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Comparison between fully-plastic solutions and the reference stress approach to evaluate *J* in circumferentially cracked pipes under bending

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Abstract

This work addresses a comparison between fully plastic solutions and the reference stress approach which are applicable to determine *J* in circumferentially cracked cylinders under pure bending. The 3-D analyses provide representative functions relating the elastic-plastic crack-tip driving forces with the applied (remote) bending moment for both *J* estimation procedures.

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1. Introduction

Fitness-for-service (FFS) assessments of cracked pipes and cylinders subjected to bending load under large plastic deformation, including reeled submarine risers, rely heavily on accurate evaluation of elastic-plastic crack driving forces, such as the *J*-integral, for circumferential surface cracks. Kumar et al. [1] developed a *J* estimation procedure for selected crack geometries based upon fully-plastic solutions for the *J*-integral which became widely known as the EPRI methodology. This original work has later been expanded by Zahoor [2] to include additional geometries for circumferentially and axially cracked pipes under tensile and bending load. However, these *J* solutions for circumferentially cracked pipes remain limited to few crack geometries and material (strain hardening) properties. Another approach to estimate *J*, most often referred to as the reference stress approach, is essentially a modification of the EPRI methodology proposed by Ainsworth [3] to reflect more closely the flow behavior of real materials, particularly high hardening materials such as austenitic stainless steels. A key feature of this approach lies

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on the evaluation of J based solely on available stress intensity factor solutions for the cracked component.

The fully plastic (EPRI) and the reference stress methods share much in common while, at the same time, proving sufficiently applicable for a broad range of crack geometries and loading modes. Further, they provide essentially similar estimates of crack driving forces for low to moderate deformation levels, as measured by J, when the material's stress-strain behavior is adequately described by a power hardening law such as the Ramberg-Osgood (R-O) model. However, this picture becomes potentially more complex as the evolving plasticity progresses from contained to fully yielded conditions, particularly for moderate to low hardening materials in cracked components under tensile loading.

Motivated by these observations, this work addresses a comparison between fully plastic solutions and the reference stress approach which are applicable to determine J in circumferentially cracked cylinders under pure bending. The presentation begins with a summary of the fully-plastic solution and the reference stress approach upon which J is derived. This is followed by the description of 3-D nonlinear analyses of a typical circumferentially cracked pipe with a surface flaw. The 3-D results provide representative functions relating the elastic-plastic crack-tip driving forces with the applied (remote) bending moment for both J estimation procedures.

2. Estimation Procedure for *J* in Circumferentially Cracked Pipes under Bending

2.1. Fully Plastic Solutions

The procedure to estimate the J Integral for a cracked component such as a circumferentially cracked pipe begins by considering the elastic and plastic contributions to the strain energy under Mode I deformation [4] given by $J = J_e + J_p$ where the elastic component, J_e , is given by

$$J_e = \frac{K_I}{E'} \quad . \tag{1}$$

Here, K_I is the elastic stress intensity factor and $E' = E$ or $E' = E/(1-\nu^2)$ whether plane stress or plane strain conditions are assumed with E representing the (longitudinal) elastic modulus. A convenient form for the elastic stress intensity factor, K_I , for a circumferential surface crack in a pipe subjected to a bending moment is given in the API procedure [5] .

The plastic component, J_p , can be evaluated from the fully plastic solution for a strain hardening material introduced by Shih and Hutchinson [6] and further validated by Kumar et al. [1]. For an elastic-plastic material obeying a Ramberg-Osgood model to describe the uniaxial true stress ($\bar{\sigma}$) vs. logarithmic strain ($\bar{\varepsilon}$) response given by

$$\frac{\bar{\varepsilon}}{\varepsilon_{ys}} = \frac{\bar{\sigma}}{\sigma_{ys}} + \alpha\left(\frac{\bar{\sigma}}{\sigma_{ys}}\right)^n \tag{2}$$

where α is a dimensionless constant, n is the strain hardening exponent, and σ_{ys} and ε_{ys} define the yield stress and strain, the fully plastic J_p for a cylinder or pipe having a circumferential surface crack under bending illustrated in Fig. 1 is expressed as [7]

$$J_p = \alpha \varepsilon_{ys} \sigma_{ys} b h_1 (a/t; D_e/t; \theta, n) \left(\frac{M}{M_0} \right)^{n+1} \tag{3}$$

where D_e is the pipe (cylinder) outer diameter, t is the wall thickness, M denotes the applied bending moment and M_0 defines the corresponding limit load for pure bending. Here, h_1 is a dimensionless factor dependent upon crack size, component geometry and strain hardening properties.

