



# 14 ISAAC Congress 2023

July 2023 | Ribeirão Preto - SP Brazil

Welcome

The ISAAC board, the Local Organizing Committee and the Department of Computer Science and Mathematics at the University of São Paulo, Campus Ribeirão Preto (Brazil), are pleased to invite you to the 14th International ISAAC Congress to be held from July 17 to July 21, 2023.

The 14th International ISAAC congress continues the successful series of meetings previously held in Delaware, USA (1997), Fukuoka, Japan (1999), Berlin, Germany (2001), Toronto, Canada (2003), Catania, Italy (2005), Ankara, Turkey (2007), London, UK (2009), Moscow, Russia (2011), Krakow, Poland (2013), Macao, China (2015), Växjö, Sweden (2017), Aveiro, Portugal (2019), Ghent, Belgium (2021).

We are looking forward to welcoming you in Ribeirão Preto.

In Memory of

The Local Organising Committee

Confirmed Plenary Speakers

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Loukas Grafakos, University of Missouri, USA  
Hubert Lacoïn, IMPA, Brazil  
Irena Lasiecka, University of Virginia, USA



Marius Mantoïu, Universidad de Chile, Chile

Anna Laura Mazzucato, Penn State University, USA  
Carlos Pérez Moreno, University of the Basque Country and BCAM, Spain  
Monica Musso, University of Bath, UK  
Gustavo Ponce, University of California, Santa Barbara, USA  
Danylo Radchenko, University of Lille, France

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# Welcome

It is a pleasure to welcome you to the *14th ISAAC Congress - 2023* and to Ribeirão Preto. We wish you a pleasant stay and that you enjoy the meetings.

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Pedro Tavares Paes Lopes & Marcone Corrêa Pereira, (IME-USP, Brazil): Evolution Equations and Dynamical Systems.

Mitsuru Sugimoto (Nagoya University, Japan) & Baoxiang Wang (Peking University, China): Function Spaces and their Applications to Nonlinear Evolutional Equations.

Michael Ruzhansky (Ghent University, Belgium), Jens Wirth (University of Stuttgart, Germany) & Vladimir Georgiev (University of Pisa, Italy): Harmonic Analysis and Partial Differential Equations.

Lucas Oliveira (UFRGS, Brazil) & Tiago Picon (USP, Brazil): Harmonic Analysis and Related Topics.

Zouhaïr Mouayn (Université Sultan Moulay Slimane de Beni-Mellal, Morocco): Integral Transforms and Reproducing Kernels.

Anahit Galstyan (University of Texas RGV, USA), Makoto Nakamura (Osaka University, Japan) & Karen Yagdjian (University of Texas RGV, USA): Partial Differential Equations on Curved Spacetime.

Lucas Catão de Freitas Ferreira (UNICAMP, Brazil), Nikolai Vasilievich Chemetov (USP, Brazil), Gabriela Planas (UNICAMP, Brazil) & Anna Laura Mazzucato (Penn State University, USA): PDEs in Fluid Mechanics.

Man Wah Wong (York University Toronto, Canada): Pseudo Differential Operators.

Swanhild Bernstein (TU Bergakademie Freiberg, Germany), Uwe Kähler (University of Aveiro, Portugal), Irene Sabadini (Politecnico di Milano, Italy) & Franciscus Sommen (Ghent University, Belgium): Quaternionic and Clifford Analysis.

Marcello D'Abbicco (University of Bari, Italy), Michael Reissig (TU Bergakademie Freiberg, Germany) & Alessandro Palmieri (University of Bari, Italy): Recent Progress in Evolution Equations.

Benedetta Ferrario (University of Pavia, Italy), Rafael Andres Rosales Mitrowsky (USP, Brazil), Fernanda Cipriano (Universidade Nova de Lisboa, Portugal) & Fernando Piguard de Almeida Prado (USP, Brazil): Stochastic Processes.

Ademir Pastor (UNICAMP, Brazil), Felipe Linares (IMPA, Brazil), Luiz Gustavo Farah (UFMG, Brazil) & Mahendra Panthee (UNICAMP, Brazil): Trends In Nonlinear Dispersive Equations.

Keiko Fujita ( University of Toyama, Japan) & Akira Morimoto (Osaka Kyoiku Toyama, Japan): Wavelet theory and its Related Topics

## Address

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**14 ISAAC Congress 2023**  
**Chapter**

# Maps

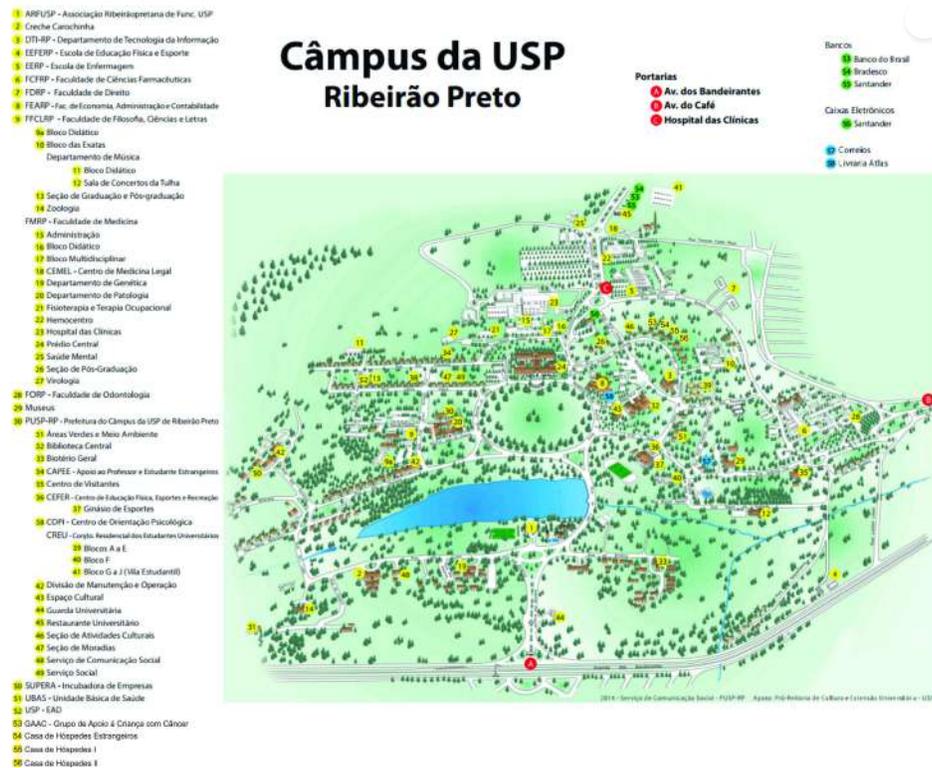


Figure 1: Campus map

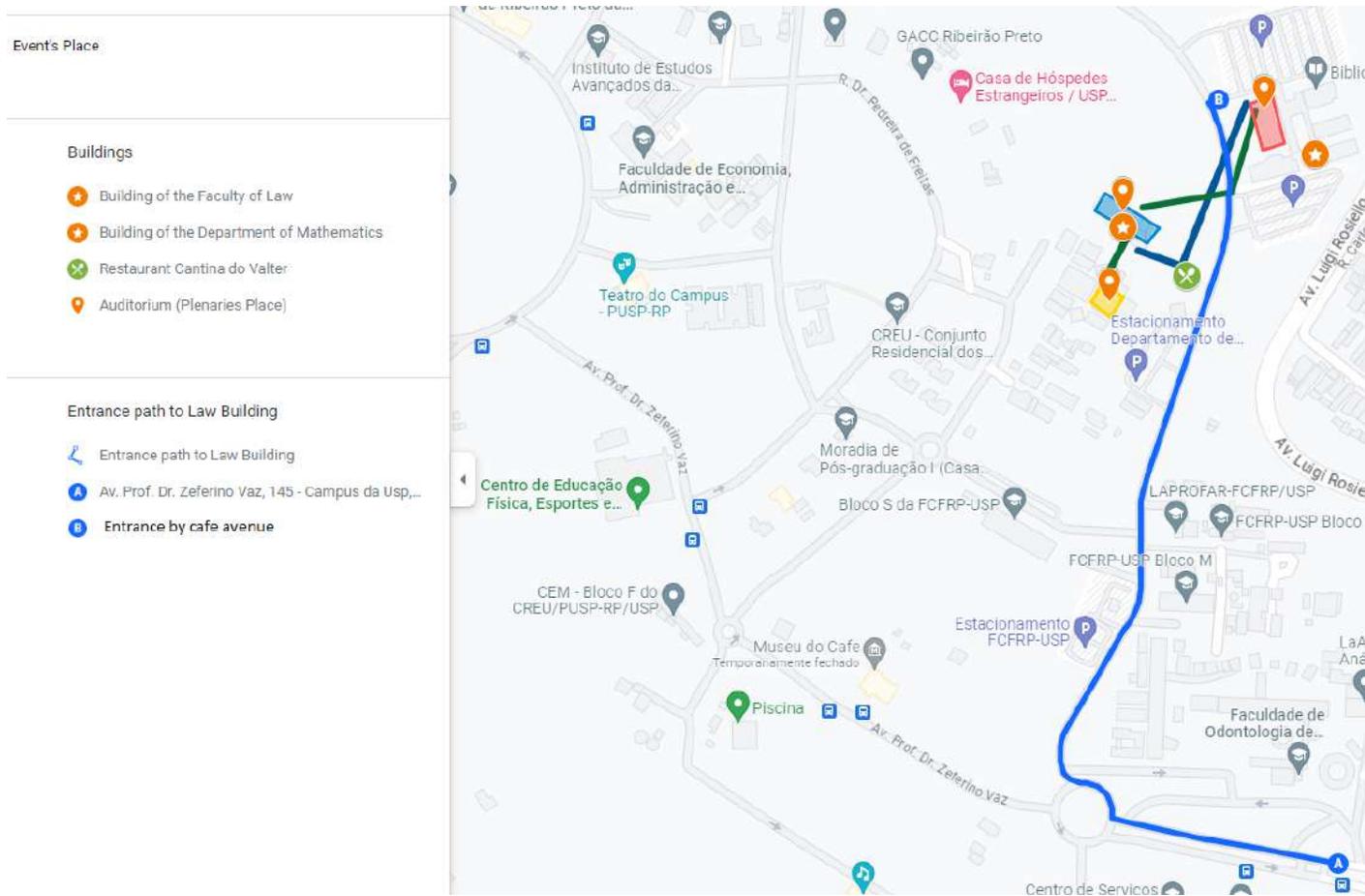


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General Information

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## Conference site

The meeting will take place in the Faculty of Law, in the Department of Computer Science and Mathematics (Building *B1*), and in the Lectures rooms of the Exact Sciences (Building *B4*). These buildings are located in zones 7 and 10 in the map on page 6.

Plenary talks will take place at the Auditorium of the Faculty of Law.

The Thematic sessions will take place in the Buildings *B1* and *B4*.

The Coffee Room is in front of the Auditorium.

## Registration

The Events Office will set up a help desk at the entrance of the Auditorium of the Faculty of Law and will be at your disposal for any questions and information.

The registrations will be made in the following schedule:

*On Monday, July 17, from 8:00 to 10:30 am.*

Those who cannot register on Monday can also do it during the mornings coffee break.

We will provide you with a badge at registration. Please wear your badge at the event to access the event rooms.

## Registration Fees

ISAAC members: 90 euros.

Non-members: 100 euros.

Student Fee and participants from developing countries: 40 euros (200 reais).

## Financial support

The financial support from the local organizing committee will be available on Wednesday, July 19 from 1 pm to 3 pm, at the help desk at the entrance of the Auditorium of the Faculty of Law. In order to receive your support, it is mandatory to completely fill out the on-line registration form available at [https://dcm.ffclrp.usp.br/isaac/pg\\_registration.php](https://dcm.ffclrp.usp.br/isaac/pg_registration.php)

## Meals and refreshments

There is a canteen "do Valter" on the campus, located between the main Auditorium and the Thematic session rooms, where you can have either snacks or lunch. Moreover, there are several restaurants near the campus at Avenida do Café.

## Social events

*Wednesday, July 19:* Photo of the meeting at 11:30 am.

*Wednesday, July 19:* Conference Banquet at 8:00 pm at Cabaña Restaurant - Argentinian Restaurant at Praça Boaventura Ferreira da Rosa, 218.

## Health emergencies

In case of accidents or health emergencies call 192 (SAMU).

## Money exchanges

In case you need to exchange your money, we recommend:

- Travelex Confidence Câmbio at Ribeirão Shopping . The working hours are from 10:00 am to 8:00 pm (Mon-Fri) and from 10:00 am to 6:00 pm (Sat).

## Smoking

Smoking is prohibited inside any of the USP buildings also in the canteen and on the ground floor of the library.

## Computer and wireless LAN use

The University provides access to wireless internet connection via **Eduroam**.

If you do not possess an Eduroam account you can use the wireless connection USPnet. A personal User Name and Password will be given during the Registration (included in your badge).

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Plenary Lectures

## PLENARY LECTURES

### **Mixing in fluids: irregular transport, enhanced dissipation, and applications**

**Anna Mazzucato**

Pennsylvania State University

Stirring and mixing in fluids, specifically incompressible fluids, have important consequences on many physical and biological processes, from dispersal of pollutants to transport of nutrients. From a mathematical point of view, mixing can be studied in different contexts, from ergodic theory to homogenization.

In this talk, I will present a quantitative approach to mixing that arises in the analysis of partial differential equations. In this context, mixing is related to irregular transport by non-Lipschitz vector fields and, when combined with diffusion, it may lead to enhanced dissipation. A variety of techniques have been employed in the literature to study these mechanisms, from geometric analysis to optimal transport to spectral theory and probability.

I will first discuss examples of incompressible flows that mix optimally in time. Then, I will show how these examples lead to loss of regularity for solutions of transport equations. Lastly, I will discuss enhanced dissipation and examples of flows that lead to enhanced dissipation for advection-(hyper)diffusion equations using resolvent estimates.

### **Extensions of Sobolev inequalities through Harmonic Analysis**

**Carlos Pérez Moreno**

University of the Basque Country and BCAM

In the first part of the lecture I will discuss a different approach to derive Poincaré-Sobolev estimates on cubes which contains the classical John-Nirenberg theorem. Poincaré-Sobolev estimates yield immediately global estimates but, in the second part of the lecture, I will present some new extensions of the global classical Sobolev type inequalities for linear and non-linear operators which are not available at local level.

### **From energy minimization to Fourier uniqueness pairs**

**Danylo Radchenko**

University of Lille

I will talk about the recent results on Fourier interpolation and uniqueness pairs that have appeared in connection with the sphere packing and energy minimization problems. These Fourier interpolation formulas allow to reconstruct an arbitrary Schwartz function from discrete samples of the function and its Fourier transform. Some such interpolation formulas with very good properties can be constructed with the help of modular forms, and I will explain how they apply to the Cohn-Kumar conjecture on energy minimization problem in 8 and 24 dimensional Euclidean spaces. Then I will put these results into a more general context of Fourier uniqueness pairs, and survey recent constructions of minimal Fourier uniqueness pairs and general analytic results.

## The asymptotic behavior of solution to the BO equation

Gustavo Ponce, Ricardo Freire, Felipe Linares, Claudio Muñoz  
University of California-Santa Barbara

We consider the long time dynamics of large solutions to the Benjamin-Ono equation. Using virial techniques, we describe regions of space where every solution in a suitable Sobolev space must decay to zero along sequences of times. Moreover, in the case of exterior regions, we prove complete decay for any sequence of times. The remaining regions not treated here are essentially the strong dispersion and soliton regions.

## Anisotropic motion by curvature obtained as a scaling limit of Glauber dynamics.

Hubert Lacoïn, François Simenhaus, Fabio Toninelli  
IMPA

In this talk we will study a random dynamic in discrete time on the set of spin configuration in a square box of diameter  $L$ ,  $\Omega_L := \{\sigma : \{1, \dots, L\}^2 \rightarrow \{-1, 1\}\}$  It has the following rules of evolution: -At each step, the spin of a random site is updated. -The update of a spin at a given site is made by looking at the spin of its neighbors. The updated site assumes the same spin value as the majority of its neighbors. Ties are broken by throwing a fair coin.

Starting from a all minus configuration with plus boundary condition, we want to investigate the time at which the last minus spin disappears. This leads us to the study of the scaling limit of the set of minus spins.

