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INTRODUCTION

Let G be a finite group and let $\mathbb{Z}G$ denote the group ring of G over the ring $\mathbb{Z}$ of rational integers. As usual, we shall denote by $\varepsilon: \mathbb{Z}G \to G$ the *augmentation function* i.e. the map given by $\varepsilon(\Sigma\alpha_g g) = \Sigma\alpha_g$.

If we denote by $U(\mathbb{Z}G)$ the group of units of $\mathbb{Z}G$ then the set $V(\mathbb{Z}G) = \{\alpha \in U(\mathbb{Z}G) \mid \varepsilon(\alpha) = 1\}$ is called the group of *normalized units* of $\mathbb{Z}G$. The elements of the form $\pm g$, with $g \in G$ are the *trivial units* of $\mathbb{Z}G$.

G. Higman, in a classical paper on the units of group rings [4] has shown that if G is abelian then all units of finite order in $\mathbb{Z}G$ are trivial. When G is not abelian, an obvious way to exibit new torsion units in $\mathbb{Z}G$ is to compute conjugates of the form $\gamma^{-1}g\gamma$, with $g \in G$ and $\gamma \in V(\mathbb{Z}G)$. One can also allow γ to belong to $\mathbb{Q}G$ provided that $\gamma^{-1}g\gamma \in \mathbb{Z}G$. H. J. Zassenhaus has conjectured that all torsion units in $\mathbb{Z}G$ can be constructed in this way. More precisely we have:

Zassenhaus Conjecture. Let $r \in \mathbb{Z}G$ be a normalized unit of finite order. Then, there exists an invertible element $\alpha \in \mathbb{Q}G$ and an element $g \in G$ such that $\alpha^{-1}r\alpha = g$.

It has been shown that the conjecture holds for some families of groups (see, for example [5]).

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Non associative generalizations of group rings have been considered in recent literature. Let L be a finite loop; then, the loop ring RL of L over an associative ring R can be defined in precisely the same way a group ring is. Since alternative algebras are close to associative algebras, it is only natural to consider those loops whose loop rings are alternative. In this paper, we shall show that the corresponding analogue to the Zassenhaus Conjecture holds for this type of ring.

1. Some Basic Facts

An RA *loop* is a loop, which is nöt a group, whose loop rings over rings of characteristic other than 2 are alternative. We list a few facts about RA loops which can be found in [2] and [3].

1.1. Proposition - Let L be an RA loop. Then:

(i) The nucleus and the center of L coincide.

(ii) The commutator subloop L' is generated by a central element e of order 2.

(iii) There exists a non abelian group $G \subset L$ and an element $u \in L$ such that $L = G \cup Gu$. The center of G coincides witn the center of L and shall be denoted by Z. For every element $g \in L$ we have that $g^2 \in Z$.

(iv) The map $*: L \to L$ given by

$$g^* = \begin{cases} g \text{ if } g \in Z \\ eg \text{ if } g \notin Z \end{cases}$$

is an involution of L which extends linearly to RL

(v) Every element $r \in RL$ can be written uniquely in the form $r = x+yu$ with $x,y \in RG$: If we set $g_o = u^2$ the involution and multiplication in RL are given respectively by:

$$(x + yu)^* = x^* + eyu$$
$$(x+yu)(z+wu) = (xz+g_o w^* y)+(wx+yz^*)u.$$

It is easy to see that the center of RL is:

$$Z(RL) = \{x+yu \mid x,y \in Z(RG),\ y = ey\}$$

so, it follows that $r = x+yu \in Z(RL)$ if and only if $r = r^*$ and hence, for every element $r \in RL$ we have that both $r+r^*$ and rr^* are central.

Now, we turn our attention to augmentation ideals. Let N be a normal subloop of an RA loop L. Then, just as is done in the case of group rings, the natural map $L \to L/N$ can be extended to a ring homomorphism $w: \mathbb{Z}L \to \mathbb{Z}L/N$ and it can be easily shown that ker(w) is spanned over $\mathbb{Z}L$ by the elements of the set $\{n-1 \mid n \in N\}$.

