 2017; Moravcova et al. 2021). In general, measurements of  $v_n\{2k\}$  depend on irreducible  $2k$ -particle correlations, so that non-collective contributions to  $v_n$  are suppressed by a combinatorial factor, which decreases with  $k$ . The four-particle  $v_n\{4\}$ , for instance, is given by

$$-v_n\{4\}^4 = \langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2 \quad (34)$$

where 2-particle contributions are subtracted by the second term (see, for instance Giacalone et al. 2017).

Non-flow contributions set aside, differences between multi-particle cumulants are also sensitive to how the coefficients  $v_n$  fluctuate event-by-event (Aad et al. 2013, 2014b). In the regime in which these coefficients scale linearly with anisotropies of the initial state, i.e.  $v_n \propto \varepsilon_n$  (Teaney and Yan 2011; Gardim et al. 2012, 2015; Niemi et al. 2013; Teaney and Yan 2012; Qiu and Heinz 2011; Betz et al. 2017), where  $\varepsilon_n$  quantifies the initial-state anisotropy, ratios between different cumulants depend only on fluctuations of the initial anisotropies, characterized by cumulants  $\varepsilon_n\{2k\}$  (Ma et al. 2016; Giacalone et al. 2017). For instance,





**Fig. 23** All charged particles collective flow from HADES in Au+Au collisions at  $\sqrt{s_{NN}} = 2.4$  GeV. Image reproduced with permission from Adamczewski-Musch et al. (2020a), copyright by APS

$$\frac{v_n\{2k\}}{v_n\{2\}} \approx \frac{\varepsilon_n\{2k\}}{\varepsilon_n\{2\}} \quad (35)$$

where the proportionality factor relating initial-state and flow anisotropy cancels out (Yan and Ollitrault 2014), so that multi-particle flow measurements can be employed to isolate information on initial-state fluctuations from information on the response of the medium (Giacalone et al. 2017; Bhalerao et al. 2019a, b). However, in peripheral collisions (Noronha-Hostler et al. 2016b), small systems (Sievert and Noronha-

Hostler 2019), differential flow (Hippert et al. 2020a), and to a much lesser extent lower beam energies (Rao et al. 2021) this linear scaling begins to break down, and it requires non-linear response as well. Multiparticle cumulants and fluctuations of flow coefficients have been measured at  $\sqrt{s_{NN}} = 130$  GeV by the STAR collaboration (Adler et al. 2002), across BES-I  $\sqrt{s_{NN}} = 7.7 - 200$  GeV from the STAR collaboration (Abdallah et al. 2022b), and at  $\sqrt{s_{NN}} = 2.76 - 5.02$  TeV by ATLAS, CMS and ALICE in Pb+Pb collisions (Aad et al. 2013, 2014b; Sirunyan et al. 2019b, 2018c; Acharya et al. 2018d), by ALICE, CMS, and ATLAS in p-Pb collisions (Abelev et al. 2014b; Khachatryan et al. 2015a, 2017; Sirunyan et al. 2018c; Aaboud et al. 2017b; Acharya et al. 2019b), and by ALICE in p+p and Xe+Xe collisions (Acharya et al. 2019b).

While multi-particle cumulants of a single flow coefficient can be used as proxies of its fluctuations, correlations between different coefficients can be studied with so-called symmetric cumulants (Bilandzic et al. 2014)

$$SC(n, m) = \langle v_n^2 v_m^2 \rangle - \langle v_n^2 \rangle \langle v_m^2 \rangle \quad (36)$$

or normalized symmetric cumulants  $sc(m, n) = SC(n, m) / (\langle v_n^2 \rangle \langle v_m^2 \rangle)$  (Bhalerao et al. 2011, 2019b; Moravcova et al. 2021), which can also be generalized to higher orders (Mordasini et al. 2020). For symmetric cumulants, it is important to ensure that multiplicity weighing and centrality binning are taken into account in order to reproduce experimental data (Gardim et al. 2017).

Normalized symmetric cumulants are related to event-plane correlations (Bhalerao et al. 2015a; Giacalone et al. 2016), which can be quantified by examining how combinations of event planes for different harmonics,  $\Psi_{n_1, n_2, \dots, n_k} \equiv a_1 \Psi_{n_1} + a_2 \Psi_{n_2} + \dots + a_m \Psi_{n_k}$  fluctuate (Jia and Mohapatra 2013; Jia and Teaney 2013; Bhalerao et al. 2011; Qin and Muller 2012). Taking the Fourier series of the distribution  $dN/d\Psi_{n_1, n_2, \dots, n_k}$ , one finds coefficients

$$V_{n_1, n_2, \dots, n_k}^j \equiv \langle \cos(j \Psi_{n_1, n_2, \dots, n_k}) \rangle \quad (37)$$

where  $n$ -fold symmetry of the event-plane angle  $\Psi_n$  requires that  $a_i = n_i c_i$ ,  $c_i \in \mathbb{Z}$ , and invariance of correlations under global rotations imposes that  $\sum_i n_i c_i = 0$ . The event-plane correlations have been suggested as a method to possibly constrain viscous effects (Niemi et al. 2016).

The PHENIX collaboration has investigated event-plane correlations in Au+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV at RHIC (Adare et al. 2011). Results for symmetric cumulants and normalized symmetric cumulants in Au+Au collisions at center-of-mass energies of  $\sqrt{s_{NN}} = 27$  GeV, 39 GeV, 54.4 GeV and 200 GeV were also published by STAR in Adam et al. (2018), Aboona et al. (2023a), where they are compared to LHC results. ATLAS and ALICE have performed measurements of event-plane correlations in Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV (Aad et al. 2014a; Acharya et al. 2017b), while ALICE, CMS, and ATLAS have published measurements of symmetric cumulants (Adam et al. 2016c; Sirunyan et al. 2018b; Aaboud et al. 2019; Acharya et al. 2021d).

### 6.3.6 Differential flow and scalar products

Another important tool for constraining QGP properties is the study of flow anisotropies *differentially* in transverse momentum  $p_T$  or (pseudo-rapidity  $\eta$ ) (Luzum and Romatschke 2008; Schenke et al. 2012; Nijs et al. 2021). In theory this amounts to considering the  $p_T$  in Eq. (20), so that the flow coefficients become  $v_n = v_n(p_T)$ . In practice, this entails binning particles of interest according to their momenta. However, a good determination of the reference event plane  $\psi_n$  demands that particles of reference be defined over a wider and suitably chosen momentum range. The differential flow coefficients can thus be found from the scalar product of Eq. (32) (see e.g. Bilandzic et al. 2011 for details).

Flow harmonics in different  $p_T$  regions reveal different physics. In the low  $p_T$  region, differential flow observables allow for the investigation of the hydrodynamic response of the QGP as a function of transverse momentum. As discussed above, the mass ordering of elliptic flow at low  $p_T$ , with lighter particles presenting stronger flow, has been regarded as a signature of hydrodynamic expansion, while the NCQ scaling of flow harmonics at intermediate  $p_T$  has been viewed as a signature of quark coalescence (Huovinen et al. 2001; Molnar and Voloshin 2003; Gyulassy 2004; Gyulassy and McLerran 2005; Huovinen and Ruuskanen 2006; Adams et al. 2005d; Abelev et al. 2008a, a). At higher  $p_T$ , flow harmonics encode information on energy loss (Gyulassy et al. 2001; Wang 2001).

Differential flow harmonics have been measured for a variety of systems and beam energies. For Pb+Pb collisions, at the LHC differential flow measurements have been made at  $\sqrt{s_{NN}} = 2.76$  TeV from ALICE (Aamodt et al. 2011b), CMS (Chatrchyan et al. 2012) and ATLAS (Aad et al. 2012b), and at  $\sqrt{s_{NN}} = 5.02$  GeV from ALICE (Adam et al. 2016b; Acharya et al. 2018b, 2023a) and CMS (Sirunyan et al. 2018a, 2020). Measurements have also been performed at RHIC at high beam energies for Au+Au (Adams et al. 2005a; Adler et al. 2003; Afanasiev et al. 2009; Adler et al. 2004; Back et al. 2005a), Cu+Cu (Adare et al. 2015a) and Cu+Au (Adare et al. 2016c) collisions at  $\sqrt{s_{NN}} = 200$  GeV and 62.4 GeV (Adare et al. 2015a), and Au+Au collisions at  $\sqrt{s_{NN}} = 130$  GeV (Ackermann et al. 2001). A comprehensive analysis of p+Au, d+Au, Cu+Cu and Cu+Au collisions at  $\sqrt{s_{NN}} = 200$  GeV, as well as U+U collisions at  $\sqrt{s_{NN}} = 193$  GeV, from the STAR collaboration can be found in Adam et al. (2019a). In the context of the BESII there have been measurements from  $\sqrt{s_{NN}} = 7.7 - 200$  GeV (Adamczyk et al. 2013c, 2016b; Abelev et al. 2008a).

At very low beam energies, sometimes differential flow measurements are reported in terms of the transverse mass  $m_T$  such that this is related to  $p_T$  through

$$m_T = \sqrt{m^2 + p_T^2} \quad (38)$$

where  $m$  is the mass of the particle(s) considered. If only one particle species is considered in the flow, it is possible to translate between  $p_T$ -dependent flow and  $m_T$ -dependent flow. However, if multiple identified particles are considered, it is not possible to make this translation without more information from the experiment.

Fixed target measurements have been made at the STAR fixed target program for  $\sqrt{s_{NN}} = 3$  GeV (Abdallah et al. 2022c) (in this work it was also normalized by the number of quarks such that  $N_q = 2$  for mesons and  $N_q = 3$  for baryons) for  $\sqrt{s_{NN}} = 4.5$  GeV (Adam et al. 2021a). At HADES, using Au-Au collisions at  $\sqrt{s_{NN}} = 2.4$  GeV (Adamczewski-Musch et al. 2020a), differential flow was measured in terms of  $m_T$ . Differential flow provides a more stringent test for models of strongly interacting matter, with transport models, represented by SMASH (Mohs et al. 2022), currently unable to reproduce results from HADES (Kardan 2019)—although this may be fixed with further improvements (Sorensen et al. 2024).

An interesting consequence of the  $p_T$  dependence of flow coefficients is that one may investigate how anisotropies over different  $p_T$  ranges correlate with one another (Kikola et al. 2012; Gardim et al. 2013; Heinz et al. 2013). In particular, 2-particle correlations are given by the covariance matrix

$$V_{n\Delta}^{ab} \equiv V_{n\Delta}(p_T^a, p_T^b) = \langle V_n^*(p_T^a) V_n(p_T^b) \rangle \quad (39)$$

where  $p_T^{(a,b)}$  denote the transverse momentum in two  $p_T$  bins  $a$  and  $b$ , and we generalize the complex  $V_n = v_n e^{i\Psi_n}$  of Eq. (21) for differential flow. In the absence of flow fluctuations or non-flow effects, independent particle emission from the fluid implies that the covariance matrix “factorizes” as  $V_{n\Delta}^{ab} = v_n(p_T^a) v_n(p_T^b)$ . The breaking of this factorization can signal non-flow contributions (Kikola et al. 2012; Jia 2011; Adare 2011; Aad et al. 2012a) or, more interestingly, event-by-event flow fluctuations (Gardim et al. 2013), and can be quantified by the factorization-breaking coefficient

$$r_n(p_T^a, p_T^b) \equiv \frac{V_{n\Delta}(p_T^a, p_T^b)}{\sqrt{V_{n\Delta}(p_T^a, p_T^a) V_{n\Delta}(p_T^b, p_T^b)}} \leq 1 \quad (40)$$

with the inequality being saturated for the case of perfect factorization. The same coefficient can be defined for pseudo-rapidity instead of transverse-momentum bins. Equation (40) is such that, in the regime of linear response to initial anisotropies, the response of the medium cancels out and  $r_n$  is determined mainly by initial-state fluctuations. This coefficient, and its counterpart for  $\eta$ -differential flow, has been measured by CMS for both Pb+Pb collisions at  $\sqrt{s_{NN}} = 2.76$  TeV and p-Pb collisions at  $\sqrt{s_{NN}} = 5.02$  TeV (Khachatryan et al. 2015b; Sirunyan et al. 2017), and was found to be generally close to, yet below 1.

