Buckling Optimization in Variable Stiffness Composite Laminated

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Abstract. *Composite laminates of variable stiffness are widely used in aeronautical structures because they have low specific mass and good mechanical properties in relation to isotropic materials. In this work, a methodology is made to optimize the critical buckling load by orienting the fibers of a plate on a rectangular plate subjected to various loading conditions using the finite element method.*

1. INTRODUCTION

Composite materials are the result of technological advancement in the field of materials, generally applied in the aeronautic, aerospace and automotive areas. They are distinguished by their characteristic of low specific mass and high strength. According to [1], biphasic composite materials can be classified as particle reinforced fiber reinforced and structural, where the last item is divided into sandwich and laminate.

The high-performance composite (structural laminate), the matrix is designed to hold the reinforcements together, protect the fibers and transmit the load to the reinforcement. The latter gives rigidity to the laminate and has the function of supporting the load transmitted by the matrix in the fiber directions [2]. Due to the modernization of manufacturing processes, it is now possible to create laminates with curved fibers, where they can adapt to the required geometry with better results. These laminates, having curved fibers have variable stiffness unlike laminates that have fibers with constant orientation. These laminates are often defined as Variable Stiffness Composite Laminated (VSCL) panels in literature [3].

In Figure 1, one can see the plate of a normalized size laminate in which its curves, in blue, represent fibers and the angles θ_1 and θ_2 their orientations at the beginning and end of the plate respectively.

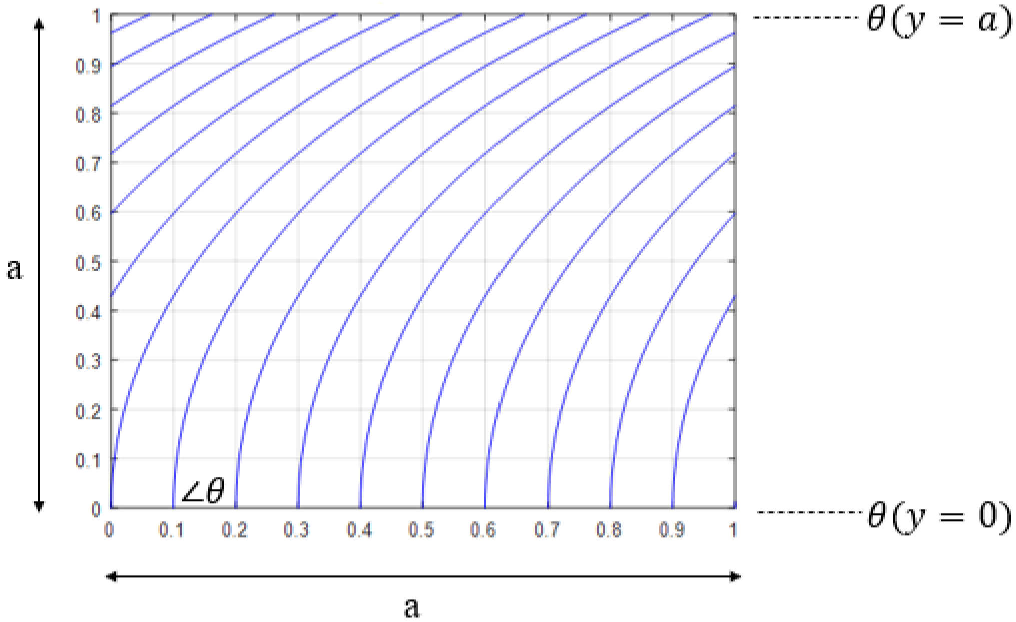

Figure 1 - Plate with its fibers

This work aims to analyze the cited plate subjected to a static compression effort, using Abaqus® software, where it is also not allowed to determine its critical load for its first buckling mode.

Using the GA (Genetic Algorithm) function of Matlab® software, it is possible to vary angles θ_1 and θ_2 by iterating with Abaqus® software to determine the highest critical buckling load for the laminate, where it is related to the eigenvalues of the plate by equation (1):

$$N_{cr} = \lambda N_a \tag{1}$$

Equation (1) relates the applied load N_a and the first eigenvalue λ and its respective critical buckling load N_{cr}.

During the manufacturing process the fibers are folded in the plane, such a process can generate the occurrence of local buckling. The fact can be avoided if the fiber deposition path meets the curvature constraint, assumed by Blom [3] as $(K = 1/25\ in)$, such restriction is expressed mathematically as a function of the fiber path:

$$\frac{f''(y)}{\left[1 + \left(f'(y)\right)^2\right]^{3/2}} < K \tag{2}$$

Where $f(y)$ it is simply the equation that defines the fiber path. By mathematical calculations, it is observed that $\frac{df(y)}{dy} = \tan(\theta(y))$, where $\theta(y)$ is the angle tow along the plate.

