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0- INTRODUCTION

The dissipative mechanical systems are second order vector fields on the tangent space of a given compact Riemannian manifold M and are obtained by the addition of a dissipative force to a conservative (Lagrangian) mechanical system. The dissipative forces are velocity dependent and slow down the system in a such way that the mechanical energy decreases strictly along the non trivial integral curves, making the non-wandering set a coletion of critical points. A first study on the geometric theory of dissipative mechanical systems was published by Shashahani in 1972 [11], see also [1], where dissipative systems with constraints are considered.

The dissipative mechanical systems are parametrized by a pair (V,D) where V, the potential, is a smooth real function defined on M, which represents the conservative component, and D is the dissipative force. Among the dissipative mechanical systems there are the strongly dissipative ones for which V is a Morse function and D satisfies a strongly dissipative condition (see Def. 1.3); they have very simple properties that we will describe below.

There are two well known results in the geometric theory of dynamical systems (see [7]) the so called theorem of Kupka and Smale ([5],[12]) and the theorems of Palis and Smale ([6], [8]) on the structural stability of the Morse-Smale systems (including gradient systems). More recently Takens [13], using some perturbation tecniques, obtained other generic results on gradient systems with a fixed Riemannian metric and on mechanical (conservative) systems in the special case of a Riemannian metric of zero curvature.

In the present paper the main results deal with generic properties and structural stability of dissipative mechanical systems. Theorem 1.4 proves that the strongly dissipative mechanical systems have only hyperbolic critical points and gives a description of the invariant manifolds. Usual topologies are considered in the set of all dissipative mechanical systems and Theorem 1.5 shows that the subset of all strongly dissipative ones such that the invariant manifolds are in general position is open and dense. Theorem 1.6 proves that the systems of the above open and dense set are structurally stable. In proving Theorem 1.5 one sees that it is possible to put the invariant manifolds in general position perturbing D with fixed V. Nevertheless we considered the parameters (V,D) for the systems because we believe that it is possible to obtain transversality between the invariant manifolds just fixing D and perturbing V.

1 - STATEMENTS OF THE RESULTS

Throughout the paper (M,<,>) will be a C^∞ compact connected Riemannian manifold, $\partial M = \phi$. We call M the configuration space. The C^∞ Riemannian metric <,> defines the kinetic energy $K: TM \longrightarrow R$ by $K(v_p) = \frac{1}{2}\langle v_p, v_p\rangle$, $v_p \in T_pM$. The induced Levi-

Civita covariant derivative will be denoted by ∇. A potential V is a C^{r+1} function $V: M \longrightarrow R$ and the mechanical energy is $E: TM \longrightarrow R$ defined by $E(v_p) = K(v_p) + V(\pi(v_p)) = K(v_p) + V(p)$, (TM,π,M) being the tangent bundle of M. Let $(TM)_0$ denote the zero section of this bundle.

DEFINITION 1.1 - *A dissipation force is a* C^r *map* $D:TM \longrightarrow TM$ *which preserves each fiber and such that:*

1) *for all* $p \in M$ *and* $0_p \in (TM)_0$ *one has* $D(0_p) = 0$;
2) *for all* $v_p \in TM-(TM)_0$, $\langle D(v_p), v_p \rangle < 0$.

As a matter of fact property 1) follows from property 2) and the continuity of D.

DEFINITION 1.2 - *A dissipative mechanical system on the configuration space* M *is a pair* (V,D) *of a* C^{r+1} *potential* V *and a* C^r *dissipation force* D, $r \geq 1$. *The pair* (V,D) *parametrizes a second order* C^r *vector field on* TM *(sometimes denoted also by* (V,D)*) defined by*

$$\nabla_{\dot{q}}\dot{q} = -\mathrm{grad}\, V(q) + D(\dot{q}),$$

where $\dot{q}$ *denotes the derivative of a motion* $q = q(t)$ *and* grad V *denotes the conservative field of forces characterized by*

$$dV(v_p) = \langle \mathrm{grad}\, V(p), v_p \rangle, \quad \forall v_p \in TM.$$

REMARK - The mechanical energy decreases along non trivial integral curves of the vector field induced on TM; in fact

$$\frac{d}{dt}E(\dot{q}(t)) = \frac{d}{dt}\left[\frac{1}{2}\langle \dot{q},\dot{q} \rangle + V \cdot \pi(\dot{q})\right] = \langle D(\dot{q}),\dot{q} \rangle$$

which shows that E decreases on all solutions not reduced to a fixed point. Note also that since this vector field (V,D) is of second order, its integral curves are derivatives of C^{r+2} curves on M and its critical points lie on the zero section $(TM)_0$. Moreover O_p is a critical point if and only if grad $V(p) = 0$.

In what follows we denote by $\mathcal{V}\times\mathcal{D}$ the set of all dissipative mechanical systems with the topologies induced by $C^{r+1}(M,R)$ and $C^r(TM,TN)$, $1\le r\le\infty$; in other words $\mathcal{V}\times\mathcal{D}$ is the product of $\mathcal{V}$ with the C^{r+1}-topology and $\mathcal{D}$ with the induced compact open C^r-topology or the induced Whitney C^r-topology. In the first case we will say that $\mathcal{V}\times\mathcal{D}$ has the C^r-topology and in the second one $\mathcal{V}\times\mathcal{D}$ has the Whitney C^r-topology.

<u>DEFINITION</u> 1.3- *A dissipative mechanical system* (V,D) *is said to be strongly dissipative if:*

1) V *is a Morse function;*

2) D *satisfies the following condition: for all* $p\in M$ *and all*

$$v_p\in TM-(TM)_0$$

one has $\langle\delta D(O_p)v_p,v_p\rangle < 0$ *where* δD *denotes the vertical derivative of* D.

Remark that condition 2) is equivalent to $\langle\delta D(O_p)v_p,v_p\rangle < 0$ for all v_p such that $\|v_p\| = 1$ and all $p\in M$.