Factorization breaking can also be understood in terms of the spectral decomposition and principal component analysis of  $V_{n\Delta}^{ab}$ , with the case of perfect factorization corresponding to a single nonzero eigenvalue. The principal component analysis of flow fluctuations was proposed in Bhalerao et al. (2015b) as a more transparent alternative to  $r_n$ , and measured by CMS, in Pb+Pb collisions, at  $\sqrt{s_{NN}} = 2.76$  TeV (Sirunyan et al. 2017). However, it has been suggested that the original prescription for carrying out this measurement led to the contamination of subleading anisotropic flow modes by fluctuations of radial flow (Hippert et al. 2020b). Results from hydrodynamic simulations indicate that the factorization-breaking coefficient and subleading components of the PCA of  $V_{n\Delta}$  are capable of probing initial-state

fluctuations at smaller length-scales, compared to the initial transverse size of the system (Kozlov et al. 2014; Gardim et al. 2018; Hippert et al. 2020a).

## 6.4 HBT

Hanbury Brown–Twiss (HBT) interferometry (Heinz and Jacak 1999; Wiedemann and Heinz 1999; Lisa et al. 2005) provides a way to probe the space-time evolution of heavy-ion collisions using correlations between pairs of identical particles emitted from these collisions, such as pions or kaons.<sup>7</sup> By analyzing the resulting two-particle correlation functions at small relative momentum, one finds that their inverse widths (the ‘HBT radii’  $R_{ij}^2$ ) correspond to the average separation between the emission points of any two identical particles, and, therefore, provide a way of quantifying the space-time structure of the particle emission process. Insofar as the bulk of particle production occurs towards the end of the system’s lifetime (Plumberg and Heinz 2015; Müller and Schäfer 2022), one expects that HBT measurements should be sensitive to the entire history of the fireball, and, therefore, should depend on the EoS as well.

This expectation turns out to be correct. Previous studies have established at least five different connections between HBT and the nuclear EoS. First, the space-time information extracted from HBT can be used to estimate the volume of the system at freeze out. The freeze-out volume can then be used to estimate the peak energy density and, thereby, obtain evidence for the onset of deconfinement (Heinz and Jacak 1999; Lisa et al. 2005). Second, the specific observables  $R_o^2 - R_s^2$  and  $R_o/R_s$  are sensitive to the emission duration and rate of expansion of the collision system (Heinz and Jacak 1999; Pratt 2009; Adamczewski-Musch et al. 2019a); measuring these quantities as functions of the pair momentum  $K_T$  can constrain the existence or absence of a first-order phase transition at different beam energies and chemical potentials (Lisa et al. 2005; Li et al. 2023). For the same reason, these observables have also been identified as potential signatures of the QCD critical point, by analyzing their scaling with beam energy  $\sqrt{s_{NN}}$  to signal a softening in the EoS (Lacey 2015). A third connection between HBT and the nuclear equation state uses the moments of collision-by-collision fluctuations in the HBT radii to isolate the geometric effects of critical fluctuations on the system’s evolution (Plumberg and Kapusta 2017), and thereby provides an independent way of probing the QCD critical point. Fourth, HBT is sensitive to the speed of sound as a function of temperature. HBT measurements in different collision systems and beam energies (Au+Au at RHIC vs. Pb+Pb at the LHC) have been shown in a multi-observable Bayesian analysis (Pratt et al. 2015; Sangaline and Pratt 2016) to directly constrain a parameterized form of  $c_s^2(T)$ . Fifth, the multiplicity dependence of the HBT radii in different size collision systems—specifically, the slope of  $R_{ij}$  versus  $(dN_{ch}/d\eta)^{1/3}$  in p+p, p+Pb, and Pb+Pb—may also reflect the influence of  $c_s^2$  on the rate of the system’s expansion (Plumberg 2020).

<sup>7</sup> The more general term *femtoscopy* typically may include, in addition to HBT, non-identical particle correlations and coalescence analyses as well, which may yield additional insights into the space-time dynamics of heavy-ion collisions (Lisa et al. 2005).

In addition to these examples, the global shape of the fireball at freezeout is sensitive to the EoS in at least two ways. First, the HBT radii can be analyzed experimentally as functions of the azimuthal pair emission angle  $\Phi_K$ , which indicates the direction of a given particle pair's average momentum in the transverse plane. By expanding this azimuthal dependence in a Fourier series and studying the extracted coefficients as functions of  $\sqrt{s_{NN}}$ , one acquires sensitivity to the EoS via the freezeout eccentricity  $\varepsilon_F$  (roughly, the normalized second-order Fourier coefficients of the  $R_{ij}^2$ ), which measures the extent to which the initial elliptic geometry of the collision system is inverted by the subsequent dynamical expansion (Adamczyk et al. 2015; Adamczewski-Musch et al. 2020b). Second, one can also observe sensitivity to the EoS in the transition from prolate to oblate freezeout configurations, which is done by correlating transverse and longitudinal HBT radii as a function of collision energy. The resulting non-monotonic trend observed in the data (Adam et al. 2021a) is attributed to a transition from dynamical evolution dominated by nucleon stopping at low energies to boost-invariant evolution at higher energies.

HBT measurements have been carried out in a vast number of different collision systems and over a wide range of collision energies and particle species. We focus here on HBT with Bose-Einstein correlations, primarily using charged pion pairs. Important recent measurements include: azimuthally sensitive analyses in Pb+Pb at 2.76 TeV (Adamova et al. 2017; Acharya et al. 2018c), p+Pb at 5.02 TeV (ATLAS Collaboration 2017), Cu+Au and Au+Au (Lisa et al. 2000; Adams et al. 2004, 2005e; Adare et al. 2014; Khyzhniak 2020a), and Pb+Au at 40, 80, and 158 GeV (Adamová et al. 2003; Adamova et al. 2008). The majority of studies present results for azimuthally averaged analyses. At the LHC, HBT has been measured: by ALICE, in Pb+Pb at 2.76 TeV (Aamodt et al. 2011d), p+p at 0.9 and 7 TeV (Aamodt et al. 2011a; Acharya et al. 2019a); by CMS, in p+p, p+Pb, and Pb+Pb at various energies 0.9, 2.76, 5.02, and 7 TeV; and by ATLAS in p+Pb at 5.02 TeV (Aaboud et al. 2017a). At RHIC, STAR has further measured HBT: in U+U collisions at 193 GeV (Campbell 2018); in Au+Au at collision energies of  $\sqrt{s_{NN}} = 4.5, 7.7, 9.2, 11.5, 19.6, 27, 39, 62.4, 130$ , and 200 GeV (Adler et al. 2001b; Abelev et al. 2010b; Anson 2011; Adamczyk et al. 2015; Zbroszczyk 2022); in p+Au and d+Au at 200 GeV (Khyzhniak 2020b); and in p+p collisions at 200 GeV (Aggarwal et al. 2011). PHENIX has conducted measurements in d+Au and Au+Au at 200 GeV (Ajitanand et al. 2014) and in Au+Au at 7.9, 19.6, 27, 39, 62, and 200 GeV (Soltz 2014). Au+Au collisions have been further studied by the HADES collaboration at 2.4 GeV (Adamczewski-Musch et al. 2019a, 2020b; Greifenhagen 2020). The majority of analyses study two-pion correlations for statistical reasons, but HBT has also been studied for kaon pairs by various collaborations at both RHIC and the LHC (Adamczyk et al. 2013b; Nigmatkulov 2016; Acharya et al. 2017a, 2019c) and for multi-pion correlations in various systems (Adare et al. 2015b; Adam et al. 2016e).

## 7 Experimental constraints: low-energy nuclear physics

In the following section, we will review various empirical observations of constraints relevant to low-energy nuclear physics for isospin-symmetric and asymmetric matter.

### 7.1 Isospin symmetric matter at saturation density

As neutron stars are in many ways like giant nuclei with  $\sim 10^{57}$  nucleons (Glendenning 1997), at densities which are typical of nuclei they should reproduce the same properties. Therefore, astrophysicists use laboratory measurements of nuclear experiments to calibrate neutron stars (or dense matter in general) properties. Low-energy laboratory conditions can be considered at  $T \sim 0$  when describing fermions, as their order of chemical potential and mass are  $10^5$  times larger than the temperature of fully evolved neutron stars ( $10^{-2}$  MeV  $\sim 10^8$  K) and even more when compared to laboratories. At effectively zero temperature, antifermions do not contribute and the relevant degrees of freedom at  $n_{\text{sat}}$  are nuclei (composed of neutrons and protons), which are approximately isospin symmetric, with charge fraction  $Y_Q \sim 0.5$ . The minimal condition on any dense matter theory is to reproduce the experimental results of zero temperature, isospin symmetric nuclear matter at saturation: saturation density  $n_{\text{sat}}$ , binding energy per nucleon  $B/A$ , and compressibility  $K$ . Additionally, for models that contain exotic degrees of freedom, hyperon potentials  $U_H$  and  $\Delta$ -baryon potential  $U_\Delta$  can be used.

These empirical values are known within uncertainties and are extracted using extrapolation within phenomenological models, since infinite nuclear matter does not exist. Here, we enlist various empirical investigations of the above-mentioned properties. The values for the observables related to isospin-symmetric nuclear matter are summarized in Table 5, those related to exotic matter in Table 6.

**Table 5** Experimental constraints related to isospin-symmetric nuclear matter at saturation

Constraints	Value	References
Saturation density, $n_{\text{sat}}$ ( $\text{fm}^{-3}$ )	$0.17 \pm 0.03$	Haensel et al. (1981)
	$0.148 - 0.185$	Gross-Boelting et al. (1999)
	$0.148 \pm 0.0038$	Adhikari et al. (2021)
Binding energy per nucleon, $B/A$ (MeV)	$-15.677$	Myers and Swiatecki (1966)
	$-16.24$	Myers and Swiatecki (1996)
Compressibility, $K_\infty$ (MeV)	$240 \pm 20$	Colo et al. (2014), Todd-Rutel and Piekarewicz (2005), Colo et al. (2004), Agrawal et al. (2003)
	$210 - 270$	Khan and Margueron (2012)
	$251 - 315$	Stone et al. (2014)

**Table 6** Empirical constraints related to isospin-symmetric exotic matter at saturation

Constraints	Value	References
Hyperon potentials (MeV)	$U_{\Lambda} = -28$	Inoue (2019), Millener et al. (1988)
	$U_{\Lambda} = -30$	Gal et al. (2016)
	$U_{\Lambda} = -26.7 \pm 1.7$	Friedman and Gal (2023)
	$U_{\Sigma} = 15$	Inoue (2019)
	$U_{\Sigma} = 10 - 50$	Gal et al. (2016)
	$U_{\Xi} = -14$	Gal et al. (2016)
	$U_{\Xi} = -21.9 \pm 0.7$	Friedman and Gal (2021)
	$U_{\Xi} = -4$	Inoue (2019)
Delta baryon potential (MeV)	$U_{\Delta} \sim U_N$	Horikawa et al. (1980)
	$U_{\Delta}(n_B) = -75n_B(r)/n_{\text{sat}}$	Koch and Ohtsuka (1985)
	$-90 < U_{\Delta} < -50$	Drago et al. (2014)

### 7.1.1 Saturation density

The saturation density (actually the number density, as it defines the number of baryons per volume) indicates that there is a size saturation in atomic nuclei, preventing them from expanding or collapsing as a result of the nuclear force's powerful attraction and repulsion. This is the most important constraint, as often other constraints are determined at this density (Fig. 24).

The simplest way to calculate a given number density is by dividing the number of baryons in a nucleus over the volume of the nucleus, as (Haensel et al. 1981)

$$n_{\text{sat}} = \frac{A}{4\pi R^3/3} = \frac{3}{4\pi(R/A^{1/3})^3} = \frac{3}{4\pi R_0^3} = 0.17 \pm 0.03 \text{ fm}^{-3} \quad (41)$$

where it was assumed that the value of the nuclear radius constant is  $R = R_0 A^{1/3} = 1.04 - 1.17 \text{ fm}$ . The error bar reflects the uncertainty on  $R_0$ , obtained from electron scattering and  $\mu$ -mesic atom experiments (Myers 1977).

Recently, in the elastic scattering of longitudinally polarized electrons extracted from  $^{208}\text{Pb}$ , the PREX collaboration reported a precise measurement of the parity-violating asymmetry  $A_{\text{PV}}$  term (Adhikari et al. 2021), where the interior baryonic density at saturation was derived from the measured interior weak density  $n_W^0$  i.e.

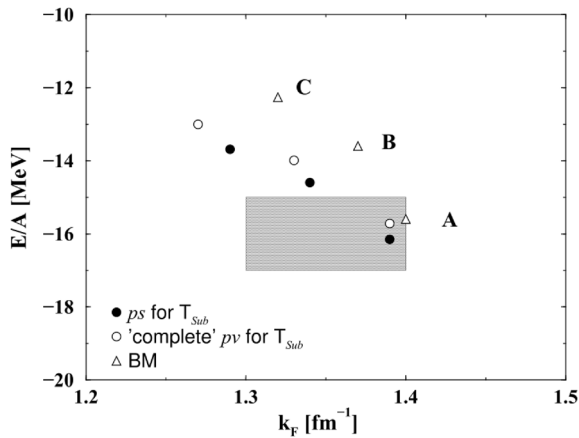
$$n_{\text{sat}} = 0.1480 \pm 0.0038 \text{ fm}^{-3}. \quad (42)$$

Here, the uncertainty contains both theoretical and experimental contributions.