In the case where L is a group G, we denote $\ker(w) = \Delta(G:N)$ and indicate simply by $\Delta(G)$ the ideal $\Delta(G:G)$ which is precisely the kernel of the augmentation function and

If $N \subset Z(L)$ then, clearly, we have that $N^* = N$. If N contains a non-central element n then, there exists an element $g \in L$ such that $g^{-1}ng = en \in N$ and hence $e \in N$. It follows that $N^* \subset N$ also in this case. .

Now, write $L = G \cup Gu$ where G is as in proposition 1.1 and assume that $N \subset G$. Let $x+yu$ be any element in $\mathbb{Z}L$, with $x,y \in \mathbb{Z}G$ and set $n \in N$. We have that:

$$(x+yu)(n-1) = x(n-1) + y(n-1)^*u \in \Delta(G:N) + \Delta(G:N)u .$$

These remarks show that the following holds.

1.2. <u>Proposition</u>: With the notations above, we have that:

(i) An element $x+yu \in \mathbb{Z}L$ belongs to $\ker(w)$ if and only if $x,y \in \Delta(G:N)$

(ii) $\Delta(G:N)^* \subset \Delta(G:N)$

2. Torsion Units

In this section, we note that a well-known property of torsion units in group rings also holds in the case of an alternative loop ring and then derive consequences peculiar to the non-associative situation. Throughout the rest of this paper, L will always denote a finite RA loop.

2.1. <u>Proposition</u>. Let $r = \sum_{g \in L} \alpha(g)g$ be a unit of finite order in the integral loop ring of L. If $\alpha(1) \neq 0$ then $r = \alpha(1) = \pm 1$.

<u>Proof.</u> - For a given element $r \in \mathbb{Z}L$ we can define a linear map $R_r: \mathbb{Z}L \to \mathbb{Z}L$ by $R_r(x) = xr$, $\forall\, x \in \mathbb{Z}L$. For each fixed element $x \in \mathbb{Z}L$ the subring generated by r and x is associative so, we see that $R_r^n = R_{r^n}$.

By considering a matrix of R_r we can apply step by step the well-known argument which is used in the case of group rings to obtain the desired conclusion. See [10, Theorem II.1.1].

2.2. <u>Corollary</u>: If $r = \sum_{g \in L} \alpha(g)g$ is a torsion unit in $\mathbb{Z}L$ and $\alpha(g) \neq 0$ for some $g \in Z(L)$ then $r = \pm g$.

2.3. <u>Corollary</u>. Let r be a torsion unit in $\mathbb{Z}L$. Then, $r^2 \in Z$.

<u>Proof:</u> Denote $S = \text{supp}(r)$ so that $r = \sum_{g \in S} \alpha(g)g$ with $\alpha(g) \neq 0$ for all $g \in S$. Then we can write

$$r^2 = \sum_{g \in S} \alpha^2(g)g^2 + \beta + \gamma$$

with

$\beta = 2\Sigma\alpha(g)\beta(h)gh$ where the sum runs over all pairs (g,h) such that $gh = hg$

$$\gamma = \Sigma\alpha(g)\beta(h)(gh+hg)$$

where the sum runs over those pairs (g,h) such that $gh \neq hg$.

Note that both $\varepsilon(\beta)$ and $\varepsilon(\gamma)$ are even integers while $\varepsilon(r^2) = 1$ since r is a unit. Therefore, there must exist an element $g_1 \in S$ such that $\alpha(g_1)$ is odd.

Since g_1^2 is central, it cannot be equal to an element gh in the support of γ (note that this would imply $gh = hg$) and, obviously, the term $\alpha^2(g_1)g_1^2$ cannot cancel with a term of the form $2\alpha(g)\beta(h)gh$. Hence, $g_1^2 \in \mathrm{supp}(r^2)$ and now corollary 2.2. shows that $r^2 = \pm g_1^2$. Finally, since $\varepsilon(r^2) = 1$ we must have that $r^2 = g_1^2 \in Z$. □

Notice that the proof of the corollary above actually shows that r^2 is equal to the square of an element in L. We wish to determine this element in a precise way.