<u>THEOREM</u> 1.4 - *Let* (V,D) *be a strongly dissipative mechanical system. Then the following properties hold:*

i) *The critical points of* (V,D) *are hyperbolic;*

ii) *The stable and unstable manifolds* $W^s(O_p)$ *and* $W^u(O_p)$ *of a critical point* O_p *are properly imbedded;*

iii) dim $W^u(O_p)$ *is the Morse index of* V *at* p, *and*

iv) $\dim W^u(O_p)\le \dim M\le \dim W^s(O_p)$.

THEOREM 1.5 - *The set of all strongly dissipative mechanical system such that the stable and unstable manifolds of critical points are in general position is a dense open set of $\mathscr{V}\times\mathscr{D}$ in the considered above topologies.*

As usually, we say that $(V,D)\in\mathscr{V}\times\mathscr{D}$ is structurally stable if there exist a neighborhood $\mathscr{W}$ of (V,D) and a continuous map h from $\mathscr{W}$ into the set of all homeomorphisms of TM with the compact open topology, such that:

1) $h(V,D)$ is the identity of TM;
2) $h(\bar{V},\bar{D})$ takes orbits of (V,D) to orbits of $(\bar{V},\bar{D})$, for all $(\bar{V},\bar{D})\in\mathscr{W}$, that is, $h(\bar{V},\bar{D})$ is a topological equivalence between (V,D) and $(\bar{V},\bar{D})$.

If the topological equivalence $h(\bar{V},\bar{D})$ preserves the time, that is, if X_t (resp. Y_t) is the flow of (V,D) (resp. $(\bar{V},\bar{D})$) and $h(\bar{V},\bar{D})\cdot X_t = Y_t\cdot h(\bar{V},\bar{D})$ for all $t\in\mathbb{R}$, then we say that $h(\bar{V},\bar{D})$ is a conjugacy between (V,D) and $(\bar{V},\bar{D})$.

THEOREM 1.6 - *Any strongly dissipative mechanical system such that all the stable and unstable manifolds of critical points are in general position is structurally stable and the topological equivalence is a conjugacy.*

The theorems 1.5 and 1.6 have also the flavor of an interesting theorem proved by D. Henry [3,4] for a dynamical system in infinite dimensions. On the Sobolev space

$$H_0^1 = H_0^1(]0,\pi[,\mathbb{R})$$

he considered the following parabolic PDE:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \lambda f(u)$$

where $f: R \longrightarrow R$ is a smooth function such that $f(0) = 0$, $f'(0) = 1$, $tf''(t) < 0$ if $t \neq 0$ and λ is a real positive parameter.

THEOREM (D. Henry) - *If $\sqrt{\lambda}$ is not a positive integer, then all stable and unstable manifolds of the flow defined on H_0^1 by above* PDE *are in general position.*

The time-one map f of the flow considered by D. Henry is a Morse-Smale map in the sense of [2] and as a consequence is stable relatively to the union of all unstable manifolds of the fixed points of f (see Theorem 10.27 in [2]).

We finish this section with the following example of a strongly dissipative mechanical system which does not satisfy the conclusions of Theorem 1.5. Consider the motion of a (mass one) particle constrained on the surface M of a symmetric vertical torus of R^3 obtained by the rotation, around the x-axis, of a circle defined by the equations $y = 0$ and $x^2 + (z-3)^2 = 1$. The potential V is proportional to the height function of the torus and the dissipation force D is given by $D(v_p) = -c \cdot v_p$, $c>0$ (a viscosity coeficient). These datas define a strongly dissipative mechanical system with the torus M as the configuration space. The metric of M is the usual one induced by R^3 and the potential is a Morse function. The symmetry shows that the unstable manifold of dimension 1 of one saddle is contained in the stable manifold of dimension 3 of another saddle and hence they are not in general position, since $\dim TM = 4$.

2 - PROOF OF THEOREM 1.4

Let p be a point of M and U an open neighborhood of p in M such that there exists a trivialization of TM over U, i. e. $\phi: \pi^{-1}(U) \to U\times \mathbb{R}^n$. Let x and v the projections onto U and $\mathbb{R}^n$. The vector field has the following expression on $U\times\mathbb{R}^n$:

$$\begin{cases} \frac{dx}{dt} = v \\ \frac{dv}{dt} = -\text{grad } V(x) + D(x,v) - \Gamma(x;v)\cdot v, \end{cases}$$

where $\Gamma: U\times\mathbb{R}^n \to \text{End}(\mathbb{R}^n)$ is the difference between the Levi-Civita connection and the trivial connection of ϕ; Γ is linear in v. Then it is clear that the critical points of (V,D) are the 0_p such that grad $V(p) = 0$. In such a point the linear part of the system is $L: T_pM\times\mathbb{R}^n \longrightarrow T_pM\times\mathbb{R}^n$ given by

$$L = \begin{bmatrix} 0 & I \\ -H & \Delta \end{bmatrix}$$

where $I: \mathbb{R}^n \to T_pM$ is the canonical isomorphism defined by the trivialization, H is the Hessian of V at p and Δ is the vertical derivative $\delta D(0_p)$ of D. The first statement of Theorem 1.4 follows from the next lemma:

LEMMA 2.1- *Let* $L: \mathbb{R}^n\times\mathbb{R}^n \longrightarrow \mathbb{R}^n\times\mathbb{R}^n$ *be a linear map given by*

$$\bar{L} = \begin{bmatrix} 0 & Id \\ -\bar{H} & \bar{\Delta} \end{bmatrix}$$

with $\bar{H}$ *symmetric*, det $\bar{H} \neq 0$, *and* $\bar{\Delta}$ *contracting*: $(\bar{\Delta}v,v) < 0$ *for all* $v\in\mathbb{R}^n$, $v\neq 0$ *((,) is the scalar product of* $\mathbb{R}^n$*). Then the eigenvalues of* $\bar{L}$ *have real* *parts different from zero.*

PROOF - If $i\beta \neq 0$ is on eigenvalue of $\bar{L}$ there exists $u \in \mathbb{C}^n$, $u = v + iw \neq 0$, $v, w \in \mathbb{R}^n$ such that

$$(i\beta)^2 u - (i\beta)\bar{\Delta}u + \bar{H}u = 0$$

or equivalently

$$\begin{cases} -\beta^2 v + \beta\bar{\Delta}w + \bar{H}v = 0 \\ -\beta^2 w - \beta\bar{\Delta}v + \bar{H}w = 0. \end{cases}$$

The symmetry of $\bar{H}$ implies

$$\beta \cdot [(\bar{\Delta}v, v) + (\bar{\Delta}w, w)] = 0$$

which is a contradiction since the bracket is negative.