### 7.1.2 Binding energy per nucleon at saturation

Binding energy ( $B$ ) is the energy required to separate a nucleus, and is given by the semi-empirical mass formula





**Fig. 24** Binding energy per nucleon (in this review, referred to as  $B/A$ ) versus Fermi momentum  $k_f$ , with the shaded box representing saturation. The Bonn potentials shown via letters A, B, and C are used for bare nucleon–nucleon interaction. In addition, the three legends describe the T-matrix calculations using a subtraction scheme with the  $ps$  representation (top), the  $pv$  representation (middle), and Brockmann and Machleidt,  $BM$  (bottom) (Brockmann and Machleidt 1990). Image reproduced with permission from Gross-Boelting et al. (1999), copyright by Elsevier

$$B = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(N-Z)^2}{A} + \delta(A, Z) \quad (43)$$

where

$$\delta(A, Z) = \begin{cases} +a_p A^{-1/2} & \text{for even } Z, \text{ even } N \\ 0 & \text{for odd } Z, \text{ even } N \text{ or even } Z, \text{ odd } N \\ -a_p A^{-1/2} & \text{for odd } Z, \text{ odd } N \end{cases} \quad (44)$$

In Eq. (43), the first term representing the volume effects ( $B_{\text{vol}}$ ), the second term denoting the surface effects ( $B_{\text{surf}}$ ), the third term describing the Coulomb interactions ( $B_{\text{Coul}}$ ), the fourth term describing the effect of isospin asymmetry ( $B_{\text{asym}}$ ), and the last term covering the effects of the pairing ( $B_p$ ) (Martin and Shaw 2019). The exponent of  $A$  in the pairing term is derived from experimental binding-energy data. While it was previously commonly assumed to be  $-3/4$ , more recent experimental data suggest that a value closer to  $-1/2$  is more accurate (Myers 1977). Furthermore, the coefficients for various terms are provided as  $a_v = 15.56$  MeV,  $a_s = 17.23$  MeV,  $a_c = 0.697$  MeV,  $a_a = 93.14$  MeV, and  $a_p = 12$  MeV. These values constitute one of the datasets employed to adjust the binding energy curve for  $A > 20$  (Martin and Shaw 2019). Since there are no surface effects and Coulomb effects are also ignored in infinite isospin-symmetric nuclear matter, the net binding energy is approximated as the binding energy's volume term ( $B \approx B_{\text{vol}}$ ). In Myers and Swiatecki (1966), the volume term of the binding energy per nucleon  $B/A = -15.677$  MeV at  $n_{\text{sat}} = 0.16146 \text{ fm}^{-3}$  was obtained from a non-relativistic semi-empirical four-parameter mass formula with coefficients from the liquid-drop model

using the experimental masses of 49 heavy nuclei. In addition, using experimental data from 1654 ground state masses of nuclei with  $N, Z \geq 8$ ,  $B/A = -16.24$  MeV at  $n_{\text{sat}} = 0.16114 \text{ fm}^{-3}$  was obtained from non-relativistic semi-empirical four-parameter mass formula with coefficients from shell-corrected Thomas–Fermi model (Myers and Swiatecki 1996).

### 7.1.3 Compressibility at saturation

The incompressibility, usually referred to as compressibility, or compression modulus  $K_{\infty} = 9 \frac{dP}{dn_B} \Big|_{n_{\text{sat}}}$  of infinite nuclear matter at saturation is considered one of the vital constraints for dense matter, as it determines the stiffness of the EoS.<sup>8</sup> In finite nuclear systems, the isoscalar giant monopole resonance (ISGMR) establishes a direct empirical link between the finite nucleus compressibility,  $K_A$  and the centroid energy  $E_{\text{ISGMR}}$  (Colo et al. 2014),

$$E_{\text{ISGMR}} = \sqrt{\frac{\hbar^2 K_A}{m \langle r^2 \rangle_m}} \quad (45)$$

where  $\langle r^2 \rangle_m$  is the mean square mass-radius in the ground state and  $m$  is the nucleon mass.

Furthermore, the finite compressibility can be expressed in terms of the liquid-drop mass formula (Blaizot 1980)

$$K_A = K_{\text{vol}} + K_{\text{surf}} A^{-1/3} + K_{\text{asym}} \left( \frac{N - Z}{A} \right)^2 + K_{\text{Coul}} \frac{Z^2}{A^{4/3}} \quad (46)$$

where  $K_{\text{vol}}$ ,  $K_{\text{surf}}$ ,  $K_{\text{asym}}$  and  $K_{\text{Coul}}$  define the volume, surface, asymmetry and Coulomb terms, respectively. The  $K_{\text{vol}}$  term is related to the nuclear matter properties such that  $K_{\text{vol}} \approx K_{\infty}$  (Blaizot 1980). Inelastic scattering of isoscalar probes can be used to determine the ISGMR strength distribution experimentally. The  $\alpha$  particle has been the most widely used and emerges as effective probe for such observations. The discussion of ISGMR collective nucleon excitations from  $^{90}\text{Zr}$  and  $^{208}\text{Pb}$  nuclei suggests  $K_{\infty} = 240 \pm 20$  MeV (Colo et al. 2014; Todd-Rutel and Piekarewicz 2005; Colo et al. 2004; Agrawal et al. 2003), but there might be a softening in this value caused by pairing (Cao et al. 2012; Vesely et al. 2012).

Khan and Margueron (2012) argue that properties of nuclei do not set constraints on the EoS at saturation density, but rather at an average density of  $\sim 0.11 \text{ fm}^{-3}$ , that they design as crossing density. Considering that the giant monopole resonance of a chain of nuclei constrains the third derivative of the energy per unit of volume at this density, they arrive at  $K_{\infty} = 230 \pm 40$  MeV with 17% uncertainty.

In another work, the authors reviewed in detail the methods of analysis of giant monopole resonance (GMR) data, as well as values obtained using different techniques and theories between 1961 and 2016 covering a range  $K_{\infty} = 100\text{--}380$

<sup>8</sup> (R3 #14) At saturation density one can also write  $K_{\infty} = 9 \left[ n_B^2 \frac{d^2 E/A}{dn_B^2} \right]_{n_{\text{sat}}}$ .

MeV (see references within Table 1 of Stone et al. (2014)), with a trend to higher values in relativistic than in non-relativistic mean-field models. Without any microscopic model assumptions, except (marginally) the Coulomb effect,  $250 < K_\infty < 315$  MeV was obtained and it was shown that surface characteristics have a crucial influence in vibrating nuclei (Stone et al. 2014).

### 7.1.4 Hyperon potentials

In this section, we list various experimental results relevant to hyperon potentials. In relativistic models, the hyperon potential is defined as  $U_H = \text{vector interaction} + \text{scalar interaction}$ , while in non-relativistic models it is defined as  $U_H = E_H - T_H - m_H$ , the difference between single-particle energy and kinetic energy (plus mass), both usually discussed at saturation. Although more constraining data exists for the potential of the  $\Lambda$  hyperon, data also exists for the  $\Sigma$  and  $\Xi$  potentials.

The analysis of data from the level spectra of  $\Lambda$  hypernuclei from  $\pi^+ K^+$  and  $K^- \pi^-$  reactions produced at emulsion and bubble chambers showed that the single-particle energies of  $\Lambda$ -hypernuclei vary smoothly with number of nucleons  $A$  and are well reproduced considering the potential at nuclear saturation density  $U(\Lambda - N) \equiv U_\Lambda = -28$  MeV (Millener et al. 1988). Considering a slightly different renormalization of the data obtained from the KEK 12-GeV PS and superconducting kaon spectrometer (SKS, Hasegawa et al. 1996), the value  $U_\Lambda = -30$  MeV was obtained in Gal et al. (2016). In Shen et al. (2006), using a relativistic mean-field approach, in particular the TM1 model (Sugahara and Toki 1994), the data of Hasegawa et al. (1996) have been used to fit the  $\sigma$ - $\Lambda$  coupling, while considering that the  $\Lambda$ -vector mesons couplings are determined from the SU(6) quark model. A  $\Lambda$ -hyperon potential in nuclear matter at saturation density equal to  $U_\Lambda = -30$  MeV was obtained. Similar values,  $U_\Lambda = -30$  to  $-32$  MeV, have been obtained with other relativistic mean-field models using the same constraints (Fortin et al. 2018).

As discussed in Gal et al. (2016), measurements at KEK (the High Energy Accelerator Research Organization in Japan) of the  $\Sigma^-$  spectrum (Noumi et al. 2002; Saha et al. 2004) have indicated that the  $\Sigma$ -nucleon potential is strongly repulsive. A  $\Sigma$  potential in symmetric nuclear matter considered reasonable is of the order  $U_\Sigma = 30 \pm 20$  MeV (Gal et al. 2016). Note that the  $\Sigma$ -hyperon is predicted by some models, such as the QMC model, not to appear in dense matter for the regime relevant for neutron stars (Stone et al. 2021).

Concerning the  $\Xi$ -nucleon potential, the measurement of  $^{12}_{\Xi}\text{Be}$  (Khaustov et al. 2000) was described with a Wood-Saxon potential of the order of  $U_\Xi = -14$  MeV (Gal et al. 2016). The more recent Kiso event (Nakazawa et al. 2015) for  $^{15}_{\Xi}\text{Ca}$ , if interpreted as a  $1p$  state, seems to indicate a deeper potential. Relativistic mean field models have been constrained by this measurement and a depth of the order of  $U_\Xi = -15$  to  $-19$  MeV was obtained (Fortin et al. 2020). Recently, considering five two-body  $\Xi$  capture events to two single  $\Lambda$ -hypernuclei obtained by KEK (Aoki et al. 2009; Nakazawa et al. 2015) and J-PARC (the Japan Proton Accelerator Research

Complex, Hayakawa et al. (2021), Friedman and Gal (2021) have calculated an attractive  $\Xi$ -nucleon interaction with a depth  $U_{\Xi} \gtrsim 20$  MeV ( $U_{\Xi} = 21.9 \pm 0.7$  MeV).

Note, however, that a much less attractive  $\Xi$  potential has been calculated by the HAL-QCD collaboration (Inoue 2019). Using hyperon interactions extracted from a (2+1) lattice QCD in Brueckner–Hartree–Fock (BHF) calculation with a statistical error of approximately  $\pm 2$  MeV related with the QCD Monte Carlo simulation, the HAL QCD collaboration anticipated  $U_{\Lambda} = -28$  MeV,  $U_{\Sigma} = +15$  MeV, and  $U_{\Xi} = -4$  MeV (Inoue 2019) at  $n_{\text{sat}}$ , which agrees with  $p - \Xi^{-}$  correlation functions from the ALICE collaboration using 3-momenta measured at  $s = \sqrt{13}$  TeV (Fabbietti et al. 2021; ALICE Collaboration 2020).

In a recent study, Friedman and Gal (2023) conducted a direct optical potential analysis, examining the binding energies of both  $1s_{\Lambda}$  and  $1p_{\Lambda}$  states across the periodic table, covering nuclei from  $A = 12$  to  $A = 208$ . This analytical approach relied on nuclear densities constrained by the charge root-mean-square (r.m.s.) radii. They investigated the three-body  $\Lambda NN$  (repulsive) interactions and found  $U_{\Lambda}^{(3)} = 13.9 \pm 1.4$  MeV. The combined effect on  $U_{\Lambda}$ , computed as the sum of  $U_{\Lambda}^{(2)}$  and  $U_{\Lambda}^{(3)}$ , was calculated to be  $-26.7 \pm 1.7$  MeV at the saturation density.

### 7.1.5 $\Delta$ -baryon potential

Here, we list the empirical observations of spin 3/2  $\Delta$  baryon-nucleon potential at saturation, which is a helpful quantity when constraining its values in dense matter models. As already discussed,  $\Delta$ 's can replace hyperons without softening the EoS as much, which can be a solution to the hyperon puzzle (Bednarek et al. 2012).