Let $\omega : \mathbb{Z}L \to \mathbb{Z}L/L'$ denote the homomorphism induced by the natural map $L \to L/L'$ and let $r \in V(\mathbb{Z}G)$. Then, $\bar{r} = \omega(r)$ is a unit of finite order in the integral group ring $\mathbb{Z}L/L'$ and is thus trivial (see [10, Corollary II.1.8.]). Hence, either $\omega(r) = \omega(g')$ or $\omega(r) = \omega(g'u)$ for some $g' \in G$. If $\omega(r) = \omega(g')$ we can write r in the form:

$$r = g' + \delta + \delta_2 u \ , \quad \text{where} \quad \delta, \delta_2 \in \Delta(G:G')$$

Using the so-called Whitcomb argument [10,p.103] it can be shown that $g' + \delta = g + \delta_1$ for some $g \in G$ and some $\delta_1 \in \Delta(G)\Delta(G')$ where $\Delta(G')$ denotes the augmentation ideal of $\mathbb{Z}G'$. Consequently, we can write r in the form:

$$r = g + \delta_1 + \delta_2 u \quad \text{with} \quad \delta_1 \in \Delta(G)\Delta(G'), \ \delta_2 \in \Delta(G;G').$$

In a similar way, if $\omega(r) = \omega(g'u)$ we can write r in the form:

$$r = (g+\delta_1)u + \delta_2 \quad \text{with} \quad \delta_1 \in \Delta(G)\,\Delta(G') \quad \text{and} \quad \delta_2 \in \Delta(G:G') \quad .$$

2.4 <u>Proposition</u>. Let r be a normalized unit of finite order in $\mathbb{Z}L$: Then either $r^2 = g^2$ or $r^2 = (gu)^2$ where $g \in G$ is an element satisfying one of the equalities above.

<u>Proof</u>: Assume first that r can be written in the form $r = g+\delta_1+\delta_2 u$ with δ_1 and δ_2 as above. Then, it is easy to see that r^2 can be written as:

$$r^2 = g^2 + \delta_1' + \delta_2' u \quad \text{with} \quad \delta_1' \in \Delta(G)\Delta(G') \quad \text{and} \quad \delta_2' \in \Delta(G:G').$$

We know from corollary 2.3 that $r^2 \in Z \subset G$ so we must have that $r^2 = g^2 + \delta_1'$ and $\delta_2' u = 0$. Hence, using [10, Lemma VI.5.1] whe have:

$$g^{-2}r^2 = 1 + \delta_2'' \in G \cap (1+\Delta(G:G')) = G'' = 1 \quad , \text{hence, } r^2 = g^2.$$

If $r^2=(g+\delta_1)u + \delta_2$ it also follows, in a similar way, that $r^2 = (gu)^2$. □

In what follows, we shall prove the Zassenhaus Conjecture by showing that a normalized torsion unit $r \in \mathbb{Z}L$ is conjugate in $\mathbb{Q}G$ to the element g or gu found above. J. Ritter and S.K. Sehgal have shown in [8] that something similar happens when working with integral group rings of nilpotent class 2 groups and observed, by means of a counterexample that this need not be the case in general.

3. A reduction

In this section, we wish to show that the Zassenhaus Conjecture can be reduced to the question of conjugacy in the complex loop algebra $\mathbb{C}L$. The argument is essentially due to C. Polcino Milies and S.K. Sehgal [6, lemma 5] and we go through it again here to show that the techniques work equally well in the alternative case.

3.1. Lemma. Let $k \subset K$ be infinite fields and let L be a finite loop whose loop algebra over K is both semisimple and alternative. If two given elements $\alpha, \beta \in kL$ are conjugate in KL then they are also conjugate in kL.

Proof: Let $\alpha, \beta \in kL$ be as in our statement. Then, the equation

$$\alpha X = X\beta$$

has a solution in KL.

Let L_α and R_β denote the linear transformations of KL defined as left multiplication by α and right multiplication by β respectively.

