RT-MAT 2003-14

THE ODD PART OF A N-KOSZUL ALGEBRA

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Julho 2003

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ABSTRACT. The so called n-Koszul algebras have been studied in ,[2, 3]. They are natural generalizations of Koszul algebras, however, Koszul duality is not well understood in this case, some partial results in this direction were obtained in [3] , where the following result is proved:

Given a n-Koszul algebra with Yoneda algebra $E(\Lambda)$, the even part, $e(E(\Lambda))$ of $E(\Lambda)$ is a Koszul algebra, and given a n-Koszul module M, the even part $e(E(M))$ of the module $E(M) = \underset{k\geq 0}{\oplus} Ext^k_\Lambda(M,\Lambda_0)$ is a Koszul module over $e(E(M))$.

The following question was raised in [3]. Is the odd part of $E(M)$ also a Koszul module over $e(E(M))$? The aim of this note is to prove that this is indeed the case if we assume further that the orthogonal algebra $\Lambda^!$ is also n-Koszul.

1. Koszul algebras.

It was shown in [3], that the even part of the Ext-algebra of a n-Koszul algebra is Koszul. Since a Koszul algebra Λ is 2-Koszul, is follows that the even part of the Yoneda algebra Γ is Koszul, but Λ is isomorphic to the Yoneda algebra $E(\Gamma)$ of the Koszul algebra Γ ,[1, 5, 6] it follows, that the even part $e(\Lambda) = \underset{j\geq o}{\oplus} \Lambda_{2j}$ is Koszul. (This was generalized in [4] showing that if Λ is a Koszul algebra then $\sum_{k\in N} \Lambda_{(kn)}$ is a Koszul algebra, (after regrading).)
Using Koszul duality, we also obtain that the even part $e(M) = \underset{j\geq 0}{\oplus} M_{2j}$ of a Koszul module M is a Koszul $e(\Lambda)-$module.

We know by [1, 5] , that JM, is a Koszul module, where $JM = \underset{j\geq 1}{\oplus} M_j$ is the graded radical of M. Hence $e(JM[1]) = \underset{m\geq 0}{\oplus} e(JM[1])_{2m+1} = \underset{m\geq 1}{\oplus} M_{2m+1}$. It follows that the odd part, $o(E(M)) = \underset{m\geq 1}{\oplus} M_{2m+1}$ of M is also a Koszul module over $e(\Lambda)$.

The even part of a graded algebra can be interpreted as follows:

Let $\Lambda = \underset{j\geq 0}{\oplus} \Lambda_j$ be a graded quiver $K-$algebra, with K a field of characteristic different from 2. We define an automorphism: $\sigma : \Lambda \to \Lambda$

Key words and phrases. Koszul, n-Koszul.
The authors thank CNPq and CONACYT for funding the reserach project.

as follows: let $x \in \Lambda_j$ be a homogeneous element, define $\sigma(x) = (-1)^j x$ and extend it to $\sigma(\sum_{j\geq 0} x_j) = \sum(-1)^j x_j$.

It is easy to check, that σ is a graded $K-$algebra automorphism with $\sigma^2 = 1$ and the fixed ring Λ^σ is isomorphic to $e(\Lambda)$. By the above remarks, Λ^σ is Koszul.

Example: Let $\Lambda = K[x,y]$ be the polynomial algebra in two variables, with K a field of characteristic different from 2. The even part, $e(\Lambda) = K[x^2,xy,y^2]$ is isomorphic to $K[x,y,z]/(xy - z^2)$. Hence; $K[x,y,z]/(xy-z^2)$ is a Koszul algebra. The algebra: $K[x^2,xy,y^2]$ is the ring of invariants $K[x,y]^G$, where G is the subgroup $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\} \cong Z_2$ of the special linear group $Sl(2,K)$.
If follows by Watanabe´s theorem [8, 9], $K[x^2,xy,y^2]$ is a Gorenstein ring.

In the general case, for a field of characteristic different from 2, the group $G = \left\{ \begin{bmatrix} 1 & 0 & . & 0 \\ . & 1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 & . & 0 \\ 0 & -1 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & -1 \end{bmatrix} \right\} \cong Z_2$ acts on the polynomial ring
$\Lambda = K[x_1, x_2 ... x_n]$ and the fixed ring $K[x_1,x_2...x_n]^G$ is isomorphic to the even part of Λ. It follows that $K[x_1,x_2...x_n]^G$ is a Koszul algebra, which is Cohen-Macaulay, by Hochester-Eagen´s theorem [7] . Moreover, when n is even, it is Gorenstein.

