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SCALING INEQUALITIES FOR ORIENTED PERCOLATION

by

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SCALING INEQUALITIES FOR ORIENTED PERCOLATION

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Abstract

In this paper we look at seven critical exponents associated with two dimensional oriented percolation. Scaling theory implies that these quantities satisfy four equalities. We prove five related inequalitites.

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1. Introduction

We begin by describing the model. Although it can be defined for any dimension, we will restrict ourselves to the two dimensional case. Let $\mathcal{L} = \{(m,n) \in Z^2:$ $m+n$ is even, $n \geq 0\}$. From each $z \in \mathcal{L}$ there is an oriented arc to $z + (1,1)$ and to $z + (-1,1)$. Each arc, also called a bond, is independently open with probability p and closed with probability 1-p. We think of an open bond as allowing us to go along it in the direction of orientation. With this in mind we define

$x \to y$ (y can be reached from x) if there is an open path from x to y, that is, there is a sequence $x = x_0, x_1, \ldots, x_m = y$ of points in $\mathcal{L}$ such that for each $k \leq m$ the arc from x_{k-1} to x_k is open.

$C_0 =$ (the cluster containing the origin (0,0)) $= \{x: 0 \to x\}$.
$\Omega_\infty = \{|C_0| = \infty\} =$ " percolation occurs ".

Here $|A|$ denotes the cardinality of A.

The event Ω_∞ has zero probability when p is small and positive probability when p is close to 1. As the value of p increases, the system undergoes a "phase transition" at $p_c = \inf\{p: P_p(\Omega_\infty) > 0\}$ (see [2]). Here, we study the critical exponents associated with the phase transition. To define these quantities we start with β, the exponent associated with the percolation probability. Intuitively β measures the rate at which $P_p(\Omega_\infty)$ decreases to zero as p approaches p_c. We expect that

$$P_p(\Omega_\infty) \sim C\,(p-p_c)^\beta,$$

where ~ means

$$\lim_{p \downarrow p_c} \frac{P_p(\Omega_\infty)}{C(p-p_c)^\beta} = 1.$$

But following common practice we use the weaker definition

$$P_p(\Omega_\infty) \approx (p-p_c)^\beta,$$

where $\approx$ means

$$\lim_{p \downarrow p_c} \frac{\log P_p(\Omega_\infty)}{\beta \log(p-p_c)} = 1.$$

The second critical exponent γ concerns the mean cluster size $E_p|C_0|$.

$$E_p|C_0| \approx (p_c-p)^{-\gamma} \text{ as } p \uparrow p_c.$$

To extend the definition to the supercritical case we restrict to the event that the cluster is finite and then define γ' by

$$E_p\{|C_0|, |C_0| < \infty\} \approx (p-p_c)^{-\gamma'} \text{ as } p \downarrow p_c.$$

The definitions above are analogous to the ones in the theory of ordinary (unoriented) percolation. The next quantity has no analogue in that theory. Let

$$\xi_n = \{x: \text{there is a } y \le 0 \text{ so that } (y,0) \to (x,n)\}.$$

In words, ξ_n is the state at time n starting from $\xi_0 = \{0,-2,-4,\dots\}$. Let

$$\overline{r_n} = \sup \xi_n.$$

It is known (see [2], pp. 1005-1006) that

$$\frac{\bar{r}_n}{n} \to \alpha(p) \qquad \text{almost surely} \qquad \text{as } n \to \infty$$

and that

$$p_c = \inf\{ p : \alpha(p) > 0 \}.$$

We define the critical exponent σ associated with the "edge speed" $\alpha(p)$ by

$$\alpha(p) \approx (p-p_c)^{\sigma} \qquad \text{as } p \downarrow p_c.$$

The quantities we have defined so far concern the behavior of the system as p approaches the critical probability p_c. The next two concern the behavior at the critical value p_c. Let

$$\xi_n^0 = \{ x : (0,0) \to (x,n) \}.$$

We define the critical exponent for the survival probability by

$$P_{cr}(\xi_n^0 = \varnothing) \approx n^{-1/\delta_r},$$

where the subscript cr indicates we are considering $p=p_c$. The r here is for radius (of the cluster) and is included to make our definition match the one for ordinary percolation (see [9]).