Set $n = |L|$. If we identify X with its coordinate vector $X = (x_1,\ldots,x_n)$ in K^n relative to the basis L of KL, then the equation above can be written in matrix form as $XM = 0$, where M is the matrix of the linear transformation $L_\alpha - R_\beta$.

Clearly, M is a n×n singular matrix with entries in k and hence the matrix equation has also solutions in k^n. Let $\{v_1,\ldots,v_t\}$ be a basis for $\ker(L_\alpha - R_\beta)$ in k^n.

We wish to show that there exists at least one vector $v \in \ker(L_\alpha - R_\beta)$ which is invertible, since it would then be such that $v^{-1}\alpha v = \beta$.

We claim that it is enough to prove that it is not a zero divisor since the simple components of the semisimple alternative algebra KL are either simple associative algebras or Cayley-Dickson algebras. In the first case it is clear that non zero divisors are invertible; in the second case this also follows since an element a is invertible if and only if $a\bar{a} \neq 0$, where $\bar{a}$ denotes the image of a under the canonical involution in a Cayley-Dickson algebra.

So, assume that for every choise of scalars $\lambda_1,\ldots,\lambda_t \in k$ the element corresponding to $x = \lambda_1 v_1 + \ldots + \lambda_t v_t$ is a zero divisor in kL or, equivalently, that the matrix of R_x is singular. Then, $\phi(\lambda_1,\ldots,\lambda_t) = \det R_x = \det(\sum_{i=1}^{t} \lambda_i R_{v_i}) \in k[\lambda_1,\ldots,\lambda_t]$ is a polynomial over the infinite field k which vanishes for all possible values of the variables and thus it is the zero polynomial. Hence, R_x is singular also for all $x \in \Sigma K v_i$, a contradiction.

Consequently, there must exist a solution of the equation $XM = 0$ whose corresponding element in kL is invertible and thus the result follows. □

As an immediate consequence we obtain:

3.2. Corollary. If two elements $\alpha, \beta \in \mathbb{Q}L$ are conjugate in $\mathbb{C}L$ then they are also conjugate in $\mathbb{Q}L$.

Because of this result, we shall work in $\mathbb{Q}L$ throughout the rest of this paper. If we write $\mathbb{C}L = A_1 \oplus \ldots \oplus A_n$, where A_i, $1 \leq i \leq n$, denote the simple alternative components of $\mathbb{C}L$, then $A_i \cap A_i^*$ is a non zero ideal contained in A_i, (for example, the center of A_i^* is fixed by the involution of $\mathbb{C}L$) so $A_i \cap A_i^* = A_i$, $1 \leq i \leq n$.

Given an element $x \in \mathbb{C}L$, we shall denote by x_i its component in A_i. To prove that two elements $\alpha, \beta \in \mathbb{Q}L$ are conjugate in $\mathbb{C}L$ it suffices to show that their respective components α_i, β_i are conjugate in A_i, $1 \leq i \leq n$.

We now define a map $\theta\colon \mathbb{C}L \to \mathbb{C}L$ by $\theta(r) = r + r^*$, $\forall\, r \in \mathbb{C}L$, and call $\theta(r)$ the *trace* of r. Notice that θ is a linear map whose values lie in the center of $\mathbb{C}L$. Also,

$\theta(r) = \theta(r^*)$ for any $r \in \mathbb{C}L$ and, if $a \in \mathbb{C}L$ is central then $\theta(a) = 2a$. Since $A_i^* \subset A_i$, $1 \leq i \leq n$, each simple component of $\mathbb{C}L$ is invariant under θ.

3.3. Lemma. Let $\delta \in \Delta(G:G')$. Then $\theta(\delta_i u_i) = 0$ and, if $\mathrm{supp}(\delta) \cap Z(G) = \emptyset$ then also $\theta(\delta_i) = 0$.

Proof: First we note that, since e is central, its component e_i lies in $Z(A_i)$, the center of A_i, which is a field. Since $e^2 = 1$ we readily get that $e_i = \pm 1$ in A_i, $1 \leq i \leq n$.