The introduction of a phenomenological  $\Delta$ -nucleus spin-orbit interaction was used to improve the fit to the experimental  $\pi$ -nucleus angular distributions for  $\pi-^{16}\text{O}$  at 114 and 240 MeV,  $\pi-^4\text{He}$  at 220 and 260 MeV and  $\pi-^{12}\text{C}$  at 180 and 200 MeV (Horikawa et al. 1980). It was anticipated that the strength of the  $\Delta$ -nucleus spin-orbit interaction term is similar (attractive) to the nucleon–nucleon one, i.e.  $U_{\Delta} \approx U_N$  (Horikawa et al. 1980). In another study, the cross-section measurement of electron-nucleus scattering observed in 2.5 GeV Synchrotron at Bonn gives a density-dependent average binding potential  $U_{\Delta}(n_B) \simeq -75 n_B(r)/n_{\text{sat}}$  MeV (Koch and Ohtsuka 1985). Furthermore, a range of uncertainty is estimated from the electron-nucleus, pion nucleus, scattering and photoabsorption experiments i.e.  $-30$  MeV  $+U_N < U_{\Delta} < U_N$ , with  $U_N \simeq -(50 - 60)$  MeV, which leads to the constraint  $-90$  MeV  $< U_{\Delta} < -50$  MeV (Drago et al. 2014).

## 7.2 Symmetry energy $E_{\text{sym}}$ and derivative $L$

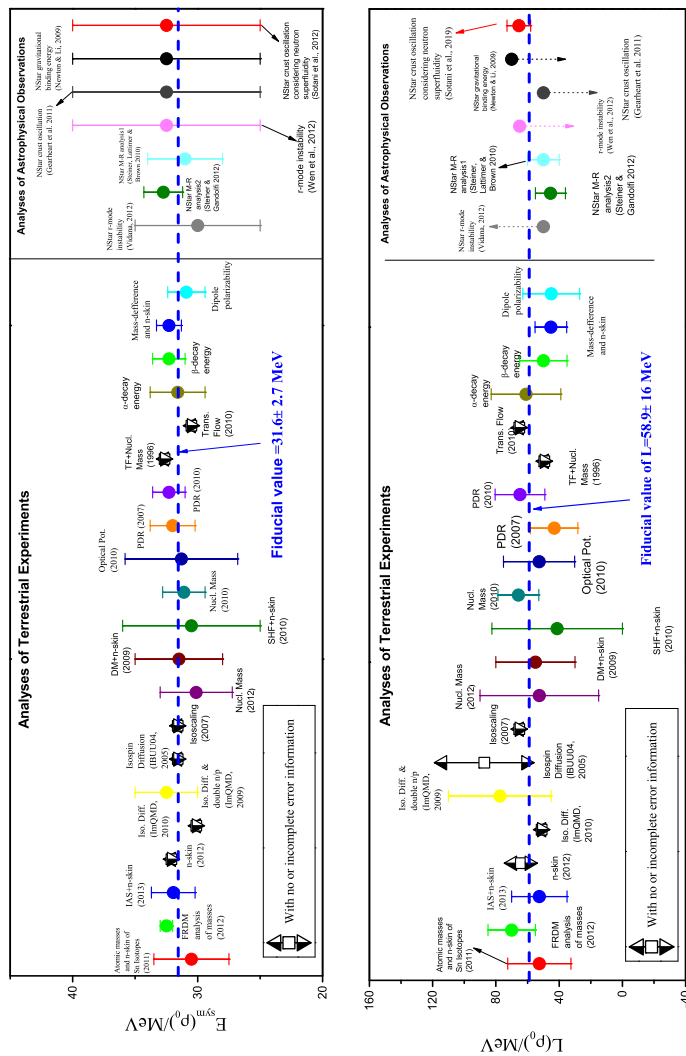
To translate between isospin symmetric matter and matter in neutron stars, with very low charge fraction, we make use of the symmetry energy  $E_{\text{sym}}$ . The determination of the saturation density  $E_{\text{sym}}$ ,  $L$ ,  $K_{\text{sym}}$ , and  $J$  parameters is very challenging, and involves large experimental and theoretical efforts (Estee et al. 2021). At low densities, we have some fundamental understanding of the symmetry energy but, at large densities, the uncertainty becomes extremely large. At subsaturation densities,

nuclear structure probes can generally confine the symmetry energy most effectively, whereas astronomical observations and heavy-ion collisions are two significant methods for constraining the symmetry energy from subsaturation to suprasaturation densities (Zhang and Chen 2015). The main source of uncertainty in the  $E_{\text{sym}}$  parameters are the poorly understood many-nucleon interactions.

Within nuclei, the distributions of neutrons and protons differ, and change with  $Z$  and  $A$ . As a result, nuclear property measurements, particularly for neutron-rich nuclei, hold promise for restricting nuclear symmetry energy parameters. It is shown that, in addition to the Fermi momentum and the isospin asymmetry parameter, the neutron skin thickness ( $R_{\text{skin}} = R_n - R_p$ ) of asymmetric semi-infinite nuclear matter is a function of the Coulomb energy,  $E_{\text{sym,sat}}$ ,  $L_{\text{sat}}$ , and  $K_{\text{sym,sat}}$  (see Suzuki 2022 and references therein). The neutron skin thickness of heavy nuclei such as  $^{208}\text{Pb}$  has been shown to correspond linearly to the slope parameter  $L$ , which can also be written as  $L = 3n_B \frac{dE_{\text{sym}}}{dn_B}$ , regulating the density dependence of  $E_{\text{sym}}$  around the saturation density  $n_{\text{sat}}$ . As a result, the high-accuracy measurement of  $R_{\text{skin}}$  is a significant limit on the density dependence of  $E_{\text{sym}}$  at subnormal densities.

### 7.2.1 $E_{\text{sym}}$ and $L$ at saturation

In this section, we review various experiments to determine the symmetry energy parameters at saturation density. Li et al. (2019) illustrated a comprehensive list of  $E_{\text{sym}}$  and  $L$  at saturation density from 28 model assessments of terrestrial nuclear tests and astrophysical data with the fiducial values of  $(31.6 \pm 2.7)$  MeV and  $(58.9 \pm 16)$  MeV, respectively (see Fig. 25). The S $\pi$ RIT collaboration from the Radioactive Isotope Beam Factory (RIBF) at RIKEN measured the spectra of charged pions produced by colliding rare isotope tin (Sn) beams with isotopically enriched Sn targets to restrict their contributions at suprasaturation densities. The calculated slope of the symmetry energy is  $(42 < L < 117)$  MeV using ratios of charged pion spectra observed at high transverse momentum (Estee et al. 2021). Furthermore, using the available experimental nuclear masses of heavy nuclei,  $E_{\text{sym}}$  was determined, which was employed further to extract  $L = (50.0 \pm 15.5)$  MeV at  $n_{\text{sat}} = 0.16 \text{ fm}^{-3}$  (Fan et al. 2014). As discussed earlier, the thickness of the neutron skin of  $^{208}\text{Pb}$  offers a strict laboratory restriction on the symmetry energy. Using the strong correlation between  $R_{\text{skin}}$  and  $L$ , the updated Lead Radius Experiment (PREX-II) reported a large  $L = (106 \pm 37)$  MeV based on both theoretical and actual data, consistently overestimating present limits (Reed et al. 2021). In another PREX-II study, for  $^{208}\text{Pb}$  the parity-violating asymmetry  $A_{\text{PV}}$  was studied using non-relativistic and relativistic energy density functionals (Reinhard et al. 2021). A neutron skin thickness of  $R_{\text{skin}}^{208} = (0.19 \pm 0.02)$  fm, and a small value of derivative  $L = (54 \pm 8)$  MeV were obtained. Recent studies have considered both CREX and PREX+II measurements in their analysis. In Lattimer (2023), the symmetry energy slope was determined to be  $L = 53 \pm 13$  MeV. However, Reinhard et al. (2022) have shown the PREX and CREX measurements cannot be simultaneously described by nuclear models, defined in the framework of an energy density functional theory, which are also consistent with a



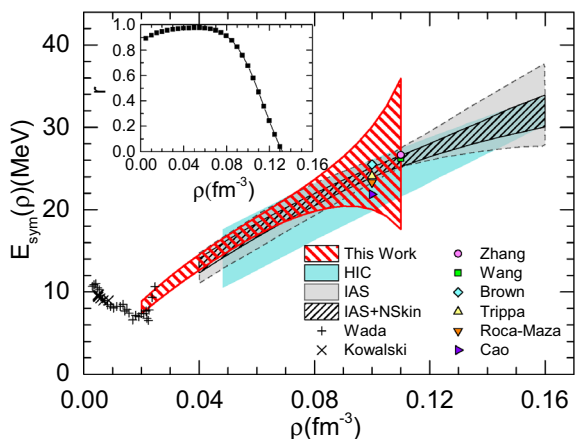
**Fig. 25** Constraints on saturation values of the symmetry energy (upper panel) and slope (lower panel) at saturation from terrestrial and astrophysical experiments. Image reproduced with permission from Li et al. (2019), copyright by SIF/Springer

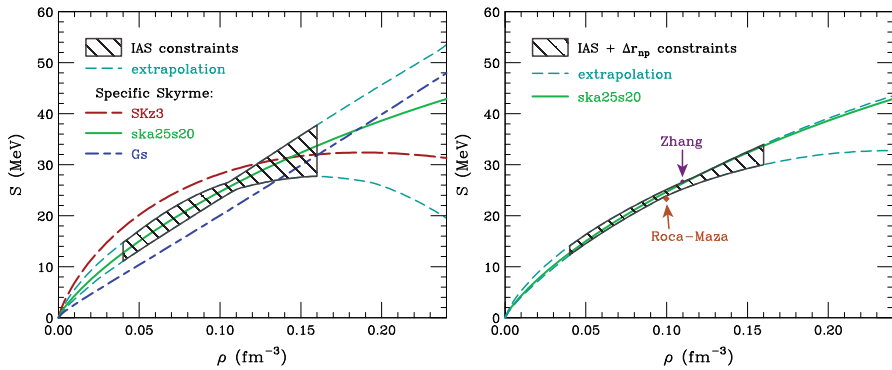
pool of global nuclear properties. The two panels of Fig. 25 are in line with earlier astrophysical estimates and show considerably lower values for the symmetry energy and its slope at saturation than those recently published using a specific set of relativistic energy density functionals to describe PREXII as in Reed et al. (2021).

### 7.2.2 $E_{\text{sym}}$ and $L$ below saturation

Constraining the symmetry energy parameters at subsaturation density also helps to restrict the EoS of the neutron-star matter. Here, we discuss experimental measurements for  $E_{\text{sym}}$  and  $L$  below saturation density. The moderate-temperature nuclear gases formed in the collisions of  $^{64}\text{Zn}$  projectiles with  $^{92}\text{Mo}$  and  $^{197}\text{Au}$  target nuclei showed a large degree of  $\alpha$  particle clustering at low densities. From isoscaling analyses of the yields of nuclei with  $A \leq 4$ , the temperature and density-dependent symmetry energy coefficients of these gases at densities  $n_B = 0.01 n_{\text{sat}} - 0.05 n_{\text{sat}}$  were evaluated as  $E_{\text{sym}} = 9.03 - 13.6$  MeV (see Table 1 of Kowalski et al. 2007). From the measured excitation energies of the isovector giant quadrupole resonance (IVGQR) in  $^{208}\text{Pb}$ , the symmetry-energy value  $E_{\text{sym}} = (23.3 \pm 0.6)$  MeV at  $n_B = 0.1 \text{ fm}^{-3}$  was estimated (Roca-Maza et al. 2013). Furthermore, the magnitude of the symmetry energy  $E_{\text{sym}} = (26.65 \pm 0.20)$  MeV at a subsaturation cross density  $n_B \approx 0.11 \text{ fm}^{-3}$  was determined by the binding energy difference  $\Delta E$  between a heavy isotope pair (Zhang and Chen 2013). Using the available experimental nuclear masses of heavy nuclei  $E_{\text{sym,sat}}$  was determined, which was employed further to extract the density dependent symmetry-energy value,  $E_{\text{sym}} = (25.98 \pm 0.01)$  MeV and the derivative,  $L = (49.6 \pm 6.2)$  MeV at  $n_B = 0.11 \text{ fm}^{-3}$  (Fan et al. 2014). The magnitude of the symmetry energy  $E_{\text{sym}}$  at densities around  $n_{\text{sat}}/3$  was determined uniquely by the electric dipole polarizability  $\alpha_D$  in  $^{208}\text{Pb}$ . A stringent constraint of  $E_{\text{sym}} = (15.91 \pm 0.99)$  MeV was observed at  $n_B = 0.05 \text{ fm}^{-3}$  and for the range  $n_B = 0.02 - 0.11 \text{ fm}^{-3}$ , results are illustrated by the red band in Fig. 26 (Zhang and Chen 2015). In addition, the nuclear symmetry coefficients were extracted using

**Fig. 26** Empirical constraints on symmetry energy versus baryon density. Image reproduced with permission from Zhang and Chen (2015), copyright by APS