The second statement of Theorem 1.4 follows from the fact that the energy E decreases along non trivial solutions. For the last statement one consider a path of matrices:

$$\mu \begin{bmatrix} 0 & I_d \\ -\bar{H} & -I_d \end{bmatrix} + (I-\mu) \begin{bmatrix} 0 & I_d \\ -\bar{H} & \bar{\Delta} \end{bmatrix} = \begin{bmatrix} 0 & I_d \\ -\bar{H} & -\mu I_d + (1-\mu)\bar{\Delta} \end{bmatrix}.$$

Since $-\mu I_d + (1-\mu)\bar{\Delta}$ is contracting for all μ, $0 \le \mu \le 1$, the continuity of the spectrum able us to consider the case

$$\begin{bmatrix} 0 & I_d \\ -H & -I_d \end{bmatrix}. \qquad \blacksquare$$

3 - PROOF OF THEOREM 1.5

We remark now that any trajectory of a strongly dissipative mechanical system meets $(TM)_0$ at most a discrete set of times. Also, the stable and unstable manifolds are embedded submanifolds of TM.

Let us denote by (SDMS) the set of all strongly dissipative mechanical systems and by G the set of all $(V,D) \in$ (SDMS) such that the stable and unstable manifolds of critical points are in general position.

LEMMA 3.1 - (SDMS) *is a dense open set of* $\mathcal{V} \times \mathcal{D}$.

PROOF - Since the set of Morse functions in open and dense in $C^{r+1}(M,R)$ and

$$"\langle \delta D(O_p)v_p, v_p\rangle < 0 \text{ on } A = \{v_p \in TM \mid \|v_p\| \le 1"$$

is an open condition one sees that the openness of (SDMS) is trivial. We only have to prove the density of (SDMS) in the Whitney C^r-topology. Given any neighborhood of $D \in \mathcal{D}$ in that topology we construct $\bar{D}$ which is equal to $D-\delta I$ on the compact set A and equal to D outside of a neighborhood of A choosing a C^∞ bump function and a small $\delta>0$, properly. ▼

From now on the set (SDMS) is supposed to be endowed with one of the two topologies induced by $\mathcal{V} \times \mathcal{D}$. As a consequence of next lemma 3.2 one can show that the set G contains a residual subset of (SDMS) which implies that G is dense in (SDMS).

LEMMA 3.2 - *For any pair* (z,X) *of a point* $z \in TM-(TM)_0$ *and a strongly dissipative mechanical system* $X \in$ (SDMS) *, there exist a neighborhood* N *of* z *and an open neighborhood* U *of* X *in* (SDMS) *such that the set of all* $Y \in U$ *for which the stable and unstable manifolds are in general position at all the points of* N *is open and dense in* U.

The openness in Lemma 3.2 is trivial and the density follows from arguments below whose statements require some classical concepts that we recall for a sake of completeness.

Fix an element $X \in$ (SDMS) and $z_0 \in TM-(TM)_0$. A flow box H of the system X, centered at z_0, with time function $t: H \longrightarrow \mathbb{R}$ and basis H_0, is the following submanifold with boundary of $TM-(TM)_0$: t is a C^r-function such that the Lie derivative under X is equal to 1 and $t(z_0) = 0$; H_0 is a ball of $t^{-1}(0)$ centered at z_0 of dimension (2n-1) and H is precisely the set of all z such that $|t(z)| \leq 1$ and the trajectory pf z meets H_0. Denote also by H_+ (resp. H_-) the set of all $z \in H$ such that $t(z) = 1$ (resp. $t(z) = -1$). (see [7] p. 40 for the existence of H). We may assume that H is a flow box with coordinates $(x = y_1, y_2, \ldots, y_{2n})$ and such that if σ_i and σ_j are two critical points of X, $H \cap W^u(\sigma_j)$ and $H \cap W^s(\sigma_i)$ have, each one, just one connected component. Let us call:

$$U_+(X) = H_+ \cap W^u(\sigma_j) \quad \text{and} \quad S_+(X) = H_+ \cap W^s(\sigma_i).$$

The first step is the construction of a vector field $\tilde{X}$ on TM (which may not be of second order in H) such that given $\varepsilon > 0$

and $v \in R^{2n-1}$, $\|v\|$ sufficiently small, one obtains $\|\tilde{X} - X\| < \varepsilon$ in H and

a) $\tilde{X} = X$ outside H;

b) $\tilde{X}$ is C^∞ in H and for all $y \in (H_0/4)$, the trajectory of $\tilde{X}$ with initial condition of coordinates $(-1,y)$ meets H_+ in the point of coordinates $(+1,y+v)$.

We may choose $v \in R^{2n-1}$ such that $S_+(X)$ is transversal to $U_+(X)+v$ (see [7] Cor. 1, pg. 25). In those hypothesis one sees that $S_+(X)$ is the intersection of $W^s(\sigma_i;\tilde{X})$ with H_+ and $U_+(X)+v$ is the intersection of $W^u(\sigma_j;\tilde{X})$ with $H_+/4$. This implies that the manifolds $W^u(\sigma_j,\tilde{X})$ and $W^s(\sigma_i;\tilde{X})$ are transversal in the points of H corresponding to $[-1,+1]\times(H_0/4)$. In order to construct $\tilde{X}$ we proceed as in [7] (see [7] Lemma 2.4, pg.101), considering two C^∞ functions ψ: $[-1,+1] \to \mathbb{R}^+$ and ϕ: $\mathbb{R}^{2n-1} \to \mathbb{R}^+$ such that:

$$\psi(x) = 0 \text{ for } x \in [-1,-1/2] \cup [1/2,1];$$

$$\psi(x) > 0 \text{ for } x \in (-1/2,+1/2);$$

$$\phi(y) = 0 \text{ for } \|y\| > 3/4;$$

$$\phi(y) = 1 \text{ for } \|y\| < 1/2.$$

In the above coordinates, $\tilde{X}$ is given in H by:

$$\begin{cases} \dfrac{dx}{dt} = 1, \\ \dfrac{dy}{dt} = \rho\cdot\phi(y)\cdot\psi(x)\cdot v, \rho^{-1} = \displaystyle\int_0^2 \psi(s-1)ds. \end{cases}$$