2. N-KOSZUL ALGEBRAS.

We will recall some definitions and basic results about n-Koszul algebras which appear in [2, 3].

We will consider positively graded $K-$algebras, $\Lambda = \bigoplus_{j\geq 0} \Lambda_j$ such that the following three conditions hold:

i) $\Lambda_0 = K \times K ... K$

ii) $\dim_K \Lambda_i < \infty$ for all i.

iii) $\Lambda_i \Lambda j = \Lambda_{i+j}$ for all pairs i,j.

We will call algebras satisfying these three conditions, graded quiver algebras, because for such algebras there exists a finite quiver Q and a graded ideal I of KQ, in the grading given by path length, such that $I \subset \sum_{j\geq 2}(KQ)_j$ and $KQ/I \cong \Lambda$.

Definition 1. *Given a graded module M over a graded algebra Λ then M is called $n-$Koszul if M has a minimal graded projective resolution: $\to P_k \to P_{k-1} \to ...P_1 \to P_0 \to M \to 0$ such that there exists a positive integer $d \geq 2$ where each module P_k is finitely generated in degree $\delta(k)$ with*

$$\delta(k) = \begin{cases} \frac{k}{2}n & if \quad k \quad is \quad even \\ \frac{k-1}{2}n+1 & if \quad k \quad is \quad odd \end{cases}$$

If $n = 2$, then a $2-$Koszul M is just a Koszul module. If Λ_0 is n-Koszul, then we say that Λ is a $n-$ Koszul algebra.

We have the following characterization of $n-$ Koszul algebras:

Theorem 1. [3] *Let $\Lambda = KQ/I$ be an indecomposable graded quiver algebra. Then the following conditions two are equivalent:*

i) The algebra Λ is a $n-$ Koszul algebra.

ii) a) For some integer $n \geq 2$ the ideal I can be generated by elements of $(KQ)_n$

b) The Yoneda algebra $E(\Lambda)$ of Λ is generated in degrees: $0, 1, 2$, in the ext-degree grading.

The inspiration of this note is the following theorem:

Theorem 2. [3] *Let Λ be a n-Koszul $K-$algebra and M a n-Koszul module. If $E(\Lambda) = \underset{k\geq 0}{\oplus} Ext^k_\Lambda(\Lambda_0, \Lambda_0)$ is the Yoneda algebra of Λ, then the even part $eE(\Lambda) = \underset{k\geq 0}{\oplus} E(\Lambda)_{2k}$ is, after regrading, a Koszul algebra and the even part eE(M) $= \underset{k\geq 0}{\oplus} E(M)_{2k}$ of $E(M) = \underset{k\geq 0}{\oplus} Ext^k_\Lambda(M, \Lambda_0)$ is after regrading a Koszul $eE(\Lambda)-$ module.*

In the classical situation [1, 5], a Koszul algebra $\Lambda = KQ/I$ is quadratic, that is: the ideal I is generated by I_2. We define a bilinear form: $< -, - >: V \times V^{op} \to K$, where $V = (KQ)_2$, by: $< \alpha\beta, \beta'\alpha' > =$

$$\begin{cases} 0 & if \quad \alpha \neq \alpha' \quad or \quad \beta \neq \beta' \\ 1 & if \quad \alpha = \alpha' \quad and \quad \beta = \beta' \end{cases}$$

and let L_2 be the orthogonal of I_2 under the bilinear form. It was proved [1, 5, 6] , that the Yoneda algebra $E(\Lambda)$ is also Koszul and isomorphic to the orthogonal algebra $\Lambda^! = KQ^{op}/ < L_2 >$.