**Fig. 27** Symmetry energy versus baryon density without (left:) and with (right:) neutron-skin constraints. Image reproduced with permission from Danielewicz and Lee (2014), copyright by Elsevier

**Table 7** Empirical constraints related to isospin asymmetric matter for the symmetry energy and slope parameter

Constraints	Value	$n_B$	References
$E_{\text{sym}}$ (MeV)	$31.6 \pm 2.7$	$n_{\text{sat}}$	Li et al. (2019)
	9.03–13.6	$0.01\text{--}0.05 \text{ fm}^{-3}$	Kowalski et al. (2007)
	$15.91 \pm 0.99$	$0.05 \text{ fm}^{-3}$	Zhang and Chen (2015)
	$23.3 \pm 0.6$	$0.1 \text{ fm}^{-3}$	Roca-Maza et al. (2013)
	$26.65 \pm 0.20$	$0.11 \text{ fm}^{-3}$	Zhang and Chen (2013)
	$25.98 \pm 0.01$	$0.11 \text{ fm}^{-3}$	Fan et al. (2014)
$L$ (MeV)	$32.20 \pm 2.4$	$0.04 \leq n_B \text{ (fm}^{-3}\text{)} \leq 0.13$	Danielewicz and Lee (2014)
	$58.16 \pm 16$	$n_{\text{sat}}$	Li et al. (2019)
	$50 \pm 15.5$	$n_{\text{sat}}$	Fan et al. (2014)
	$54 \pm 8$	$n_{\text{sat}}$	Reinhard et al. (2021)
	$106 \pm 37$	$n_{\text{sat}}$	Reed et al. (2021)
	$49.6 \pm 6.2$	$0.11 \text{ fm}^{-3}$	Fan et al. (2014)

excitation energies to isobaric analog states (IAS) and charge invariance. A narrow constraint of  $E_{\text{sym}} = 32.2 \pm 2.4$  MeV was obtained at saturation and for the range of density ( $0.04 \lesssim n_B \lesssim 0.13$ )  $\text{fm}^{-3}$ , the behavior is illustrated in the left panel of Fig. 27. In addition, inclusion of the skin constraints narrows down the constraints for the same density range (see right panel of Fig. 27) and at saturation  $E_{\text{sym}}$  becomes  $(32.2 \pm 1.1)$  MeV (Danielewicz and Lee 2014). Table 7 summarizes the empirical constraints on  $E_{\text{sym}}$  and  $L$ .



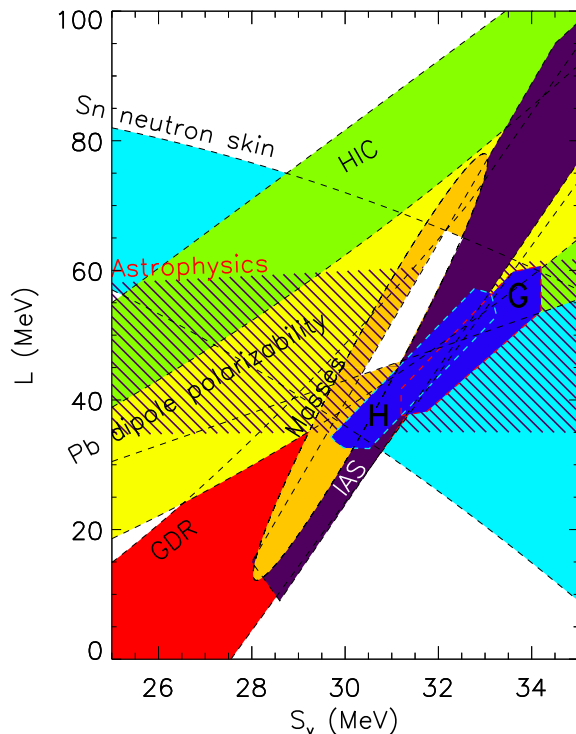
### 7.2.3 Correlation of symmetry energy and slope parameter

As discussed earlier, the symmetry energy and its slope play an important role in constraining the EoS of neutron-star matter. Thus, the correlation between both parameters can be useful to understand their interdependence. Lattimer and Steiner (2014) illustrate the experimentally determined values of  $E_{\text{sym}}$  and  $L$  parameters. Figure 28 contains different regimes coming from the observations of low-energy heavy-ion collisions, astrophysics, neutron skin, giant dipole resonance (GDR), isobaric analog states (IAS), and dipole polarizability.

### 7.2.4 Heavy-ion collision measurements of neutron skin of $^{197}\text{Au}$ and $^{238}\text{U}$

Recently, from heavy-ion collisions it was discovered that it is possible to measure the nucleus charge radius by using the energy dependence of the Breit–Wheeler process (Wang et al. 2023). Additionally, the STAR experiment has also found the total matter radius determined using the diffractive photoproduction in ultra-peripheral collisions to observe a unique spin interference pattern in the angular distribution of  $\rho \rightarrow \pi^+ + \pi^-$  decays (Abdallah et al. 2023c). Combining these two measurements, they found that for  $^{197}\text{Au}$  there was a neutron skin of  $0.17 \pm 0.03$  (stat.)  $\pm 0.08$  (syst.) fm and a for  $^{238}\text{U}$  there was a neutron skin of  $0.44 \pm 0.05$  (stat.)  $\pm$

**Fig. 28** Correlation between symmetry energy and slope at saturation. Image reproduced from Lattimer and Steiner (2014), copyright by SIF/ Springer



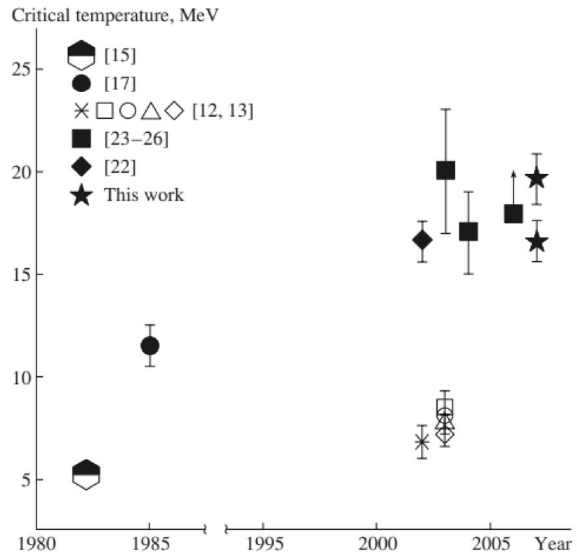
0.08 (syst.) fm (Abdallah et al. 2023c). To date these results have not yet been used in theoretical calculations but they do appear to be consistent with PREXII results.

### 7.3 Isospin-symmetric nuclear matter liquid–gas critical point

The similarity between nucleon–nucleon and Van der Waals interactions suggested the possibility of also having a liquid–gas phase transition in nuclear matter if the effects of Coulomb and limited size are ignored. If that is the case, the existence of a coexistence line associated with a first order transition stopping at a critical point is expected. Indeed in the liquid–gas phase diagram, the surface tension of a nucleus diminishes as its temperature rises, eventually disappearing at a point known as the critical point (Landau and Lifshitz 2000). The liquid–gas phase transition line divides two separate nuclear phases, i.e., nuclei (towards low density) and bulk matter (towards high density). In many models, if just nuclear matter is taken into account and Coulomb interactions are disregarded, this phase transition is anticipated as first-order (Soma and Bozek 2009; Fiorilla et al. 2012) whereas in astrophysical environments, the production of nuclei involves the Coulomb and other interactions that occur in finite nuclei, where some characteristics of the first order phase transition may be retained (Buyukcizmeci et al. 2013).

In this section, we review the different nuclear reaction experiments relevant to the liquid–gas phase diagram of symmetric nuclear matter. The critical temperature found in the studies on finite nuclei (Karnaukhov et al. 2003; Elliott et al. 2013; Natowitz et al. 2002; Karnaukhov et al. 2008) is in the range  $T_c \approx 17 - 20$  MeV. In a study from empirical observations of limiting temperatures, the critical temperature of infinite nuclear matter was found to be  $T_c = 16.6 \pm 0.86$  MeV (Natowitz et al. 2002). Also, in p + Au collisions at 8.1 GeV, the charge distribution of the intermediate mass fragments was analyzed in the framework of the statistical multifragmentation model, and the critical temperature was found to be  $T_c = 20 \pm 3$  MeV (Karnaukhov et al. 2003). Furthermore, in another study, a critical temperature  $T_c > 15$  MeV was anticipated for the nuclear liquid–gas phase transition from fission and multifragmentation data (see Fig. 29 for a list of  $T_c$  obtained from different studies (Karnaukhov et al. 2008)). In the latest study, six different sets of experimental data obtained from the Lawrence Berkeley National Laboratory 88-inch cyclotron, Indiana Silicon Sphere Collaboration and Equation of State Collaboration indicated that infinite nuclear matter has critical rms values for temperature ( $T_c = 17.9 \pm 0.4$ ) MeV, density  $n_{Bc} = (0.06 \pm 0.01)$  fm<sup>-3</sup>, and pressure  $P_c = (0.31 \pm 0.07)$ , MeV fm<sup>-3</sup> (Elliott et al. 2013). The critical temperature is essentially a parameter that affects how quickly the surface tension falls when the nuclei heat up. The values mentioned in this work are useful to constrain nuclear matter models at finite temperature. They are summarized in Table 8.

**Fig. 29** Constraints on the critical temperature of the liquid–gas phase transition. Image reproduced with permission from Karnaukhov et al. (2008), copyright by Pleiades



**Table 8** Empirical constraints for the critical temperature, pressure, and baryon density related to liquid–gas critical point

Constraints	Value	References
$T_c$ (MeV)	$16.6 \pm 0.86$	Natowitz et al. (2002)
	$20 \pm 3$	Karnaukhov et al. (2003)
	$>15$	Karnaukhov et al. (2008)
	$17.9 \pm 0.4$	Elliott et al. (2013)
$P_c$ (MeV fm $^{-3}$ )	$0.31 \pm 0.07$	Elliott et al. (2013)
$n_{Bc}$ (fm $^{-3}$ )	$0.06 \pm 0.01$	Elliott et al. (2013)

## 8 Observational constraints: astrophysics

Measurements of stellar masses, radii and tidal deformabilities play a central role in establishing a link between neutron stars' microscopic and macroscopic properties. These observables depend upon the internal structure and composition of the matter that makes up the neutron star. This microphysical information is relayed through the equations of stellar structure by the EoS, which cannot be directly measured in laboratory experiments at the densities, temperatures, and isospin asymmetries relevant for neutron star cores. In the astrophysical context, the EoS is probed by a variety of astronomical observations of neutron stars, particularly electromagnetic observations in the radio and X-ray bands, as well as gravitational waves from the coalescence of compact binaries. These probes assume general relativity is the correct description of nature, as otherwise, observables would also depend on the theory of gravity that is considered (see e.g. Yagi and Yunes 2013a, b).

Neutron stars, whether in isolation or in binaries systems, are described by the Einstein equations, which require, as input, the EoS to connect the pressure to the

energy density and temperature inside the stars. When in isolation and when considering non-rotating stars, the Einstein equations reduce to the Tolman–Oppenheimer–Volkoff (TOV) equations (Tolman 1939; Oppenheimer and Volkoff 1939), whose solution determines the mass and radius of the non-rotating star for a given central energy density. Choosing a sequence of central densities yields a sequence of pairs of masses and radii, which together form a “mass-radius curve.” For slowly-rotating stars, one can expand the Einstein equations in small rotation through the Hartle–Thorne approximation (R4 #17) (Hartle 1967; Paschalidis and Stergioulas 2017), and the solution to the expanded equations yields the moment of inertia at first order in rotation, the quadrupole moment and a mass correction at second order. When a neutron star is not in isolation, but instead is perturbed by some external object (like a binary companion), the perturbations can also be studied by solving the linearized Einstein equations about a neutron star background, which yields the tidal deformabilities of the star. In this way, one can construct various curves, including the mass-radius, moment of inertia-mass, quadrupole moment-mass, tidal deformability-mass, and so on, that represent neutron stars of various central densities. These curves can change shape depending on the EoS,<sup>9</sup> there is a single set of such curves that correctly describes Nature. Astrophysical observations can determine which set is the correct one, and thus, which is the correct EoS to describe neutron stars.

In the next subsections, we discuss one by one the empirical macroscopic constraints obtained from various astronomical observations. We provide summaries of current constraints with only brief explanations of the underlying measurements. See, e.g., Chatziioannou (2020) for a more thorough review.