## Can we control a flutter in flow-structure interactions? How and where?

Irena Lasiecka  
University of Memphis

Flow-structure interactions are ubiquitous in nature. Problems such as attenuation of turbulence or flutter in an oscillating structure [Tacoma bridge], flutter in tall buildings, fluid flows in flexible pipes, in nuclear engineering flows about fuel elements and heat exchanger vanes -are prime examples of relevant applications. Mathematically, the models are represented by a 3 D compressible, irrotational Euler Equation coupled to a **nonlinear** dynamic elasticity on a 2 D manifold. Strong boundary-type coupling at the interface between the two media is at the center of the analysis. This provides for a rich mathematical structure, opening the door to several unresolved problems in the area of nonlinear PDE's, dynamical systems, related harmonic analysis and differential geometry. This talk aims at providing a brief overview of recent developments in the area along with a presentation of some recent advances addressing the issues of control and long time behavior of such models.

Part of this talk is based on recent work with D. Bonheur, F. Gazzola and J. Webster :*Annales de l'Institut Henri Poincaré Analyse*, 2022 and with A. Balakrishna, J. Webster *Math. Models Methods Appl. Sciences*, 2023. The work was partially performed while the author was a member of the MSRI program "Mathematical problem in fluid dynamics" at the University of California Berkeley during the Spring 2021 semester (NSF DMS -1928930).

## Maximal averages with respect to balls and spheres

Loukas Grafakos, Georgios Dosidis  
University of Missouri

We study a family of maximal operators that provides a continuous link between the Hardy-Littlewood maximal function and the spherical maximal function. For this family of operators we obtain bounds between Lebesgue spaces in the optimal range of exponents.

## Leapfrogging for Euler equations

Monica Musso  
University of Bath

We consider the Euler equations for incompressible fluids in 3-dimension. A classical question that goes back to Helmholtz is to describe the evolution of vorticities with a high concentration around a curve. The work of Da Rios in 1906 states that such a curve must evolve by the so-called "binormal curvature flow". Existence of true solutions whose vorticity is concentrated near a given curve that evolves by this law is a long-standing open question that has only been answered for the special case of a circle travelling with constant speed along its axis, the thin vortex-rings, and of a helical filament, associated to a translating-rotating helix. In this talk I will consider the case of two vortex rings interacting between each other, the so-called leapfrogging. The results are in collaboration with J. Davila (U. of Bath), M. del Pino (U. of Bath) and J. Wei (U. of British Columbia).

## Stochastic wave equations with constraints: well-posedness and Smoluchowski-Kramers diffusion approximation

Zdzisław Brzezniak  
University of York

I will discuss the well-posedness of a class of stochastic second-order in time damped evolution equations in Hilbert spaces, subject to the constraint that the solution lies on the unitary sphere. A specific example is provided by the stochastic damped wave equation in a bounded domain of a  $d$ -dimensional Euclidean space, endowed with the Dirichlet boundary conditions, with the added constraint that the  $L^2$ -norm of the solution is equal to one. We introduce a small mass  $\mu > 0$  in front of the second-order derivative in time and examine the validity of the Smoluchowski-Kramers diffusion approximation. We demonstrate that, in the small mass limit, the solution converges to the solution of a stochastic parabolic equation subject to the same constraint. We further show that an extra noise-induced drift emerges, which in fact does not account for the Stratonovich-to-Itô correction term. This talk is based on joint research with S. Cerrai (Maryland).

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Thematic Sessions

## APPLICATIONS OF DYNAMICAL SYSTEMS THEORY IN BIOLOGY

**Organizer:** Torsten Lindström (Linnaeus University, Sweden)

### Global dynamics of delay equations

**Ábel Garab**

University of Szeged, Hungary

In this talk we mainly focus on the global dynamics of delay difference equations of the form

$$x_{k+1} = g(x_{k-d}, x_k).$$

Many discrete-time population dynamical models, such as the ones by Ricker, Pielou, Mackey–Glass, Wazewska–Lasota, and Clark, fit into this framework.

First we consider some special cases, when a unique positive equilibrium exists and its local stability implies its global stability. The proofs are based on a combination of analytic and rigorous, computer-assisted methods.

In the general setting we give sufficient criteria for the existence of the global attractor and obtain some structural results of it. More precisely, we give a so-called Morse decomposition of the global attractor based on an integer valued Lyapunov function introduced by J. Mallet-Paret and G. Sell.

We also show some analogues of the latter results for (systems of) delay differential equations.

### Piecewise contractions of the interval and its applications

**Benito Pires**

University of Sao Paulo

Many discrete-time continuous-state real-world systems have their dynamics governed by a piecewise contraction of the interval. The dynamics of such maps is very rich and it consists of finitely many periodic attractors and Cantor attractors. Such maps may also present sensitive dependence on initial conditions. In this talk, the author will provide a panorama of the dynamics of piecewise contractions and its applications.

### Biased movements of mixed populations: the influence of convection in the existence of wavefronts

**Andrea Corli**, Diego Berti, Luisa Malaguti

Ferrara

We investigate a model describing the movement of a biological population consisting of isolated and grouped organisms. By the introduction of biases in the movements one obtains a scalar reaction-diffusion equation, which includes a convective term as a consequence of the biases, of the following form:

$$u_t + f(u)_x = (D(u)u_x)_x + g(u), \quad t \geq 0, x \in \mathbf{R},$$

for suitable diffusivity  $D$ , reaction term  $g$  and convection  $f$ . The focus is on the case the diffusivity makes the parabolic equation of *forward-backward-forward* type and the reaction term models a *strong*

*Allee effect*, with the Allee parameter lying between the two internal zeros of the diffusion. Several other cases can be treated as well.

In the case under consideration, the unbiased equation (i.e., without convection) possesses *no* smooth traveling-wave solutions; on the contrary, in the presence of convection, traveling-wave solutions are shown to exist for some significant choices of the parameters. The study of the sign of the traveling-wave speeds is also taken into account; it provides information on the long term behavior of the population, namely, its survival or extinction.

Our results are proved by applying suitable techniques about the existence of traveling waves for parabolic equations with negative diffusivity that we developed in previous papers; they will be briefly presented in the talk.

This is a joint work with Diego Berti (Department of Mathematics, University of Turin, Italy) and Luisa Malaguti (Department of Sciences and Methods for Engineering, University of Modena and Reggio Emilia, Italy).

## Hopf bifurcation made simple for some scalar DDEs

Gergely Röst, István Balázs  
University of Szeged

We show that for a class of scalar delay differential equations, that includes for example the delayed logistic equation, the criticality of Hopf-bifurcations reduces to a very simple condition and we do not need to calculate the complicated first Lyapunov coefficient. We characterize all possible bifurcation sequences as we increase the delay. Moreover, estimates are given for the period of the bifurcating periodic solutions. We also show that all Hopf bifurcations of the Nicholson's blowfly equation are supercritical.

## On the stochastic engine of transmittable diseases in exponentially growing populations

Torsten Lindström  
Linnaeus University

The purpose of this paper is to analyze the mechanism for the interplay of deterministic and stochastic models for transmittable diseases. Deterministic models for transmittable diseases are prone to predict global stability. Small natural birth and death rates in comparison to disease parameters like the contact rate and the removal rate ensures that the globally stable endemic equilibrium corresponds to a tiny average proportion of infected individuals. Asymptotic equilibrium levels corresponding to low numbers of individuals invalidate the deterministic results.

Diffusion effects force frequency functions of the stochastic model to possess similar stability properties as the deterministic model. Particular simulations of the stochastic model predict, however, oscillatory patterns. Small and isolated populations show longer periods, more violent oscillations, and larger probabilities of extinction.

We prove that evolution maximizes the infectiousness of the disease as measured by the ability to increase the proportion of infected individuals. This holds provided the stochastic oscillations are moderate enough to keep the proportion of susceptible individuals near a deterministic equilibrium.

We close our paper with a discussion of the herd-immunity concept and stress its close relation to vaccination-programs.

## COMPLEX GEOMETRY

**Organizer:** Alexander Schmitt (Freie Universität Berlin, Germany)

### **New results on hyperpolygons and moduli space of parabolic Higgs bundles.**

**Alessia Mandini**

Universidade Federal Fluminense

Hyperpolygons spaces are a family of hyperkähler quiver varieties that can be obtained by hyperkähler reduction of a finite number of  $SU(2)$ -coadjoint orbits. Jointly with L. Godinho, we showed that these spaces are isomorphic to moduli spaces of rank 2, holomorphically trivial parabolic Higgs bundles over  $P^1$ , with fixed determinant and trace-free Higgs field, when a suitable condition between the parabolic weights and the spectra of the coadjoint orbits is satisfied. In this talk I will describe some recent works, including on-going work, that generalize this construction in several ways. The talk is based on joint works with L. Godinho, and with I. Biswas and C. Florentino and L. Godinho.

### **Harder-Narasimhan theory for gauged maps**

**Andres Fernandez Herrero**, Daniel Halpern-Leistner

Columbia University

Many objects of interest in algebraic geometry are parametrized by algebraic varieties, called moduli spaces.

In this talk I will discuss recent techniques developed to construct moduli spaces for the moduli of decorated principal bundles on a fixed compact Riemann surface. Using these techniques, we construct a Harder-Narasimhan stratification, which can be used to obtain a generalization of the Verlinde formula in the context of decorated principal bundles.

This talk is based on joint work with Daniel Halpern-Leistner.

### **Some results about the moduli spaces $\mathcal{M}_g$ and $\mathcal{A}_g$**

**Anita M. Rojas**, Sebastián Reyes-Carocca

Universidad de Chile

Moduli spaces of Riemann surfaces  $\mathcal{M}_g$  and of abelian varieties  $\mathcal{A}_g$  are central objects of research in Complex Geometry. Since they are related by the Torelli map, it is natural to search for results about  $\mathcal{A}_g$  using what is known for  $\mathcal{M}_g$ . This approach is particularly fruitful when considering group actions. There are famous bounds for several concepts regarding the geometry of group actions on compact Riemann surfaces of genus  $g \geq 2$ : The well known Hurwitz bound  $84(g-1)$ , which is attained for infinite genera, and also in infinitely many others not. Another remarkable bounds are Wiman's bound  $4g+2$ , which is attained for every  $g$ , and the Accola-Maclachlan bound  $8g+8$ . Following Accola-Maclachlan idea, other geometrically meaningful bounds are obtained; such as  $4g+1$  and  $4g-4$ . In this talk we will explain the question that motivates the search of these bounds and discuss extensions of these ideas.

Specifically, we consider a compact Riemann surface  $X$  of genus  $g \geq 2$  and  $T \in \text{Aut}(X)$  an automorphism of large prime order  $q > g$ . It is known that either  $q = 2g+1$ , and this gives

Lefschetz surfaces, or  $q = g + 1$ . We are interested in the understanding of Riemann surfaces, and in their corresponding Jacobian varieties, in this second case. That is to provide a classification of those Riemann surfaces and some descriptions of the corresponding Jacobian varieties, in terms of decompositions and other properties. In particular, we compute the Riemann matrix of the Accola-Maclachlan curve of genus 4.

Part of this work is in collaboration with Sebastián Reyes-Carocca, from Universidad de Chile.

## G-bundles on singular curves and conformal blocks

**Avery Wilson**

Kenyon College

I will talk about the problem of compactifying spaces of  $G$ -bundles on a projective nodal curve. The stack of  $G$ -bundles on a singular curve is not "complete," and it is unclear what types of objects should be used to fill out the boundary. I will discuss one approach to compactification using conformal blocks, which are representation theoretic objects arising from infinite dimensional Lie algebras. In recent work I showed that, when  $G$  is a type A or C simple Lie group, the Proj of the conformal blocks algebra is a projective variety closely related to Schmitt and Muñoz-Castañeda's moduli space of singular  $G$ -bundles.

## Degenerations of moduli spaces of principal bundles over projective curves

**Angel Luis Muñoz Castañeda**

Universidad de León

The study of degenerations of moduli spaces of bundles over projective curves goes back to Igusa and have encounter applications in theoretical physics, like in F- Theory (Witten, Friedman and Morgan) and in Theory of Conformal Blocks (Tsuchiya, Ueno and Yamada).

The approach to the degeneration of moduli spaces of bundles on projective curves,  $\text{Bun}(X)$ , can be presented in three different ways. 1) Given a flat family of stable curves of genus  $g$ ,  $C$ , over a DVR,  $A$ , whose generic fiber  $X$  is smooth, find a flat algebraic variety  $\overline{\text{Bun}}(C)$  over  $A$  whose generic fiber is  $\text{Bun}(X)$ . 2) Given  $g > 2$ , find an algebraic variety  $\overline{\text{Bun}}_g$  proper and flat over  $\overline{M}_g$  whose fibers on smooth curves  $X$  coincide with  $\text{Bun}(X)$ . 3) If a compactification  $\overline{\text{Bun}}(X)$  of  $\text{Bun}(X)$  is known for any stable curve  $X$ , find an algebraic variety  $\overline{\text{Bun}}_g$  over  $\overline{M}_g$  whose fibers over smooth curves  $X$  coincide with  $\text{Bun}(X)$  and whose fibers on singular curves  $X$  coincide with  $\overline{\text{Bun}}(X)$ .

Although much is known in the case of vector bundles, little were known for principal  $G$ -bundles ( $G$  being an arbitrary semisimple algebraic group) until very recently. The first two problems have been tackled by Balaji and Wilson respectively, while the third has been addressed by the speaker and Schmitt on the basis of the theory of singular principal bundles. In this talk, I will expose the results concerning this last problem and, if time permits, raise some questions regarding this topic.

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2. Muñoz Castañeda, A. L. "On the moduli spaces of singular principal bundles on stable curves". Advances in Geometry **20**, pp. 573-584 (2020).

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## A Kobayashi-Hitchin correspondence for $\rho$ -coherent systems

**Cesare Goretti**  
Freie Universität

$\rho$ -coherent systems arise as a natural generalization of coherent system, which had been deeply studied in the early 2000. Using the techniques developed by Bradlow, Garcia-Prada and Mundet i Riera for principal pairs we are going to prove a Kobayashi-Hitchin correspondence for  $\rho$ -coherent systems, with that is meant that there is a link between existence of solutions to a set of differential equations and stability of the  $\rho$ -coherent system. Furthermore, we are going to show that this notion of stability is equivalent to the one that arises from GIT, which has been defined recently by Schmitt.

## Complex Convexity

**Christer Oscar Kiselman**  
Uppsala University

Complex convexity is a variant of the more well-known concept of real convexity. I will present complex convexity, in particular lineal convexity, which I learnt about from André Martineau (1930–1972). There are no less than eleven variants of lineal convexity, all different. A lineally convex set can be the union of an increasing family of lineally convex sets with smooth boundary, while others are not of that kind. It is of interest to find sufficient conditions for this to happen, and also to find necessary conditions—ideally a condition which is both necessary and sufficient.

## Finiteness properties of the subalgebra of invariant elements

**Francisco J Plaza-Martin**, Jesús Martín Ovejero, Ángel Luis Muñoz Castañeda  
Universidad de Salamanca

Hilbert's 14th problem is concerned with the properties of the sub algebra of invariant elements  $A^G$  of an  $\mathbb{k}$ -algebra acted on by an algebraic group  $G$ . This is a crucial problem in classical invariant theory that has been studied by several authors (e.g. Shur, Weil, Nagata, Seshadri, etc.) and that, thanks to Nagata's result, has unveiled the relevance of reductive groups and its role in geometric invariant theory. In this paper, we study this problem in a relative and non-noetherian context.

Let us be more precise. Let  $R$  denote an algebra over a ring  $\mathbb{k}$ ,  $T$  an  $R$ -algebra,  $M$  a finitely generated projective  $R$ -module and  $N$  a  $T$ -module. Let  $G$  be a linearly reductive group scheme over  $\mathbb{k}$  together with a representation  $\rho : \underline{G}$

Then, we focus on two issues: first, determine under which conditions the graded  $T$ -algebra  $A$  is finitely generated, finitely presented or flat; and second, determine under which

## Atiyah sequences, connections and Chern-Weil theory for principal bundles over smooth stacks

**Frank Neumann**  
University of Pavia

We present a theory of general and integrable connections on smooth stacks, as well as on principal bundles over them using Atiyah exact sequences of vector bundles associated to transversal tangential distributions. Finally, we develop the corresponding Chern-Weil theory and describe associated characteristic classes. Joint work with I. Biswas (TIFR Mumbai), S. Chatterjee (IISER Kerala) and P. Koushik (IISER Pune).