The second quantity at criticality is related to the mean cluster size as function of the time n:

$$E_{cr}|\xi_n^0| \approx n^{\eta} \qquad 0 \leq \eta \leq 1.$$

While the definition of δ_r is analogous to the one for ordinary percolation, the definition of η is different from its counterpart:

$$P_{cr}(0 \to x) \approx |x|^{2-d-\eta'} \quad \text{as} \quad |x| \to \infty$$

where $|x| = \max(|x_1|,|x_2|)$ if $x = (x_1,x_2)$. To relate the two definitions observe that

$$\sum_{x:|x|=n} P_{cr}(0 \to x) \approx n^{1-\eta'}.$$

Hence our η is like $1-\eta'$.

Last but not least we come to the correlation lengths. We use the definitions introduced and explained in the companion paper [5]. If we let $\tau^A = \inf\{ n : \xi_n^A = \varnothing \}$ and write τ^0 when $A = \{0\}$ then the parallel correlation length $L_\parallel(p)$ can be defined in the subcritical case by

$$[L_\parallel(p)]^{-1} = \lim_{n\to\infty} [-(1/n) \log P_p(\tau^0 > n)].$$

The associated critical exponent $\nu_\parallel$ is defined by

$$L_\parallel(p) \approx (p_c - p)^{-\nu_\parallel}.$$

Let r_n^0 denote the rightmost site in ξ_n^0, i.e.,

$$r_n^0 = \sup \xi_n^0 \qquad (= -\infty \text{ if } \xi_n^0 = \varnothing)$$

and let

$$R^0 = \sup_n r_n^0.$$

The perpendicular correlation length $L_\perp(p)$ for the subcritical case is defined by

$$[L_\perp(p)]^{-1} = \lim_{n\to\infty} [(-1/n) \log P_p(R^0 > n)]$$

and $\nu_\perp$ by

$$L_\perp(p) \approx (p_c - p)^{-\nu_\perp}.$$

For the supercritical case there are also two correlation lengths. First, the parallel one. $L_\parallel(p)$ is defined by

$$[L_\parallel(p)]^{-1} = \lim_{n\to\infty} [(-1/n) \log P_p(n < \tau^0 < \infty)]$$

and $\nu_\parallel'$ by

$$L_\parallel(p) \approx (p - p_c)^{-\nu_\parallel'}.$$

(The prime on $\nu_\parallel$ is to indicate that we are now looking at the limit as $p \downarrow p_c$.)

Extrapolating from the first three definitions the reader might expect the last one to be

$$[L_\perp(p)]^{-1} = \lim_{n\to\infty} [(-1/n) \log P_p(R^0 > n, \tau^0 < \infty)].$$

For the results we will prove below it is convenient to use

$$[L_\perp^\tau(p)]^{-1} = \lim_{n\to\infty} [(-1/n) \log P_p(\tau^{(-2n,0)} < \infty)]$$

instead. This is supported by (1.9) in [5] which together with Lemma 3 in [4] gives us $L_\perp(p) \le L_\perp^\tau(p) \le 2 L_\perp(p)$. The associated critical exponent $\nu_\perp'$ is defined by

$$L_\perp^\tau(p) \approx (p - p_c)^{-\nu_\perp'}.$$

Having introduced the critical exponents we turn now to the results. Scaling theory predicts that

(1.1) $\gamma = \nu_\parallel' + \nu_\perp' - 2\beta$

(1.2) $\sigma = \nu_\parallel' - \nu_\perp'$

(1.3) $\beta = \nu_\parallel' / \delta_r$

(1.4) $\gamma = (\eta + 1)\,\nu_\parallel$.

The first three equalities can be found in [7], while the last one is in [1]. Those papers use the notation of Reggeon Field Theory so to get the results above one has to change variables:

$\delta_r = 1/\delta$ $\quad \nu_\parallel = \nu$ $\quad \nu_\perp = (z/2)\,\nu$ $\quad \alpha = \nu$.

The main purpose of this work is to prove some inequalities related to (1.1)-(1.4). In Section 2 we show that

(1.5) $E_p|C_0| \le 10\, L_\parallel(p) L_\perp(p)$ when $L_\parallel(p), L_\perp(p) \ge 1$

or in terms of critical exponents,

(1.6) $\gamma \le \nu_\parallel + \nu_\perp$.

Comparing this with (1.1) shows -2β is missing from the right hand side.

In Section 3 we show that

(1.7) $L_\perp^\epsilon(p) \le \alpha(p)\, L_\parallel(p)$

which implies

(1.8) $\sigma \le \nu_\parallel' - \nu_\perp'$.