For a general n-Koszul algebra Λ we do not have such a nice relation between the Yoneda algebra $E(\Lambda)$ and the orthogonal algebra $\Lambda^! = KQ^{op}/ < L_n >$ where L_n is the orthogonal of I_n under the corresponding bilinear form in $V = (KQ)_n$. However, $E(\Lambda)$ can still be constructed from $\Lambda^!$. This is the content of the following theorem:

Theorem 3. [3] *Let $\Lambda = KQ/I$ be an indecomposable n-Koszul algebra and $\Lambda^! = KQ^{op}/ < L_n >$, where L_n is the orthogonal of I_n under the corresponding bilinear form in $V = (KQ)_n$,. We will call this algebra the orthogonal algebra. Define a graded algebra $B = \underset{j\geq 0}{\oplus} B_j$ as follows: $B_k = \Lambda^!_{\delta(k)}$ as $K-$vector spaces and define the product $B_kB_m = 0$ if both k and m are odd and the product in $\Lambda^!$ otherwise. Then the Yoneda algebra $E(\Lambda)$ is isomorphic to B as graded algebras.*

For a general n- Koszul algebra is not true that $\Lambda^!$ is again n-Koszul.

Example 1. *Using the characterization of monomial n-Koszul algebras given in [2, 3] it is easy to see that the $K-$ algebra $\Lambda = KQ/I$ with quiver*
$Q: \xrightarrow{\alpha_1}\xrightarrow{\alpha_2}\xrightarrow{\alpha_3}\xrightarrow{\alpha_4}\xrightarrow{\alpha_5}$ *and ideal* $I = <\alpha_3\alpha_2\alpha_1, \alpha_4\alpha_3\alpha_2>$ *is* $3-$*Koszul with orthogonal algebra* $\Lambda^! = KQ/L$ *with quiver*
$Q: \xrightarrow{\alpha_1}\xrightarrow{\alpha_2}\xrightarrow{\alpha_3}\xrightarrow{\alpha_4}\xrightarrow{\alpha_5}$ *and ideal* $L = <\alpha_5\alpha_4\alpha_3>$, *again 3-Koszul.*

Example 2. *Consider now the $K-$algebra $\Lambda = KQ/I$ with quiver*
$Q: \xrightarrow{\alpha_1}\xrightarrow{\alpha_2}\xrightarrow{\alpha_3}\xrightarrow{\alpha_4}\xrightarrow{\alpha_5}$ *and ideal* $I = <\alpha_4\alpha_3\alpha_2>$ *. It is* $3-$ *Koszul with orthogonal algebra:* $\Lambda^! = KQ/L$ *with quiver*
$Q: \xrightarrow{\alpha_1}\xrightarrow{\alpha_2}\xrightarrow{\alpha_3}\xrightarrow{\alpha_4}\xrightarrow{\alpha_5}$ *and ideal* $L = <\alpha_3\alpha_2\alpha_1, \alpha_5\alpha_4\alpha_3>$, *which is not 3-Koszul.*

3. The odd part of a n-Koszul algebra.

It was conjectured in [3] , that the odd part $oE(\Lambda) = \underset{k\geq o}{\oplus} E(\Lambda)_{2k+1}$ of the Yoneda algebra $E(\Lambda)$ of a n-Koszul algebra Λ is a Koszul module over the even part $eE(\Lambda) = \underset{k\geq 0}{\oplus} E(\Lambda)_{2k}$. This statement is still a conjecture. The purpose of this note is to prove this conjecture for a particular case, our result is the following:

Theorem 4. *Let $\Lambda = KQ/I$ be an indecomposable n-Koszul algebra and*
$\Lambda^! = KQ^{op}/ < L_n >$, where L_n is the orthogonal of I_n under the corresponding bilinear form in $V = (KQ)_n$,be the orthogonal algebra and assume $\Lambda^!$ is also n-Koszul. Then the odd part $oE(\Lambda) = \underset{k\geq o}{\oplus} E(\Lambda)_{2k+1}$ of the Yoneda algebra $E(\Lambda)$ of Λ is a Koszul module over the even part $eE(\Lambda) = \underset{k\geq 0}{\oplus} E(\Lambda)_{2k}$.

Proof. By hypothesis, there exits a minimal graded projective resolution of the right $\Lambda^!-$module $\Lambda^!_0$ of the form:
$\rightarrow (V_k)^*\otimes\Lambda^![-\delta(k)] \rightarrow (V_{k-1})^*\otimes\Lambda^![-\delta(k-1)] \rightarrow ...(V_1)^*\otimes\Lambda^![-\delta(1)] \rightarrow (V_0)^* \otimes \Lambda^![\delta(0)] \rightarrow 0$

where each V_k is a semisimple left Λ_0 module concentrated in degree zero and $(V_k)^*$ is the dual with respect to Λ_0.

