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# SOME RESULTS ON OUTER LINEARIZATION IN THE PRESENCE OF CONCAVITY

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# SOME RESULTS ON OUTER LINEARIZATION IN THE PRESENCE OF CONCAVITY

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**Abstract**

The theory and usage of outer linearizations is usual in conjunction with assumptions of convexity, as in Bender's type of methods [1] or in cutting plane algorithms, in the sense introduced by Eaves and Zangwill [2]. In the later case, the focal point is the dropping of cuts (outer linearizations), and, in certain specific conditions (algorithm 4), the maximal number of cuts, in each interation, can be shown to be bounded by the dimension of the space.

The usage of outer linearization in the presence of concavity is not usual, although we show that this leads to Kuhn–Tucker stationary points, using just one outer linearization in each iteration. Moreover, this result is exploited for a class of problems for which the resulting stationary point is a global minimum of the original problem.

## 1. Introduction

The classical nonlinear programming problem (MP) is stated as:

"Given a non empty set $X^o (X^o \subset R^n)$,

$f : X^o \to R$,

$g : X^o \to R^m$,

find an $\bar{x} \in X$, if it exists, such that

$$f(\bar{x}) = \min\{f(x) \mid x \in X\}, \text{ where } X = \{x \in X^o \mid g(x) \le 0\}."$$

In the main body of the literature, the results are centered on the case where $f(\cdot)$ and $g(\cdot)$ are convex functions over the convex set $X^o$, or some technical generalization of such assumption (quasi convexity, pseudoconvexity). Even when these assumptions are not explicit, they appear implicitly (like subdifferentiability at every point of $X^o$).

This work is centered upon the cases:

(f) $f(\cdot)$ is concave and $g(\cdot)$ is convex;

(g) $f(\cdot)$ is convex and $g(\cdot)$ is concave.

It will be clear that the presented ideas can be easily extrapolated to the case when both $f(\cdot)$ and $g(\cdot)$ are concave.

In order to simplify the presentation we shall constrain this work to the case "$X^o$ open set, $f(\cdot)$ and $g(\cdot)$ differentiable", although similar results can be derived using sub and/or superdifferentiability.

We shall use as subproblems, the ones obtained by the outer linearization of the concave functions, i.e., given $x^k \in X$, we define:

$$(MPL)_k^f \begin{cases} \min f(x^k) \quad + < \nabla f(x^k), x - x^k > \quad (= f_L^k(x)). \\ s.t. \qquad g(x) \leq 0 \\ \qquad\qquad x \in X^o \; ; \end{cases}$$

$$(MPL)_k^g \begin{cases} \min \quad f(x) \\ s.t. \quad g_i(x^k) + < \nabla g_i(x^k), x - x^k > \leq 0, \quad i = 1, \ldots, m, \\ \qquad x \in X^o \; . \end{cases}$$

It is quite clear that if $(MPL)_k^{\cdot}$ [1] has an optimal solution $x^{k+1}$, this solution is a feasible pont for $(MP)$, i.e., $x^{k+1} \in X$. For $(MPL)_k^g$, we can assert that every local minimum is global and that the reverse constraint qualification holds [9]. For $(MPL)_k^f$, the global minimality property also holds and we shall assume the validity of a constraint qualification (for instance, Slater's).

In order to guarantee the existence of solutions to the linearized problems, we shall assume throughout this work, the following assumption:

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[1] The notation $(MPL)_k^{\cdot}$ is used in the sense "both for $(MPL)_k^f$ and $(MPL)_k^g$".

**Definition 1.1)** The *compactness assumption* (CA) holds for $(MP)$ if and only if

$$\forall \alpha \in \mathbf{R}, \quad \{x \in X \mid f(x) \leqq \alpha\} \quad \text{is compact.}$$ ■

The assumption (CA) besides guaranteeing that there is a solution $x^{k+1}$, for $(MPL)^{\cdot}_{k}$, also is determinant to the fact that if a sequence $\{x^k\}_{k\in \mathbf{N}}$ is generated by recursively solving $(MPL)^{\cdot}_{k}$, then this sequence is compact.