In Section 4 we will introduce yet another definition of the parallel correlation length in the supercritical case: $L_\parallel^\epsilon(p)$ = the smallest length for which the renormalized bond construction of [3] works. This definition is analogous to the definition in terms of sponge crossings for ordinary percolation (see [9]). We would like to show that this definition is (up to constants) the same as $L_\parallel(p)$ but all we can show is

(1.9) $L_\parallel^\epsilon(p) \ge 2 \log 2\, L_\parallel(p)$.

In Section 5 we use this definition to prove that if $L = L_\parallel^\epsilon(p)$

(1.10) $P_p(\Omega_\infty) \ge (1/2)\, P_p(\xi_L^0 \ne \emptyset)$.

The last inequality says that percolation is almost the same as surviving up to the correlation length. The proof of Lemma 4.1 in [11] also works in this case to show (1.10) when we take $L = 4d\, L_\parallel(p)|\log L_\parallel(p)|$. In fact the result in [11] is more general in the sense that it is true for any finite dimension d. In terms of critical exponents (1.10) says

(1.11) $\beta \le \nu_\parallel^\epsilon / \delta_r$.

An extension of the proof of (1.10) gives

(1.12) $P_p(x \in \xi^0_{2L}) \asymp [P_p(\Omega_\infty)]^2$ for $|x| \le 1.5\alpha(p) L$.

where $\asymp$ means the ratio of these quantities is bounded above and away from 0 by constants independent of p, and again we have written L for $L^{\varepsilon}_{\gamma}(p)$. If we define η' by

$$E_p|\xi^0_{2L}| \approx [L]^{-\eta'} \approx (p - p_c)^{\nu_\gamma'\eta'}$$

(trusting here that quantities at the correlation length are, up to constants, the same as at p_c) then (1.12) leads to

(1.13) $\nu_\gamma'\eta' \ge \nu_\gamma' - \sigma - 2\beta$.

The last result is one half of

(1.14) $\nu_\gamma'\eta' = \nu_\gamma' - \sigma - 2\beta$

a relationship which follows from (1.1), (1.2) and (1.4).

Finally, we have

(1.15) $\gamma \le (\eta + 1)\nu_\gamma$.

If the reader remembers $\eta = 1 - \eta'$ then he will recognize this as Fisher's inequality (see [6]). If one defines the connectivity radius as the random variable R_γ whose distribution is given by

$$P_p(R_\gamma = n) = \frac{\sum_x P_p((0,0) \to (x,n))}{E_p|C_0|}$$

and defines exponents ν_k by

$$\{ E_p(R_\parallel)^k \}^{1/k} \approx (p_c - p)^{-\nu_k}$$

then one can use ideas of [10] to show

$$\lim_{k\to\infty} \nu_k = \nu_\parallel$$

and

$$\gamma \le (\eta + 1)\, \nu_\parallel .$$

No new ideas are needed so the proof is omitted.

2. Scaling inequality for the subcritical process

In this section we will prove

(2.1) For $p < p_c$ and $L_\perp(p) \ge 1$, $E_p| C_0 | \le 10\, L_\perp(p)\, L_\parallel(p)$.

This relationship is natural if we notice that $L_\parallel(p)$ and $L_\perp(p)$ give the height and width of a typical cluster while $| C_0 |$ gives its volume. To see where the missing 2β in the associated exponent inequality (1.6) should come from look at (1.12).

<u>Proof</u>: From the definitions of the correlation lengths we have that

$$P_p(\, (0,0) \to (x,n)\,) \le \min \{ \exp(-n/L_\parallel(p)), \exp(-|x|/L_\perp(p)) \}.$$

Set $c = L_\perp(p)/L_\parallel(p)$. Let $A = \{ (x,n) : |x| \le cn \}$ where $\exp(-n/L_\parallel(p))$

$\leq \exp(-|x|/L_\perp(p))$ and $B = A^c \cap \{ (x,n) : n \geq 0 \}$ where the opposite inequality holds. Then

$$E_p|C_0| = \sum_n \sum_x P_p((0,0)\to(x,n)) = \sum_A P_p((0,0)\to(x,n)) + \sum_B P_p((0,0)\to(x,n))$$

$$\leq \sum_n (2[cn]+1)\exp(-n/L_\parallel(p)) + \sum_x [c^{-1}|x|]\exp(-|x|/L_\perp(p))$$

where [a] is the largest integer $\leq$ a. Using the trivial inequality

$$\sum_{k=1}^{\infty} k e^{-ak} \leq \int_0^{\infty} (x+1)e^{-ax}dx = a^{-2} + a^{-1}$$

we see that the expression above is

$$\leq 1 + 2c(\{L_\parallel(p)\}^2 + L_\parallel(p)) + L_\parallel(p) + 2c^{-1}(\{L_\perp(p)\}^2 + L_\perp(p))$$

$$\leq 10\, L_\parallel(p) L_\perp(p) \qquad \text{when} \quad L_\perp(p),\, L_\parallel(p) \geq 1. \qquad |||$$

(2.2) Corollary: $\gamma \leq \nu_\parallel + \nu_\perp$.