Assuming an approximately linear variation between two points with distance Δy, the restriction of equation (2) is equivalent:

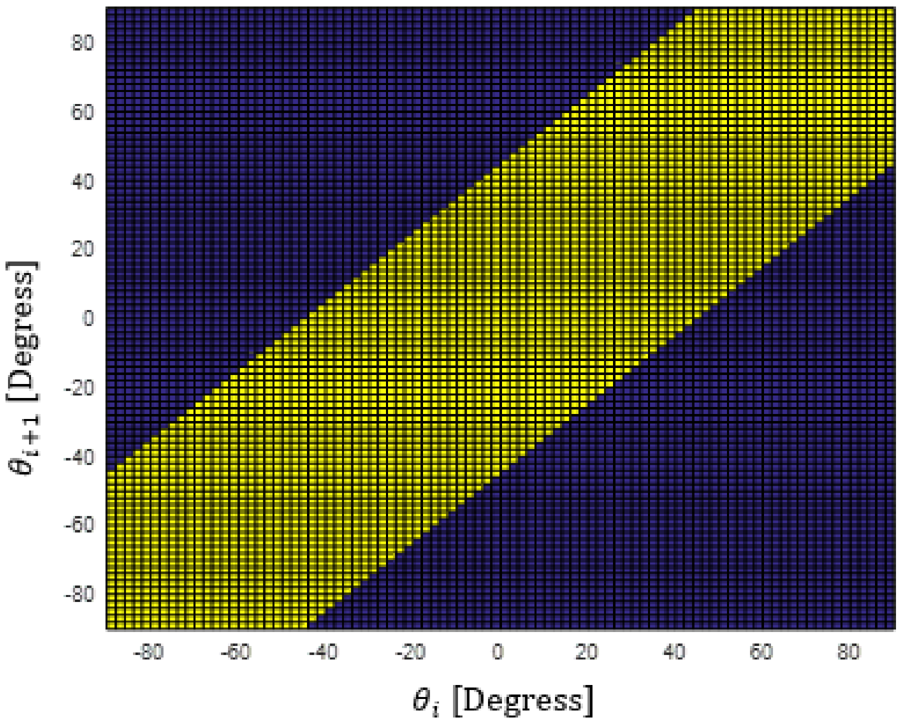

Figure 2 - Manufacturing Restrictions

Figure 2 shows a mesh with the angle values θ_i e θ_{i+1} between consecutive points y_i e y_{i+1}, where the yellow values respect the maximum allowed curvature constraint and the purple ones do not. Such values were obtained numerically.

2. FINITE ELEMENT MODELING AND OPTIMIZATION PARAMETERS

To perform the analysis of fig. 1, a model was built using the CAE software ABAQUS^{TM} where the lamina thickness is $h = 0.2\,mm$ is divided into five equal parts along the length 'a', varying the orientation of its fibers independently as shown in fig. 3 (a).

Then the boundary conditions champed were applied in $(x, y = 0)$ with compressive loading $N_y = 1\,N/mm$ at the other end $(x, y = a)$, as shown in fig. 3 (b).

The boundary conditions shown in the previous paragraph were used in the first three optimization cases, where the main difference between them is in the way in which the variation of the angles of the fibers is made.