An element $\delta \in \Delta(G:G')$ is of the form $\delta = \alpha(e-1)$ with $\alpha \in \mathbb{C}G$. If $e_i = 1$ it is clear that $\theta(\delta_i) = 0$. So, we may assume, from now on, that $e_i = -1$.

On the one hand, we have that $\theta((\delta_i u_i)^*) = \theta(\delta_i u_i)$; on the other hand, since $(\delta_i u_i)^* = e_i \delta_i u_i$, we get that $\theta((\delta_i u_i)^*) = -\theta(\delta_i u_i)$ and thus we must have $\theta(\delta_i u_i) = 0$, $1 \leq i \leq n$ and hence $\theta(\delta_i u_i) = 0$.

Now, assume that $\mathrm{supp}(\delta) \cap Z = \emptyset$. Writing $\delta = (\Sigma\alpha_g g)(e-1)$ it is easy to see that we must have $\alpha_g = 0$ for every $g \in Z$. Now $\theta(\delta_i) = -2\Sigma\alpha_g\theta(g_i)$ and, when g is not central, we have that $\theta(g_i) = \theta(g_i^*) = \theta(e_i g_i) = -\theta(g_i)$ so $\theta(g_i) = 0$ and hence $\theta(\delta_i) = 0$, $1 \leq i \leq n$. □

3.4. Lemma. Let r and s be units in $\mathbb{C}L$ such that $\mathrm{supp}(r) \cap Z = \mathrm{supp}(s) \cap Z = \emptyset$. Then r_i is central in A_i if and only if s_i is central, $1 \leq i \leq n$. Furthermore, if this is the case and if $\theta(r_i) = \theta(s_i)$ then $r_i = s_i$.

Proof. Our hypothesis on the supports of r and s implies that $r^* = er$ and $s^* = es$ hence, $(rs + sr)^* = e^2(rs+sr) =$ $= rs + sr$. Thus, $rs+sr$ is central and, consequently, $r_i s_i + s_i r_i$ is central in A_i, $1 \le i \le n$. Since both r_i and s_i are units, it is clear that r_i is central if and only if s_i is central.

Also, if $\theta(r_i) = \theta(s_i)$ and r_i and s_i are central, we have that $2r_i = \theta(r_i) = \theta(s_i) = 2s_i$ and thus $r_i = s_i$. □

4. The Conjecture

We are now ready to prove the main result of this paper.

Theorem. Let L be a finite RA loop and let r be a normalized torsion unit in $\mathbb{Z}L$. Then, there exists a unit $\alpha \in \mathbb{Q}L$ and an element $w \in L$ such that $\alpha^{-1}r\alpha = w$.

Proof. If $\text{supp}(r) \cap Z(G) \neq \emptyset$ then Corollary (2.2) shows that actually $r = w$, for some $w \in L$. So, we may assume that $\text{supp}(r) \cap Z = \emptyset$.

We can write r either in the form $r = g+\delta_1+\delta_2 u$ or in the form $r = gu + \delta_1 + \delta_2 u$, with $\delta_1 \in \Delta(G)\Delta(G')$ and $\delta_2 \in \Delta(G:G')$. We consider these two cases separately.

Case 1 - We assume that r is of the form $r = g+\delta_1+\delta_2 u$, so $r^2 = g^2$ as shown by Proposition (2.4). Notice that since $supp(r) \cap Z = \emptyset$ it follows that $g \notin Z$ and $supp(\delta) \cap Z = \emptyset$. Hence, Lemma (3.3) shows that $\Theta(r_i) = \Theta(g_i)$, $1 \leq i \leq t$. Then Lemma (3.4) shows that we should study only those components where r_i and g_i are non-central . Also if $e_i = 1$ it is clear that $r_i = g_i$ (because $(\delta_1)_i = (\delta_2)_i = 0$) so we may assume that $e_i = -1$.