## Instanton bundles on contact Fano manifolds

**Gaia Comaschi**  
IMECC-UNICAMP

Instanton bundles originally appeared in the context of Yang-Mills gauge theory. In their seminal work Atiyah, Drinfeld, Hitchin and Manin established a correspondence between the Both twistor geometry and Yang Mills theory can be generalized to a  $4n$ -dimensional *Quaternion Kähler* manifold  $M$ ; this allows us to define the notion of instanton bundle on the twistor space  $Z$ , a so called *contact Fano* manifold. In this talk I will recall some features of instantons on contact Fano manifolds and present some properties of these bundles and of their moduli in the case  $M = G_2/SO(4)$  and  $Z = G_2/U(1) \cdot SU(2)$ .

## The field of moduli of plane curves

**Giulio Bresciani**  
Scuola Normale Superiore di Pisa

It is a classical fact going back to F. Klein that an elliptic curve  $E$  over  $\bar{\mathbb{Q}}$  is defined by a homogeneous polynomial in 3 variables with coefficients in  $\mathbb{Q}(j_E)$ , where  $j_E$  is the  $j$ -invariant of  $E$ , and  $\mathbb{Q}(j_E)$  is the *field of moduli* of  $E$ . The general definition of field of moduli goes back to T. Matsusaka and G. Shimura: loosely speaking, it is the smallest field where we might hope that the curve is defined.

We prove that every smooth plane curve of degree prime with 6 is defined by a homogeneous polynomial with coefficients in the field of moduli. Furthermore, we show that most curves in arbitrary degree, and more generally most algebraic cycles with finite automorphism group, descend to a Brauer-Severi surface over the field of moduli.

## On the splitting type of the holomorphic vector bundles on Riemann sphere

**Grigori Giorgadze**  
Tbilisi State University

We prove that for any Fuchsian system of differential equations on the Riemann sphere, there exists a rational matrix function whose partial indices coincide with the splitting type of the canonical vector bundle induced from the Fuchsian system. From this, we obtain solution of the Riemann-Hilbert boundary value problem for piecewise constant matrix function in terms of holomorphic sections of vector bundle and calculate the partial indices of the problem.

Using this technique we indicate relationship between Holder continuous matrix functions and moduli space of vector bundles on the Riemann sphere. For second order systems with three singular points we give complete characterization of corresponding vector bundles by invariants of Fuchsian system.

The talk based on recently papers [1], [2].

**Acknowledgment.** The research supported in part by GNSF project "Problem of factorization and invariants of holomorphic bundles on Riemann surfaces", Grant agreement ID: FR-22-354

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2. G.Giorgadze. G.Gulagashvili. Riemann-Hilbert Boundary Value Problem with Piecewise Constant Transition Function. *Journal of Dynamical and Control Systems*. 28(8), 1-11, 2022. <https://doi.org/10.1007/s10883-020-09524-z>

## On slice regular Bergman space and fiber bundle theory

**José Oscar González Cervantes**

Instituto Politécnico, Escuela Superior de Física y Matemáticas

Recently, the theory of fiber bundle has been used to explain several phenomena in the theory of slice regular functions, e.g., the slice regular functions is the base space of a fiber bundle, the slice regular functions are defined on the total space of some sphere bundle and finally the theory of slice regular functions in several quaternionic variables is well-defined using the theory fiber bundle. The purpose of this talk is to explain some properties of the Bergman space of slice regular functions in terms of the fiber bundle theory.

## A Donaldson-Uhlenbeck type compactification of the Moduli Space for Singular Principal G-Bundles

**Juan Martin Perez Bernal**

Freie Universität

In this talk, we will discuss our work on the construction and projectivity of the Donaldson-Uhlenbeck moduli space for singular principal G-bundles. We will adapt Langton's argument to the setting of singular principal G-bundles to prove its projectivity. Additionally, we will explore the technical difficulties concerning the Quot-Chow morphism, more precisely, the non-connectedness of the fibres of this morphism is the biggest obstacle for defining the classical Donaldson-Uhlenbeck moduli space.

## **$BC_n$ Matroids and torus invariant subvarieties of the Symplectic Grassmannian of Isotropic planes.**

**Pedro Luis del Ángel Rodríguez**, E. Javier Elizondo H., Cristhian Garay López, Felipe Zaldivar Cruz  
CIMAT

Let  $\text{SpG}(2, 2n)$  be the complex symplectic Grassmannian of affine isotropic planes in  $2n$ -space. We study the problem of characterizing the set of  $T$ -invariant algebraic subvarieties of  $\text{SpG}(2, 2n)$  for the maximal torus  $T$ .

As a by-product we obtain irreducibility of the thin symplectic Schubert cells as well as some representability criterion for  $BC_n$ -matroids.

## **Seiberg-Witten differentials on the Hitchin base**

**Peter Dalakov**, Ugo Bruzzo  
American University in Bulgaria

In this talk I will report on a recent work with Ugo Bruzzo (SISSA). We describe explicitly, in terms of Lie theory and cameral data, the covariant (Gauss–Manin) derivative of the Seiberg–Witten differential defined on the weight-one variation of Hodge structures that exists on a Zariski open subset of the base of the Hitchin fibration.

## **On group actions on Riemann surfaces and Weierstrass points**

**Sebastián Reyes-Carocca**, Pietro Speziali  
Universidad de Chile

In this talk, we will address the general problem of finding compact Riemann surfaces with automorphism group acting transitively on the set of Weierstrass points. We will discuss known results and open problems, and some recent progress.

## **Ricci-flat metrics on canonical bundles**

**Ugo Bruzzo**  
SISSA

Calabi described all (possibly singular) extremal Kähler metrics on compact surfaces with underlying  $S^2 \times S^2$  topology. They include a 2-parameter family of singular Kähler-Einstein metrics, which was rediscovered by Gibbons and Pope and further studied by Gauntlett, Martelli, Sparks and Waldram. Abreu gave a neat description of Calabi's full 4-parameter family using symplectic techniques. In this talk we describe an enhancement of Abreu's formalism and apply it to study the 2-parameter family of Kähler-Einstein metrics together with their degenerations and special cases, also revisiting Calabi's trick to construct Ricci-flat metrics on the total spaces of the canonical bundles of (possibly singular) Kähler-Einstein surfaces. The formalism is quite effective and produces explicit formulas.

## COMPLEX VARIABLES AND POTENTIAL THEORY

**Organizer:** Tahir Aliyev Azeroglu (Istanbul Arel University, Turkey) & Massimo Lanza de Cristoforis (University of Padua, Italy) & Anatoly Golberg (HIT Holon Institute of Technology, Israel) & Sergiy Plaksa (Institute of Mathematics of the National Academy of Sciences of Ukraine, Ukraine)

### Homeomorphisms of finite area distortion

**Anatoly Golberg**, Elena Afanas'eva  
Holon Institute of Technology

We discuss the interplay of mappings of finite area distortion (FAD) with finitely bi-Lipschitz mappings, ring and lower  $Q$ -homeomorphisms, and absolutely continuous homeomorphisms of the class  $AC_{\Lambda}^{n,p}$  on Riemannian manifolds. Some additional relations to the hyper  $Q$ -homeomorphisms,  $\eta$ -quasisymmetric and  $\omega$ -quasimöbius mappings are also presented. As applications of the above results, we provide several extension conditions to the weakly flat and strongly accessible boundaries under FAD-homeomorphisms.

### On the order of growth of ring $Q$ -homeomorphisms

**Bogdan Klishchuk**, Ruslan Salimov  
Institute of Mathematics of NAS of Ukraine

Let  $\Gamma$  be a family of curves  $\gamma$  in  $\mathbb{R}^n$ ,  $n \geq 2$ . A Borel measurable function  $\rho : \mathbb{R}^n \rightarrow [0, \infty]$  is called *admissible* for  $\Gamma$ , (abbr.  $\rho \in \text{adm } \Gamma$ ), if

$$\int_{\gamma} \rho(x) ds \geq 1$$

for any curve  $\gamma \in \Gamma$ . Let  $p \in (1, \infty)$ . The quantity

$$M_p(\Gamma) = \inf_{\rho \in \text{adm } \Gamma} \int_{\mathbb{R}^n} \rho^p(x) dm(x)$$

is called  *$p$ -modulus* of the family  $\Gamma$ .

For arbitrary sets  $E$ ,  $F$  and  $G$  of  $\mathbb{R}^n$  we denote by  $\Delta(E, F, G)$  a set of all continuous curves  $\gamma : [a, b] \rightarrow \mathbb{R}^n$ , that connect  $E$  and  $F$  in  $G$ , i.e., such that  $\gamma(a) \in E$ ,  $\gamma(b) \in F$  and  $\gamma(t) \in G$  for  $a < t < b$ .

Let  $D$  be a domain in  $\mathbb{R}^n$ ,  $n \geq 2$ ,  $x_0 \in D$  and  $d_0 = \text{dist}(x_0, \partial D)$ . Set

$$\mathbb{A}(x_0, r_1, r_2) = \{x \in \mathbb{R}^n : r_1 < |x - x_0| < r_2\},$$

$$S_i = S(x_0, r_i) = \{x \in \mathbb{R}^n : |x - x_0| = r_i\}, \quad i = 1, 2.$$

Let a function  $Q : D \rightarrow [0, \infty]$  be Lebesgue measurable. We say that a homeomorphism  $f : D \rightarrow \mathbb{R}^n$  is ring  $Q$ -homeomorphism with respect to  $p$ -modulus at  $x_0 \in D$ , if the relation

$$M_p(\Delta(fS_1, fS_2, fD)) \leq \int_{\mathbb{A}} Q(x) \eta^p(|x - x_0|) dm(x)$$

holds for any ring  $\mathbb{A} = \mathbb{A}(x_0, r_1, r_2)$ ,  $0 < r_1 < r_2 < d_0$ ,  $d_0 = \text{dist}(x_0, \partial D)$ , and for any measurable function  $\eta : (r_1, r_2) \rightarrow [0, \infty]$  such that

$$\int_{r_1}^{r_2} \eta(r) dr = 1.$$

Denote by  $\omega_{n-1}$  the area of the unit sphere  $\mathbb{S}^{n-1} = \{x \in \mathbb{R}^n : |x| = 1\}$  in  $\mathbb{R}^n$  and by  $q_{x_0}(r) = \frac{1}{\omega_{n-1} r^{n-1}} \int_{S(x_0, r)} Q(x) d\mathcal{A}$  the integral mean over the sphere  $S(x_0, r) = \{x \in \mathbb{R}^n : |x - x_0| = r\}$ ,

here  $d\mathcal{A}$  is the element of the surface area. Let  $L(x_0, f, R) = \sup_{|x-x_0| \leq R} |f(x) - f(x_0)|$ .

**Theorem.** *Let  $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$  be a ring  $Q$ -homeomorphism with respect to  $p$ -modulus at the point  $x_0$  for  $p > n$ , where  $x_0$  is some point in  $\mathbb{R}^n$ . If for some numbers  $c > 0$ ,  $0 \leq \kappa \leq p$ ,  $r_0 > 0$  the condition*

$$\int_{\mathbb{A}(x_0, r_0, R)} Q(x) \psi^p(|x - x_0|) dm(x) \leq c I^\kappa(r_0, R) \quad \forall R > r_0$$

holds, where  $\psi(t)$  is a nonnegative Lebesgue measurable function at  $(0, +\infty)$  such that

$$0 < I(r_0, R) = \int_{r_0}^R \psi(t) dt < \infty \quad \forall R > r_0,$$

then

$$\liminf_{R \rightarrow \infty} L(x_0, f, R) I^{\frac{\kappa-p}{p-n}}(r_0, R) \geq \left(\frac{p-n}{p-1}\right)^{\frac{p-1}{p-n}} \left(\frac{\omega_{n-1}}{c}\right)^{\frac{1}{p-n}},$$

where  $\omega_{n-1}$  is the area of the unit sphere  $\mathbb{S}^{n-1}$  in  $\mathbb{R}^n$ .

## Harmonic measure distribution functions in various geometries

**Christopher Green**

Wichita State University

Consider releasing a Brownian particle from a basepoint  $z_0$  in a planar domain  $\Omega \cup \mathbb{C}$ . What is the chance, denoted  $h_{\Omega, z_0}(r)$ , that the particle's first exit from  $\Omega$  occurs within a fixed distance  $r > 0$  of  $z_0$ ? The function  $h_{\Omega, z_0}(r) : [0, \infty) \rightarrow [0, 1]$  is called the harmonic measure distribution function, or  $h$ -function, of  $\Omega$  with respect to  $z_0$ . It can also be formulated in terms of a Dirichlet problem on  $\Omega$  with suitable boundary values. For simply connected domains  $\Omega$ , the theory of  $h$ -functions is now quite well-developed, and in particular the  $h$ -function can often be explicitly computed, making use of the Riemann mapping theorem. However, until recently, for multiply connected domains the theory of  $h$ -functions has been almost entirely out of reach. In this talk, it will be shown how to construct explicit formulae for  $h$ -functions of symmetric multiply connected slit domains whose boundaries consist of an even number of colinear slits, and how these formulae can be generalized to compute  $h$ -functions

for multiply connected slit domains on a spherical surface. Special function theory and conformal mapping are judiciously combined to this end.

## On mappings with inverse Poletsky inequality on Riemannian manifolds

Sevost'yanov Evgeny

Zhytomyr Ivan Franko State University

Let  $\mathbb{M}^n$  and  $\mathbb{M}_*^n$  are Riemannian manifolds of dimension  $n$  with geodesic distances  $d$  and  $d_*$ , respectively,  $dv(x)$  and  $dv_*(x)$  are volume measures on  $\mathbb{M}^n$  and  $\mathbb{M}_*^n$ , respectively. Let  $x_0 \in D$ , and the number  $r_0 > 0$  be such that the ball  $B(x_0, r_0)$  lies in some normal neighborhood  $U$  of the point  $x_0$  with its closure. Given sets  $E, F$  and  $G$  in  $\mathbb{M}^n$ , we denote by  $\Gamma(E, F, G)$  the family of all paths  $\gamma: [a, b] \rightarrow \mathbb{M}^n$ , joining  $E$  and  $F$  in  $G$ . Denote  $S_i = S(x_0, r_i)$ ,  $i = 1, 2$ , geodesic spheres centered at the point  $x_0$  and radii  $r_1$  and  $r_2$ . If  $y_0 \in f(D)$  and  $0 < r_1 < r_2 < d_0 = \sup_{y \in f(D)} d_*(y, y_0)$ , we denote by  $\Gamma_f(y_0, r_1, r_2)$

the family of all paths  $\gamma$  in the domain  $D$  such that  $f(\gamma) \in \Gamma(S(y_0, r_1), S(y_0, r_2), A(y_0, r_1, r_2))$ . Let  $Q: \mathbb{M}_*^n \rightarrow [0, \infty]$  be a measurable function with respect to the volume measure  $v_*$ , and let  $M(\cdot)$  be a modulus of families of paths. We say that  $f$  satisfies the inverse Poletskii inequality at the point  $y_0 \in f(D)$ , if the relation  $M(\Gamma_f(y_0, r_1, r_2)) \leq \int_{A(y_0, r_1, r_2) \cap f(D)} Q(y) \cdot \eta^n(d_*(y, y_0)) dv_*(y)$  holds for

any Lebesgue measurable function  $\eta: (r_1, r_2) \rightarrow [0, \infty]$  such that  $\int_{r_1}^{r_2} \eta(r) dr \geq 1$ . In what follows,  $q_{x_0}(r) = \frac{1}{r^{n-1}} \int_{S(x_0, r)} Q(x) dA$ , where  $dA$  is the area element of  $S(x_0, r)$ .

For domains  $D \subset \mathbb{M}^n$ ,  $D_* \subset \mathbb{M}_*^n$ ,  $n \geq 2$ , and a function  $Q: \mathbb{M}_*^n \rightarrow [0, \infty]$ ,  $Q(x) \equiv 0$  for  $x \notin D_*$ , denote by  $\mathfrak{R}_Q(D, D_*)$  the family of all open discrete mappings  $f: D \rightarrow \mathbb{M}_*^n$ ,  $f(D) = D_*$ , for which  $f$  satisfies the inverse Poletsky inequality at each point  $y_0 \in D_*$ . The following result holds.