With these remarks in mind, it is easy to verify

**Lemma 1.2)** Under the assumptions

(2.1) The compactness assumption is valid;

(2.2) $X^o$ non-empty open subset of $R^n$, $f(\cdot)$ and $g(\cdot)$ are continuously differentiable on $X^o$;

(2.3) $x^k \in X = \{x \in X^o \mid g(x) \leqq 0\}$;

(2.4) either $f(\cdot)$ is convex and $g(\cdot)$ is concave $[(MPL)^g_k]$ or

$f(\cdot)$ is concave and $g(\cdot)$ is convex $[(MPL)^f_k]$;

(2.5) for $(MPL)^f_k$ a constraint qualification holds,

it can be asserted that

(i) $\forall x^k \in X$, $(MPL)^{\cdot}_k$ has solution $x^{k+1}$;

(ii) $\forall x^k \in X$, $f(x^k) \geqq VO((MPL)^{\cdot}_k) \geqq f(x^{k+1}) \geqq VO(MP)$,

where $VO(P)$ = optimal value of problem $P$;

(iii) $f(x^k) = VO((MPL)^{\cdot}_k) \Leftrightarrow x^k$ is a Kuhn–Tucker stationary point for $(MP)$. ■

This result, whose straightforward demonstration is omitted, motivates the idea of the usage of outer linearization techniques, for the problem in consideration.

## 2. Outer linearization method.

The suggested method is:

**Method 2.1).** Given $x^1 \in X$, construct the following sequence, starting with $k = 1$:

(i) solve $(MPL)_k$. If $VO(MPL)_k = f(x^k)$, stop; otherwise

(ii) Using $x^{k+1}$, construct $(MPL)_{k+1}$; $k \leftarrow k+1$; return to (i). ■

Clearly, in the case where the method (2.1) generates a finite number of points, the last generated one is a Kuhn–Tucker stationary point (lemma 1.2). So, from now on, this work will be concerned with the case where a sequence of points is generated. For this case, we can assert, **under the assumptions** of (1.2):

**Fact 2.2)** The sequence $\{f(x^k)\}_{k\in\mathbf{N}}$ is a convergent sequence, and $(MP)$ is bounded.

**Proof:** Trivial, as by (1.2), it is a monotonic decreasing sequence with lower bound. This lower bound is the optimal value of $(MP)$, whose existence is guaranteed by (CA) and the continuity assumptions on $f(\cdot)$ and $g(\cdot)$. ■

**Fact 2.2.a)** Under (CA), $\{x^k\}_{k\in\mathbf{N}}$ is a compact sequence.

**Proof:** Trivial. ■

**Theorem 2.3)** Every convergent subsequence generated by method (2.1), converges to a Kuhn–Tucker stationary point of $(MP)$, if the Kuhn–Tucker multipliers $\{M^k\}_{k\in\mathbf{N}}$ form a compact sequence, i.e., if they are bounded by above.

**Proof:** Let $\{x^k\}_{k\in K}$ be a convergent subsequence of $\{x^k\}_{k\in\mathbf{N}}$, with limit $\overline{x} \in X$.

Consider $\{x^{k+1}\}_{k\in K}$, that is a compact sequence, so $\exists K' \subset K$, such that $\{x^{k+1}\}_{k\in K'}$ converges to $\hat{x}$.