The trajectory defined by $x(0) = -1$, $y(0) = y_0 \in (H_0/4)$ is given by

$$\begin{cases} x(t) = t-1 \\ y(t) = y_0 + \rho\left[\int_0^t \phi(y(s))\psi(s-1)ds\right]\cdot v. \end{cases}$$

For $\|v\|$ small enough, $\|y(t)\| < 1/2$ $\forall t\in[0,2]$, and then $\phi(y(s)) = 1$, which implies, finally,

$$y(t) = y_0 + \rho\left[\int_0^t \psi(s-1)ds\right]\cdot v$$

and the trajectory meets H_+ in the point of coordinates $(+1, y_0+v)$. We proceed, analogously, with all pairs (σ_i, σ_j) of critical points of X.

The second step is to transform $\tilde{X}$ in H in order to obtain a second order vector field Y which is equal to X outside H and in H differs of $\tilde{X}$ by a C^r-diffeomorphism h close to the identity (see [10], pg. 267). We may assume that the flow box H is contained in a natural chart of TM,

$$(q_1, q_2, \ldots, q_n, \dot{q}_1, \dot{q}_2, \ldots, \dot{q}_n),$$

and the last n coordinates of all points of H are are constant sign.

If

$$\lambda^y(t) = (\lambda_1^y(t), \ldots, \lambda_{2n}^y(t)),\ t\in[-1,+1],$$

is a trajectory of $\tilde{X}$ in the flow box H, the trajectories of Y will be given by $\gamma^y(t) = (\gamma_1^y(t), \ldots, \gamma_{2n}^y(t))$ such that:

(i) $d\gamma_i^y(t)/dt = \gamma_{n+i}^y(t)$, $i = 1,2,\ldots,n$, $t \in [-1,+1]$

(ii) $\gamma^y(\pm 1) = \gamma^y(\pm 1) = P^{\pm} = (P_1^{\pm}, P_2^{\pm}, \ldots, P_{2n}^{\pm})$

(iii) $d^j\gamma_i^y(t)/dt^j \Big|_{t=\pm 1} = d^j\lambda_i^y(t)/dt^j \Big|_{t=\pm 1}$, $i=1,2,\ldots,2n$ and $1 \le j \le r$.

Use the family of flat functions:

$$\phi_\tau(s) = 1 + \tau \cdot \exp[s^2/s^2-1)], \; s \in (-1,+1),$$

$\tau \in [-1,+1]$, and $\phi_\tau(s) = 1$ elsewhere.

Let τ_i, $i = 1,2,\ldots,n$, be numbers in $[-1,+1]$ given by

$$P_i^+ - P_i^- = \int_{-1}^{+1} \phi_{\tau_i}(s)\lambda_{n+1}^y(s)ds$$

or

$$\tau_i = \frac{(P_i^+ - P_i^-) - \int_{-1}^{+1} \lambda_{n+i}^y(s)ds}{\int_{-1}^{+1} \lambda_{n+1}^y(s)\exp[s^2/(s^2-1)]ds};$$

Define

$$\gamma_i^y(t) = P_i^- + \int_{-1}^{t} \phi_{\tau_i}(s)\lambda_{n+i}^y(s)ds,$$

$$\gamma_{n+i}^y(t) = \phi_{\tau_i}(t) \cdot \lambda_{n+1}^y(t).$$

and check conditions (i), (ii) and (iii). The map $h: H \longrightarrow TM$ given by

$$(\lambda_1^y(t), \ldots, \lambda_{2n}^y(t)) \longrightarrow (\gamma_1^y(t), \ldots, \gamma_{2n}^y(t))$$

has its range in H for $\tilde{X}$ close to X and h is a C^r map close to the identity then a C^r-diffeomorphism.

DENSITY OF G:

We remark, easily, that $G = \{Y \in (SDMS) \mid$ the transversality holds in $\tilde{TM} = TM - TM_0\}$. Let $K_n = \left\{ v_p \in TM \,\middle|\, \frac{1}{n} \le \|v_p\| \le n \right\}$. Each K_n is compact and $\tilde{TM} = \bigcup_{n \ge 1} K_n$. Let G_n be defined by

$$G_n = \left\{ Y \in (SDMS) \,\middle|\, \pitchfork \text{ holds in } K_n \right\}$$

and it is clear that $G = \bigcap_{n \ge 1} G_n$. It is enough to prove that each G_n is open and dense in (SDMS). The openness of G_n is trivial. Let us prove that G_n is dense in (SDMS). Take $Y \in (SDMS)$ and U neighborhood of Y in (SDMS); we have to prove that $U \cap G_n \ne \phi$. For each $(Z,z) \in U \times K_n$, there exist neighborhoods $U(Z,z) \subset U$ of $Z \in U$ and $N(Z,z)$ of $z \in K_n$ such that $G(Z,z) = \left\{ X \in U(Z,z) \,\middle|\, \pitchfork \text{ holds in } N(Z,z) \right\}$ is open and dense in $U(Z,z)$, by Lemma 3.2. Fix Z and cover K_n by a finite number of neighborhoods $N(Z,z_i)$, each $G(Z,z_i)$ open and dense in $U(Z,z_i)$. It is clear that $\bigcap_i U(Z,z_i) \subset U$ is a neighborhood of Z. The set $I(Z) = \bigcap_i G(Z,z_i)$ is open in $\bigcap_i U(Z,z_i)$ and is contained in U and it is easy to see that

$$I(Z) = G_n \cap \left[\bigcap_i U(Z,z_i) \right].$$

But $I(Z)$ is dense in $\bigcap_i U(Z,z_i)$ since each $G(Z,z_i) \cap \left[\bigcap_i U(Z,z_i) \right]$ is open and dense in

$$\bigcap_i U(Z,z_i).$$

Then $I_Z \ne \phi$ that is, $U \cap G_n \ne \phi$.