In the above expression, the uncracked ligament is given by $b = t - a$, the surface crack length is described by the angle $\theta = \pi c / 2D_e$ (see Fig. 1) where c is the circumferential crack half-length and the limit load M_0 is conventionally given by [5]

$$M_0 = 2\sigma_{ys} R_m^2 t \left(2\sin\beta - \frac{a}{t}\sin\theta \right) \tag{4}$$

in which R_m denotes the mean radius given by $(R_e + R_i)/2$, where R_e and R_i are the external and internal radius, and parameter β is defined as

$$\beta = \frac{\pi}{2}\left[1 - \left(\frac{\theta}{\pi} \right)\left(\frac{a}{t} \right) \right] \quad . \tag{5}$$

The limit solution for the bending moment given by Eq. (4) is applicable in the range $(\theta + \beta) \le \pi$ [5].

2.2. Reference Stress Approach

The reference stress approach, is essentially a modification of the EPRI methodology proposed by Ainsworth [3] to reflect more closely the flow behavior of real materials, particularly high hardening materials such as austenitic stainless steels. By defining a reference stress, σ_{ref}, for a cracked component in the form

$$\sigma_{ref} = \frac{P}{P_0}\sigma_{ys} = \frac{M}{M_0}\sigma_{ys} \tag{6}$$

where P is the applied load and P_0 is the corresponding limit load, Ainsworth [3] noticed that factor h_1 appearing in previous Eq. (3) defining J_p becomes relatively insensitive to material properties.

Following further manipulation of $J = J_e + J_p$ and noting that the elastic component, J_e, can be expressed in terms of previous Eq. (3) with $\alpha = 1$ and $n = 1$ (*i.e.*, elastic regime), evaluation of J is simply accomplished by the expression

$$J = \frac{K_I^2}{E'}\left(\frac{E\varepsilon_{ref}}{\sigma_{ref}} + \frac{1}{2}\frac{\sigma_{ref}^3}{\sigma_{ys}^2 E\varepsilon_{ref}}\right) \tag{7}$$

where the reference strain, ε_{ref} , is defined as the (uniaxial) strain corresponding to the (uniaxial) stress, σ_{ref} . The above expression to estimate J does not require a specific description of the material's stress-strain behavior, such the Ramberg-Osgood, and can thus be applied to any material response.

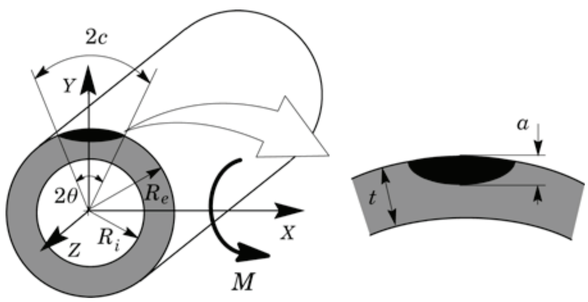

Figure 1. Pipe configuration and defect geometry adopted in the numerical analyses.

3. Computational Procedures and Finite Element Models

Nonlinear 3-D finite element analyses are conducted on circumferentially cracked pipes with external surface flaws subjected to bending. The analyzed pipe models have wall thickness $t = 20.6$ mm with outside diameter $D_e = 206$ mm ($D_e/t = 10$) which typifies current trends in high pressure, high strength pipelines. The analysis matrix considers surface flaws with varying crack depth (*a*) and crack length (2*c*) as defined by $a/t = 0.1$ to 0.5 with increments of 0.05 and $\theta/\pi = 0.04$, 0.08, 0.12, 0.16 and 0.20 (see Fig. 1). The finite element models employ a conventional mesh configuration having a focused ring of elements surrounding the crack front; here, the small initial root radius at the crack tip is $\rho_0 = 0.005$ mm. A typical half-symmetric model for the cracked pipes has approximately 16000 elements and 19000 nodes with appropriate constraints imposed on nodes defining the symmetry planes. The crack front is described by 15 (circumferential) layers defined over the crack half-length (*c*); the thickest layer is defined at the deepest point of the crack with thinner layers defined near the free surface to accommodate the strong gradient in the stress distribution along the crack front. The finite element code WARP3D [8] provides the numerical solutions for the 3-D analyses reported here. Evaluation of the *J*-integral derives from a domain integral procedure [8] which yields *J*-values retaining strong path independence for domains defined outside the highly strained material near the crack tip.

The elastic-plastic constitutive model follows conventional Mises plasticity in small geometry change (SGC) setting. The finite element analyses consider material flow properties covering typical structural, steels with $E = 206$ GPa, $\nu = 0.3$, $\alpha = 1$ with three hardening levels as defined by the Ramberg-Osgood exponent: *i*) $n = 10$ and $E/\sigma_{ys} = 500$ (moderate hardening material), *ii*) $n = 5$ and $E/\sigma_{ys} = 800$ (high hardening material) and *iii*) $n = 20$ and $E/\sigma_{ys} = 300$ (low hardening material).