**Theorem.** Assume that,  $\overline{D}$  and  $\overline{D}_*$  a compact sets in  $\mathbb{M}^n$  and  $\mathbb{M}_*^n$ , respectively,  $\overline{D}_* \neq \mathbb{M}_*^n$  and, in addition,  $\mathbb{M}_*^n$  is connected. Suppose also that the following condition is satisfied: for each point  $y_0 \in \overline{D}_*$  there is  $r_0 = r_0(y_0) > 0$  such that  $q_{y_0}(r) < \infty$  for each  $r \in (0, r_0)$ . Then the family  $\mathfrak{R}_Q(D, D_*)$  is equicontinuous in  $D$ .

## Fast computation of high-dimensional volume potentials

Flavia Lanzara, Vladimir Maz'ya, Gunther Schmidt  
Sapienza University of Rome, Italy

Many integral operators of mathematical physics are convolutions with singular kernel functions, for example with fundamental solutions of partial differential operators. Because of the singularity of the integrand, the numerical computation of those integrals by standard methods is an involved and time consuming task. The use of the quasi-interpolants introduced in the framework of *Approximate Approximations* (V. Maz'ya, *A New Approximation Method and its Applications to the Calculation of Volume Potentials, Boundary Point Method*, 1991) with adapted basis functions can be very advantageous. This approach, combined with separated representations, makes the method fast and effective also in high dimensions.

In this talk we show how this procedure can be applied for the cubature of potentials of the three dimensional equations in static and quasi-static uncoupled thermoelasticity.

This is a joint work with Vladimir Maz'ya (Linköping University, Sweden) and Gunther Schmidt (Wias, Berlin, Germany).

## Rearrangement universality of the Dirichlet type series in a complex field

George Giorgobiani, George Chelidze, Vaja Tarieladze  
MICM at Georgian Technical University

**Abstract.** In our talk presented at the ISAAC 13th congress we showed that for any complex number  $s, 0 < \operatorname{Re}(s) \leq 1$  the Dirichlet series  $\sum_n \frac{(-1)^{n-1}}{n^s}$  is universal in  $\mathbb{C}$ , i.e. its sum range under the rearrangements is the whole complex number field  $\mathbb{C}$ . In this talk we shall discuss the question of validity of a similar result for Dirichlet type series  $\sum_n \frac{(-1)^{n-1}}{\lambda_n^s}$ , where  $(\lambda_n)_{n \in \mathbb{N}}$  is an increasing sequences of natural numbers.

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## An extremal problem on non-overlapping domains containing ellipse points

Iryna Denega  
Institute of Mathematics of NAS of Ukraine

An extremal problem of geometric function theory of a complex variable for the maximum of product of the inner radii on a system of  $n$  mutually non-overlapping multiply connected domains  $B_k$  containing the points  $a_k, k = \overline{1, n}$ , located on an arbitrary ellipse  $\frac{x^2}{d^2} + \frac{y^2}{t^2} = 1$  for which  $d^2 - t^2 = 1$ , is considered. Based on joint work with Ya. Zabolotnyi.

## On the correctness and maximal regularity of a second-order differential equation with unbounded coefficients

Kordan Ospanov  
L.N. Gumilyov Eurasian National University

We consider the following differential equation:

$$-s(x)(\rho(x)y')' + r(x)y' + q(x)y = f(x), \quad (1)$$

where  $x \in R = (-\infty, +\infty)$  and  $f \in L_2(R)$ . We assume that  $s > 0$  and  $\rho > 0$  are twice continuously differentiable,  $r > 0$  is continuously differentiable, and  $q$  is a continuous function.

Let  $L$  be the closure in  $L_2(R)$  of the differential operator  $L_0 y = -s(x)(\rho(x)y')' + r(x)y' + q(x)y$ , with  $D(L_0) = C_0^{(2)}(R)$ . The function  $y$  is called a solution of equation (1) if  $y \in D(L)$  and  $Ly = f$ .

We will discuss some conditions for the coefficients of (1) such that for any  $f \in L_2(R)$  there exists a unique solution  $y$  and the following estimate holds:

$$\|s(\rho y')'\|_2 + \|r y'\|_2 + \|q y\|_2 \leq C \|Ly\|_2, \quad (2)$$

where  $\|\cdot\|_2$  is the norm in  $L_2(R)$ .

We assume that the potential  $q$  may not be of constant sign, and the growth of other coefficients near infinity does not depend on  $q$ . Although we require that the coefficients be differentiable, we do not impose any restrictions on their derivatives.

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## Conjugate Complex Harmonic Functions

Luis Manuel Tovar, Lino Feliciano Reséndis, Yesenia Bravo  
Instituto Politecnico Nacional (MEXICO)

In this talk we discuss properties of the components of a bicomplex holomorphic function: We know that  $F = F_1 + jF_2$  is a bicomplex holomorphic function on a domain of BC, if and only if, the harmonic components  $F_1$  and  $F_2$  are related, among them, by a Cauchy-Riemann type condition, very similar to the complex case. If we express  $F$  through its idempotent form  $F = G_1 e + G_2 e^\dagger$ , we show how the Cauchy-Riemann conditions of  $F_1$  and  $F_2$  determine the kind of conditions between  $G_1$  and  $G_2$ . We also show how determine the conjugate harmonic functions  $F_2$  associate to the harmonic function  $F_1$

## On the logarithmic asymptotic of solutions of the nonlinear Cauchy-Riemann-Beltrami type equation

Mariia Stefanchuk, Ruslan Salimov  
Institute of Mathematics of the NAS of Ukraine

Let  $\sigma: G \rightarrow \mathbb{C}$  be a measurable function and  $m \geq 0$ . Consider in the polar coordinate system  $(r, \theta)$  the following equation:

$$f_r = \sigma(re^{i\theta}) |f_\theta|^m f_\theta, \quad (1)$$

where  $f_r$  and  $f_\theta$  are partial derivatives of the mapping  $f$  of  $r$  and  $\theta$ , respectively. The equation (1) may be written in a complex form:

$$f_z = \frac{z}{\bar{z}} \frac{\sigma(z) |z| i |z f_z - \bar{z} f_{\bar{z}}|^m - 1}{\sigma(z) |z| i |z f_z - \bar{z} f_{\bar{z}}|^m + 1} f_z. \quad (2)$$

The mapping  $f: G \rightarrow \mathbb{C}$  is called *regular at the point*  $z_0 \in G$ , if  $f$  has a complete differential and its Jacobian  $J_f = |f_z|^2 - |f_{\bar{z}}|^2 \neq 0$  at this point. The homeomorphism  $f$  of the Sobolev class  $W_{loc}^{1,1}$  is called *regular* if  $J_f > 0$  a.e. A *regular homeomorphic solution* of the equation (2) is called *regular homeomorphism*  $f: G \rightarrow \mathbb{C}$ , that satisfies the equation (2) a.e. in  $G$ .

Later on we use the following notations

$$\gamma_r = \{z \in \mathbb{C} : |z| = r\}, \quad \mathbb{B} = \{z \in \mathbb{C} : |z| < 1\}.$$

We denote

$$e_1 = e, e_2 = e^e, \dots, e_{k+1} = e^{e^k},$$

$$\ln_1 t = \ln t, \ln_2 t = \ln \ln t, \dots, \ln_{k+1} t = \ln \ln_k t,$$

where  $k \geq 1$  are integer.

**Theorem.** Let  $f: \mathbb{B} \rightarrow \mathbb{C}$  be a regular homeomorphic solution to equation (2) of Sobolev class  $W_{loc}^{1,2}$  satisfying  $f(0) = 0$ . Suppose that for some  $q > 0$  and  $\varepsilon_0 \in (0, \frac{1}{e_n})$ ,

$$I_{m,\sigma}(t) = \left( \int_{\gamma_t} \frac{ds}{|z| (\operatorname{Im} \bar{\sigma}(z))^{\frac{1}{m+1}}} \right)^{m+1} \leq q t \ln_1 \frac{1}{t} \ln_2 \frac{1}{t} \dots \ln_n \frac{1}{t}$$

for almost all  $t \in (0, \varepsilon_0)$ , then

$$\liminf_{z \rightarrow 0} |f(z)| \left( \ln_{n+1} \frac{1}{|z|} \right)^{\frac{1}{m}} \leq c_0 q^{\frac{1}{m}} < \infty,$$

where  $c_0 = (2\pi)^{-\frac{m+1}{m}} m^{-\frac{1}{m}}$ .

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## A survey on the boundary behavior of the double layer potential in Schauder spaces in the frame of an abstract approach

Massimo Lanza de Cristoforis

Universita' degli Studi di Padova

We provide a summary of the continuity properties of the boundary integral operator corresponding to the double layer potential associated to the fundamental solution of a *nonhomogeneous* second order elliptic differential operator with constant coefficients in Hölder and Schauder spaces on the boundary of a bounded open subset of  $\mathbb{R}^n$ . The purpose is two-fold. On one hand we try present in a single paper all the known continuity results on the topic with the best known exponents in a Hölder and Schauder space setting and on the other hand we show that many of the properties we present can be deduced by applying results that hold in an abstract setting of metric spaces with a measure that satisfies certain growth conditions that include non-doubling measures as in a series of papers by García-Cuerva and Gatto in the frame of Hölder spaces and later by the author.

## On the structure of regular generalized analytic functions

N. Manjavidze, G. Giorgadze, G. Makatsaria

Iliia State University

In the present work the Carleman-Vekua equations on the complex plane with the regular coefficients are considered. An intensive study of such equations began in the middle of the last century (see [Vekua I. Generalized Analytic Functions. Oxford: Pergamon, 1962]) and has not lost its relevance to this day, since the solution of a number of important theoretical and practical problems is directly reduced to such equations. In this direction, the most principal and profound results have been obtained by various authors, but a number of interesting problems remain open, the solution of which is important not only from a theoretical point of view, but for practical reasons. We have introduced special classes of solutions of the mentioned equations, which we call generalized meromorphic functions, and we study them both from the point of view of the pure theory of functions, as well as from the point of view of the analysis of boundary value problems of the theory of functions. Sufficiently important information about the structure of the generalized meromorphic function in the neighborhood of the point at infinity has been determined. On the basis of these results, it is possible to correctly pose natural boundary value problems for generalized meromorphic functions and carry out their (in some sense) complete analysis. It should be also mentioned that the obtained results are new for the classical meromorphic functions. All the above results are essentially based on the iterative transformation of the meromorphic functions studied by the authors. The obtained results continue the research started in the monograph [Akhalaiia G., Giorgadze G., Jikia V., Kaldani N., Makatsaria G., Manjavidze N. Elliptic Systems on Riemann Surfaces. Bulletin of TICMI. 13 (2012), 1-154].

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## On compact classes of solutions of Dirichlet problem in simply connected domains

Dovhopiatyi Oleksandr, Sevost'yanov Evgeny  
Zhytomyr Ivan Franko State University

Let  $D$  be a domain in  $\mathbb{C}$ . In what follows, a mapping  $f : D \rightarrow \mathbb{C}$  is assumed to be *sense-preserving*, moreover, we assume that  $f$  has partial derivatives almost everywhere. Put  $f_{\bar{z}} = (f_x + if_y)/2$  and  $f_z = (f_x - if_y)/2$ . The *complex dilatation* of  $f$  at  $z \in D$  is defined as follows:  $\mu(z) = \mu_f(z) = f_{\bar{z}}/f_z$  for  $f_z \neq 0$  and  $\mu(z) = 0$  otherwise. The *maximal dilatation* of  $f$  at  $z$  is the following function:  $K_\mu(z) = K_{\mu_f}(z) = \frac{1+|\mu(z)|}{1-|\mu(z)|}$ .

Consider the following Cauchy problem:

$$\begin{aligned} f_{\bar{z}} &= \mu(z) \cdot f_z, \\ \lim_{\zeta \rightarrow P} \operatorname{Re} f(\zeta) &= \varphi(P) \quad \forall P \in E_D, \end{aligned}$$

where  $\varphi : E_D \rightarrow \mathbb{R}$  is a predefined continuous function. In what follows, we assume that  $D$  is some simply connected domain in  $\mathbb{C}$ . The solution of this problem is called *regular*, if one of two conditions is fulfilled: or  $f(z) = \text{const}$  in  $D$ , or  $f$  is an open discrete  $W_{\text{loc}}^{1,1}(D)$ -mapping such that  $J(z, f) \neq 0$  for almost any  $z \in D$ .

Given  $z_0 \in D$ , a function  $\varphi : E_D \rightarrow \mathbb{R}$ , a function  $\Phi : \overline{\mathbb{R}^+} \rightarrow \overline{\mathbb{R}^+}$  and a function  $\mathcal{M}(\Omega)$  of open sets  $\Omega \subset D$ , we denote by  $\mathfrak{F}_{\varphi, \Phi, z_0}^{\mathcal{M}}(D)$  the class of all regular solutions  $f : D \rightarrow \mathbb{C}$  of the Cauchy problem that satisfy the condition  $\operatorname{Im} f(z_0) = 0$  and, in addition,  $\int_{\Omega} \Phi(K_\mu(z)) \cdot \frac{dm(z)}{(1+|z|^2)^2} \leq \mathcal{M}(\Omega)$

for any open set  $\Omega \subset D$ .

**Theorem.** Let  $D$  be some simply connected domain in  $\mathbb{C}$ , and let  $\Phi : \overline{\mathbb{R}^+} \rightarrow \overline{\mathbb{R}^+}$  be a continuous increasing convex function which satisfies the condition  $\int_{\delta}^{\infty} \frac{d\tau}{\tau \Phi^{-1}(\tau)} = \infty$

for some  $\delta > \Phi(0)$ . Assume that the function  $\mathcal{M}$  is bounded, and the function  $\varphi$  in Cauchy problem is continuous. Then the family  $\mathfrak{F}_{\varphi, \Phi, z_0}^{\mathcal{M}}(D)$  is compact in  $D$ .

## Monogenic functions and harmonic vectors

Sergiy Plaksa

Institute of Mathematics of the National Academy of Sciences of Ukraine

We consider special topological vector spaces with a commutative multiplication for some of elements of the spaces and monogenic functions taking values in these spaces. Monogenic functions are understood as continuous and differentiable in the sense of Gâteaux functions. We describe relations between the mentioned monogenic functions and harmonic vectors in the three-dimensional real space and establish sufficient conditions for infinite monogeneity of functions. Unlike the classical complex analysis, it is done in the case where the validity of the Cauchy integral formula for monogenic functions remains an open problem.

## Biharmonic problem in an angle and monogenic functions

Serhii Gryshchuk, Sergiy Plaksa

Institute of Mathematics of NAS of Ukraine

We consider a piecewise continuous biharmonic problem in an angle and the corresponding Schwartz-type boundary-value problem for monogenic functions in a commutative biharmonic algebra. These problems are reduced to a system of integral equations on the positive semiaxis. It is

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shown that, on each segment of this semiaxis, the set of solutions of the system coincides with the set of solutions of a certain system of Fredholm integral equations.

### **$\sigma$ -monogenic functions in commutative algebras**

**Vitalii Shpakivskyi**

Institute of Mathematics of the NAS of Ukraine

In finite-dimensional commutative associative algebra, the concept of  $\sigma$ -monogenic function is introduced. Necessary and sufficient conditions for  $\sigma$ -monogeneity have been established. In some low-dimensional algebras, with a special choice of  $\sigma$ , the representation of  $\sigma$ -monogenic functions is obtained using holomorphic functions of a complex variable. We proposed the application of  $\sigma$ -monogenic functions with values in two-dimensional biharmonic algebra to representation of solutions of two-dimensional biharmonic equation.

### **Some estimates of the products of some powers of the inner radii of multiconnected domains**

**Yaroslav Zabolotnyi**

Institute of Mathematics of NAS of Ukraine

We consider two problems of geometric function theory about the extreme partition of the complex plane. In the first problem some estimates of maximum of the product of inner radii of  $n$  disjoint domains with respect to  $n$  arbitrary points of complex plane are obtained. The exact solutions to this problem are currently known only for cases  $n = 2, 3, 4$ . In the second problem some estimates of maximum of the product of some positive powers  $\alpha_k$  of inner radii of  $n$  disjoint domains with respect to  $n$  arbitrary points of complex plane are obtained. This problem does not have a solution for all possible configurations of  $\alpha_k$ . For these problems we find estimates that can be applied in various problems of the geometric theory of functions.

