Corrigendum to “Supercritical regime for the Kissing polynomials” [JAT Volume 255]

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Abstract: We correct a mistake in the statement of Theorem 2.4.

The authors regret that there is a mistake in the statement of Theorem 2.4. To state its corrected version, we replace the set Θ^* (which is introduced right before Theorem 2.4) by a new set

$$\Theta^*_\varepsilon := \{(n,\lambda) : \operatorname{dist}(2n\kappa(\lambda) - c(\lambda), 2\pi\mathbb{Z}) < \varepsilon\}.$$

where $\varepsilon > 0$ is any fixed small number, and the functions $\kappa(\lambda)$ and $c(\lambda)$ are given in Equations (2.9) and (6.17), respectively.

For comparison, in the original version the set Θ^* was obtained from the condition $2\kappa(\lambda) - c(\lambda) \notin 2\pi\mathbb{Z}$, so with a factor $2\kappa(\lambda)$ instead of $2n\kappa(\lambda)$ as it should be.

Then the correct statement of Theorem 2.4 is

Theorem 0.1 (Theorem 2.4 in the original paper)**.** *Fix $\varepsilon > 0$ and suppose $\lambda > \lambda_c$. For n sufficiently large, and $(n,\lambda) \notin \Theta^*_\varepsilon$ in case n is odd, the kissing polynomial p_n^λ in (1.4) uniquely exists as a monic polynomial of degree exactly n, and the weak asymptotics of its zeros $z_1, \dots, z_n$ is given by the weak limit*

$$\lim_{n\to\infty} \frac{1}{n}\sum_{k=1}^{n} \delta_{z_k} \overset{*}{=} \mu_*, \tag{0.1}$$

where δ_z is the atomic measure with mass 1 at z.

Furthermore, as $n \to \infty$ the asymptotic formulas

$$\begin{aligned} p_{2n}^\lambda(z) &= \Psi_{n,0}(z) e^{2n\left(\frac{i\kappa}{2} - \frac{i\lambda}{2}z - l - \phi(z)\right)} \left(1 + \mathcal{O}(n^{-1})\right), \\ p_{2n+1}^\lambda(z) &= e^{(2n+1)\left(\frac{i\kappa}{2} - \frac{i\lambda}{2}z - l - \phi(z)\right)} \left(\Psi_{n,1}(z) + \mathcal{O}(n^{-1})\right) \end{aligned} \tag{0.2}$$

hold true uniformly in compacts of $\mathbb{C} \setminus (\gamma_1 \cup \gamma_2)$ and $\mathbb{C} \setminus (\gamma_1 \cup \widehat{\gamma} \cup \gamma_2)$, respectively, where the functions $\Psi_{n,0}$ and $\Psi_{n,1}$ have the following properties.

(i) $\Psi_{n,0}$ is holomorphic on $\mathbb{C} \setminus (\gamma_1 \cup \gamma_2)$, whereas $\Psi_{n,1}$ is holomorphic on $\mathbb{C} \setminus (\gamma_1 \cup \widehat{\gamma} \cup \gamma_2)$, where $\widehat{\gamma}$ is a contour connecting $-\overline{z}_*$ and z_*, and they remain bounded in compacts of their domains of definition as $n \to \infty$.
(ii) $\Psi_{n,0}$ does not have zeros.
(iii) The function $\Psi_{n,1}$ has a unique zero at a point $a_* = a_*(n,\lambda)$, which is simple and located on the imaginary axis.

In the original version, the essential difference is that we claimed that the functions $\Psi_{n,0}$ and $\Psi_{n,1}$ above were independent of n, but in fact they are not.

The mistake in the paper happened in Corollary 6.6: therein, the constant in the first equation in (6.21) is wrong, and its correct version reads

$$\oint_A \Omega_n^{(k)} = 2n\kappa i.$$

As a consequence, the meromorphic differential $\Omega_n^{(k)}$ also depends on n, not only on its parity as it was claimed in the original version. The proof of Corollary 6.6 remains exactly the same (but with this new value of constant $2n\kappa i$)

This mistake in Corollary 6.6 has impact in the following places:

1) In Corollary 6.6, the pole $a_*^\nu = a_*^{(\nu)}(n)$ of $\Omega_n^{(k)}$ also depends on n.
2) The functions $u_1^{(k)}$ and $u_2^{(k)}$ are still defined exactly as in (6.22) and (6.23), but they now depend on n as well.
3) Proposition 6.7, concerning $u_1^{(k)}$ and $u_2^{(k)}$, remains exactly the same.
4) The proof and conclusions of Theorem 6.8 remain the same, with the exception that now the function $M_{1,1}$ depends on n.
5) In the discussion in Section 6.5, we replace the expression $2\kappa - c$ by $2\kappa n - c$, and everything goes through as is, except that now the condition "$\lambda \notin \Theta^*$" has to be replaced by "$(n, \lambda) \notin \Theta_\varepsilon^*$". Also, as an extra statement to Theorem 6.9 we have to add "Furthermore, the entries of M remain bounded on compacts as $n \to \infty$ with n even or n odd with $(n, \lambda) \notin \Theta_\varepsilon^*$". The proof of this added statement is immediate from the construction with this new set Θ_ε^* and the discussion in Section 6.6.
6) Right after (7.25), we have to add "with n even or with n odd and $(n, \lambda) \notin \Theta_\varepsilon^{(*)}$.
7) In formula (8.1), only the first identity is true, but not the second. Nevertheless, the first identity therein is enough to conclude the proof of Theorem 2.4 as stated above.

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