As the Kuhn–Tucker conditions are necessary for the linearized problems we can assert:

$$
\text{(i)} \begin{cases} \nabla f(x^k) + \Sigma M_i^k \nabla g_i(x^{k+1}) = 0, & \text{for } (MPL)_k^f, \\ \nabla f(x^{k+1}) + \Sigma M_i^k \nabla g_i(x^k) = 0, & \text{for } (MPL)_k^g \, ; \end{cases}
$$

(ii) $M^k \geq 0$;

$$
\text{(iii)} \begin{cases} < M^k, g(x^{k+1}) >= 0, & \text{for } (MPL)_k^f, \\ M_i^k.(g_i(x^k) + < \nabla g_i(x^k), x^{k+1} - x^k >) = 0, & \text{for } (MPL)_k^g \, ; \end{cases}
$$

$$
\text{(iv)} \begin{cases} g(x^k) \leq 0; \\ g(x^{k+1}) \leq 0; \\ g_i(x^{k+1}) \leq g_i(x^k) + < \nabla g_i(x^k), x^{k+1} - x^k > \leq 0, & \text{for } (MPL)_k^g \, . \end{cases}
$$

If $\{M^k\}_{k \in K'}$ is bounded by above, i.e., $\exists \widehat{M} : \forall\, k \in K', \;\; M^k \leq \widehat{M}$, this sequence is compact. So taking a convergent subsequence $\{M^k\}_{k \in K''}$, with limit $\overline{M}$, from continuity, it follows that

(v) $(\hat{x}, \overline{M})$ is a Kuhn–Tucker stationary point for $(MP)$ linearized around $\overline{x}$.

Moreover, by fact 2.2, we can assert

(vi) $f(\overline{x}) = f(\hat{x})$.

From (v) and (vi), applying lemma (1.2), with $x^k = \overline{x}$, the result follows. ∎

The theorem (2.3), although quite natural from a linearization viewpoint is not a trivial one with respect to the usual results in the literature. This comment sounds strange, but it is easily clarified.

If we assume that the sequence $(x^k, M^k)$ is convergent the result is a trivial one by continuity, and in this sense it is natural. But this assumption holds false in a number of cases. For instance, in the purely convex case (2.3) does not hold, as can be seen using $f(x) = x^2$, $X = \{x \in \mathrm{R} \mid -1 \leq x \leq 1\}$. In this case (convexity), the best results are

A critique to this result is that it states the convergence to a stationary point and not to an optimal solution. In order to dimish the impact of this relevant critique, we assert:

**Corollary 2.4)** In the conditions of lemma 1, if for $(MP)$ holds the property of $f(\cdot)$ uniqueness, i.e.,

$$((\bar{x},\bar{u}),(\hat{x},\hat{u})) \text{ are Kuhn–Tucker stationary points of } (MP) \Rightarrow f(\bar{x}) = f(\hat{x}),$$

then method (3.2) converges to an optimal solution of $(MP)$, in the sense of (2.3)[2]

**Proof:** It follows from the facts:

(a) (CA) and continuity $\Rightarrow$ existence of optimal solution $x^*$;

(b) constraint qualification $\Rightarrow$ necessity of Kuhn–Tucker conditions at $x^*$;

(c) $f(\cdot)$ unicity $\Rightarrow$ $[(\hat{x},\overline{M})$ is a Kuhn–Tucker stationary point $\Rightarrow g(\hat{x}) \leq 0$ and $f(\hat{x}) = f(x^*)]$.

For details, see [7]. ■

**3. An illustrative example.**

This work was motivated by the continuous capacity assignment problem in store-and-forward computer networks ([8], [5]). This problem can be stated as:

"Given a set of flows $\{\overline{f}_i\}_{i=1}^m$, find capacities (transmission speeds) $\{c_i\}_{i=1}^m$, that minimize the cost (delay) under the constraints of a maximum admissible delay (cost) and that $c_i > \overline{f}_i$, $i = 1, 2, \ldots, m$.".

The cases of interest have the following structure:

(i) the cost $D(c)$ is the sum of individual costs, i.e.,

$$D(c) = \sum^m d_i(c_i),$$

where $d_i(\cdot)$ is a concave monotonic increasing function;

(ii) the delay $T(\overline{f}, c)$ has a product form, i.e.,

$$T(\overline{f}, c) = \sum_{i=1}^{m} t_i(\overline{f}_i, c_i),$$

where $t_i(\overline{f}_i, \cdot)$ is strictly convex monotonic decreasing function;

$\lim_{x \to \overline{f}_i^+} t_i(\overline{f}_i, x) = +\infty$;

$\lim_{x \to +\infty} t_i(\overline{f}_i, x) = 0$.