3. One scaling inequality for the edge speed

We begin with some definitions. Set

$$l_n^0 = \inf \xi_n^0 \qquad \inf \varnothing = +\infty$$

$$r_n^0 = \sup \xi_n^0 \qquad \sup \varnothing = -\infty$$

$$G_n = \{\, l_n^0 \geq -(1+\delta)\,\alpha(p)\,n,\ r_n^0 \leq (1+\delta)\,\alpha(p)\,n \,\}$$

where $\delta > 0$. Let

$$\xi_n^{(A,m)} = \{\, y : \text{for some } x \in A,\ (x,m) \to (y,n) \,\}$$

$$H_n = \{\, \tau^{[-(1+\delta)\alpha(p)n,\,(1+\delta)\alpha(p)n]} < \infty \,\}.$$

From the definitions above it should be clear that

$$P(\, n < \tau^0 < \infty \,) \geq P(\, \xi_n^0 \neq \emptyset,\ G_n,\ \xi_m^{(\xi_n^0, n)} \text{ dies out} \,)$$

$$\geq P(\, \xi_n^0 \neq \emptyset,\ G_n)\ P(\, H_n).$$

In [2] it was shown that

$$P(\, G_n \mid \xi_n^0 \neq \emptyset) \to 1 \qquad \text{as } n \to \infty,$$

so we have

$$P(\, \xi_n^0 \neq \emptyset,\ G_n) \to P(\, \Omega_\infty) > 0.$$

On the other hand

$$\frac{1}{n} \log P(\, H_n\,) \to -\alpha(p)(1+\delta)/\, L_\perp^\tau(p).$$

and since δ is arbitrary it follows that

$$-1/L_\parallel(p) \geq -\alpha(p)/\, L_\perp^\tau(p).$$

or rearranging

(3.1) $$L_\perp^\tau(p) \leq \alpha(p)\, L_\parallel(p).$$

In terms of critical exponents, we have obtained

(3.2) $$\sigma \le \nu_{\parallel}' - \nu_{\perp}' .$$

4.Renormalized Bond Construction

The first thing to do is to describe a construction due to Durrett and Griffeath [3]. We follow the version in [2]. Let G be the graph with vertices in $\mathcal{L}_0 = \{(m,n): m+n \text{ is even}, n \ge 0\}$ and with oriented bonds connecting each $(m,n) \in \mathcal{L}_0$ to $(m+1,n+1)$ and to $(m-1,n+1)$. Consider the "renormalized lattice" $\mathcal{L}_0$ to be mapped into the upper half plane $\Re \times [0,\infty)$ by $\phi(m,n) = (aLm, Ln)$, where a is a special constant and L is a large number, both to be chosen below.

To each $z \in \mathcal{L}_0$ we associate a random variable $\eta(z)$ such that $\eta(z) = 1$ if a certain "good event" happens near $\phi(z)$ in our original percolation process and $\eta(z) = 0$ otherwise. This procedure generates a 1-dependent, oriented site percolation process with $\eta(z) = 1$ meaning that the site z is open and $\eta(z) = 0$ meaning that the site z is closed. We call this new process the rescaled process.

The choices of the constants and of the "good events" are made in a such a way that

(i) the random variables $\eta(z), z \in \mathcal{L}_0$ are 1-dependent, i.e., if we let $\|(m,n)\|_{\mathcal{L}} = (|m|+|n|)/2$ and $z_1,\dots,z_m$ are points with $\|z_i - z_j\|_{\mathcal{L}} > 1$ for $i \ne j$ then $\eta(z_1),\dots,\eta(z_m)$ are independent.

(ii) if L is large then the probability that $\eta(x) = 1$ is close to 1.

(iii) if percolation occurs in the η-process starting from the origin, then the same thing happens in the original process starting from some point near the origin.

To introduce the "good event" let A be the parallelogram with vertices

$u_0 = (-.1\alpha L, 0)$ $\quad\quad$ $v_0 = (.1\alpha L, 0)$

$u_1 = ((1-.05)\alpha L, (1+.05)L)$ $\quad\quad$ $v_1 = ((1+.15)\alpha L, (1+.05)L)$.