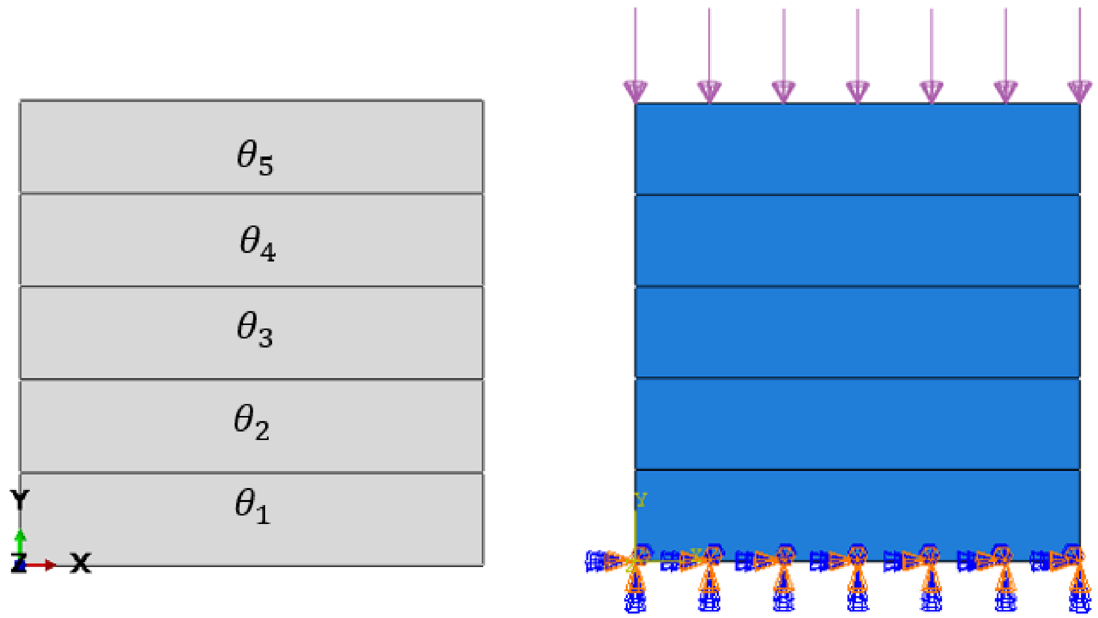

Figure 3 - (a) Parts and angles; (b) Boundary condition case 1 ,2 and 3

In the first simulation, the angles of each of the five parts shown in fig. 2 were varied independently using an optimization algorithm called a genetic algorithm.

This search algorithm is based on the mechanics of natural selection, through reproductive, mutation and selective strategies, it seeks to increase the fitness of a given population subject to certain restrictions and select the best individual [4].

For the second simulation, the maximum values between the angles of two consecutive parts were limited, due to some difficulties in manufacturing fibers with variable orientation, trying to respect the restriction of fig. 2, equivalent to $(\theta_{i+1} - \theta_i) \leq 31.5^0$.

In the third case, only the angle in the crimped line was used as variables in the genetic algorithm. (T_0) the angle (T_1) in $(y = a)$ to intermediate parts with variations according to equation (3) defined based on [5], but with variation of the angle along y.

$$\theta(y) = \frac{(T_1 - T_0)}{a} y + T_0 \tag{3}$$

In a fourth test, a transverse loading was applied $N_{xy} = 1\ \text{N/mm}$ distributed on the top surface of the plate, as shown in fig. 4 in addition to the compression loading conditions already applied in previous experiments with the same angle conditions adopted in case 2.

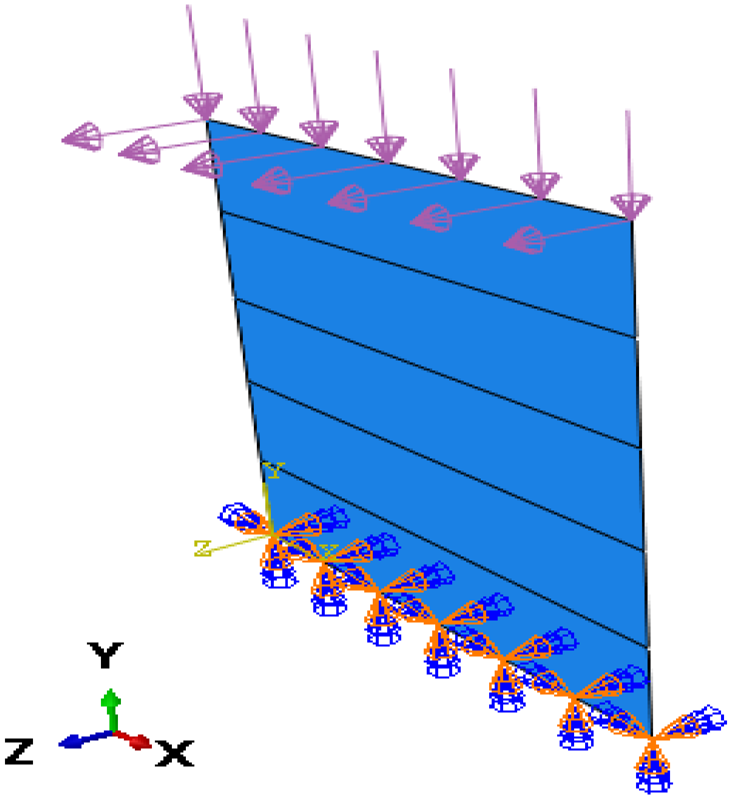

Figure 4 - boundary condition case 4

Each case shown previously was simulated using the finite element program ABAQUS™ where the angles of each part of the blade are given as an input parameter and it returns the eigenvalue corresponding to the critical buckling load on each plate, this eigenvalue is optimized by a GA using the Matlab® software.