## EVOLUTION EQUATIONS AND DYNAMICAL SYSTEMS

**Organizer:** Pedro Tavares Paes Lopes & Marcone Corrêa Pereira, (IME-USP, Brazil)

### Bound states in quantum waveguides

**Alessandra Verri**  
UFSCar

Let  $-\Delta_{\Omega}^D$  be the Dirichlet Laplacian in a three dimensional waveguide  $\Omega$ . If  $\Omega$  is bounded, it is known that the spectrum of  $-\Delta_{\Omega}^D$  is purely discrete. The situation changes when  $\Omega$  is unbounded. For example, if  $\Omega$  is a straight infinite waveguide, the operator  $-\Delta_{\Omega}^D$  does not have discrete eigenvalues. However, local deformations in  $\Omega$  can create them. In this talk some constructions for  $\Omega$  will be presented to show how its geometry can affect the spectral behavior of  $-\Delta_{\Omega}^D$ .

### Bifurcation and hyperbolicity for a nonlocal quasilinear parabolic problem.

**Alexandre N. Carvalho**, José Arrieta, Estefani M. Moreira, José Valero  
Universidade de São Paulo

We study a one-dimensional nonlocal quasilinear problem of the form  $u_t = a(\|u_x\|^2)u_{xx} + \nu f(u)$ , with Dirichlet boundary conditions on the interval  $[0, \pi]$ , where  $0 < m \leq a(s) \leq M$  for all  $s \in \mathbb{R}^+$  and  $f$  satisfies suitable conditions. We give a complete characterization of the bifurcations and of the hyperbolicity of the corresponding equilibria. With respect to the bifurcations we extend the existing results to the case when  $a(\cdot)$  is not necessarily monotone increasing and show that bifurcations may be pitchfork or saddle-node, subcritical or supercritical. We also give a complete characterization of hyperbolicity specifying necessary and sufficient conditions for its presence or absence.

### The existence of isolating blocks for multivalued semiflows

**Estefani M. Moreira**, José Valero  
Universidade de São Paulo

Our aim is to present a theory of isolating blocks for multivalued semiflows in which we understand such a neighborhood of a weakly isolated invariant set in the same way as we understand it for invariant sets in the single-valued scenario.

We will apply this abstract result to a differential inclusion in order to show that we can construct isolating blocks for each equilibrium of the problem.

### Theory of attractors for a class of impulsive systems

**Everaldo de Mello Bonotto**, José Manuel Uzal  
Universidade de São Paulo

In this lecture, we present a study of the theory of attractors for a class of dynamical systems under perturbations. A continuous dynamical system can be related to an impulsive dynamical system and a discrete dynamical system. We will present a relationship among the attractors of these systems.

## Large diffusion phenomena for autonomous evolution equations with variable exponents

Jacson Simsen

UNIFEI (Federal University of Itajubá)

In this talk I will present an overview about results on continuity of solutions with respect to initial conditions and parameters and as well upper semicontinuity of global attractors for autonomous evolution equations with variable exponents and large diffusion.

## A Necessary and Sufficient Conditions for the Global Existence of Solutions to Fractional Reaction-Diffusion Equations on $\mathbb{R}^N$

Jaeho Hwang, Soon-Yeong Chung  
Sogang University

A necessary-sufficient condition for the existence of global solutions to the following fractional reaction-diffusion equations

$$\begin{cases} u_t = \Delta_\alpha u + \psi(t)u^p, & \text{in } \mathbb{R}^N \times (0, \infty), \\ u(\cdot, 0) = u_0 \geq 0, & \text{in } \mathbb{R}^N, \end{cases}$$

where  $\Delta_\alpha = -(-\Delta)^{\alpha/2}$  and  $p > 1$ , has not been known and remained as an open problem for a few decades. The purpose of this talk is to resolve this problem completely, for more general source term  $\psi(t)f(u)$  as follows:

There is a global solution to the equation if and only if

$$\int_0^\infty \psi(t) \frac{f(\|S_\alpha(t)u_0\|_\infty)}{\|S_\alpha(t)u_0\|_\infty} dt < \infty$$

for some nonnegative and nontrivial  $u_0 \in L^\infty(\mathbb{R}^N)$ .

Here,  $(S_\alpha(t))_{t \geq 0}$  is the fractional heat semigroup on  $\mathbb{R}^N$ .

## Homogenization of the non-isothermal, non-Newtonian fluid flow in a thin domain with oscillating boundary

Jean Carlos Nakasato, Jean Carlos Nakasato, Igor Pazanin  
University of Sao Paulo

We study the flow of a non-Newtonian fluid through a thin three-dimensional domain with an oscillating boundary. Starting from a nonlinear coupled system describing a non-isothermal regime of the flow, we rigorously derive three different effective models, depending on the period of the boundary roughness. We employ a homogenization technique based on the adaption of the unfolding method and deduce the roughness-induced effects on the fluid flow.

## Rate of convergence of attractors for evolution equations

Leonardo Pires, Leonardo Pires  
Universidade Estadual de Ponta Grossa

In this talk, we discuss the main results of the theory of the rate of convergence of attractors for evolution equations. We present old and new results and discuss the main applications in diffusion reaction equations under perturbations of the domain and equation. One of our arguments consists of using invariant manifolds to reduce the problem to a finite dimension. We use the structural stability of Morse-Smale flows in a finite dimension to obtain the corresponding result in an infinite dimension. As a consequence, we obtain the optimal rate of convergence of the attractors

## The spectrum of a nonlocal Dirichlet problem

**Marcone C. Pereira**, Rafael D. Benguria, Mariel Sáez  
Universidade de São Paulo

In this talk we study the continuity of the spectrum of a nonlocal Dirichlet problem obtaining a Hadamard type formula for simple and multiple eigenvalues. We also obtain an analog to the Rayleigh-Faber-Krahn inequality. The Hadamard formula is computed allowing domain perturbations given by embeddings of bounded sets while the Rayleigh-Faber-Krahn inequality is shown by rearrangement techniques.

## Critical Domains for Eigenvalues of the Grushin Laplacian on Cartesian Product Manifolds

**Marcus Marrocos**, Naisa C Garcia, Wanessa F Tavares  
UFAM

In this talk we investigate properties of the sequences of extremal values that could be achieved by the eigenvalues of the Grushin operator  $\Delta_G := \Delta_{\mathbb{R}^k} + |x|^{2s} \Delta_N$  on bounded domains  $\Omega$  of  $\mathbb{R}^k \times N$  under Dirichlet and Neumann boundary conditions, respectively. We analyze locally extremal domains for the first nontrivial eigenvalue with respect to volume-preserving domain perturbations and show a relation with overdetermined boundary problems. The main technique used is based on an extension of Hadamard shape derivative formula for the eigenvalues.

## An Abstract lagrangian framework for computing shape derivatives

**Pedro T. P. Lopes**, Antoine Laurain, Jean C. Nakasato  
Universidade de São Paulo

We present the results of a recent paper, in which we study an abstract framework for computing shape derivatives of functionals subject to PDE constraints.

We develop this abstract framework using implicit function theorem. It yields practical formulae to compute the derivative of a shape functional, the material derivative of the state and the adjoint state. Applications in linear elliptic, nonlinear elliptic, parabolic PDEs and distributions will be presented.

Pedro T. P. Lopes was partially supported by grant 2019/15200-1, São Paulo Research Foundation (FAPESP)

## A Necessary and Sufficient Conditions for the Global Existence of Solutions to Reaction-Diffusion Systems with Time-dependent Sources

**Soon-Yeong Chung**, Jaeho Hwang  
Sogang University

In this talk, we discuss the global existence of solutions to the following reaction-diffusion systems with time-dependent reactions

$$(S) \begin{cases} u_t = \Delta u + \psi(t)v^p & \text{in } \Omega \times (0, \infty), \\ v_t = \Delta v + \psi(t)u^q & \text{in } \Omega \times (0, \infty), \\ u = v = 0 & \text{in } \partial\Omega \times (0, \infty), \\ u(\cdot, 0) = u_0 \geq 0, v(\cdot, 0) = v_0 \geq 0 & \text{in } \Omega, \end{cases}$$

where  $\Omega$  is a bounded domain in  $\mathbb{R}^N$  with a smooth boundary  $\partial\Omega$ ,  $p \geq q > 0$  with  $pq > 1$ , and  $\psi$  is a nonnegative continuous function on  $[0, \infty)$ . Also, the initial data  $u_0$  and  $v_0$  are nonnegative  $C_0(\Omega)$ -functions such that  $(u_0, v_0)$  is nontrivial.

In fact, it is shown that the system (S) has a global solution for sufficiently small data  $(u_0, v_0)$  if and only if

(i) when  $p = q$ ,

$$\int_0^\infty \psi_1(t) dt < \infty.$$

(ii) when  $p > q$ ,

$$\int_0^\infty \psi_1(t) dt < \infty \text{ or } \int_0^\infty \psi_1^{p+1}(t) dt < \infty.$$

Here,  $\psi_1(t) := \psi(t)e^{-\frac{pq-1}{p+1}\lambda_1 t}$  and  $\lambda_1$  is the first Dirichlet eigenvalue of the Laplace operator  $\Delta$  in  $\Omega$ .

## FUNCTION SPACES AND THEIR APPLICATIONS TO NONLINEAR EVOLUTIONAL EQUATIONS

**Organizer:** Mitsuru Sugimoto (Nagoya University, Japan) & Baoxiang Wang (Peking University, China)

### The Keller-Segel model with fractional diffusion

**Crystianne Lilian de Andrade**, Anne C. Bronzi  
Unicamp

The Keller-Segel model is a system of partial differential equations that models chemotactic motion, that is the mechanism by which cells convert chemical signals into motion behavior. This system models the chemotactic aggregation phenomenon in cellular systems, mathematically viewed as a blow-up in finite time. It is known that the classical parabolic-parabolic system has blow-up solutions for sufficiently large initial conditions in dimensions  $d \geq 2$ , but all solutions are regular in dimension one. We consider a modified model in which the cellular dispersion and the chemical dispersion are modeled by a fractional nonlocal operator with different exponents. We prove the existence, uniqueness, and temporal decay of the local mild solution and the global mild solution under the smallness initial assumptions in an  $L^p$  space. This is a joint work with Anne C. Bronzi.

### CMO spaces associated to Schrödinger operators: characterizations and applications

**Liangchuan Wu**  
Anhui University

Let  $L$  be a Schrödinger operator of the form  $L = -\Delta + V$  acting on  $L^2(\mathbb{R}^n)$ , where the nonnegative potential  $V$  belongs to the reverse Hölder class  $\text{RH}_q$  for some  $q \geq (n+1)/2$ . Let  $\text{CMO}_L(\mathbb{R}^n)$  denote the function space of vanishing mean oscillation associated to  $L$ . In this talk, we discuss the Dirichlet problem for Schrödinger equation in the Upper Half-Space with the CMO trace. Concretely, we show that a function  $f$  of  $\text{CMO}_L(\mathbb{R}^n)$  is the trace of the solution to  $\mathbb{L}u = -u_{tt} + Lu = 0$ ,  $u(x, 0) = f(x)$ , if and only if,  $u$  satisfies a kind of vanishing Carleson condition. This continues the lines of the previous characterizations by Duong, Yan and Zhang, and Jiang and Li for the  $\text{BMO}_L$  spaces, which were founded by Fabes, Johnson and Neri for the classical BMO space. For this purpose, we will prove two new characterizations of the  $\text{CMO}_L(\mathbb{R}^n)$  space, in terms of mean oscillation and the theory of tent spaces, respectively. This is a joint work with Prof. Liang Song.

### Local well-posedness for the kinetic derivative nonlinear Schrödinger equation on the real line

**Nobu Kishimoto**, Yoshio Tsutsumi  
Kyoto University

The kinetic derivative nonlinear Schrödinger equation (KDNLS) is a nonlinear Schrödinger equation with a nonlocal cubic derivative nonlinear term, which has dissipative nature. The gauge transformation is known to be effective for the standard derivative NLS or even for KDNLS in the periodic

setting, but it does not work well in the nonperiodic setting. In this talk, we apply the short-time  $U^p-V^p$  method to prove local well-posedness of the Cauchy problem for KDNLS on  $\mathbb{R}$  without size restriction, in Sobolev space of some regularity lower than  $3/2$  (above which the classical energy method works). The point is how to carry out the energy estimates under the presence of dissipation (i.e., under restricted symmetry).

## Estimates on modulation spaces for solutions to Schrödinger equations with magnetic fields

Ryo Muramatsu, Keiichi Kato  
Tohoku University

In this talk, we study the estimate of the solutions of the Schrödinger equation in a magnetic field by initial data in the modulation space. For the Schrödinger equation with free or scalar potentials, estimates on the modulation space of the solutions have been obtained (Bényi-Gröchenig-Okoudjou-Rogers (2007), Cordero-Gröchenig-Nicola-Rodino (2013), Kato-Kobayashi-Ito (2014), etc.). However, for the Schrödinger equations in magnetic fields, it has been challenging to estimate the solutions using conventional methods due to the increase in frequency components caused by first-order derivative term of the solutions. In this presentation, we discuss the estimate of the  $M^{p,p}$  norm of the solutions by the  $M^{p,p}$  norm of the initial data in the cases of spatially decaying magnetic fields and spatially uniform magnetic fields, along with its proof method. Additionally, we present results for the case where the magnetic field can be considered “short-range”. This talk is based on the joint work with Professor Keiichi Kato (Tokyo University of Science).

## Decoupling inequality and its application to the periodic Zakharov system

Shinya Kinoshita, Shohei Nakamura, Akansha Sanwal  
Tokyo Institute of Technology

In this talk, we consider the Cauchy problem of the Zakharov system on the multidimensional torus  $\mathbb{T}^d$ . The aim is the well-posedness of the Zakharov system in low regularity Sobolev spaces. In the 2 and the higher dimensional cases, Kishimoto established the well-posedness results. In particular, he proved the sharp well-posedness result on  $\mathbb{T}^2$ . We improve his results when  $d \geq 3$ . The key ingredient is the linear decoupling inequality for paraboloid under thin annulus constraint. This talk is based on the joint work with Shohei Nakamura (Osaka) and Akansha Sanwal (Innsbruck).

## Multilinear pseudo-differential operators with limited smooth $S_{0,0}$ class symbols

Tomoya Kato  
Gunma University

In this talk, we consider the boundedness of the multilinear pseudo-differential operators with symbols in the multilinear Hörmander class  $S_{0,0}$ . The aim of this paper is to discuss smoothness conditions for symbols to assure the boundedness between local Hardy spaces. A key idea is to consider the operators in Wiener amalgam spaces introduced by Feichtinger (1980) and Triebel (1983).

## HARMONIC ANALYSIS AND PARTIAL DIFFERENTIAL EQUATIONS

**Organizer:** Michael Ruzhansky (Ghent University, Belgium) & Jens Wirth (University of Stuttgart, Germany) & Vladimir Georgiev (University of Pisa, Italy)

### Discrete time-dependent wave equation for the schrodinger operator with discrete spectrum

**Abhilash Tushir**, Aparajita Dasgupta, Shyam Swarup Mondal, Michael Ruzhansky  
Indian Institute of Technology Delhi

In this article, we investigate the semiclassical version of the wave equation for the discrete Schrödinger operator,  $\mathcal{H}_{\hbar,V} := -\hbar^{-2}\mathcal{L}_{\hbar} + V$  on the lattice  $\hbar\mathbb{Z}^n$ , where  $\mathcal{L}_{\hbar}$  is the discrete Laplacian, and  $V$  is a non-negative multiplication operator. We prove that  $\mathcal{H}_{\hbar,V}$  has a purely discrete spectrum when the potential  $V$  satisfies the condition  $|V(k)| \rightarrow \infty$  as  $|k| \rightarrow \infty$ . We also show that the Cauchy problem with regular coefficients is well-posed in the associated Sobolev type spaces and very weakly well-posed for distributional coefficients. Finally, we recover the classical solution as well as the very weak solution in certain Sobolev type spaces as the limit of the semiclassical parameter  $\hbar \rightarrow 0$ .

### Reverse integral Hardy inequality on metric measure spaces

**Aidyn Kassymov**, Michael Ruzhansky, Durvudkhan Suragan  
Institute of Mathematics and Mathematical Modeling

In this talk, we obtain a reverse version of the integral Hardy inequalities on metric measure space with two negative exponents and  $q < 0 < p < 1$ . Also, as for applications we show the reverse Hardy-Littlewood-Sobolev and the Stein-Weiss type inequalities with two negative exponents on homogeneous Lie groups and with arbitrary quasi-norm, the result which appears to be new already in the Euclidean space and for  $q < 0 < p < 1$ .