4. Crack Driving Forces Estimates

Figure 2 provides typical results for the h_1-factors in circumferentially cracked pipes with varying geometries and material properties; here, h_1 follows from solving Eq. (3) upon computation of the plastic component of the J integral, J_p, with the applied bending moment, M. For all sets of analyses, the results reveal that factor h_1 displays a rather strong sensitivity to crack geometry and strain hardening behavior. For shallow crack sizes ($a/t \le 0.2 \sim 0.3$), the h_1-values are fairly insensitive to crack length (defined by parameter θ/π) for all hardening levels; here, the evolution of factor h_1 with crack depth essentially falls onto a single curve particularly for $a/t \le 0.2$. In contrast, the h_1-factors for deeply cracked pipes ($a/t \ge 0.4$) depend rather strongly on θ/π for all hardening levels, particularly for shorter crack lengths ($\theta/\pi \le 0.12$). It should be noted that the h_1-factors for the low hardening material are strictly valid only for $a/t \le 0.25$ (see Chiodo and Ruggieri [7] for further discussion and additional results).

Figure 3 compares compare the variation of J with applied bending moment normalized by the limit bending moment (M_0) for the cracked pipe specimen with $a/t = 0.25$, $\theta/\pi = 0.12$ and varying hardening properties. In this plot, the *J*-values determined directly from the finite element analysis (which are based upon a domain procedure - see [8]) provide a baseline value against which the *J*-values evaluated from the reference stress approach are compared. For all hardening levels considered, the *J*-values defined by Eq. (7) agree well with the finite element computations. The results for the $n = 20$ material reveal that the reference stress approach provide relatively unconservative estimates of J which are nevertheless within an acceptable range of crack driving force estimates.

Acknowledgements

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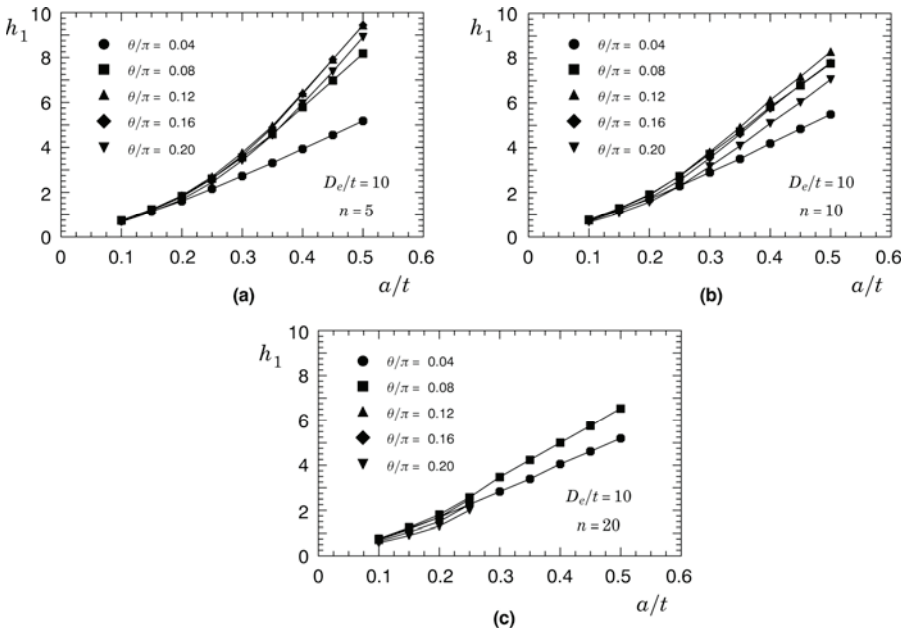

Figure 2. Variation of factor h_1 with varying hardening properties.

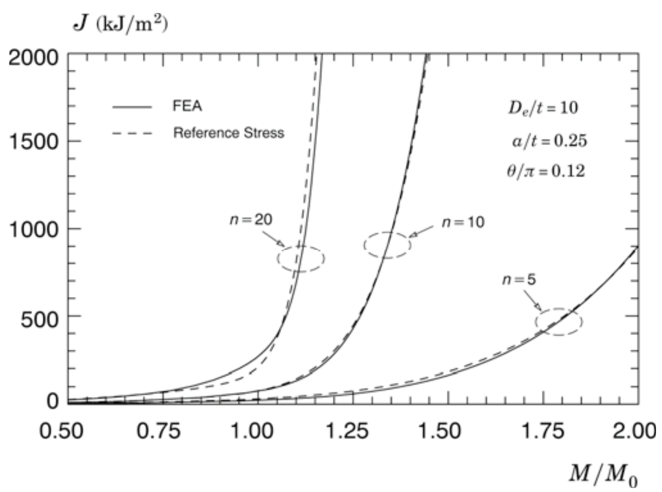

Figure 3. Comparison between J derived from the reference stress approach with corresponding finite element results.