OPENNESS OF G

We start the proof of openness observing that if $(V,D)\in\mathscr{V}\times\mathscr{D}$ then all bounded solutions are defined for all $t\in\mathbb{R}$.

LEMMA 3.3 - *Let* $(V,D)\in\mathscr{V}\times\mathscr{D}$ *and* $\mathscr{A}=\mathscr{A}(V,D)$ *be the set* $\mathscr{A}=\{v\in TM \mid$ *the solution of* (V,D) *through* v *is bounded*$\}$. *Then:*

i) $\mathscr{A}$ *is connected and is the largest compact invariant set;*

ii) $\mathscr{A}$ *is uniformly asymptotically stable set for the flow on* TM;

iii) $\mathscr{A}(V,D)$ *is an upper semicontinuous function of* (V,D) *in* $\mathscr{V}\times\mathscr{D}$;

iv) *If* $f=X_{t=1}$ *is the time one map associated to* (V,D) *and*

$$\mathscr{B}_a=\{v\in TM \mid E(v)<a\}$$

for a sufficiently large $a>0$, *then*

$$\mathscr{A}=\bigcap_{n\geq 0} f^n(\mathscr{B}_a);$$

v) *The map* $\pi/\mathscr{A}:\mathscr{A}\longrightarrow M$ *is surjective;*

vi) *If* $(V,D)\in(\mathrm{SDMS})$, *that is,* (V,D) *is strongly dissipative, then* $\mathscr{A}$ *is the union of the unstable manifolds of all (finite number) critical points.*

(For a proof see [1]).

To prove now that the set

$$G=\left\{(V,D)\in(\mathrm{SDMS}) \mid \pitchfork \text{ holds in } TM\right\}$$

is open in (SDMS) we remark that

$$G=\left\{(V,D)\in(\mathrm{SDMS}) \mid \pitchfork \text{ holds in } \mathscr{A}(V,D)\right\}.$$

Since $\mathscr{A}(V,D)$ is compact, there exist a neighborhood W of $\mathscr{A}(V,D)$

(W with compact closure) and neighborhood U of (V,D) in (SDMS) such that for all $(\bar{V},\bar{D})\in U$ one has $\mathscr{A}(\bar{V},\bar{D})\subset W$ and the transversality holds in W. Then $U\subset G$ and G is open. This finishes the proof of Theorem 1.5. ∎

4 - PROOF OF THEOREM 1.6

We recall now Lemma 4.1 which we quote from [6] (see also [2]). Let Crit (V,D) be the set of critical points of $(V,D)\in$(SDMS). By the implicit function theorem, given $(V,D)\in$(SDMS) there exist neighborhoods W of (V,D) and U_i each $Q_i\in$Crit (V,D) such that for any $(\bar{V},\bar{D})\in W$, there exists in U_i one only $Q_i^*\in$Crit $(\bar{V},\bar{D})$ near Q_i.

LEMMA 4.1 - *Let* $(V,D)\in G$, $P\in$Crit (V,D) *and* dim $W^u(P) = m$. *Fix a* m-*disc* B_u^m *centered at* P *contained in* $W_{loc}^u(P)$. *Given* $\varepsilon>0$, *there exist neighborhoods* U *of* P *and* W *of* (V,D) *in* (SDMS) *such that if* $(\bar{V},\bar{D})\in W$, $Q\in$Crit (V,D) *and* $Q^*\in$Crit $(\bar{V},\bar{D})$ *is the corresponding critical point near* Q, *and moreover, if* $W^u(Q^*)\cap U\neq\phi$, *then* $W^u(Q^*)\cap U$ *is fibered by* m-*discs* ε-C^1 *close to* B_u^m.

A partial order in the set Crit (V,D) of a strongly dissipative mechanical system (V,D) is the following (see [6], [12]):

$$P\le Q \text{ iff } \overline{W^u(Q)}\cap W^u(P)\neq\phi \quad \forall P,Q\in \text{Crit }(V,D)$$

The <u>phase diagram</u> of (V,D) is (Crit (V,D),≤). If P ≤ Q there exists a chain $(P_1 = Q, P_2,\dots,P_\ell = P)$ such that

$$W^u(P_j)\cap W^s(P_{j+1})\neq\phi,\ 1\le j\le \ell-1;$$

define depth $(Q|P)=k$ meaning that k is the maximum of the lenghts ℓ of all chains connecting Q to P; depth $(Q|P)=0$ means that $W^u(Q)\cap W^s(P)=\phi$. Remark that if Depth $(Q|P)=1$ and $G^s(P)$ is a fundamental domain ($G^s(P)$ is the boundary of a cell $B_s(P)$ centered at P and contained in $W^s_{loc}(P)$) then $W^u(Q)\cap G^s(P)$ is compact. For any $Q\in$Crit (V,D) there exists at least one maximal chain of lenght $n\geq 1$, $(P_1=Q,\dots,P_n)$, that is, P_n is a sink and depth $(P_j|P_{j+1})=1$, $j=1,2,\dots,n-1$.

The next lemma is lemma 7.3 of [7], pg. 87:

<u>LEMMA</u> 4.2 - *Let* P *be a critical point of* $(V,D)\in$(SDMS). *There exist a neighborhood* $\tilde{U}$ *of* P *and a continuous map* $\bar{\pi}$: $\tilde{U}\longrightarrow B_s$ *where*

$$B_s=B_s(P)=\bar{U}\cap W^s_{loc}(P)$$

such that:

1) $\bar{\pi}^{-1}(P)=B_u=\bar{U}\cap W^u_{loc}(P)$ *is a disc containing* P;
2) *for each* $x\in B_s$, $\bar{\pi}^{-1}(x)$ *is a* C^r*-submanifold of* TM *transversal to* $W^s_{loc}(P)$ *at the point* x;
3) $\bar{\pi}$ *is of class* C^r *except possibly at the points of* B_u;
4) *the fibration defined by* $\bar{\pi}$ *is invariant for the flow* X_t *of the vector field defined by* (V,D), *that is, if* $t\geq 0$ *then*

$$X_t(\bar{\pi}^{-1}(x))\supset\bar{\pi}^{-1}(X_t(x)),\ \forall x\in B_s.$$

In proving lemmas 4.1 and 4.2 we really have an <u>Unstable Foliation of $\tilde{U}$ at $P\in$Crit (V,D)</u>, $(V,D)\in G$, that is, a continuous foliation

$$\mathscr{F}(P,\tilde{U}):\ x\in\tilde{U}\longrightarrow \mathscr{F}_x(P,\tilde{U})=\bar{\pi}^{-1}(\bar{\pi}(x)).$$