It is important to remember that the densities relevant for the cores of neutron stars can certainly reach several times nuclear saturation density (see, e.g., Fig. 5 of Legred et al. (2021), Fig. 6 of Mroczek et al. (2023b), Fig. 7 of Pang et al. (2024), Fig. 5 of Koehn et al. (2024) for recent estimations), and connecting constraints from theoretical calculations and terrestrial experiments at lower densities (Sects. 5 and 7) to neutron star observations will almost certainly involve some extrapolation. The amount of freedom allowed in extrapolation schemes may be, but are not always, informed by physical models. Some care is, therefore, warranted when interpreting results combining low-density and high-density constraints as the conclusions often depend on the particular extrapolation scheme chosen. See, e.g., Essick et al. (2021b), Essick et al. (2021a), Legred et al. (2022) for more discussion.

## 8.1 Neutron-star maximum mass

The measurement of a neutron star’s mass sets a lower bound on the mass above which the neutron star must undergo gravitational collapse to a black hole. Most observed neutron stars spin slowly compared with their Keplerian break-up rotation rate. Hence, mass observations are often used to bound the maximum mass

<sup>9</sup> There is also the possibility of a two-family scenario, where more than one distinct EoS is used to describe neutron stars (Drago et al. 2016)

achievable by cold, non-rotating neutron stars even though spin can support up to approximately  $\sim 20\%$  additional mass (Breu and Rezzolla 2016).

The radio pulsar PSR J1614-2230 was the first massive pulsar to be measured with a mass close to two solar masses (Demorest et al. 2010). Recently, its mass was updated to  $1.908^{+0.016}_{-0.016} M_{\odot}$  (Arzoumanian et al. 2018). The pulsar PSR J1614-2230 has a low mass companion and the determination of its mass was possible through the measurement of the Shapiro delay, a retardation effect of the pulse signal that originates on the curvature of the space-time close to the companion, and is given by (van Straten et al. 2001)

$$\Delta_S = 2r \ln[1 - s \cos(\phi - \phi_0)],$$

where  $s = \sin i$  with  $i$  the inclination angle,  $r = Gm_2/c^3$  with  $m_2$  the companion mass,  $\phi$  is the orbital phase, and  $\phi_0$  is the phase where the pulsar is on the opposite side of the companion from Earth. The measurement of  $\Delta_S$  allows the determination of the companion mass.

The record for the highest precisely and reliably measured neutron star mass currently belongs to the 2.8-ms radio pulsar (PSR) J0740+6620, which is likely orbiting an ultracool white-dwarf companion (Fonseca et al. 2021b). The data sets of this study are integrated pulse arrival-time measurements, obtained using the Canadian Hydrogen Intensity Mapping Experiment (CHIME) and 100-meter Green Bank Telescope (GBT). Timing solutions for PSR J0740+6620 were produced by the GBT and CHIME/Pulsar collaborations using narrow-band and wide-band times of arrival. While comparing the credible intervals of the Shapiro delay parameters across different dispersion measures (DM) evolution models, all calculated solutions were found to be statistically compatible with different DMX models at 68.3% credibility, with a 1% fluctuation in the credible intervals of the Shapiro delay. The Shapiro delay, also known as the gravitational time delay effect, is an increase in travel time of a signal when it passes near a massive object. The estimated mass of PSR J0740+6620 is  $m_p = 2.08^{+0.07}_{-0.07} M_{\odot}$  (in solar masses) and of its companion is  $m_c = 0.253^{+0.006}_{-0.005} M_{\odot}$  (Fonseca et al. 2021b) calculated at  $1\sigma$  (68.3% credibility), which is consistent with the earlier observations of Cromartie et al. (2019) (posterior in Fonseca et al. 2021a).

The second-highest precisely and reliably measured pulsar mass belongs to PSR J0348+0432. For this pulsar, the mass is estimated to be  $2.01 \pm 0.04 M_{\odot}$  at 68.3% credibility based on a combination of radio timing of the pulsar and precise spectroscopy of the white dwarf companion, which has a mass of  $m_c = 0.173 \pm 0.003 M_{\odot}$  and is in a 2.46-h orbit around the pulsar (Antoniadis et al. 2013).

An *upper* limit on the maximum mass of nonrotating neutron stars has been placed, albeit with more substantial astrophysical uncertainties, using the properties of short gamma-ray bursts. These have long been assumed to be produced by the merger of two neutron stars, with the remnant either forming a black hole immediately or collapsing quickly to a black hole (e.g., Murguía-Berthier et al. 2014). Under this assumption, and working off of the mass distribution of double neutron star systems in our Galaxy, several authors proposed that the maximum mass  $M_{\max}$  of a nonrotating neutron star was  $\sim 2.3 M_{\odot}$  (Bauswein et al. 2013; Lawrence

**Table 9** Pulsar empirical constraints on the maximum mass of neutron stars. Pulsar timing constraints are less susceptible to modeling assumptions than current gravitational wave constraints

Neutron Star	$M_{\max} (M_{\odot})$	References
PSR J0740+6620	$\geq 2.08 \pm 0.07$	Fonseca et al. (2021b)
PSR J0348+0432	$\geq 2.01 \pm 0.04$	Antoniadis et al. (2013)

et al. 2015; Fryer et al. 2015). The gravitational wave and electromagnetic data from the binary neutron-star merger GW170817, for which it was possible to make a good estimate of the total mass, also led to an estimate of  $M_{\max} \sim 2.2 - 2.3 M_{\odot}$  (Margalit and Metzger 2017). The argument for rapid collapse to a black hole is that otherwise the rapidly spinning remnant will spin down and inject energy into the remnant, which has not been seen. A key but unproven assumption in this argument is that the merger process will generate strong and ultimately poloidal magnetic fields that will spin down the remnant.

Detailed modeling of the electromagnetic afterglow of GW170817 has led to other estimates of the maximum mass, e.g.,  $M_{\max} \simeq 2.16^{+0.17}_{-0.15} M_{\odot}$  (Rezzolla et al. 2018),  $M_{\max} \simeq 2.16 - 2.28 M_{\odot}$  Ruiz et al. (2018),  $M_{\max} \simeq 2.13^{+0.17}_{-0.11} M_{\odot}$  (Shao et al. 2020),  $M_{\max} \lesssim 2.3 M_{\odot}$  (Shibata et al. 2019),  $M_{\max} \simeq 2.21^{+0.12}_{-0.12} M_{\odot}$  (Nathanail et al. 2021), and if the assumption that the GW170817 event resulted in a black hole is relaxed,  $M_{\max} \simeq 2.43^{+0.16}_{-0.12} M_{\odot}$  Ai et al. (2020). All of these estimates are subject to numerical and modeling uncertainties, but they are promising for the future. The empirical constraints on the maximum neutron-star mass are summarized in Table 9

There have also been several claims of heavier neutron stars. The discovery of black-widow pulsars with masses estimated as  $2.13 \pm 0.04 M_{\odot}$  (Romani et al. 2021) for PSR J1810+1744 and  $2.35 \pm 0.17 M_{\odot}$  (Romani et al. 2022) (68% confidence) for PSR J0952-0607 have been reported; however, these mass measurements are less secure (due to possible systematics) than the Shapiro delay-based measurements for PSR J0740+6620 and PSR J0348+0432. Moreover, the  $2.59^{+0.08}_{-0.09} M_{\odot}$  secondary component of the compact binary merger GW190814 has been touted as a potential heavy neutron star because it is less massive than known black holes (Abbott et al. 2020b). However, its proximity in mass to the ostensible  $2.7 M_{\odot}$  black-hole remnant of GW170817, and the lack of any observed tidal effects or electromagnetic counterpart, have made people think that it may be more likely to be a black hole. However, this conclusion comes with some caveats. Tidal effects for such massive neutron stars are expected to be very small (Tan et al. 2020), and in fact, not observable with the sensitivity of ground interferometers when this event was detected (Abbott et al. 2020b). Moreover, the lack of an electromagnetic counterpart in the gamma range could be due to the short-gamma ray burst not being pointed toward Earth. An electromagnetic counterpart at other frequencies could have been missed due to the poor localization of the event in the sky through only the use of gravitational wave information. See de Sá et al. (2022) for a discussion on many

**Table 10** Empirical constraints related to mass-radii from NICER

Neutron Star	$M(M_{\odot})$	Radius (km)	References
PSR J0030+0451	$1.34^{+0.15}_{-0.16}$	$12.71^{+1.14}_{-1.19}$	Riley et al. (2019a)
PSR J0740+6620	$2.072^{+0.067}_{-0.066}$	$12.39^{+1.30}_{-1.98}$	Riley et al. (2021a)
PSR J0030+0451	$1.44^{+0.15}_{-0.14}$	$13.02^{+1.24}_{-1.06}$	Miller et al. (2019)
PSR J0740+6620	$2.08^{+0.07}_{-0.07}$	$13.7^{+2.6}_{-1.5}$	Miller et al. (2021b)

candidates whose masses that lay inside the so called mass gap (between confirmed masses of neutron stars and black holes).

In another work, the authors used binary inclination  $i$  from lightcurve modeling and observed that J1810 offers a lower limit on the NS maximum mass of  $2 M_{\odot}$  at 99.7% credibility (Romani et al. 2021). A flat, but asymmetric, light-curve maximum and a deep, narrow minimum were observed in the spectrophotometry of the companion of PSR J1810+1744 measured by the Keck telescope. A hot pole, surface winds, and severe gravity darkening (GD) around the  $L_1$  point are all indicated by the maximum, whereas the minimum denotes significant limb darkening and a low underlying temperature. Having the radial-velocity amplitude  $K_c = 462.3 \pm 2.2$  km s<sup>-1</sup> recorded by the Keck telescope provides a precise neutron star mass of  $M_{\text{NS}} = 2.13 \pm 0.04 M_{\odot}$ , a value which is very relevant to the understanding of the dense-matter EoS (Romani et al. 2021).

## 8.2 Neutron-star mass-radius regions from NICER

NICER is a soft X-ray telescope mounted on the International Space Station (ISS) in 2017. The main aim of NICER is to determine masses and radii of neutron stars using pulse-profile modeling of neighboring rotation-powered millisecond pulsars. The NICER observations have played an important role in lowering the ambiguity in the EoS of high-density matter ( $1.5\text{--}5 n_{\text{sat}}$ ) at zero temperature. See Table 10 for a summary of the results reported from NICER, which we now discuss in detail. A word of warning is due at this stage: the results presented in this table (and in other tables in this section) are a summary, and in particular, the reported masses and radius with error bars are not a square in probability space. Rather, these quantities correspond to the maximum likelihood points and the 90% confidence regions of the marginalized posterior after a careful Bayesian parameter estimation study. Two-dimensional posteriors on mass and radius are not squares, but rather complicated shapes due to the correlation between parameters.

Using Bayesian parameter estimation based on pulse-profile modeling of the NICER XTI event data for the isolated pulsar PSR J0030+0451, Miller et al. (2019) (posterior Bogdanov et al. 2019) reported a gravitational mass  $M = 1.44^{+0.15}_{-0.14} M_{\odot}$  and a circumferential radius  $R = 13.02^{+1.24}_{-1.06}$  km at 68% credibility. For the same data but using slightly different models and a different statistical sampler, Riley et al.

(2019a) found  $M = 1.34^{+0.15}_{-0.16} M_{\odot}$  and  $R = 12.71^{+1.14}_{-1.19}$  km (posterior Riley et al. 2019b and raw data Steiner 2020).

The heavy binary pulsar PSR J0740+6620 has a NICER count rate only  $\sim 5\%$  that of PSR J0030+0451 and thus NICER data alone are insufficient for precise measurements of the mass and radius. As a result, for this pulsar, radio data and X-ray data from the X-ray Multi-Mirror (XMM-Newton) satellite were analyzed jointly with the NICER data. Miller et al. (2021b) (posterior Miller et al. 2021a) found  $M = 1.97 - 2.15 M_{\odot}$  and  $R = 12.21 - 16.33$  km, both at 68% credibility, whereas Riley et al. (2021a) (posterior Riley et al. 2021b) reported  $M = 2.072^{+0.067}_{-0.066} M_{\odot}$  and  $R = 12.39^{+1.30}_{-0.98}$  km.