In order to verify the validity of the assumptions of theorem (2.3) and lemma (1.2), besides differentiability assumption, [3] it is necessary to present an argument for the compacity assumption and the compactness of the sequence of multipliers. This is easily derived using the fact bellow.

**Fact 3.1)** Under the assumptions above listed, the feasible set for the capacity assignment problem can be substituted, by its intersection with hypercube of the form $c_i^L \leq c_i \leq c_i^v$.

**Sketch of Proof)** *(The detailed proof can be found in [6]).*

The bound on delay (cost) can be used to determine a lower (upper) bound for the capacity values, assigning all the delay (cost) to an individual channel. The strict monotonicity added to the fact that the corresponding function range is $(0, +\infty)$, gives the necessary assumptions to the existence of such limits.

In a similar way, the existence of a feasible point is used to generate the remaining bounds. For instance, in cost minimization, the delay constraint generates the lower bounds, and applying all the cost of a feasible point in each channel, it generates the upper bound. ∎

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[3] It will be assumed that $d_i(\cdot)$ and $t_i(\overline{f}_i, \cdot)$ are twice continuously differentiable on $R_{++}$ and on $\{x \mid x > \overline{f}_i\}$, respectively, with $d_i(\cdot)$ continuous at 0. This suits the usual model $d_i(x) = \ell_i x^\alpha$, $(\alpha \in (0,1))$, and $t_i(\overline{f}_i, c_i) = \overline{f}_i (c_i - \overline{f}_i)^{-1}$ (*power law costs* and *M/M/1 delays*, [8]).

One of the attractive properties of this class of problems, is that the assumption of $f(\cdot)$ unicity (central in (2.4)) is verified under a mild assumption:

**Fact 3.2)** If

$$\forall\, i = 1, \ldots, m, \;\; \forall\, c_i : c_i > \overline{f}_i, \quad d_i''(c_i)t_i'(\overline{f}_i, c_i) - d_i'(c_i)t_i''(\overline{f}_i, c_i) \neq 0,$$

then $f(\cdot)$ unicity holds for the capacity assignment problem.

**Proof:** see [6].

The above condition can be justified by economic arguments associating costs to congestion levels and it is convenient to note that it holds under the usual assumption of (as in [2], see [8]), power law costs and $M/M/1$ delays.

But the most striking point of this example is that if the condition presented in (3.2) is made slightly more strict $(d''(\cdot)t'(\cdot) - t''(\cdot)d'(\cdot) < 0)$, it corresponds to a change of variables (using delays as variables, instead of capacities, what can be done due to monotonicity) and imposing that the cost function is convex with respect to delays.

This case, when the usual assumptions holds, implies in the need of several cuts in each iteration and not just one. For instance, if we were naïve enough to apply just one outer linearization directly on the capacity assignment, it would be easy to see that we would try to get an infeasible point. For instance, under delay constraints, the resulting linear programming problem would have an optimal value associated with the unfeasible solution of all delays, but one, being equal to zero.

So this case is relevant to the statement that outer linearization methods under concavity assumptions are relevant and can not be derived from the classical results with convexity.

## 4. Conclusions.

Although it must be clear to the reader that the results hereby presented can be easily generalized ($f(\cdot)$ and $g(\cdot)$ concave or $f(\cdot)$ concave, $g_I(\cdot)$ concave, $g_J(\cdot)$ convex, and so on), the striking result of the need of just one cut at each linearized problem, under convenient concavity assumptions, stands.

The fact that in general convergence is shown to a stationary point and not necessarily to an optimal solution is quite natural and maybe this point can be partially solved using the ideas of Tuy et alii [10].

The illustrative example hereby presented (section 3) is nothing but a very particular case of designing a system with concave costs ("scale economy") and congestion effects, that correspond to models of a large class of problems. Insomuch, it is expected that a good case for the interest in such results has been made.

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