We associate the sites in $\mathscr{L}_0$ with translations of A in the original percolation structure: If we let $v_{m,n} = ((\alpha-4\delta)m, n)$ for $(m,n) \in \mathscr{L}_0$ then we define the translations of A by

$$A_{m,n} = (v_{m,n} + (-4\delta\alpha, 0)).L + A$$

$$B_{m,n} = (v_{m,n} + (4\delta\alpha, 0)).L - A$$

where $x - A = \{ x-y : y \in A \}$. For a picture see Figure 4.1 below.

The "good event" for the site (m,n) happens if there are open paths from top to bottom lying entirely in $A_{m,n}$ and in $B_{m,n}$. If we denote the good event by SC(L) (for sponge crossing) then it is known (see [2], Section 9) that

(4.1) For $p > p_c$, $P_p(\, SC(L)\,) \to 1$ as $L \to \infty$.

Again by [2], now in Section 10,

(4.2) If $P_p(\, SC(L)\,) \geq 1 - 6^{-36}$ then the probability that the η-system percolates is greater than 1/2.

Let $\varepsilon_0 = 6^{-36}$ and define

$$L^{\varepsilon}_{s}(p) = \inf \{ n : P_p(\, SC(n)\,) \geq 1 - \varepsilon_0 \}.$$

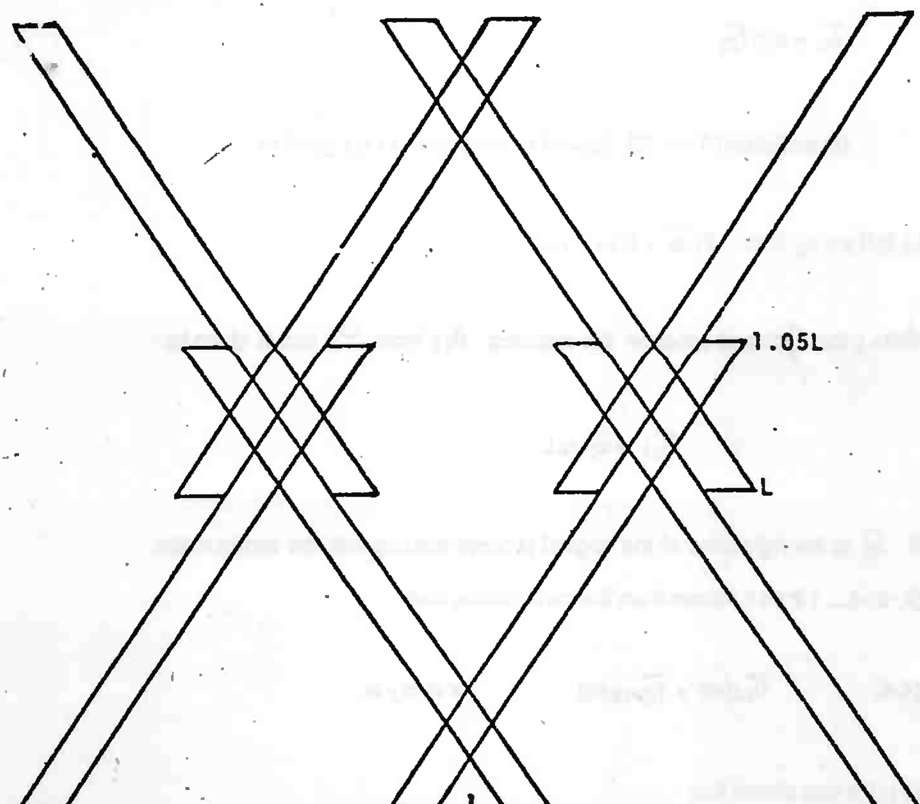

Figure 4.1.

Let

$$\xi_m = \{ x : (x,m) \in \mathscr{L}_o \text{ and there is an } y \le 0 \text{ such that } (y,0) \in \mathscr{L}_o \text{ and } (y,0) \to (x,m) \}$$

where $(y,0) \to (x,m)$ means there is an open path from (y,0) to (x,m) on the renormalized lattice. Finally, let

$$\overline{s_m} = \sup \xi_m .$$

(2) in Section 11 of [2] applied to our process $\{\eta\}$ gives us

(4.3) If $p > p_c$ then $P(\overline{s_k} \le 0) \le (1/2)^{k-1}$.