The differences between the two results for PSR J0740+6620 (e.g., Miller et al. report a  $-1\sigma$  radius of 12.21 km, compared with Riley et al.'s estimate of 11.41 km) led the two groups to explore the reasons for the difference. As discussed in Miller et al. (2021b), the primary differences are as follows. First, Miller et al. (2021b) use a relative calibration between NICER and XMM-Newton that is consistent with the results of cross-calibration tests, whereas Riley et al. (2021a) assume a much broader range of possible cross-calibration factors. When Riley et al. (2021a) apply the same cross-calibration as Miller et al. (2021b), they find a  $-1\sigma$  radius of 11.75 km rather than 11.41 km, i.e., this accounts for almost half the difference from the 12.21 km  $-1\sigma$  radius of Miller et al. (2021b). Second, Riley et al. (2021a) have a hard prior upper bound on the radius of 16 km, whereas Miller et al. (2021b) allow the radius to be anything that fits the data. When Miller et al. (2021b) eliminate all solutions with  $R > 16$  km, their  $-1\sigma$  radius drops to 12.06 km. The remaining difference, of 0.31 km (compared with the original 0.8 km), is likely to be primarily due to different choices of statistical samplers: Miller et al. (2021b) use the parallel-tempered Markov chain Monte Carlo code PT-emcee whereas Riley et al. (2021a) use the nested sampler MultiNest; Miller et al. (2021b) argue that at least in this specific case, MultiNest may underestimate the widths of the posteriors. More generally, there are many moving parts within such analyses and sometimes subtle choices on how data is analyzed or which data is analyzed can affect the resulting constraints; see Essick (2022) for an example related to data selection.

### 8.3 Other observational constraints on neutron star masses and radii

Quiescent low-mass X-ray binaries (QLMXBs) in globular clusters are promising objects for mass and radius constraints because (i) previous accretion events heat up the surface increasing their luminosity, (ii) their magnetic fields are expected (but not observed) to be relatively small, the magnetic field having been buried by the accretion, and (iii) the globular cluster permits a determination of the distance. There are several QLMXBs which have been used to obtain mass and radius constraints, see Steiner et al. (2018) for a recent analysis leading to neutron star radii between 10 and 14 km. However, there are still a significant number of potential systematics which may be important: (i) the magnetic field is not observed so the assumption of a small magnetic field is untested, (ii) the determination of the mass and radius requires a model of the neutron star atmosphere and the associated emergent X-ray spectrum,



(iii) the composition of the accreted material is not always known, (iv) the temperature is often presumed to be uniform over the entire surface, and (v) scattering by the interstellar medium leads to a reddening of the spectrum which is not fully known. A recent attempt (Al-Mamun et al. 2021) to search for systematic effects in QLMXB models compared QLMXBs to other mass and radius constraints (including those from LIGO/Virgo and NICER) and found no evidence for systematics that poison the QLMXB results. However, this result may change as the LIGO/Virgo constraints improve.

Within the supernova remnant HESS J1731-347, a star that is the centre compact object has been studied, according to Doroshenko et al. (2022), the mass and radius of this star is estimated to be  $M = 0.77^{+0.20}_{-0.17} M_{\odot}$  and  $R = 10.4^{+0.86}_{-0.78}$  km, respectively, which is based on the modeling of Gaia observations and X-ray spectrum. This result depends critically on the assumption that the entire surface of the star emits at the same temperature, based on the lack of clear modulation with the stellar rotation. With that assumption, and the assumption that the atmospheric effects of the stellar magnetic field can be neglected, a carbon atmosphere is favored over a hydrogen or a helium atmosphere. However, as shown by Alford and Halpern (2023), nonuniform emission is consistent with the data on several similar sources (and preferred for some). This means that hydrogen and helium atmospheres are possible, and these could imply standard masses well above one solar mass. According to their estimation, this star can be either the lightest neutron star ever discovered or a strange star with exotic EoS (Doroshenko et al. 2022). The examination of several SN explosions indicates that it is not feasible to form a neutron star (NS) with a mass less than approximately  $1.17 M_{\odot}$  (Suwa et al. 2018), which begs the issue of what astronomical activity might result in such a small mass. Di Clemente et al. (2022) suggests that in the case of strange quark stars, masses of the order or less than one solar mass can be found and it is conceivable to construct a cogent astrophysical hypothesis that accounts for the object's mass, radius, and gradual cooling.

#### 8.4 Neutron-star tidal deformability from gravitational waves

The phasing of the gravitational waves emitted during the inspiral of a compact binary system is sensitive to the tidal deformation experienced by each component as a result of its companion's non-uniform gravitational field. The size of the deformation is measured by the tidal deformability parameter (Flanagan and Hinderer 2008; Hinderer 2008), which depends on the neutron-star mass and the EoS. The tidal deformabilities of the individual neutron stars appear at leading order in the gravitational waveform as a mass-weighted average known as the chirp or binary tidal deformability,  $\tilde{\Lambda}$  (Favata 2014; Wade et al. 2014), namely

$$\tilde{\Lambda} = \frac{16}{13} \left[ \frac{(m_1 + 12m_2)m_1^4 \Lambda_1 + (m_2 + 12m_1)m_2^4 \Lambda_2}{(m_1 + m_2)^5} \right] \quad (47)$$

where  $m_1$  and  $m_2$  are the masses of the binary components and  $\Lambda_1$  and  $\Lambda_2$  are their (dimensionless) individual tidal deformabilities. Because an EoS predicts a unique

$m$ – $\Lambda$  relation, a measurement of  $\tilde{\Lambda}$  and the binary masses helps to determine the EoS in a manner analogous to constraints on the  $m$ – $R$  relation.

#### 8.4.1 Extraction of $\tilde{\Lambda}$ from GW170817

The first measurement of  $\tilde{\Lambda}$  was enabled by the detection of gravitational waves from the binary neutron-star merger GW170817 (Abbott et al. 2017a) by LIGO and Virgo. The binary tidal deformability was constrained simultaneously with the system's masses, spins and other source properties via Bayesian parameter estimation, in which a waveform model is compared against the data to produce a likelihood function over the waveform parameters (see e.g. Thrane and Talbot 2019). Because the tidal deformability is mass-dependent, and gravitational-wave measurements of the binary mass ratio and spins are correlated, the constraints on  $\tilde{\Lambda}$  are sensitive to prior assumptions about spin. Assuming that both neutron stars in GW170817 had low spins, in keeping with the Galactic double neutron-star population (Burgay et al. 2003; Stovall et al. 2018), LIGO and Virgo measured  $\tilde{\Lambda} = 300^{+500}_{-190}$  at 90% confidence; without this assumption, the constraint is  $\tilde{\Lambda} \leq 630$  (Abbott et al. 2019). The joint constraints on  $\tilde{\Lambda}$  and the binary masses are expressible in terms of a multi-dimensional posterior probability distribution, samples from which are available in the data release accompanying (Abbott et al. 2019).

As before, a word of caution is due at this stage. The measurements of the tidal deformabilities discussed above and below in this section result from a Bayesian parameter estimation study, and as such, they correspond to the maximum likelihood points and the 90% confidence region of the marginalized posterior. The posterior is multi-dimensional and covariances exist between the various parameters that enter the waveform model. Therefore, 2-dimensional confidence regions, like that for the chirp tidal deformability and the chirp mass, are not squares, but rather complicated shapes, which can be found in the papers we have referenced here.

The analysis of the signal (Abbott et al. 2019), and therefore, the tidal deformabilities reported above, do not require both of GW170817's compact objects to share the same EoS, implicitly leaving open the possibility that one of them is a black hole. Because of the kilonova and short gamma-ray burst counterparts to GW170817, however, it is reasonable to assume that the merger consisted of two neutron stars (De et al. 2018). The assumption that all neutron stars share the same EoS implicitly relates  $\Lambda_1$  and  $\Lambda_2$ , since the EoS predicts a unique  $m$ – $\Lambda$  relation. De et al. (2018) approximated this relation as  $\Lambda_1 = \Lambda_2 m_2^6 / m_1^6$ , and used it to further constrain  $\tilde{\Lambda} = 222^{+420}_{-138}$  at 90% confidence, assuming low neutron-star spins. An alternative to making such an approximation is to average over many different candidate EoSs drawn from a prior distribution: each EoS sample imposes an exact relation between  $\Lambda_1$  and  $\Lambda_2$ , and the EoS uncertainty encoded in the prior distribution is propagated to  $\tilde{\Lambda}$  by the averaging process. Of course, this approach requires modeling the EoS. Typically, this is done phenomenologically, either with a parameterization of the pressure-density relation—e.g., a spectral decomposition (Lindblom 2010)—or with a *Gaussian process* (Landry and Essick 2019)—i.e., a

distribution over functions described by a mean function and a covariance kernel. Caveats exist here, however, because the choice of the functional form of the EoS may not include the entire phase space of possible equations of state that can be conceived from nuclear physics.

The EoS-averaging approach was taken by Essick et al. (2020), which modeled the EoS nonparametrically as a Gaussian process and found  $\tilde{\Lambda} = 245^{+361}_{-160}$  (posterior median and 90% highest-probability-density credible region). The Gaussian process was constructed to explore the entire functional space of EoSs that obey causality (sound speed less than the speed of light) and thermodynamic stability (positive semidefinite sound speed), subject to the requirement that the EoS support neutron star masses of at least  $1.93 M_{\odot}$  to account for the existence of heavy pulsars. For comparison, when using a spectral parameterization for the EoS, one finds  $\tilde{\Lambda} = 412^{+313}_{-262}$  for the posterior median and 90% credible regions. If one instead uses the global maximum of the posterior, one finds  $\tilde{\Lambda} = 225^{+500}_{-75}$ , which is similar to that found using Gaussian processes.

Electromagnetic observations of GW170817's kilonova counterpart have also been used to constrain the binary tidal deformability. However, these bounds depend on the assumed kilonova lightcurve model and are thus subject to sizeable systematic uncertainty: for example,  $\tilde{\Lambda} \geq 197$  (90% confidence) (Coughlin et al. 2018),  $120 < \tilde{\Lambda} < 1110$  (90% confidence) (Breschi et al. 2021), and  $109 \leq \tilde{\Lambda} \leq 137$  (68% confidence) (Nicholl et al. 2021) was found. Unlike the gravitational-wave measurements of binary tidal deformability,  $\tilde{\Lambda}$  does not appear directly as a parameter in the light curve model and it must therefore be constrained via correlations with other observables. As discussed in Radice et al. (2018), these constraints come with the possibility of large errors associated with uncertainties in the mass ratio of the system (Kiuchi et al. 2019).

#### 8.4.2 Extraction of individual tidal deformabilities $\Lambda_1$ and $\Lambda_2$

The binary tidal deformability  $\tilde{\Lambda}$  is the tidal parameter that is best constrained by gravitational-wave observations. However, it is the individual tidal deformability  $\Lambda$  that is directly determined by the EoS together with the equations of stellar structure. The relation between the individual tidal deformabilities  $\Lambda_1$  and  $\Lambda_2$  of each member of a compact binary and the system's binary tidal deformability is expressed in Eq. (47). Generally, the component masses  $m_1$  and  $m_2$  are constrained simultaneously with  $\tilde{\Lambda}$  through the Bayesian parameter estimation of the gravitational-wave signal. Thus, we have one equation relating three tidal parameters: a measurement of  $\tilde{\Lambda}$  only determines  $\Lambda_1$  and  $\Lambda_2$  up to a residual degeneracy. This degeneracy is broken by a tidal parameter  $\delta\tilde{\Lambda}$  that appears at higher order in the gravitational waveform, but it is unfortunately not measurable with current detectors (Wade et al. 2014).

Nonetheless, this degeneracy is not an obstacle to translating measurements of  $\tilde{\Lambda}$  into constraints on  $\Lambda_1$  and  $\Lambda_2$ : it is simply the case that every combination of  $\Lambda_1$  and  $\Lambda_2$  that produces the same  $\tilde{\Lambda}$  is equally likely. Thus, one can build up a posterior for

$\Lambda_1$  and  $\Lambda_2$  by sampling in the individual tidal deformabilities, and assigning to each sample the likelihood of the  $\tilde{\Lambda}$  it predicts (the likelihood is the ratio of the posterior probability to the prior probability). The result of this process for GW170817 is the posterior on the component tidal deformabilities illustrated in Fig. 10 of Abbott et al. (2019); notice that there is a direction in  $\Lambda_1$ – $\Lambda_2$  plane along which there is essentially no constraint due to the degeneracy.

One can break the degeneracy by introducing an additional layer of modeling that imposes a common EoS for both components of GW170817: this augments Eq. (47) with a second equation relating the individual tidal deformabilities. As in the case of  $\tilde{\Lambda}$ , this can be done either with an approximate relation connecting  $\Lambda_1$  and  $\Lambda_2$ , or by averaging over an EoS prior distribution.

Using the Gaussian process-based EoS representation described above, Essick et al. (2020) mapped the binary tidal deformability measurement into constraints of  $\Lambda_1 = 148^{+274}_{-125}$  and  $\Lambda_2 = 430^{+519}_{-301}$  on the component tidal deformabilities, under the assumption of low neutron-star spins. Moreover, since each EoS sample from the Gaussian process prescribes an exact  $m$ – $\Lambda$  relation, this approach allows the tidal deformability to be inferred at any mass scale. Essick et al. (2020) also constrained  $\Lambda_{1.4} = 211^{+312}_{-137}$  (posterior median and 90% highest-probability-density credible region).