3. RESULTS AND DISCUSSION

In all four cases shown in the methodology, a population of 100 individuals and 30 generations was used. These numbers were sufficient for all five cases the eigenvalue (defined as the objective in GA) for the initial population converges to the eigenvalue of the best individual at the end of the last generations, as shown in fig. 6, ensuring that significant improvements for the respective case.

The material used for the plate was the carbon-epoxy composite whose properties are shown in tab. 1:

Table 1: Material properties in material direction

$E_1(GPa)$	$E_2 = E_3(GPa)$	$G_{12} = G_{13}\ (GPa)$	$G_{23}(GPa)$	$v_{12} = v_{13}$
131	8.7	5.0	3.0	0.28

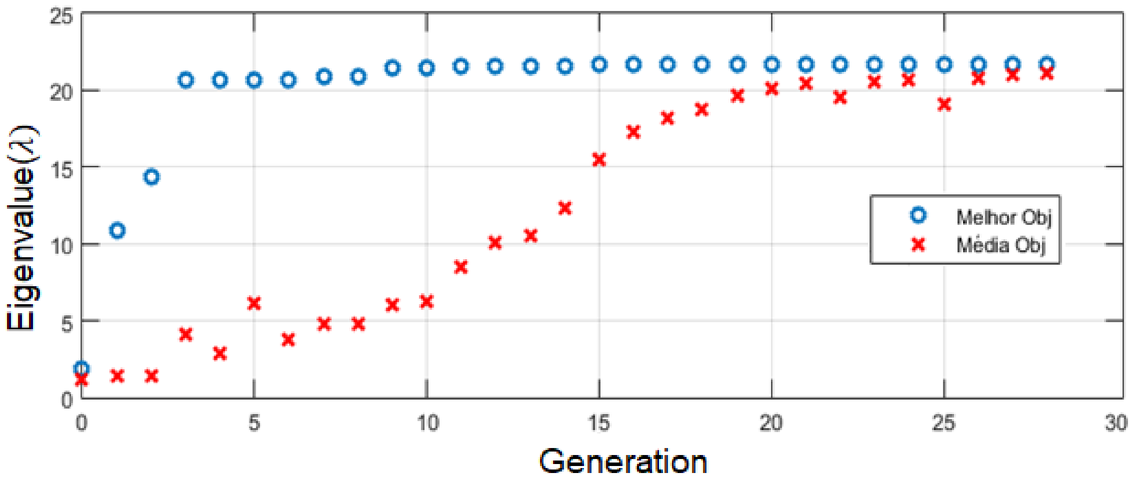

Figure 5 – Convergence objective (eigenvalue - λ) of the genetic algorithm

Table 2 shows the values of angles (in degrees) obtained for each case by GA, respecting the restrictions imposed.

For comparison, critical loads were normalized N_{cr} for equation (4):

$$\overline{N_{cr}} = \frac{a^2 N_{cr}}{E_1 h^3} \tag{4}$$

Table 2 – Angles of the best plates

Case	θ_1	θ_2	θ_3	θ_4	θ_5	Eigenvalue $(\lambda)*(10^3)$	$\overline{N_{cr}}$
1	-64.92	-89.51	-89.96	85.05	-73.78	17.1339	163.49
2	90.00	89.88	89.92	89.10	90.00	21.7135	207.19
3	90.00	89.75	89.50	89.25	89.00	21.7030	207.09
4	90.00	89.69	89.93	89.80	90.00	21.7169	207.22

Observe in case 1 sudden variations between the anglesθ_2, θ_3 end θ_4, therefore, the case does not meet the curvature restrictions due to manufacturing shown in fig. 2, which motivated the creation of restrictions for cases 2 and 3.

It was expected that case 1 would have better results compared to cases 2 and 3, as it has no design restrictions, perhaps better results will be found by increasing the number of individuals per generation.

Comparing cases 2 and 3, it is observed that the second presented the highest critical load of buckling, this happened because it has a greater number of degrees of freedom (5 degrees of freedom) compared to the third case (2 degrees of freedom).

In cases 4, it is observed that N_{xy} did not significantly vary the critical buckling load, so we can conclude that the first buckling mode is not associated with shear stresses.

4. CONCLUSION

In this work, the objective was reached to analyze the behavior of the critical buckling load on a plate manufactured with composite material using finite element programs and optimization algorithm.

It's observed that the variation in the orientation of the fibers has a lot of influence on their critical buckling load and it was possible to optimize it with the chosen parameters.

However, the evaluation of each individual using genetic algorithm together with the finite element software leaves the computational cost quite high, making it necessary for its consideration.

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