Hence; $Ext^k_{\Lambda^!}(\Lambda^!_0, \Lambda^!_0) = Hom_{\Lambda^!}((V_k)^* \otimes \Lambda^! [-k], \Lambda^!_0) \cong Hom_{\Lambda^!}((V_k)^*, \Lambda^!_0) \cong V_k$.

By theorem 4 , above $Ext^k_{\Lambda^!}(\Lambda^!_0, \Lambda^!_0) \cong \Lambda_{\delta(k)}$.

Therefore: the minimal projective resolution of $\Lambda^!_0$ has the following form:

$\to (\Lambda_{\delta(k)})^* \otimes \Lambda^! [-\delta(k)] \to (\Lambda_{\delta(k-1)})^* \otimes \Lambda^! [-\delta(k-1)] \to ...(\Lambda_{\delta(1)})^* \otimes \Lambda^! [-\delta(1)] \to (\Lambda_{\delta(0)})^* \otimes \Lambda^! [\delta(0)] \to 0$

which, omitting the arrows, can be displayed as a matrix:

$$\begin{array}{cccccc}
 & & & 0 & (\Lambda_0)^* \otimes \Lambda^!_0 & \Lambda^!_0 \\
 & & 0 & (\Lambda_1)^* \otimes \Lambda^!_0 & \Lambda^!_1 & 0 \\
 & & 0 & (\Lambda_1)^* \otimes \Lambda^!_1 & \Lambda^!_2 & 0 \\
 & & & \cdots & \cdots & \\
 & & 0 & (\Lambda_1)^* \otimes \Lambda^!_{n-2} & \Lambda^!_{n-1} & 0 \\
 & 0 & (\Lambda_n)^* \otimes \Lambda^!_0 & (\Lambda_1)^* \otimes \Lambda^!_{n-1} & \Lambda^!_n & 0 \\
0 & (\Lambda_{n+1})^* \otimes \Lambda^!_0 & (\Lambda_n)^* \otimes \Lambda^!_1 & (\Lambda_1)^* \otimes \Lambda^!_n & \Lambda^!_{n+1} & 0 \\
 & \cdots & \cdots & \cdots & \cdots & \\
(\Lambda_{2n})^* \otimes \Lambda^!_0 & (\Lambda_{n+1})^* \otimes \Lambda^!_1 & (\Lambda_n)^* \otimes \Lambda^!_n & (\Lambda_1)^* \otimes \Lambda^!_{2n-1} & \Lambda^!_{2n} & 0
\end{array}$$

If $E(\Lambda)$ denotes the Yoneda algebra of Λ, then the odd part has the following form: $oE(\Lambda) = \bigoplus_{n\geq o} Ext^{2k+1}_\Lambda(\Lambda_0, \Lambda_0) = \bigoplus_{k\geq o} \Lambda^!_{kn+1}$ and the even part $eE(\Lambda) = \bigoplus_{n\geq o} Ext^{2k}_\Lambda(\Lambda_0, \Lambda_0) = \bigoplus_{n\geq o} \Lambda^!_{kn}$.

It follows from the above matrix, that there exists an exact sequence of graded $eE(\Lambda)-$ modules:

$\to (\Lambda_{3n})^* \otimes oE(\Lambda) [-3] \to (\Lambda_{2n+1})^* \otimes eE(\Lambda) [-3] \to (\Lambda_{2n})^* \otimes oE(\Lambda) [-2] \to (\Lambda_{n+1})^* \otimes eE(\Lambda) [-1] \to (\Lambda^*_n \otimes oE(\Lambda) [-1] \to (\Lambda_1)^* \otimes eE(\Lambda) \to oE(\Lambda) \to 0$

Consider the graded modules $H_k = Ker f_{2(k-1)}$ and $K_k = Ker\ f_{2k-1}$ for $k \geq 1$. Here f_i denotes the map with domain $(\Lambda_i)^* \otimes ...$ in the sequence above.

From the exactness of the above sequence, it follows that both H_k and K_k are generated in degree k for all $k \geq 1$.

We have exact sequences:

$0 \to K_k \to (\Lambda_{\delta(2k)})^* \otimes oE(\Lambda) [-k] \to H_k \to 0$ and

$0 \to H_{k+1} \to (\Lambda_{\delta(2k+1)})^* \otimes eE(\Lambda) [-k] \to K_k \to 0$, in particular, $H_{k+1} = \Omega(K_k)$.

We claim that for any k the module $\Omega(H_k)$ is generated in degree $k+1$.