In the famous work, G.H. Hardy showed the following (direct) integral inequality:

$$\int_a^\infty \frac{1}{x^p} \left( \int_a^\infty f(t) dt \right)^p dx \leq \left( \frac{p}{p-1} \right)^p \int_a^\infty f^p(x) dx, \quad (3)$$

where  $f \geq 0$ ,  $p > 1$ , and  $a > 0$ .

The main goal of this talk is to show the reverse Hardy inequalities to general metric measure space with two negative exponents and  $q < 0 < p < 1$ . More specifically, we consider metric spaces  $\mathbb{X}$  with a Borel measure  $dx$  allowing for the following *polar decomposition* at  $a \in \mathbb{X}$ : we assume that there is a locally integrable function  $\lambda \in L^1_{loc}$  such that for all  $f \in L^1(\mathbb{X})$  we have

$$\int_{\mathbb{X}} f(x) dx = \int_0^\infty \int_{\Sigma_r} f(r, \omega) \lambda(r, \omega) d\omega_r dr, \quad (4)$$

for some set  $\Sigma_r = \{x \in \mathbb{X} : d(x, a) = r\} \subset \mathbb{X}$  with a measure on it denoted by  $d\omega$ , and  $(r, \omega) \rightarrow a$  as  $r \rightarrow 0$ .

## Fractional integral operators on Stummel spaces

Alexandre Almeida  
Universidade de Aveiro

The so-called Stummel classes seemed to appear in connection with regularity properties of solutions to PDE. In this talk we introduce general Stummel spaces with variable integrability. After discussing some connections to other known spaces, we present some results on the boundedness of fractional integral operators. This is based on joint work with Humberto Rafeiro from UAEU.

## Solvability for a class of Vekua-type operators on the torus

Alexandre Kirilov, Pedro Meyer Tokoro, Wagner Augusto Almeida de Moraes  
Federal University of Parana

In this talk, we investigate the solvability of Vekua-type equations on the  $n$ -dimensional torus  $\mathbb{T}^n$ . Specifically, we focus on the partial differential operator  $P : C^\infty(\mathbb{T}^n) \rightarrow C^\infty(\mathbb{T}^n)$  defined as

$$Pu = Lu - Au - B\bar{u},$$

where  $A, B \in \mathbb{C}$  and  $L$  is any constant-coefficient partial differential operator on  $\mathbb{T}^n$ .

Our main objective is to establish necessary and sufficient conditions for the solvability of this operator. Building upon the work of A. Bergamasco, P. Dattori, and A. Meziani, who examined the solvability of this equation in the case where  $L$  is a complex vector field, we extend the analysis to the general setting of constant-coefficient operators.

As a consequence of our results, we show that the solvability of  $P$  is equivalent to being globally hypoelliptic. At the end, we will present nice examples the classical Laplace, heat, and wave operators in the context of Vekua-type equations.

## Global Properties of First Order Differential Operators on $\mathbb{T}^{r+1} \times \mathbb{S}^{3s}$

André Pedroso Kowacs, Alexandre Kirilov, Wagner Augusto Almeida Moraes  
Universidade Federal do Paraná

The interest in studying global properties of partial differential operators on compact manifolds, such as hypoellipticity and solvability, has grown in the last few decades and has proven to be an interesting and challenging problem. Many results have been obtained for the particular case where the operator is a vector field and the manifold is a torus. In these cases, geometric conditions on the coefficients of the vector field were obtained, as well as Diophantine conditions on the symbol of the operator associated with the vector field. More recently, perturbations of these operators by lower order terms have been considered, as well as the extension of these studies to closed manifolds other than the torus.

In this talk, we present one of these possible extensions by analyzing the case of a vector field associated with an evolution operator and a 0-order perturbation defined on a product of  $r + 1$  tori and  $s$  spheres ( $\mathbb{T}^{r+1} \times \mathbb{S}^{3s}$ ), with  $r$  and  $s$  natural numbers. By varying the values of  $r$  and  $s$ , we show that it is possible to recover results already known in the literature and present new results.

The main tool used in this study is Fourier analysis, taken partially with respect to each copy of the torus and sphere, as well as Diophantine-type inequalities and connectivity of level sets associated with primitives of the operator under study.

## **Orbital stability and concentration of standing-wave solutions to a nonlinear Schrödinger system with mass critical exponent**

**Daniele Garrisi**, Tianxiang Gou  
University of Nottingham Ningbo China

For a nonlinear Schrödinger system with mass critical exponent, we prove the existence and orbital stability of standing-wave solutions obtained as minimizers of the underlying energy functional restricted to a double mass constraint. In addition, we discuss the concentration of a sequence of minimizers as its masses approach to certain critical masses.

## **Fujita exponent on stratified Lie groups**

**Durvudkhan Suragan**  
Nazarbayev University

We show that  $\frac{Q}{Q-2}$  is the Fujita exponent for a semilinear heat equation on an arbitrary stratified Lie group with homogeneous dimension  $Q$ . This covers and unifies known results on the Euclidean case and the Heisenberg group. The equation we study has a forcing term which depends only upon a group element and has a positive integral. The stratified Lie group structure plays an important role in our proofs, along with test function method and Banach fixed point theorem. We also delve into the implications of introducing a singular potential on the problem of determining the critical exponent. This talk is based on our joint work with Bharat Talwar.

## **Continuity properties for some operators arising in non-commutative harmonic analysis**

**Duvan Cardona**  
Ghent University

The non-commutative harmonic analysis on nilpotent Lie groups after the developments by Folland and Stein in the 70's has been fundamental for the analysis of hypoelliptic problems on graded Lie groups. They started the program of generalising in the setting of nilpotent Lie groups the results available in the Euclidean harmonic analysis. Several fundamental results in this area have been obtained in the last 50 years. In this setting, we review some recent results about the boundedness of oscillating Fourier multipliers, pseudo-differential operators and other operators arising in this setting. The results presented in this talk are part of my joint works with M. Ruzhansky (Ghent) and J. Delgado (Colombia).

## **Pseudo-differential calculus adapted to Grushin manifolds**

**I. Beschastnyi**, H. Quan  
Universidade de Aveiro

A Grushin metric on manifold is a singular Riemannian metric, which forces the geodesics to cross the the singular submanifold orthogonally. This is the simplest class of complete singular Riemannian manifolds, which enjoys many good properties, such as having a tubular neighbourhood around the singular submanifold adapted to its metric geometry. In this talk I will discuss a joint work with H. Quan, in which we construct a general pseudo-differential calculus adapted to those structures and give some of its applications.

## A close look at the entropy numbers of the unit ball of the Reproducing Hilbert Space of isotropic positive definite kernels.

Karina Navarro Gonzalez, Thaís Jordão  
University of São Paulo

We present accurate upper and lower bounds for the covering numbers of the unit ball for two general classes of Reproducing Kernel Hilbert Space (RKHS) on the unit sphere of  $\mathbb{R}^{d+1}$ . In both classes, the RKHS is generated by an isotropic continuous positive definite kernel. The upper and lower bounds we present carry precise information about the asymptotic constants, depending on the dimension of the sphere and the monotonic behavior of the Schoenberg/Fourier coefficients of the isotropic kernel.

## On generalized singular number of positive matrix of $\tau$ measurable operators

Madi Raikhan  
Astana IT University, Kazakhstan

Let  $(\mathcal{M}, \tau)$  be a semi-finite von Neumann algebra,  $L_0(\mathcal{M})$  be the set of all  $\tau$ -measurable operators,  $\mu_t(x)$  be the generalized singular number of  $x \in L_0(\mathcal{M})$ . We extend the related inequalities of  $2 \times 2$  positive semi-definite block matrices to  $2 \times 2$  positive matrices of  $\tau$ -measurable operators, i.e, we obtained that the following statements are equivalent:

1. if  $x, y \in L_0(\mathcal{M})$  are self-adjoint operators such that  $\pm y \leq x$ , then  $\mu_t(y) \leq \mu_t(x \oplus x)$  for any  $t > 0$ ,
2. if  $x, y, z, a, b \in L_0(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then  $\mu_t(a^*zb + b^*z^*a) \leq \mu_t((a^*xa + b^*yb) \oplus (a^*xa + b^*yb))$  for any  $t > 0$ ,
3. if  $x, y, z \in L_0(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then  $\mu_t(z + z^*) \leq \mu_t((x + y) \oplus (x + y))$  for any  $t > 0$ ,
4. if  $x, y \in L_0(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then  $\mu_t(z) \leq \mu_t(x \oplus y)$  for any  $t > 0$ .

We also proved some related symmetric norm inequalities.

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## On generalized singular number of $2 \times 2$ positive matrices of $\tau$ -measurable operators

Myrzagali Ospanov  
L.N. Gumilyov Eurasian National University

We denote the set of all  $n \times n$  complex matrices by  $\mathbb{M}_n$ . Let  $x, y \in \mathbb{M}_n$  be Hermitian matrices such that  $\pm y \leq x$ . Then the following relation holds.

$$\prod_{j=1}^k s_j(y) \leq \prod_{j=1}^k s_j(x), \quad k = 1, 2, \dots, n. \quad (5)$$

Let  $(\mathcal{M}, \tau)$  be a semi-finite von Neumann algebra. We denote by  $L_0(\mathcal{M})$  the set of all  $\tau$ -measurable operators and by  $\mu_t(x)$  the generalized singular number of  $x \in L_0(\mathcal{M})$ . We generalized (5) for operators in  $L_{\log_+}(\mathcal{M})$ . We prove the following inequalities are equivalent:

1. If  $x, y \in L_{\log_+}(\mathcal{M})$  are self-adjoint operators such that  $\pm y \leq x$ , then  $y \preceq_{\log} x$ .

2. If  $a, b \in \mathcal{M}$ ,  $x, y \in L_{\log_+}(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then

$$a^*zb + b^*z^*a \preceq_{\log} a^*xa + b^*yb.$$

3. If  $x, y \in L_{\log_+}(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then  $z^* + z \preceq_{\log} x + y$ .

4. If  $x, y \in L_{\log_+}(\mathcal{M})$  are positive operators, then  $x - y \preceq_{\log} x + y$ .

5. If  $x, y \in L_{\log_+}(\mathcal{M})$  and  $\begin{pmatrix} x & z \\ z^* & y \end{pmatrix} \geq 0$ , then  $z^* \oplus z \preceq_{\log} x \oplus y$ .

We also give some application this inequalities.

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## Hardy and Poincare identities and inequalities related to Baouendi-Grushin operator

Nurgissa Yessirkegenov

Institute of Mathematics and Mathematical Modeling, Kazakhstan

The classical Hardy inequality has been generalised for Baouendi-Grushin vector fields by Garofalo,

$$\int_{\mathbb{R}^n} \left( |\nabla_x f|^2 + |x|^{2\gamma} |\nabla_y f|^2 \right) dz \geq \left( \frac{Q-2}{2} \right)^2 \int_{\mathbb{R}^n} \left( \frac{|x|^{2\gamma}}{|x|^{2+2\gamma} + (1+\gamma)^2 |y|^2} \right) |f|^2 dz, \quad (1)$$

where  $z = (x_1, \dots, x_m, y_1, \dots, y_k) = (x, y) \in \mathbb{R}^m \times \mathbb{R}^k$  with  $n = m + k$ ,  $m, k \geq 1$ ,  $\gamma \geq 0$ ,  $Q = m + (1 + \gamma)k$  and  $f \in C_0^\infty(\mathbb{R}^m \times \mathbb{R}^k \setminus \{(0, 0)\})$ . Here,  $\nabla_x f$  and  $\nabla_y f$  are the gradients of  $f$  in the variables  $x$  and  $y$ , respectively.

In this talk, we discuss Hardy type identities for the Baouendi-Grushin operator involving radial derivatives in some of the variables leading to refinements of the above Hardy inequality. Moreover, we establish magnetic Hardy inequalities for the Baouendi-Grushin operator with Aharonov-Bohm type magnetic field. Furthermore, we discuss a sharp remainder formula for the Poincaré inequality. If time permits, we also show an application in studying blow-up properties of positive solutions to the Dirichlet initial-boundary value problem for the Baouendi-Grushin heat operator.

This talk is based on the joint research with Ari Laptev (Imperial College London, UK) and Michael Ruzhansky (Ghent University, Belgium), and with Durvudkhan Suragan (Nazarbayev University, Kazakhstan).

## Solvability in the large and boundary value problems for Mizohata type operators

Paulo Leandro Dattori da Silva, Camilo Campana

University of Sao Paulo

We will seek for (Hölder) continuous global solutions to the equation  $Lu = f$  in the plane, for  $f$  belonging to a subspace of  $L^p(\mathbb{R}^2)$ ,  $p \geq 1$ , where  $L$  is a degenerate elliptic vector field of Mizohata type. Our approach is via the study of an associated integral operator. Also, we obtain bounded Hölder continuous solutions to

Vekua type equations in the form  $Lu = au + b\bar{u} + f$ . As an application we study an associated Riemann boundary value problem.

This is a joint work with Camilo Campana (Federal University of Santa Catarina, Brazil).

## Fractional hypergeometric functions

**Praveen Agarwal**

Anand International College of Engineering

The fractional calculus of hypergeometric functions (special functions) has significant importance and applications in various fields of science and engineering. Here, we aim to discuss some extensions and generalizations of hypergeometric functions and fractional integral and differential formulas of the extended hypergeometric type functions by using fractional operators.

## On Young's inequality for the twisted convolution

**Ratnakumar P.K.**

Harish-Chandra Research Institute

The classical generalised Young's inequality says that the convolution inequality  $\|f * g\|_{L^r(\mathbb{R}^n)} \leq \|f\|_{L^p(\mathbb{R}^n)} \|g\|_{L^q(\mathbb{R}^n)}$  holds for  $1 \leq p, q, r \leq \infty$  if and only if  $\frac{1}{p} + \frac{1}{q} = 1 + \frac{1}{r}$ . We explore similar inequality for the twisted convolution on  $\mathbb{C}^n$  given by

$$f \times_{\lambda} g(z) = \int_{\mathbb{C}^n} f(z-w) g(w) e^{i\frac{\lambda}{2}\text{im}(z\cdot\bar{w})} dw, \quad 0 \neq \lambda \in \mathbb{R}$$

which has better mapping properties for  $\lambda \neq 0$ .

We prove a geometric characterisation of the triples  $(p, q, r)$ ,  $1 \leq p, q, r \leq \infty$  for the boundedness of the bi-linear  $\lambda$ -twisted convolution operator  $B_{\lambda} : (f, g) \rightarrow f \times_{\lambda} g$  from  $L^p(\mathbb{C}^n) \times L^q(\mathbb{C}^n) \rightarrow L^r(\mathbb{C}^n)$ , leading to an analogue of Young's inequality for twisted convolution.

## Restriction theorems and orthonormal Strichartz inequalities for certain operators

**Shyam Swarup Mondal**

Indian Institute of Technology Delhi

In this talk, I will discuss how restriction problems are related to Strichartz inequalities. First, I prove the restriction theorem for certain Fourier-type transforms. Further, as an application of the restriction problems, we obtain Strichartz's estimate for a system of orthonormal functions associated with certain operators, including Hermite, special Hermite, and Dunkl operators.

## Noncommutative symmetric space associated with a weight

**Turdybek Nurlybekuly (Turdebek N. Bekjan)**

Astana IT University

Let  $\mathcal{N}$  be a semifinite von Neumann algebra on the Hilbert space  $\mathcal{H}$  with a faithful normal semifinite trace  $\nu$  satisfying  $\nu(1) = a$ . Now let  $\varphi(\cdot) = \nu(D_{\varphi}\cdot)$  be a faithful normal locally finite weight on  $\mathcal{N}$  (where  $D_{\varphi}$  is the Radon-Nikodym derivative of  $\varphi$  with respect to  $\nu$ ,  $D_{\varphi}$  is locally measurable. For  $1 \leq p < \infty$  and  $\alpha \in [0, 1]$ , define

$$\mathcal{N}_{\alpha}^{\frac{1}{p}} = \{x \in \mathcal{N} : D_{\varphi}^{\frac{\alpha}{p}} x D_{\varphi}^{\frac{1-\alpha}{p}} \in L_p(\mathcal{N})\};$$

$$\|x\|_{p,\alpha} = \|D_{\varphi}^{\frac{\alpha}{p}} x D_{\varphi}^{\frac{1-\alpha}{p}}\|_p.$$

Then  $(\mathfrak{R}_\alpha^{\frac{1}{p}}, \|\cdot\|_{p,\alpha})$  is a normed space. Trunov proved its completion is isometrically isomorphic to  $L_p(\mathcal{N}, \nu)$  for all  $\alpha \in [0, 1]$ .