Write L for $L_\eta^\varepsilon(p)$ and consider the mapping $\overline{R_{\varepsilon,k}}$ from $\mathscr{L}_o$ into $\Re$ given by

$$\overline{R_{\varepsilon,k}} = \overline{s_k} .\alpha.L.$$

If $\overline{r_n}$ is the right edge of the original process starting with the configuration {0,-2,-4,.... } then it follows from the construction that

(4.4) $\overline{R_{\varepsilon,k}}(\omega) \le \overline{r_{(k+1)L}}(\omega)$ for every ω.

In [4] it was shown that

(4.5) For any $p \in [0,1]$, $\lim_{n\to\infty} \{ -\frac{1}{n} \log P_p(\overline{r_n} \le 0) \} = [2L_\eta(p)]^{-1}$.

With these results in hand we can get

(4.6) For $p > p_c$, $L_\gamma^\varepsilon(p) \geq 2\log 2\, L_\gamma(p)$.

Proof: By using (4.3) and (4.4) it is not difficult to see that

$$P_p(\overline{r}_{(k+1)L} \leq 0) \leq P_p(\overline{R}_{\varepsilon,k} \leq 0) \leq P_\varepsilon(\overline{s}_k \leq 0) \leq (1/2)^{k-1}.$$

so that

$$-\frac{1}{(k+1)L}\log P_p(\overline{r}_{(k+1)L} \leq 0) \geq \frac{\log 2}{L}\cdot\frac{k-1}{k+1}.$$

By letting $k \uparrow \infty$ and recalling (4.5) we get that

$$[2L_\gamma(p)]^{-1} \geq \frac{\log 2}{L_\gamma^\varepsilon(p)}. \qquad |||$$

5. Proofs of (1.7) and (1.10)

Throughout this section we will write L for $L_\gamma^\varepsilon(p)$.

(5.1) If $p > p_c$ then

$$(1/2)\, P_p(\xi_L^0 = \varnothing) \leq P_p(\Omega_\infty) \leq P_p(\xi_L^0 = \varnothing).$$

Proof: The right hand inequality is obvious. To prove the other one we note two things: First, that the event W = { the rescaled process percolates } has probability ≥ (1/2) by (4.2). Second, in order to have a path from zero to infinity on the event W it suffices that the process starting with configuration {0} survives until time L, since if ξ_t^0 survives until time L then it crosses out at least one of the paths involved in the event $A_{00} \cap B_{00}$ (see Figure 5.1). Using Harris-FKG inequality (see [8]) now gives