From now on, we shall work inside a fixed component A_i so we shall omit the subindex i to simplify notations. Let B be the subalgebra of A generated by r and g (which is associative in view of Artin's Theorem [7, Theorem 3.1]). Since $r^2 = g^2$ is central in B, it is a scalar, so we can find $\lambda \in \mathbb{C}$ such that $\lambda^2 = r^2 = g^2$. Also $\mu = rg+gr$ is a scalar so every element in B is a linear combination of 1,r,g and rg.

Set $f_1 = \frac{1}{2}(1+\frac{r}{\lambda})$ and $f_2 = \frac{1}{2}(1-\frac{r}{\lambda})$. Then, it is easily seen that f_1 and f_2 are othogonal idempotents, different from 0 and 1, such that $f_1+f_2 = 1$ and $r = \lambda(f_1-f_2)$. We can write $B = f_1Bf_1 + f_1Bf_2+f_2Bf_1+f_2Bf_2$ and find by straightforward computation:

$$f_{11} = f_1gf_1 = \frac{\mu}{2\lambda}f_1 \text{ , } f_{22} = f_2gf_2 = \frac{-\mu}{2\lambda}f_2, \ f_{12} = f_1gf_2 = f_1g - \frac{\mu}{2\lambda}f_1$$

$$f_{21} = f_2gf_1 = f_2g + \frac{\mu}{2\lambda} f_2 .$$

Hence f_iBf_i is one-dimensional with basis f_i, $i = 1,2$ and $\dim (f_iBf_j) \leq 1$ with equality holding if $f_{ij} \neq 0$ in which case f_iBf_j is spanned by f_{ij}.

Moreover, we see that:

$$f_{12}f_{21} = (\lambda^2 - \frac{\mu^2}{4\lambda^2})f_1 \quad \text{and} \quad f_{21}f_{12} = (\lambda^2 - \frac{\mu^2}{4\lambda^2})f_2$$

If $\lambda^2 - \frac{\mu^2}{4\lambda^2} \neq 0$ it follows readily that $B \cong M_2(\mathbb{C})$. Since the minimal polynomial of both r and g over $\mathbb{C}$ is $X^2 - \lambda^2$ each one is similar to diag $(\lambda, -\lambda)$ so r is conjugate to g in B.

If $\lambda^2 - \frac{\mu^2}{4\lambda^2} = 0$ then $\frac{\mu}{2\lambda} = \pm 1$. Assume first that $\frac{\mu}{2\lambda} = 1$. Then $f_{12} + f_{21} = (f_1 + f_2)g - \frac{1}{2}\frac{\mu}{\lambda}(f_1 - f_2) = g - r$ so $g = r + f_{12} + f_{21}$. Write $s = f_1 - f_2 + \frac{1}{2\lambda} f_{12} + \frac{1}{2\lambda} f_{21}$; then $s^2 = 1$ and $s^{-1}rs = g$.

If $\frac{\mu}{2\lambda} = -1$ we obtain that $g = f_2 + \frac{1}{2\lambda}f_{12} + \frac{1}{2\lambda}f_{21}$ and setting $t = -f_1 + f_2 + \frac{1}{2\lambda} f_{12} + \frac{1}{2\lambda} f_{21}$, we have that $t^2 = 1$ and $t^{-1}rt = -g$. Since g is non central, for some h in the projection of L in A we must have that $h^{-1}gh \neq h$ so actually $h^{-1}gh = eg = -g$ so it follows again that r is conjugate to g.

<u>Case 2</u>. Assume now that $r = \delta_1 + (\delta_2 + g)u$ and thus, that $r^2 = (gu)^2$. Since $\mathrm{supp}(r) \cap Z = \emptyset$ we easily see that $\mathrm{supp}\, \delta_1 \cap Z = \emptyset$ and hence $\theta(r_i) = \theta(g_i)$ so, once again, we may suppose that r_i and g_i are non-central in A_i and also that $e_i = -1$ in A_i. Hence we can essentially repeat the argument above to obtain the desired conclusion.

Since r and g are conjugate in $\mathbb{C}L$, lemma 3.1. shows that there exist $\alpha \in \Phi L$ and $w \in L$ such that $\alpha^{-1} r \alpha = w$.

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