Sh. A. Ayupov, V. I. Chilin and R. Z. Abdullaev extended this result to the Orlicz space associated with an  $\mathcal{N}$ -function case. We defined noncommutative quasi symmetric spaces and noncommutative quasi symmetric Hardy spaces associated with a weight and extend the above results to the case that  $E$  is a separable  $p$ -convex symmetric quasi Banach function space on  $[0, a)$  for some  $0 < p < \infty$ .

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## Global Properties for a Class of Operators on Compact Lie Groups

**Wagner Augusto Almeida de Moraes**

Universidade Federal do Paraná

In this talk I will present some results on the global hypoellipticity and the global solvability for a class of first order differential operators on compact Lie groups. In view of Greenfield's and Wallach's conjecture, about the non-existence of globally hypoelliptic vector fields on compact manifolds different from tori, we study zero-order perturbations of vector fields and we also investigate different notions of regularity weaker than global hypoellipticity.

## HARMONIC ANALYSIS AND RELATED TOPICS

**Organizer:** Lucas da Silva Oliveira (UFRGS, Brazil)& Tiago Picon (USP, Brazil)

### Existence and Ulam-type stability for a Riemann-Liouville fractional initial value problem

**Anabela Silva**, Luís Castro  
University of Aveiro

In this talk, we consider a class of nonlinear fractional differential equations, with initial conditions, involving the Riemann-Liouville fractional derivative of order  $\alpha \in (1, 2)$ . The main objectives are to obtain conditions for the existence and uniqueness of solutions (within appropriate spaces), and to analyze the stabilities of Ulam-Hyers and Ulam-Hyers-Rassias types. The results are achieved based on the analysis of an associated class of fractional integral equations and distinct fixed-point arguments. Additionally, using a Bielecki-type metric and some additional contractive arguments, conditions are also obtained to guarantee Ulam-Hyers and Ulam-Hyers-Rassias stabilities for the problems under analysis.

### The Least Quadratic Nonresidue and the Least Prime in an Arithmetic Progression through Fourier Optimization

**Antonio Pedro Ramos**, Emanuel Carneiro, Micah Milinovich, Emily Quesada-Herrera

SISSA

In this joint work with E. Carneiro, M. Milinovich and E. Quesada-Herrera, we establish new asymptotic bounds for the least quadratic nonresidue and the least prime in an arithmetic progression under the Generalized Riemann Hypothesis. We obtain these new bounds by investigating a Fourier extremal problem arising from an application of the Guinand-Weil explicit formula.

### Self-improving Poincaré-Sobolev type functionals in product spaces

**Carolina A. Mosquera**, Eugenia Cejas, Carlos Pérez, Ezequiel Rela  
Universidad de Buenos Aires

We give a geometric condition which ensures that  $(q, p)$ -Poincaré-Sobolev inequalities are implied from generalized  $(1, 1)$ -Poincaré inequalities related to  $L^1$  norms in the context of product spaces. We provide several  $(1, 1)$ -Poincaré type inequalities adapted to different geometries and then show that our self-improving method can be applied to obtain special interesting Poincaré-Sobolev estimates.

Among other results, we prove that for each rectangle  $R$  of the form  $R = I_1 \times I_2 \subset \mathbb{R}^n$ , where  $I_1 \subset \mathbb{R}^{n_1}$  and  $I_2 \subset \mathbb{R}^{n_2}$  are cubes with sides parallel to the coordinate axes, we have that

$$\left( \frac{1}{\omega(R)} \int_R |f - f_R|^{p_{\delta, \omega}^*} \omega dx \right)^{\frac{1}{p_{\delta, \omega}^*}} \leq c(1 - \delta)^{\frac{1}{p}} [\omega]_{A_{1, \mathcal{R}}}^{\frac{1}{p}} (a_1(R) + a_2(R)),$$

where  $\delta \in (0, 1)$ ,  $\omega \in A_{1, \mathcal{R}}$ ,  $\frac{1}{p} - \frac{1}{p_{\delta, \omega}^*} = \frac{\delta}{n} \frac{1}{1 + \log[\omega]_{A_{1, \mathcal{R}}}}$  and  $a_i(R)$  are bilinear analog of the fractional Sobolev seminorms  $[u]_{W^{\delta, p}(Q)}$ .

The results are based on a joint work with E. Cejas (UNL, Argentina), C. Pérez (BCAM, Spain) and E. Rela (UBA, Argentina).

## Inhomogeneous cancellation conditions and boundedness of operators in local Hardy spaces

**Claudio Machado Vasconcelos**, Tiago Picon, Galia Dafni, Chun Ho Lau  
Universidade Federal de São Carlos

In this work we present necessary and sufficient cancellation conditions for the boundedness of inhomogeneous-type Calderón–Zygmund operators in local Hardy spaces  $h^p(\mathbb{R}^n)$  for  $0 < p \leq 1$ . The sufficiency relies on a refined atomic and molecular decomposition of  $h^p(\mathbb{R}^n)$ , where the local vanishing moment condition is relaxed to an approximated one.

## On Fourier uncertainty and extremal problems

**Emily Quesada-Herrera**  
Graz University of Technology

The uncertainty principle states, broadly, that one cannot have an unrestricted control of a function and its Fourier transform simultaneously. This paradigm is related to different sorts of Fourier optimization problems, where one imposes conditions on a function and its transform and seeks to optimize a certain quantity of interest. We will discuss an uncertainty principle involving the signs of a function and its transform, some extremal problems, and connections to other fields.

## Boundedness of commutators on local Hardy spaces

**Galia Dafni**, Chun Ho Lau  
Concordia University

We study the boundedness on the local Hardy space  $h^1$  of the commutator  $[b, T]$ , where  $T$  is an inhomogeneous singular integral operator and  $b$  is in  $\text{bmo}$  (the dual of  $h^1$ ). We define an atomic space  $h_b^1$ , as was done in the case of  $H^1$  by Pérez, but using approximate cancellation conditions. When  $b$  is in  $\text{Imo}$ , this space coincides with  $h^1$ .

## Weighted Hardy spaces and ultradistributions

**Gustavo Hoepfner**, Andrew Raich, Patricia Rampazo  
UFSCar

The goal of this talk is to identify certain classes of global ultradistributions as boundary values of generalized Hardy spaces defined on cones. The ultradistributions arise as elements of dual spaces of classes of globally  $L^q$ -integrable ultradifferentiable functions defined in terms of weight functions which appeared recently and naturally to capture a very particular type of exponential decay of the  $\square_b$ -heat kernel on a class of unbounded CR manifolds of finite type.

## $L_p$ -boundedness of Fourier and Schur multipliers

**José Manuel Conde Alonso**  
Universidad Autónoma de Madrid

Fourier multipliers are operators that act by pointwise multiplication on the Fourier transform side, while Schur multipliers are operators that act on matrices by multiplication, only using the *bad student* product:

$$A = (a_{ij})_{ij}, \quad B = (b_{ij})_{ij}, \quad A \bullet B := (a_{ij}b_{ij})_{ij}.$$

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In this talk, we will explain the connections between  $L_p$ -boundedness of Fourier multipliers and Schur multipliers and we will show how Calderón-Zygmund theory can be of importance to understand the latter through the former. This is joint work with A. González Pérez, J. Parcet and E. Tablate.

### **$L^p$ boundedness of directional maximal operators in the plane: theory vs practice**

**Laurent Moonens**, Emma D’Aniello, Anthony Gauvan  
Université Paris-Saclay

Maximal directional operators in the plane are Hardy-Littlewood type maximal operators obtained by averaging, instead of on balls, on rectangles oriented along an infinite set of directions. According to a beautiful work of M. Bateman, they are either bounded on all  $L^p$  spaces for finite  $p > 1$ , or unbounded on all of them. In this talk, we shall discuss how one can, with specific examples of sets of directions, decide in which of the two above cases one falls. The talk will be based on a joint work with E. D’Aniello and A. Gauvan.

### **Extending homeomorphisms of the real line**

**Lucas da Silva Oliveira**,  
Universidade Federal do Rio Grande do Sul

In this talk we will discuss the problem of extending homeomorphisms on the real line to a homeomorphism the upper-half plane. Our basic question is: such extensions behave well under compositions of homeomorphism? We will discuss the situations where the answers are known, and present some recent advances under extra regularity of the mappings involved. Based on joint work with José Afonso Barrionuevo and Victor Medeiros.

### **Qualitative analysis of a $q$ -fractional boundary value problem with an integral condition**

**Luis Castro**  
University of Aveiro

As it is well known, the fractional  $q$ -difference calculus generalizes the notion of  $q$ -derivatives and  $q$ -integrals to non-integer orders. This allows the consideration of fractional  $q$ -derivative boundary value problems. In this talk, we will deduce different conditions to guarantee some qualitative properties of the solutions to a class of nonlinear Riemann-Liouville fractional  $q$ -derivative boundary value problems which depend on an integral condition. After equivalently translating the initial problem into a  $q$ -integral equation where the corresponding Green’s function is explicitly identified, the structure of this function is studied in detail. Then, different techniques are combined (e.g. the upper and lower solutions method and the use of non-negative continuous concave functionals on cones) to extract information about possible solutions of the problem (and, in particular, to identify conditions that ensure different numbers of positive solutions).

### **Weighted a priori estimates for solutions of uniformly elliptic systems**

**Maria Eugenia Cejas**, Ricardo Durán

Universidad Nacional de La Plata

Let us consider the following problem

$$\begin{aligned} u &= \sum_{i,j} a_{ij}(x) \frac{\partial^2 u}{\partial x_i \partial x_j} = f \text{ in } \Omega \\ u &= 0 \text{ on } \partial\Omega \end{aligned} \quad (6)$$

where  $\Omega \subseteq \mathbf{R}^n$  is a bounded domain, is an uniformly elliptic operator of order 2 defined for functions  $u : \Omega \rightarrow \mathbf{R}^n$ , in the spirit of [ADN]

In this talk we present some results obtained for the problem (6) in weighted Sobolev spaces for operators with coefficients  $a_{ij}$  in the class  $VMO$ .

We will show the proof of solvability and uniqueness of the problem (6) and the weighted a priori estimate

$$\|u\|_{W_w^{2m,p}(\Omega)} \leq C \|f\|_{L_w^p(\Omega)} \quad (7)$$

where  $w$  is a weight in the well-known  $A_p$  class. We will see that these results can be used to prove weighted a priori estimates for elliptic equation for functions  $f$  in the class of local weighted Hardy spaces  $h_w^1(\Omega)$

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## Singular Integral Operators, Boundary Value Problems, and Herz Spaces

Pedro Henrique Takemura Feitosa da Silva, Marius Mitrea  
Baylor University

The goal of this talk is to present recent progress in the direction of solving boundary value problems on rough domains with data in Herz spaces. Via a powerful extrapolation result, we obtain boundedness results for a relevant class of singular integral operators, and employ the method of layer potentials to establish a well-posedness result for the Dirichlet problem with data in Herz spaces. This is a joint work with Marius Mitrea.

## Generalization of Titchmarsh theorem in the deformed Hankel setting.

Radouan Daher ,  
Hassan II University Casablanca Morocco

In this talk, using the translation operator and harmonic analysis associated to the deformed Hankel transform we prove a generalization of Titchmarsh's theorem.  
This joint work with A. ElGargati., E. Loualid

## Uncertainty principles for $q$ -integral transforms with bounded kernels

Rita C. Guerra, L. P. Castro  
University of Coimbra

In this talk, we consider a class of  $q$ -integral transforms with bounded kernel and obtain uncertainty principles for such integral operators. The global characteristics of this class of integral transforms allow us to immediately deduce corresponding uncertainty principles for operators that are particular cases of such  $q$ -integral transforms (with bounded kernel). Among these particular cases we have well-known integral operators, namely: a  $q$ -cosine-Fourier transform, a  $q$ -sine-Fourier transform, a  $q$ -Dunkl transform and a  $q$ -Bessel-Fourier transform. Furthermore, we obtain other norm inequalities for some of those operators. This is a joint work with L. P. Castro.

## **Decay of Fourier coefficients of kernels satisfying extended Hölder conditions given by multipliers on the sphere**

**Thaís Jordão**

Universidade de São Paulo

In this talk I will present the extension of results published in 2020 concerning on the decay of Fourier coefficients of spherical kernels satisfying an abstract Hölder condition. Kernels admitting Fourier series expansion with positive coefficients will be considered satisfying an extended Hölder condition defined in terms of the function spaces of generalized smoothness on the  $d$ -dimensional unit sphere of the Euclidean space. These spaces will be defined for integrable functions and in terms of the rate of approximation of an admissible family of multipliers operators on the sphere and the majorant functions.

## **Lebesgue solvability of elliptic homogeneous linear equations with measure data**

**Victor Sandrin Biliatto**

UFSCar

In this talk we present new results on solvability of the equation  $A^*(D)f = \mu$  for  $f \in L^p(\mathbb{R}^n)$  and positive measure data  $\mu$  associated to an elliptic homogeneous differential operator  $A(D)$  of order  $m$ . Our method is based on controlling the  $(m, p)$ -energy of  $\mu$  giving a natural characterization for solutions when  $1 \leq p < \infty$ . We also obtain sufficient conditions in the limiting case  $p = \infty$  using new  $L^1$  estimates on measures for elliptic and canceling operators.

This is a joint work with Tiago Picon (FFCLRP-USP).

## INTEGRAL TRANSFORMS AND REPRODUCING KERNELS

**Organizer:** Zouhair Mouayn (Université Sultan Moulay Slimane de Beni-Mellal, Morocco)

### A finite measure with a scale over vector-valued paths and its properties

**Dong Hyun Cho**  
Kyonggi University

Let  $C^{\mathbb{B}}[a, b]$  denote an analogue of Wiener space over paths in abstract Wiener space  $\mathbb{B}$ , the space of  $\mathbb{B}$ -valued continuous functions on  $[a, b]$ . In the present talk, we introduce a positive finite measure with a scale on  $C^{\mathbb{B}}[a, b]$  which is a generalized analogue of Wiener measure. Then we investigate its various properties including the Fourier-transforms on the space and the translation of time interval  $[a, b]$  defining  $C^{\mathbb{B}}[a, b]$ . As applications of the results, we derive various relationships between the analogue of Wiener space and its product space. In particular, we express the measure on  $C^{\mathbb{B}}[a, b]$  in terms of a product measure on  $C^{\mathbb{B}}[a, s] \times C^{\mathbb{B}}[s, b]$  by dividing  $[a, b]$  as  $[a, s]$  and  $[s, b]$ . Using the results, we finally provide various applicable examples which are useful in the theory of a generalized Brownian motion process.

### Fractional Fourier transform, harmonic oscillator propagators and Strichartz estimates

**Joachim Toft**, Divyang Bhimani, Ramesh Manna  
Linnaeus University

We show that harmonic oscillator propagators and fractional Fourier transforms are essentially the same. We deduce continuity properties for such operators on modulation spaces, and apply the results to prove Strichartz estimates for the harmonic oscillator propagator when acting on modulation spaces. Especially we extend some results in [?, ?, ?, ?]. We also show that general forms of fractional harmonic oscillator propagators are continuous on suitable Pilipović spaces. Especially we show that fractional Fourier transforms of any complex order can be defined, and that these transforms are continuous on any Pilipović space and corresponding distribution space, which are *not* Gelfand-Shilov spaces.

*The talk is based on a joint work with Divyang Bhimani and Ramesh Manna.*

### Reproducing Krein Kernel Modules in the context of Clifford algebras

**Paula Cerejeiras**  
Universidade de Aveiro

Classic complex analysis is linked to elliptic operators and quadratic forms, of which the Laplace operator is a prime example. Nevertheless, many applications exist (like ultrahyperbolic equations, for example) linked to non-commutative structures which require a different approach based on indefinite inner products. In this situation Hilbert modules are not the right choice as they do not reflect the

induced geometry and so-called Clifford-Krein modules appear as a natural answer. In this talk we are going to discuss the theory of Clifford-Krein modules with reproducing kernels and present some examples.