Alternatively, one can break the degeneracy with an approximate EoS-insensitive relation connecting the individual tidal deformabilities. The so-called binary Love relations (Yagi and Yunes 2016) fit the relation between two independent linear combinations of  $\Lambda_1$  and  $\Lambda_2$  to many candidate EoSs from nuclear theory. With the neutron stars' individual tidal deformabilities related by way of this EoS-insensitive relation (Chatziioannou et al. 2018), augmented with Gaussian white noise to account for the scatter in the fit, and  $\Lambda$ 's mass dependence expanded in a series about  $1.4 M_\odot$ , Abbott et al. (2018) reduced the joint constraint on binary masses and  $\tilde{\Lambda}$  from GW170817 to an estimate of the tidal deformability of a  $1.4 M_\odot$  neutron star:  $\Lambda_{1.4} = 190^{+390}_{-120}$  at the 90% credible level, in the low neutron-star spin scenario. The empirical constraints on the tidal deformability from GW170817 are summarized in Table 11.

The results for the extracted tidal deformability posteriors from GW170817 are shown in Fig. 30. While there is a lot of overlap for the posteriors of the EoS insensitive (universal relations), spectral EoS, and Gaussian Process EoS, they are not exactly the same and this may lead to incorrect conclusions when extracting the EoS unless one is careful. For instance, most nuclear physics EoS tend towards higher values of  $\Lambda$  for a given  $M$ . Thus, the EoS insensitive posterior may seem more restricting than the spectral EoS. However, since these are 90% confidence regions, all posteriors are actually statistically consistent with each other, and one cannot simply take the edge of one posterior to draw strong conclusions without a careful Bayesian parameter estimation study.

**Table 11** Empirical constraints on the tidal deformability for GW170817 event from LIGO and Virgo

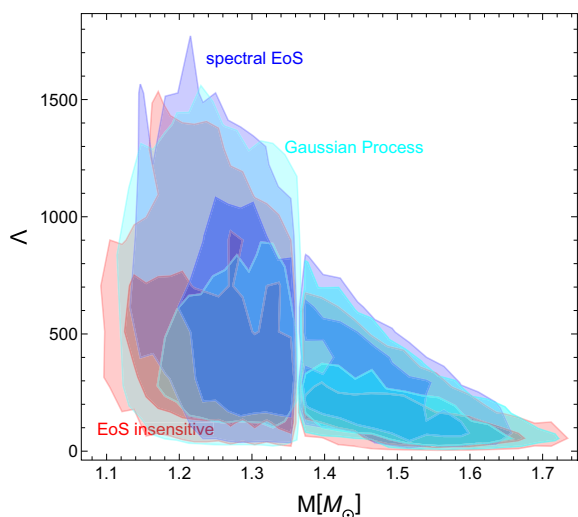
Tidal deformability	References	Confidence level
$\tilde{\Lambda} = 300^{+500}_{-190}$	Low spins (Abbott et al. 2019)	90%
$\tilde{\Lambda} \leq 630$	Minimal assumptions (Abbott et al. 2019)	90%
$\tilde{\Lambda} = 222^{+420}_{-138}$	Common EoS via analytic approximation (De et al. 2018)	90%
$\Lambda_{1.4} = 190^{+390}_{-120}$	Common EoS via binary Love relation (Abbott et al. 2018)	90%
$\tilde{\Lambda} = 245^{+361}_{-160}$	Common EoS via Gaussian process (Essick et al. 2020)	90%
$\Lambda_{1.4} = 211^{+312}_{-137}$	Common EoS via Gaussian process (Essick et al. 2020)	90%

### 8.4.3 Connecting tidal deformabilities to the mass-radius sequence

Because the neutron-star tidal deformability scales strongly with the stellar radius ( $\Lambda \sim R^5/m^5$ ), and because  $R$  is an easily interpretable parameter, gravitational-wave measurements of  $\Lambda$  have often been translated into radius constraints in the literature. Nonetheless, we stress that gravitational waves from compact binaries do not directly measure the neutron star radius—the mapping from  $\Lambda$  to  $R$  necessarily involves additional modeling. This modeling can either be done at the level of the EoS (e. g. with a spectral or nonparametric EoS representation), or at the level of the mapping itself (e.g., with an EoS-insensitive relation between  $\Lambda$  and the stellar compactness  $m/R$ ).

In Abbott et al. (2018), LIGO and Virgo mapped the joint posterior on component tidal deformabilities and masses from GW170817 (assuming a binary Love relation) into a posterior on  $m_1$ ,  $m_2$ ,  $R_1$  and  $R_2$  by adopting a spectral parameterization for the EoS. The original posterior was used to compute the likelihood of each spectral EoS

**Fig. 30** Constraints on the tidal deformability using the universal relations (EOS insensitive), the spectral EOS and an EoS constructed from Gaussian Processes. The universal-relation constraint and spectral-EoS constraint are obtained using GW170817 data. The Gaussian Process constraint is obtained by combining GW170817, PSR 1614-2230, PSR 0348+0432, and PSR 0740+6620 (mass only) data. Shaded regions are shown at the 68% and 95% confidence regions



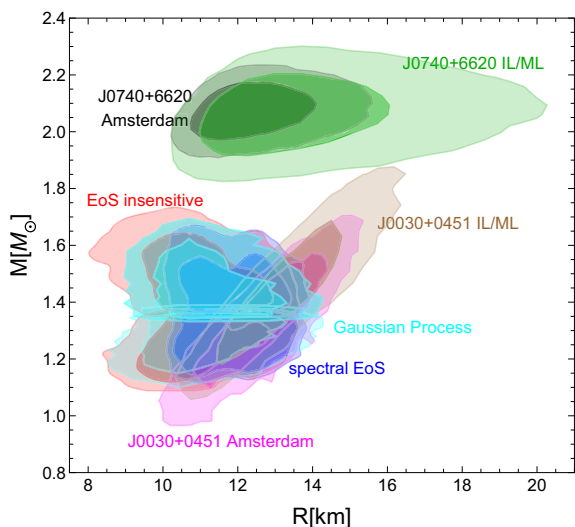
realization, and then for each component mass sample from the original posterior, the radius predicted by the EoS realization's mass-radius relation was assigned that likelihood. This procedure led to inferred radii of  $R_1 = 11.9^{+1.4}_{-1.4}$  km and  $R_2 = 11.9^{+1.4}_{-1.4}$  km.

LIGO and Virgo also implemented an alternative approach that used an EoS-insensitive  $\Lambda$ - $m/R$  relation. For each component mass and component tidal deformability sample from the original posterior,  $\Lambda$  was mapped to  $m/R$  using the relation from Yagi and Yunes (2017), and the radius was extracted using the sampled mass. The uncertainty in the EoS-insensitive relation fit was modeled as Gaussian white noise in the relation. This procedure led to inferred radii of  $R_1 = 10.8^{2.0}_{-1.7}$  km and  $R_2 = 10.7^{2.1}_{-1.5}$  km. The small differences versus the radius constraints with the spectral EoSs illustrate the systematic uncertainty that arises from the extra modeling required.

In De et al. (2018), a different EoS-insensitive relation was used to map from the tidal deformability measurement to a radius constraint. Imposing the common EoS assumption  $\Lambda_1 = \Lambda_2 m_2^6 / m_1^6$  described above and assuming that both neutron stars involved in GW170817 had the same radius  $R$ , the definition of the binary tidal deformability reduces to  $\tilde{\Lambda} \propto (R/\mathcal{M})^6$ , where  $\mathcal{M} \equiv \eta^{3/5}(m_1 + m_2)$  is the chirp mass with  $\eta \equiv m_1 m_2 / (m_1 + m_2)^2$ , in an EoS-insensitive relation that can be fit to a sample of various EoSs. The joint posterior on  $\tilde{\Lambda}$  and  $\mathcal{M}$  from GW170817 thus determines the common neutron-star radius, which was reported as  $R = 10.7^{+2.1}_{-1.6}$  km at 90% confidence.

A summary of the resulting mass-radius constraints from both NICER and GW170817 can be seen in Fig. 31. The heavier NICER pulsar (J0740+6620) has the highest posterior distribution that is centered nearly directly above the lighter NICER pulsar (J0030+0451) and the GW170817 extracted mass-radius posteriors. However,

**Fig. 31** Constraints on the mass-radius using the universal relations (EOS insensitive), the spectral EOS and an EoS sampled through Gaussian Processes. The universal-relation constraint and spectral-EoS constraint are obtained using GW170817 data. The Gaussian Process constraint is obtained by combining GW170817, PSR 1614-2230, PSR 0348+0432, and PSR 0740+6620 (mass only) data. NICER constraints from both the Amsterdam and Illinois/Maryland groups are shown for both J0030+0451 and J0740+6620. Shaded regions show the 68% and 95% confidence region



there is some probability that heavier  $M \sim 2M_{\odot}$  may bend to the right. The lighter constraints from J0030+0451 and GW170817 overlap in their posteriors as well, although GW170817 generally prefers a slightly smaller radius whereas J0030+0451's posterior prefers larger radii.

Different methods (EoS insensitive, spectral EoS and Gaussian processes) to obtain the posterior on the mass and radius with GW170817 may look different, but they all are statistically consistent with each other. In particular, we stress that one cannot use these posteriors, over-impose mass-radius curves computed with a given EoS and then determine whether the EoS is allowed or disallowed based on whether it overlaps with the contours or not. This is because each point on this plane, including those outside the 68% or the 95% confidence regions, actually has a posterior weight assigned to it (not shown in the figure). Therefore, comparison of the data to a given EoS needs to be done carefully in a Bayesian way. Overall, all posteriors are statistically consistent with each other at this time, and therefore, there is no tension between observations. Eventually, if future observations place better constraints on the mass-radius plane, one expects that a single EoS will be preferred by all data.

We emphasize that these radius constraints from GW170817 are merely approximate encapsulations of the information contained within the tidal deformability measurements reported above, which are directly constrained by the gravitational waves. When considering gravitational wave data, it is thus preferable to use the tidal deformability measurements to constrain the EoS.

#### 8.4.4 GW190425, GW200105, and GW200115

Gravitational waves also yielded a constraint on tidal deformability in the binary neutron-star merger GW190425, but because this system was heavier (such that its  $\tilde{\Lambda}$  is intrinsically smaller) and more distant (such that its signal-to-noise ratio was lower), the constraint is not competitive with GW170817. Similarly, GW200105 and GW200115 (Abbott et al. 2021) both likely contained neutron stars, but tidal effects were unmeasurably small in these systems due to the large masses of their companions and subsequent rapid orbital evolution through GW emission.

## 9 Outlook

In this work, we compiled various constraints coming from nuclear and astrophysics related to dense matter and neutron stars. We encapsulated up to date results from first principle theories and experimental observations. The usage of anticipated next-generation facilities and theoretical advancements are the most promising path for substantial improvement above current limitations. With enhanced sensitivity and expanded receiver bandwidths, several upcoming astrophysical radio observatories, including the dish Deep Synoptic Array, DSA-2000 (Hallinan et al. 2019), Canadian Hydrogen Observatory and Radio-transient Detector, CHORD (Vanderlinde et al. 2019), and the next-generation Very Large Array, ngVLA (Chatterjee 2018), will present significant opportunities in pulsar science. Also, we anticipate seeing more



multi-messenger binary neutron-star merger detections in the coming years, the INTEGRAL, Fermi, Swift, and SVOM will observe merging binary neutron stars in conjunction with GW and electromagnetic data during the fourth LIGO-Virgo-KAGRA observing run which started on May, 2023 (Patricelli et al. 2022; Fletcher et al. 2023). Future detections of neutrinos from supernova explosions in our neighborhood will provide information about larger  $Y_Q$  at large densities and intermediate temperatures, while with future LIGO and NICER runs the post-merger signal of gravitation waves from neutron-star mergers will provide information about low  $Y_Q$  at large densities and large temperatures (Lovato et al. 2022b).

Several improvements are anticipated in the next phase of BES, thanks to the performance enhancements brought on by collider and detector modifications which will help to dig deeper into the high density regime of the QCD phase diagram. Constraining the EoS is not a simple task and significant work from different communities in nuclear and astrophysics is needed to achieve this. Within the nuclear physics community, the diverse models and theories, working in a different regime of QCD phase diagram are to be updated with the modern constraints so that we can have a unified EoS for nuclei, bulk baryonic and quark matter.

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