Moreover, this unstable foliation can be easily globalized through saturation by X_t. This way we obtain a <u>global unstable foliation</u> $\mathcal{F}(P,U)$ where

$$U = \bigcup_{t\in R} X_t(\tilde{U}),$$

and a projection $\pi: U \longrightarrow W^s(P)$ given by $\pi\cdot X_t(p) = X_t\cdot\tilde{\pi}(p)$, $p\in\tilde{U}$, and such that:

a) the leaves are C^1 manifolds with tangent spaces varying continuously in the Grassmanian and

$$\mathcal{F}_P(U,P) = W^u(P);$$

b) the leaf $\mathcal{F}_x(P,U)$ containing $x\in U$ is equal to

$$\pi^{-1}(\pi(x));$$

c) $\mathcal{F}(P,U)$ is invariant for the flow X_t of (V,D); that is, $X_t(\mathcal{F}_x(P,U)) = \mathcal{F}_{X_t(x)}(P,U)$, $t\in R$, $x\in U$, or $\pi\cdot X_t = X_t\cdot\pi$ in U.

The same holds for $(\bar{V},\bar{D})$ near (V,D) in G.

For any maximal chain $(P_1,P_2,\ldots,P_n)$ on the phase diagram of (V,D) we obtain, by induction, a <u>compatible system of global unstable foliations</u>,

$$(\mathcal{F}(P_1,U_1),\ \mathcal{F}(P_2,U_2),\ldots,\ \mathcal{F}(P_n,U_n))$$

and the associated projections

$$\pi_i: U_i \longrightarrow W^s(P_i),\ \pi_i(X_t/U_i) = X_t\cdot\pi_i,\ i=1,2,\ldots,n.$$

The compatibility means that if a leaf F of $\mathcal{F}(P_k,U_k)$ intersects *a leaf $\tilde{F}$ of $\mathcal{F}(P_\ell,U_\ell)$, $k<\ell\le n$, then $F\supset\tilde{F}$; moreover, the restriction of $\mathcal{F}(P_\ell,U_\ell)$ to a leaf of $\mathcal{F}(P_k,U_k)$ is a C^1 foliation.*

Consider again $(V,D)\in G$ and fix $a>0$, sufficiently large, such that the bounded set $\mathcal{B}_a$ of Lemma 3.3 contains $(TM)_0$ and the set $\mathcal{A}(V,D)$. We know that for any small $\varepsilon>0$ there exists a neighborhood W of (V,D) in G such that $\mathcal{A}(\bar{V},\bar{D})$ is contained in the ε-neighborhood of $\mathcal{A}(V,D)$ in $\mathcal{B}_a$, for all $(V,D)\in W$. We may also assume that the vector field corresponding to $(\bar{V},\bar{D})\in W$ points inward at every point of $\partial\mathcal{B}_a$. $\mathcal{B}_a$ is a disc bundle in TM with sphere bundle $\partial\mathcal{B}_a$ and

$$\mathcal{B}_a = \bigcup_{P_i\in \mathrm{Crit}\ (VD)} W^s(P_i)\cap\mathcal{B}_a.$$

From now on, in this section, we call $W^s(P)\cap\mathcal{B}_a$ the stable manifold of P which we denote simply by $W^s(P)$. Let us denote by $\overline{W}^s(P)$ the closure of $W^s(P)$ in $\mathcal{B}_a$. The topological boundary of $W^s(P)$ in $\mathcal{B}_a$ is $\partial W^s(P)=\overline{W}^s(P)-W^s(P)$. Then $x\in\partial W^s(P)$ if and only if there exist a sequence of points y_i in a fundamental domain $G^s(P)$ and $t_i\to-\infty$ as $i\to\infty$ such that

$$x=\lim_{i\to\infty} X_{t_i}(y_i)$$

where X_t denotes the flow corresponding to (V,D). Remark also that $\partial W^s(P)$ is positively invariant. If P,Q are two distinct points of Crit (V,D) such that $\overline{W}^s(P)\cap W^s(Q)\neq\phi$, then $Q\in\overline{W}^s(P)$ and there exists $x\in W^s(P)\cap W^n(Q)$, $x\neq Q$; furthermore, by transversality condition $\dim W^s(P)>\dim W^s(Q)$.

The following sequence L_i is similar to the considered by

Shashahani [11]:

$L_0 = \phi$; L_1 is the union of all stable manifolds whose topological boundary is empty; for $i \geq 1$ one defines L_{i+1} to be the union of L_i with the union of all stable manifolds whose topological boundary is contained in L_i. It is clear that for all $i \geq 0$ L_i is closed, $L_{i+1}-L_i$ is a disjoint union of stable manifolds and $\phi = L_0 \subset L_1 \subset L_2 \subset \ldots \subset L_p = \mathscr{B}_a$.

Denote by P^* the critical point of $(\bar{V},\bar{D})$ corresponding to $P \in \mathrm{Crit}\,(V,D)$, for $(\bar{V},\bar{D})$ near $(V,D) \in G$.

We start now the construction of a homeomorphism h which will give the structural stability of (V,D).

Take any $W^s(P_1) \in L_1$ and the corresponding $W^s(P_1^*)$. Since $W^s(P_1)$ and $W^s(P_1^*)$ are εC^r-close on compact sets (see [7], pg. 75), for $(\bar{V},\bar{D})$ near (V,D) there is a diffeomorphism

$$\tilde{h}_1 : G^s(P_1) \longrightarrow G^s(P_1^*)$$

and let us extend it to the full $W^s(P_1)$ using the flows X_t and X_t^* of (V,D) and $(\bar{V},\bar{D})$. That is, if $x \in W^s(P_1)$, $x \neq P_1$, $t \in \mathbb{R}$ is the unique time t such that $X_t(x) \in G^s(P_1)$, then we define $h_1(P_1) = P_1^*$ and $h_1(x) = X_{-t}^* \cdot \tilde{h}_1 \cdot X_t(x) \in W^s(P_1^*)$. The map

$$h_1 : W^s(P_1) \longrightarrow W^s(P_1^*)$$

is a homeomorphism (a diffeomorphism on $W^s(P_1) - \{P_1\}$).