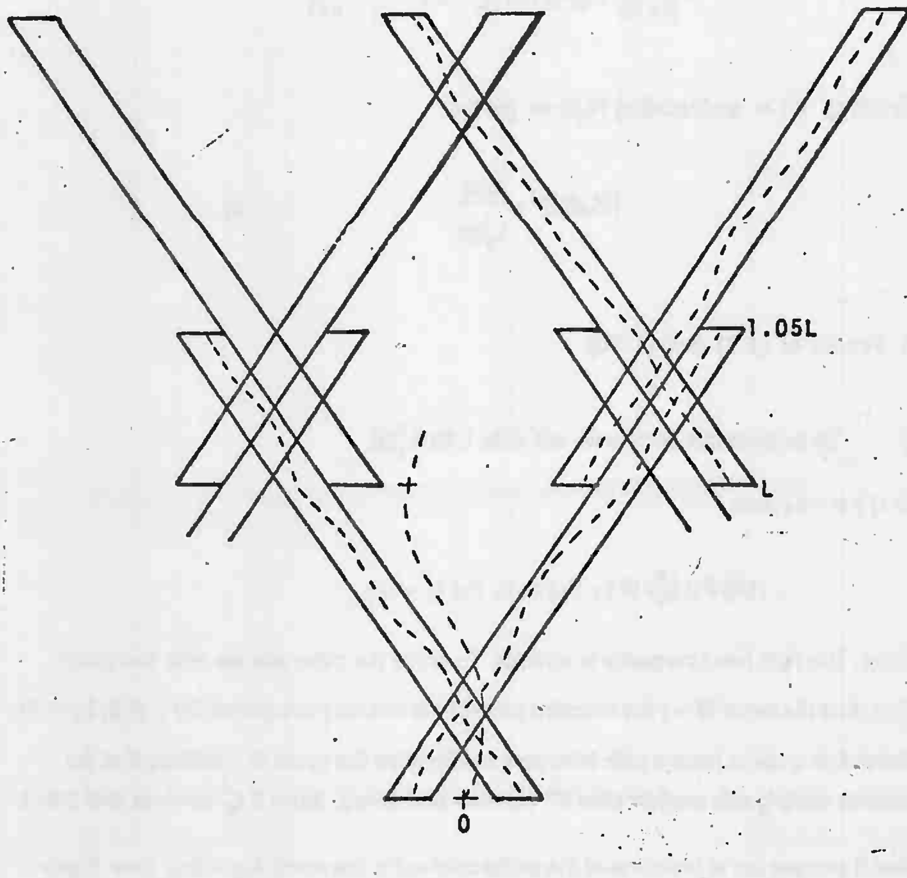

Figure 5.1.

$$P_p(\Omega_\infty) \ge P_p(\{\xi_L^0 \ne \emptyset\} \cap W) \ge P_p(\xi_L^0 \ne \emptyset)(1/2)$$

and the proof is complete. |||

(5.2) Corollary: $\beta \le \nu_\parallel' / \delta_r$.

Proof: We have that

$$P_p(\xi_L^0 \ne \emptyset) \ge P_{cr}(\xi_L^0 \ne \emptyset) \approx (L)^{-1/\delta_r} \approx (p-p_c)^{\nu_\parallel'/\delta_r}$$

and from the previous proposition it follows that

$$P_p(\xi_L^0 \ne \emptyset) \asymp P_p(\Omega_\infty) \approx (p-p_c)^\beta. \qquad |||$$

(5.3) For $p > p_c$ and $|x| \le 1.5\alpha L$ there are constants c and $C \in (0,\infty)$ so that

$$c(P_p(\Omega_\infty))^2 \le P_p(x \in \xi_{2L}^0) \le C(P_p(\Omega_\infty))^2.$$

Proof: Note that if for convenience we choose t to be even then

$$P_p(x \in \xi_t^0) = P_p(\xi_{t/2}^0 \ne \emptyset,\ \xi_t^{(\xi_{t/2}^0, t/2)}(x) = 1)$$

$$= P_p(\xi_{t/2}^0 \ne \emptyset)\, P_p(\xi_t^{(t/2, \xi_{t/2}^0)}(x) = 1 \mid \xi_{t/2}^0 \ne \emptyset) \le P_p(\xi_{t/2}^0 \ne \emptyset)\, P_p(\xi_{t/2}^Z(x) = 1).$$

Since the probability of a path from Zx{0} to (x,t/2) is the same as that of a path from (x,0) to Zx{t} (see [2], Section 8),

$$P_p(\xi^{Z}_{t/2}(x) = 1) = P_p(\xi^{\{x\}}_{t/2} \neq \varnothing) = P_p(\xi^{0}_{t/2} \neq \varnothing).$$

Combining the last two equations gives

$$P_p(x \in \xi^0_t) \leq \{P_p(\xi^0_{t/2} \neq \varnothing)\}^2.$$

By choosing t = 2L and recalling (5.1) one can get

$$P_p(x \in \xi^0_{2L}) \leq \{P_p(\Omega_\infty)/2\}^2.$$

This proves the right hand inequality. For the other half let

F = { The sites (0,0), (1,1), (-1,1) are open in the rescaled process }

$G = \{ \xi^0_L \neq \varnothing \}$

$H = \{ x \in \xi^{[Z,L]}_{2L} \}$.

We claim that if $|x| \leq 1.5\,\alpha\,L$ then on $F \cap G \cap H$, $x \in \xi^0_{2L}$. To see this look at Figure 5.2 and notice that

(i) when F occurs there are paths inside each one of the six parallelograms;

(ii) when G occurs the origin is connected to at least one of the open paths in A_{00} and B_{00};

(iii) when H occurs one can get from one of parallelograms to the point (x,2L).

Combining the observations above

$$P_p(x \in \xi^0_{2L}) \geq P_p(F \cap G \cap H) \geq P_p(F)P_p(G)P_p(H)$$

by the Harris-FKG inequality since all the three events are increasing. Since each rescaled site is closed with probability ε_0