We have a commutative exact diagram:

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 \to & & \mathrm{H}_{k+1} & \to & \Lambda^*_{\delta(2k)} \otimes \mathrm{H}_{(k+1)} & \to & \Omega\mathrm{H}_k & \to 0 & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 \to & & \Lambda^*_{\delta(2k+1)} \otimes \mathrm{eE}(\Lambda)_k & \to & \Lambda^*_{\delta(2k)} \otimes (\Lambda_1)^* \otimes \mathrm{eE}(\Lambda)_k & \to & \mathrm{S}^*_k \otimes \mathrm{eE}(\Lambda)_k & \to 0 & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 \to & & \mathrm{K}_k & \to & \Lambda^*_{\delta(2k)} \otimes \mathrm{oE}(\Lambda)_k & \to & \mathrm{H}_k & \to 0 & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

with S_k a semisimple Λ_0−module generated in degree zero. It follows that $\Omega(H_k)$ is generated in degree $k+1$.

We have an exact sequence of graded modules generated in the same degree:

$0 \to H_{k+1} \to (\Lambda_{\delta(2k)})^* \otimes H_1\,[-(k+1)] \to \Omega(H_k) \to 0$

Hence; we obtain an exact sequence of modules generated in the same degree:

$0 \to \Omega(H_{k+1}) \to \Omega((\Lambda_{\delta(2k)})^* \otimes H_1\,[-(k+1)]) \to \Omega^2(H_k) \to 0.$

It follows by induction, $\Omega^m(H_k)$ is generated in degree $m+k$ for all m and all k. Therefore: all graded modules H_k and K_k are Koszul up to shifting, in particular, $H_1 = \Omega(oE(\Lambda))$ is Koszul. It follows $oE(\Lambda)$ is Koszul. □

Our main theorem can be generalized as follows:

Theorem 5. *Let Λ be a $n-$Koszul algebra such that the orthogonal algebra $\Lambda^!$ is also $n-$Koszul. Then for any n-Koszul module M the graded module $oE(M) = \bigoplus_{k\geq 0} Ext^{2k+1}_\Lambda(M,\Lambda_0)$ is a Koszul module over $eE(\Lambda)$.*

Proof. We know from [3] , we have an exact sequence:

$0 \to Ext^{2(k-1)+1}_\Lambda(\Omega(JM),\Lambda_0) \to Ext^{2k+1}_\Lambda(M_0,\Lambda_0) \to Ext^{2k+1}_\Lambda(M,\Lambda_0) \to$ 0, with J the graded radical of Λ.

The sequence can be written as:

$0 \to E(\Omega(JM))_{2(k-1)+1} \to (M_0)^* \otimes E(\Lambda)_{2k+1} \to E(M)_{2k+1} \to 0.$

Adding the sequences, we obtain an exact sequence:

$0 \to oE(\Omega(JM))\,[-1] \to (M_0)^* \otimes oE(\Lambda) \to oE(M) \to 0.$

Hence; we obtain a commutative exact diagram:

$$
\begin{array}{ccccccccc}
 & & 0 & & 0 & & & & \\
 & & \downarrow & & \downarrow & & & & \\
0 \to & & (M_0)^* \otimes \Omega(oE(\Lambda)) & \to & (M_0)^* \otimes \Omega(oE(\Lambda)) & \to & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 \to & & \Omega(oE(M)) & \to & (M_0)^* \otimes \Lambda_1^* \otimes eE(\Lambda) & \to & oE(M) & \to 0 & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 \to & & oE(\Omega(JM))\,[-1] & \to & (M_0)^* \otimes oE(\Lambda) & \to & oE(M) & \to 0 & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}
$$

The modules $(M_0)^* \otimes \Omega(oE(\Lambda))$ and $oE(\Omega(JM))\,[-1]$ are generated in degree one. It follows, $\Omega(oE(M))$ is generated in degree one.

We know by [3] , $\Omega(JM)$ is n- Koszul, hence; considering this module instead of M it follows by induction, $oE(\Omega(JM))$ is Koszul. Using the fact $oE(\Lambda)$ is Koszul and the sequence:

$0 \to (M_0)^* \otimes \Omega(oE(\Lambda)) \to \Omega(oE(M)) \to oE(\Omega(JM))\,[-1] \to 0$ is exact, it follows $oE(M)$ is a Koszul $eE(\Lambda)-$ module. □

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