Do the same for all stable manifolds of L_1.

The second step is to define a homeomorphism h_2 from

$$W^s(P_2) \in L_2 - L_1$$

onto the corresponding $W^s(P_2^*)$ in a such way that h_2 will be compatible with the defined above h_1, for the case in which $\overline{W}^s(P_2) \cap W^s(P_1) \neq \phi$. The manifolds $W^u(P_1)$ and $W^u(P_1^*)$ are εC^r-close on compact sets and we have depth $(P_1|P_2) = 1$. Then the set $V_{12} = G^s(P_2) \cap W^u(P_1)$ is a compact manifold and also $W^s(P_2)$ and $W^s(P_2^*)$ are εC^r-close on compact sets. By the transversality conditions of the invariant manifolds of (V,D) and of $(\overline{V},\overline{D})$ near (V,D), there exists a diffeomorphism h_2' from V_{12} onto $V_{12}^* = G^s(P_2^*) \cap W^u(P_1^*)$.

Let $\pi_1 \colon U_1 \longrightarrow W^s(P_1)$ and $\pi_1^* \colon U_1^* \longrightarrow W^s(P_1^*)$ be the projections associated to the global instable foliations $\mathcal{F}(P_1,U_1)$ and $\mathcal{F}(P_1^*,U_1^*)$. The transversality conditions imply that we may consider $\pi_{12} = \pi_1/TV_{12}$ and $\pi_{12}^* = \pi_1^*/TV_{12}^*$ for suitable tubular neighborhoods

$$(TV_{12},\sigma_2,V_{12}) \text{ of } V_{12} \text{ in } G^s(P_2)$$

and

$$(TV_{12}^*,\sigma_2^*,V_{12}^*) \text{ of } V_{12}^* \text{ in } G^s(P_2^*),$$

chosen in a such way that the open maps $h_1 \circ \pi_{12}$ and π_{12}^* have the same image in $W^s(P_1^*)$. The maps

$$(\pi_{12} \times \sigma_2) \colon TV_{12} \longrightarrow W^s(P_1) \times V_{12}$$

$$(\pi_{12}^* \times \sigma_2^*) \colon TV_{12}^* \longrightarrow W^s(P_1^*) \times V_{12}^*$$

and the homeomorphism

$$(h_1 \times h_2') \colon W^s(P_1) \times V_{12} \longrightarrow W^s(P_1^*) \times V_{12}^*$$

able us to define, uniquely, $h_2'': TV_{12} \longrightarrow TV_{12}^*$, such that the diagram below is commutative:

$$\begin{array}{ccc} TV_{12} & \xrightarrow{h_2''} & TV_{12}^* \\ \Big\downarrow{\scriptstyle(\pi_{12}\times\sigma_2)} & & \Big\downarrow{\scriptstyle(\pi_{12}^*\times\sigma_2^*)} \\ W^s(P_1)\times V_{12} & \xrightarrow{(h_1\times h_2')} & W^s(P_1^*)\times V_{12}^* \end{array}$$

Remark that $h_2''/(TV_{12}-V_{12})$ is a diffeomorphism.

We have to repeat the same construction of h_2'' for all Q_1 such that $W^s(Q_1)\in L_1$ and $\overline{W}^s(P_2)\cap W^s(Q_1)\neq\phi$. Using properly the Isotopy Extension Theorem (IET) for diffeomorphisms (see [2], pg. 133 for a statement) we extend all the $h_2'': TV_{12} \longrightarrow TV_{12}^*$ to $G^s(P_2)$ and obtain a homeomorphism $\tilde{h}_2: G^s(P_2) \longrightarrow G^s(P_2^*)$ which is a diffeomorphism except for the points of the compact manifolds V_{12} above considered. Finally $h_2: W^s(P_2) \longrightarrow W^s(P_2^*)$ is constructed by $h_2(z) = X_{-t}^*\cdot h_2\cdot X_t(z)$ for $z\neq P_2$, where $t\in\mathbb{R}$ is the unique time such that $X_t(z)\in G^s(P_2)$, and $h_2(P_2)=P_2^*$. The second step is finished if we do the same for all $W^s(Q_2)$ of $L_2 - L_1$. Consider the union $h_1\cup h_2$ defined on the union of all stable manifolds of L_2.

Thus it remains to prove the continuity of $h_1\cup h_2$. The only points to check continuity are those $x\in\partial W^s(P_2)$ such that, say, $x\in W^s(P_1)$. We may (and will) assume that x is sufficiently close to P_1. Recall that h_2 takes leaves of $\mathscr{F}(P_1,U_1)$ near $W^u(P_1)$ to leaves of $\mathscr{F}(P_1^*,U_1^*)$. Takes a sequence $x_n\in W^s(P_2)$, $x_n \longrightarrow x$. The leaf through $h_2(x_n)$ converges to the leaf through

$(h_1 \cup h_2)(x) = h_1(x)$. It remains to prove that $h_2(x_n)$ converges to $W^s(P_1^*)$. But this happens since the sequence of times t_n such that $X_{t_n}(h_2(x_n)) \in G^s(P_2^*)$ tends to infinity.