$$P_p(F) \geq 1-3\varepsilon_0.$$

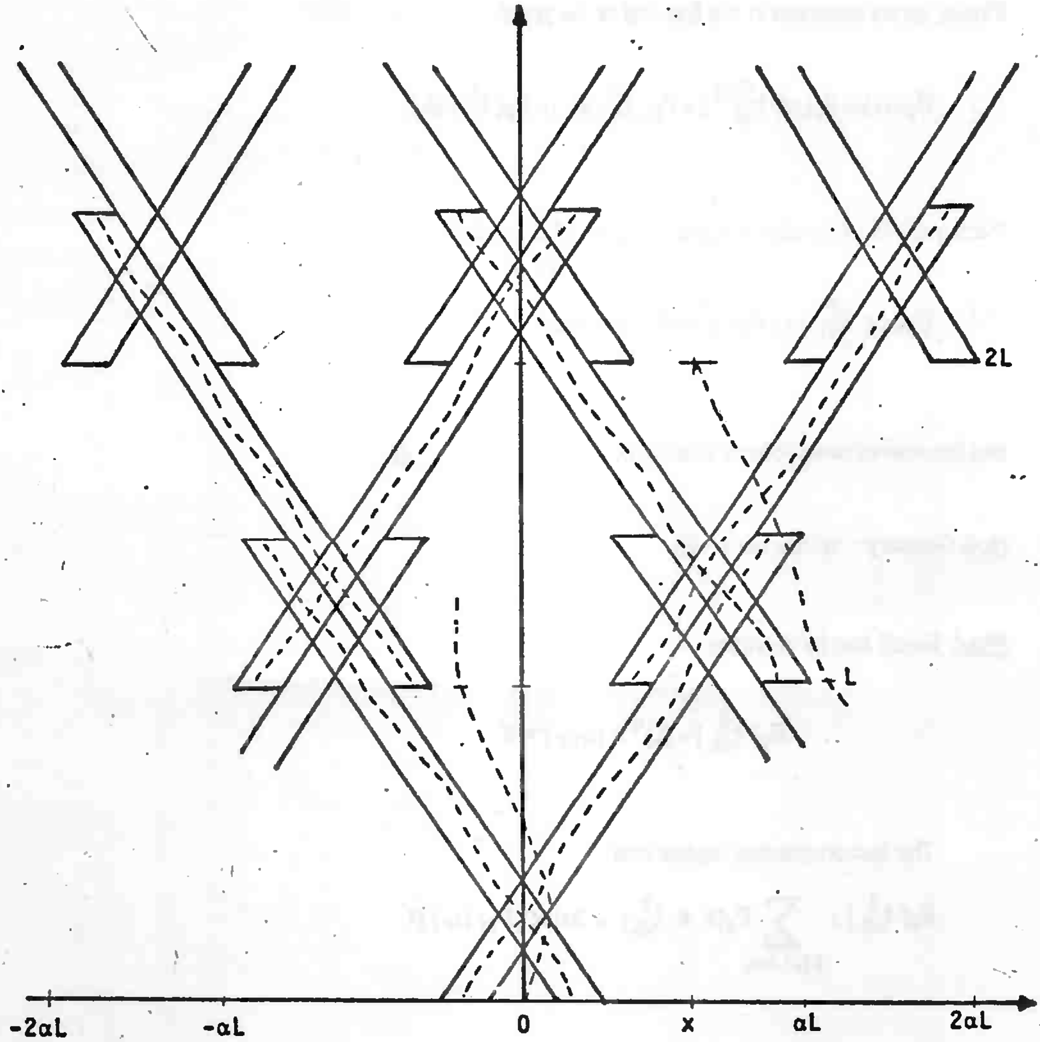

Figure 5.2.

By definition

$$P_p(G) = P_p(\xi_L^0 \neq \varnothing).$$

Finally, as we observed in the first half of the proof

$$P_p(H) = P_p(x \in \xi_{2L}^{(Z,L)}) = P_p(\xi_L^x \neq \varnothing) = P_p(\xi_L^0 \neq \varnothing).$$

Putting the last four observations together it follows that

$$P_p(x \in \xi_{2L}^0) \geq \{P_p(\xi_L^0 \neq \varnothing)\}^2(1-3\varepsilon_0).$$

and the desired result follows from (5.1). |||

(5.4) Corollary: $v_\parallel' \eta' \geq v_\parallel' - \sigma - 2\beta$.

Proof: Recall that by definition

$$E_p|\xi_{2L}^0| \approx (L)^{\eta'} \approx (p-p_c)^{-v_\parallel'\eta'}.$$

The last proposition implies that

$$E_p|\xi_{2L}^0| \geq \sum_{|x| \leq 1.5\alpha L} P_p(x \in \xi_{2L}^0) \geq 3\alpha L.c\{P_p(\Omega_\infty)\}^2$$

which implies the result stated in the corollary. |||

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