The next (third) step is the consideration of P_3 such that $W^s(P_3) \in L_3 - L_2$ and we will construct a homeomorphism h_3 from $W^s(P_3)$ onto the corresponding $W^s(P_3^*)$ in a such way that h_3 will be compatibly with h_1 and h_2. The fact that $W^s(P_3) \in L_3 - L_2$ implies that there exist at least one point $P \in \mathrm{Crit}\,(V,D)$ such that depth $(P|P_3) \leq 2$. For each critical print Q_1 such that depth $(Q_1|P_3) = 1$, $W^s(Q_1) \in L_1$ and h_1 is defined on $W^s(Q_1)$; we proceed as in the second step and construct germs of diffeomorphisms h_3'', defined (locally) on $G^s(P_3)$, exactly as we did before when we did construct h_2''. For points P_1 such that depth $(P_1|P_3) = 2$ one considers a sequence (P_1, P_2, P_3) such that depth $(P_1|P_2) =$ depth $(P_2|P_3) = 1$. That implies that the manifolds $W^u(P_2)$ (resp. $W^s(P_3)$) and $W^u(P_2^*)$ (resp. $W^s(P_3^*)$) are εC^r-close on compact sets. By the transversality conditions $V_{23} = G^s(P_3) \cap W^u(P_2)$ is a compact manifolds and there is a diffeomorphism h_3' from V_{23} onto $V_{23}^* = G^s(P_3^*) \cap W^u(P_2^*)$. Let $\pi_2 \colon U_2 \longrightarrow W^s(P_2)$ and $\pi_2^* \colon U_2^* \longrightarrow W^s(P_2^*)$ be the projections associated to $\mathcal{F}(P_2, U_2)$ and $\mathcal{F}(P_2^*, U_2^*)$. The transversality conditions imply that we may consider

$$\pi_{23} = \pi_2 / TV_{23} \quad \text{and} \quad \pi_{23}^* = \pi_2^* / TV_{23}^*$$

for suitable tubular neighborhoods

$$(TV_{23}, \sigma_3, V_{23}) \text{ of } V_{23} \text{ in } G^s(P_3)$$

and

$$(TV_{23}^*, \sigma_3^*, V_{23}^*) \text{ of } V_{23}^* \text{ in } G^s(P_3^*),$$

such that the open maps $h_2 \circ \pi_{23}$ and π_{23}^* have the same image in $W^s(P_2^*)$. As we did before we construct h_3'' such that the following diagram is commutative:

$$\begin{array}{ccc} TV_{23} & \xrightarrow{\;h_3''\;} & TV_{23}^* \\ \downarrow{\scriptstyle (\pi_{23}\times\sigma_3)} & & \downarrow{\scriptstyle (\pi_{23}^*\times\sigma_3^*)} \\ W^s(P_2)\times V_{23} & \xrightarrow[(h_2\times h_3')]{} & W^s(P_2^*)\times V_{23}^* \end{array}$$

The construction shows us that h_3'' takes leaves of $\mathcal{F}(P_2,U_2)\cap TV_{23}$ to leaves of $\mathcal{F}(P_2^*,U_2^*)\cap TV_{23}^*$. But moreover, since h_2 takes leaves of $\mathcal{F}(P_1,U_1)$ near $W^u(P_1)$ to leaves of $\mathcal{F}(P_1^*,U_1^*)$ and by the compatibily of the system of foliations we see that h_3'' takes leaves of $\mathcal{F}(P_1,U_1)\cap TV_{23}$, to leaves of $\mathcal{F}(P_1^*,U_1^*)\cap TV_{23}^*$.

We have to repeat the same construction of the last h_3'' for all sequences (P_1,P_2',P_3) sach that

$$\text{depth } (P_1|P_2') = \text{depth } (P_2'|P_3) = 1$$

with P_1 fixed. We assume also that we did the same for all P_1 such that depth $(P_1|P_3) = 2$. Using properly the (IET) for difeomorphisms we extend to $G^s(P_3)$ all the h_3'' constructed in the second step and obtain a homeomorphism $\tilde{h}_3\colon G^s(P_3) \longrightarrow G^s(P_3^*)$. Finally we extend $\tilde{h}_3$ to $W^s(P_3)$ using the flows X_t and X_t^* and obtain $h_3\colon W^s(P_3) \longrightarrow W^s(P_3^*)$ by $h_3(u) = X_{-\tau}^*\cdot\tilde{h}_3\cdot X_\tau(u)$ for $u \neq P_3$, where $\tau\in\mathbb{R}$ is the unique such that $X_\tau(u)\in G^s(P_3)$, and $h_3(P_3) = P_3^*$.

The third step is finished if we do the same for all $W^s(Q_3)$ of $L_3 - L_2$. Consider the union $h_1 \cup h_2 \cup h_3$ defined on the union of all stable manifolds in L_3. The continuity of $h_1 \cup h_2 \cup h_3$ is proved in the same way as we did in the second step. The induction procedure is now evident. ∎

REFERENCES

[1] - G.FUSCO & W.M.OLIVA, Dissipative systems with constraints. To appear in *J.Diff. Equations*.

[2] - J.K.HALE, L.T.MAGALHÃES & W.M.OLIVA, "An introduction to infinite dynamical systems - Geometric Theory", *Springer Applied Math. Sciences*, 47 (1984).

[3] - D.HENRY, Some infinite dimensional Morse Smale systems defined by parabolic differential equations. To appear in *J.Diff.Equations*.

[4] - D.HENRY, Semilinear one-dimensional parabolic equations and Morse-Smale maps, *17º Sem. Bras. Análise* (SBM), (1983), 177-181.

[5] - I.KUPKA, Contribution à la théorie des champs génériques, *Contrib. Diff. Equations*, 2(1963).

[6] - J.PALIS, On Morse-Smale dynamical systems, *Topology* 8(1969), 385.

[7] - J.PALIS & W.DE MELO, *Geometric theory of dynamical systems — An introduction*, Springer-Verlag (1982).

[8] - J.PALIS & S.SMALE, Structural stability theorems, *Global Anal.Proc. Symp. Pure Math.*, A.M.S. 14, (1970).

[9] - C.ROBINSON, C^r structural stability implies Kupka-Smale, *Dynamical Systems*, ed. M.M.Peixoto, Ac. Press, (1973).

[10] - S.SHASHAHANI, Second order ordinary differential equations on differentiable manifolds, *Global Anal.Proc. Symp. Pure Math.*, AMS, 14, (1970).

[11] - S.SHASHAHANI, Dissipative systems on manifolds, *Invent. Math.* 16 (1972), 177.

[12] - S.SMALE, Differentiable dynamical systems, *Bull. Amer. Math. Soc.*, 73, (1967).

[13] - F.TAKENS, Mechanical and gradient systems; local perturbations and generic properties, *Bol. Soc. BRas. Mat.* v.14, 2(1983), 147.

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