## Reproducing Kernel Cartan Subalgebra

**Anoh Yannick Kraidi**, Kangni Kinvi  
Félix Houphouët Boigny University

Let  $\mathfrak{g}$  be a semi-simple Lie algebra,  $\mathfrak{j}$  a Cartan subalgebra of  $\mathfrak{g}$ ,  $\mathfrak{j}^*$  the dual of  $\mathfrak{j}$ ,  $\mathfrak{j}^\vee$  the bidual of  $\mathfrak{j}$  and  $B(\cdot, \cdot)$  the restriction to  $\mathfrak{j}$  of the Killing form of  $\mathfrak{g}$ . In this work, we define the inversion formula on  $\mathfrak{j}^*$  using a linear map  $L$  on  $\mathfrak{j}^*$  and the reproducing kernel defined on the bidual  $\mathfrak{j}^\vee$  of  $\mathfrak{j}$ . When we consider the transform of the reproducing kernel obtained from the image of a representation of  $\mathfrak{g}$  on  $\mathfrak{j}^*$ , we get a relationship between these two kernels. We give a generalization of the theorem of decreasing principle for operators in the classical case. After that, we will prove some properties of the reproducing kernel Cartan subalgebra.

## Heat coefficients for magnetic Laplacians on the complex projective space $\mathbb{P}^n(\mathbb{C})$

**Z. Mouayn**, K. Ahbli, A. Hafoud  
Sultan Moulay Slimane

We denote by  $\Delta_\nu$  the Fubini-Study Laplacian perturbed by a uniform magnetic field whose strength is proportional to  $\nu$ . When acting on bounded functions on the complex projective  $n$ -space, this operator has a discrete spectrum consisting on eigenvalues  $\beta_m$ ,  $m \in \mathbb{Z}_+$ . For the corresponding eigenspaces, we give a new proof for their reproducing kernels by using Zarembo's expansion directly. These kernels are then used to obtain an integral representation for the heat kernel of  $\Delta_\nu$ . Using a suitable polynomial decomposition of the multiplicity of each  $\beta_m$ , we write down a trace formula for the heat operator associated with  $\Delta_\nu$  in terms of Jacobi's theta functions and their higher order derivatives. Doing so enables us to establish the asymptotics of this trace as  $t \searrow 0^+$  by giving the corresponding heat coefficients in terms of Bernoulli numbers and polynomials. The obtained results can be exploited in the analysis of the spectral zeta function associated with  $\Delta_\nu$ .

## PARTIAL DIFFERENTIAL EQUATIONS ON CURVED SPACETIMES

**Organizer:** Anahit Galstyan (University of Texas RGV, USA) & Makoto Nakamura (Osaka University, Japan) & Karen Yagdjian (University of Texas RGV, USA)

### **On a blow-up result with critical nonlinearities for a wave equation in the expanding de Sitter spacetime**

**Alessandro Palmieri**  
University of Bari

We consider a semilinear wave equation in the expanding de Sitter spacetime focusing on blow-up results and upper-bound estimates for the lifespan of local solutions to the corresponding Cauchy problem. The technique in the proofs consists in studying the growth of the time-dependent function given by the spatial average of a local solution. A special emphasis is given to a threshold case for the parameters appearing in the nonlinear term: by combining an integral representation formula by Yagdjian-Galstian with a technique borrowed from the critical case for the classical wave equation we prove that local solutions blow up even in this threshold case. Based on joint papers with Hiroyuki Takamura (Tohoku University).

### **The self-interacting Dirac fields in FLRW spacetime**

**Anahit Galstian**  
University of Texas RGV

In this talk we study solutions of the semilinear Dirac equation in the curved spacetime of the FLRW models of cosmology. We describe the relationship between the mass term, scale factor, nonlinear term, and initial function, which provides a global in time existence or an estimate on the lifespan of the solution of the Dirac equation in the expanding universe. The conditions on the imaginary part of mass will be discussed by proving nonexistence of the global solutions if certain relation between scale factor and the mass are fulfilled. Based on joint paper with Karen Yagdjian.

### **Singularity formation for the critical Zakharov system**

**Joachim Krieger**  
EPFL

I will discuss a recent result, obtained jointly with T. Schmid, which describes the formation of finite time singularities for the energy critical Zakharov system arising in Plasma physics. The method combines an approach due to Krieger-Schlag-Tataru as well as one due to G. Perelman.

### **Global (in time) existence of solutions for semilinear damped wave equations in Friedmann-Lemaître-Robertson-Walker spacetime**

**Jorge Marques, Marcelo Ebert**  
University of Coimbra

We study the Cauchy problem for the semilinear massless damped wave equation:

$$u_{tt}(t, x) - (1+t)^{-2\ell} \Delta u(t, x) + \frac{\beta}{1+t} u_t(t, x) = |u|^p, t \geq 0, x \in \mathbf{R}^n, \quad (8)$$

with  $0 < \ell < 1$ ,  $\beta > 0$  and  $p > 1$ . This wave equation is represented in the family of FLRW spacetime, which is endowed with a metric written in the form

$$ds^2 = -dt^2 + \alpha^2(t)(dx_1^2 + \dots + dx_n^2)$$

with an appropriate scale factor  $\alpha(t)$ . If  $\frac{d}{dt}\alpha(t) > 0$  and  $\frac{d^2}{dt^2}\alpha(t) < 0$  we can say that the universe is in decelerating expansion. Our model (8) is derived from  $\alpha(t) = (1+t)^\ell$  with  $\ell = \frac{2}{\gamma n}$  satisfying  $\frac{2}{n} < \gamma \leq 2$ ,  $n \geq 2$ . We also have  $\alpha(t) = (1+t)^{2/3}$  for the Einstein- de Sitter 4-dimensional spacetime, so that it is well known as a particular case of FLRW spacetime in a decelerating expansion universe model. The case in which  $\beta = 2$ ,  $n = 3$  and  $\ell = \frac{2}{3}$  in (8) was studied by Galstian and Yagdjian and it is called the covariant massless field in the Einstein- de Sitter spacetime with non-singular (at  $t = 0$ ) coefficients. Our main goal is to prove global (in time) existence of solutions in the case of decelerating expansion universe with small initial data belongs to  $L^1(\mathbf{R}^n) \cap H^{k-1}(\mathbf{R}^n)$ ,  $k \geq 1$ . We also find the critical exponent  $p_c(n, \ell) = 1 + \frac{2}{n(1-\ell)}$ , which is of Fujita type. For a given range of the values of  $\beta$  and  $\ell$  and  $p > p_c(n, \ell)$ , the global (in time) solution has the same long time behavior of  $u^{lin}$ , which is the solution of the corresponding linear Cauchy problem. Using the representation of the solution through by Duhamel's formula and optimal  $L^p - L^q$  estimates of  $u^{lin}$  we apply a fixed point argument for a nonlinear operator in a suitable evolution space to prove our results. This is a joint work with Marcelo Ebert (University of São Paulo, Brazil).

## Integral transform approach to solving partial differential equations in curved space-times

Karen Yagdjian

University of Texas Rio Grande Valley

In the talk we present integral transform approach to solving partial differential equations of quantum field theory in some curved space-times of cosmology. We briefly review some early results obtained by that approach such as the fundamental solutions to the Klein-Gordon equation, to Dirac and generalized Dirac operators, the Huygens' and incomplete Huygens' principles, the problem of the existence of global in time solutions for equations in the curved space-times. Then we present the new results obtained by that approach to some cosmological model.

## Global solutions of Klein-Gordon equation under the quartic potential in the de Sitter spacetime

Makoto Nakamura

Osaka University

The Cauchy problem for the Klein-Gordon equation under the quartic potential is considered in the de Sitter spacetime. The existence of global solutions for small rough initial data is shown based on the mechanism of the spontaneous symmetry breaking for the small positive Hubble constant. The effects of the spatial expansion and contraction on the problem are considered.

## Stability of standing waves for cubic-quintic NLS with delta potential

Masahito Ohta

Tokyo University of Science

We consider a nonlinear Schrödinger equation with the cubic-quintic combination of repulsive and attractive nonlinearities, and an attractive delta potential in one space dimension. The existence and stability of standing wave solutions are studied.

## Regularity theory and global existence of small data solutions to semi-linear de Sitter models with power non-linearity

Michael Reissig, Marcelo Rempel Ebert

Technical University Bergakademie Freiberg

In the talk we discuss different methods to treat the Cauchy problem for semi-linear de Sitter models with power non-linearity. The model of interest is

$$\phi_{tt} - e^{-2t} \Delta \phi + n \phi_t + m^2 \phi = |\phi|^p, \quad (\phi(0, x), \phi_t(0, x)) = (f(x), g(x)),$$

where  $m^2$  is a non-negative constant and  $n$  is the dimension. We study the global (in time) existence of small data solutions. In particular, we show the interplay between the power  $p$ , admissible data spaces and admissible spaces of solutions (Sobolev solutions, energy solutions or classical solutions). For this reason we distinguish between models with dominant dissipation, models with dominant mass and models with balanced mass and dissipation.

Finally, we turn to the open question of critical regularity.

## Asymptotic behavior of solutions to a system of nonlinear Schrodinger equations with cubic dissipative nonlinearity

Naoyasu Kita, Yoshihisa Nakamura, Yuji Sagawa

Kumamoto University

This is a joint work with Y. Nakamura (Kumamoto Univ.) and Y. Sagawa (Kumamoto Univ.), and supported by JSPS Grant-in-Aid for Scientific Research (C) No.23K03168.

We consider the initial value problem of a system of nonlinear Schrödinger equations:

$$\begin{cases} i \partial_t \mathbf{u} + \frac{1}{2} \partial_x^2 \mathbf{u} = \mathbf{f}(\mathbf{u}), \\ \mathbf{u}(0, x) = \mathbf{u}_0(x), \end{cases} \quad (9)$$

where  $(t, x) \in [0, \infty) \times \mathbb{R}$ ,  $i$  is the imaginary unit and  $\mathbf{u} = \mathbf{u}(t, x) = (u_1, u_2, \dots, u_n)^t$  is  $\mathbb{C}^n$ -vector valued unknown function. The nonlinearity  $\mathbf{f}(\mathbf{u})$  is a map from the set of  $\mathbb{C}^n$ -vectors to itself, described as

$$\mathbf{f}(\mathbf{u}) = (f_1(\mathbf{u}), f_2(\mathbf{u}), \dots, f_n(\mathbf{u}))^t.$$

We assume that the nonlinearity satisfies

$$(A.1) \text{ (gauge invariance) } \mathbf{f}(e^{i\theta} \mathbf{u}) = e^{i\theta} \mathbf{f}(\mathbf{u}) \text{ for any } \theta \in \mathbb{R}.$$

$$(A.2) \text{ (cubic nonlinearity) } \mathbf{f}(\lambda \mathbf{u}) = \lambda^3 \mathbf{f}(\mathbf{u}) \text{ for any } \lambda > 0.$$

(A.3) (dissipative structure) There exists some  $\rho > 0$  such that

$$\operatorname{Im} \left\{ \bar{\mathbf{p}}^t \cdot \left( \frac{\partial \mathbf{f}(\mathbf{u})}{\partial \mathbf{u}} \mathbf{p} \pm \frac{\partial \mathbf{f}(\mathbf{u})}{\partial \bar{\mathbf{u}}} \bar{\mathbf{p}} \right) \right\} \leq -\rho \sum_{j=1}^n |u_j|^2 |p_j|^2$$

for any  $\mathbf{u} = (u_1, u_2, \dots, u_n)^t$ ,  $\mathbf{p} = (p_1, p_2, \dots, p_n)^t \in \mathbb{C}^n$ , where  $\bar{\mathbf{p}} = (\bar{p}_1, \bar{p}_2, \dots, \bar{p}_n)^t$ .

The equation (9) is a generalization of a model in the nonlinear optics. Our interest lies on the asymptotic behavior of  $\mathbf{u}(t, x)$  as  $t \rightarrow \infty$ . Unlike the case of single equation, it is usually difficult to detect the explicit asymptotic leading term of  $\mathbf{u}$  for the system. However we will prove the  $L^\infty$ -decay estimate of the solution, and show that the asymptotic leading term satisfies an associated system of ordinary differential equations.

## On local well-posedness for critical NLS with power nonlinearity in higher spatial dimensions

Takeshi Wada  
Shimane University

In this paper we consider the following Cauchy problem for the nonlinear Schrödinger equation with power nonlinearity:

$$i\partial_t u + \Delta u = f(u) \equiv \lambda |u|^\alpha u, \quad u(0) = \phi,$$

where  $u$  is a complex-valued function defined on the spacetime  $\mathbf{R}^{1+N}$  with  $N \geq 3$ ,  $\Delta$  is the Laplace operator on  $\mathbf{R}^N$ ,  $\lambda \in \mathbf{C}$ , and  $\alpha > 0$ . We prove time local well-posedness in  $H^s(\mathbf{R}^N)$  in the case where the nonlinear term is critical from the scaling point of view, namely  $\alpha = 4/(N - 2s)$ , and has limited regularity so that the nonlinear term does not belong to  $C^s(\mathbf{R}^2; \mathbf{R}^2)$ .

## PDES IN FLUID MECHANICS

**Organizer:** Lucas Catão de Freitas Ferreira (UNICAMP, Brazil) & Nikolai Vasilievich Chemetov (USP, Brazil), Gabriela Planas (UNICAMP, Brazil) & Anna Laura Mazzucato (Penn State University, USA)

### Statistical solutions to evolution equations

**Anne Bronzi**, Cecília Mondaini (Drexel University, USA), Ricardo Rosa (UFRJ, Brazil)  
IMECC-Unicamp

In this talk, we will present an abstract notion of statistical solutions that generalizes the concept of statistical solutions initially introduced for the three-dimensional incompressible Navier-Stokes equations to a wide range of evolution equations. The main results are the existence of statistical solutions for initial value problems and the convergence of statistical solutions of regularized problems to the statistical solutions of the original problem. We will illustrate the applicability of the theory with the very incompressible Navier-Stokes equation, a reaction-diffusion equation, a non-linear wave equation and the inviscid limit of the Navier-Stokes equations to the Euler equations in both two- and three-dimensional cases.

### Existence of solution for a solidification model with convection

**Bianca Morelli Rodolfo Calsavara**, Francisco Guillén González  
UNICAMP

In this work, we introduce a PDE problem modeling a solidification/melting process in bounded 3D domains, coupling a phase-field equation and a free-boundary Navier-Stokes-Boussinesq system, where the latent heat effect is considered via a modification of the Caginalp model. Moreover, the convection in the non-solid regions is treated via a phase-dependent viscosity of the material that degenerates in the solid phase, letting only rigid motions in this phase. Then, we prove existence of global in time weak solutions for a regularized model, by means of the convergence of non-degenerate problems furnished truncating the viscosity.

### Micro-rotation and Vorticity in Micropolar Flows

**Cilon V Ferreira Perusato**, Robert Guterres, Wilberclay Melo, César Niche, Paulo Zingano  
Universidade Federal de Pernambuco

The spontaneous synchronization between micro-rotation and vorticity in micropolar flows in  $\mathbb{R}^n$  ( $n = 2, 3$ ) is investigated and used to explain the faster decay by  $t^{-1/2}$  of the angular velocity of the fluid particles' microrotation, as well as establishing its optimality. This synchronization effect remained elusive for sixty years of investigations in micropolar fluid flows and was revealed after important upper and lower bounds for the solutions in  $\dot{H}^m(\mathbb{R}^n)$  were obtained by the authors using monotonicity ideas. Several related results of independent interest are also given along the discussion. It is a joint work with R. Guterres (UFRGS), C. Niche (UFRJ), W. Melo (UFS) and Paulo Zingano (UFRGS).

## Local and global analysis in Besov-Morrey spaces for inhomogeneous Navier-Stokes equations

**Daniel Ferreira Machado**, Lucas Catão de Freitas Ferreira  
Universidade Estadual de Campinas - UNICAMP

In this work we consider the incompressible inhomogeneous Navier-Stokes equations in the whole space with dimension  $n \geq 3$ . We present local and global well-posedness results in a new framework for inhomogeneous fluids, namely Besov-Morrey spaces that are Besov spaces based on Morrey ones. In comparison with the previous works in Sobolev and Besov spaces, our results provide a larger initial-data class for both the velocity and density, constructing a unique global-in-time flow under smallness conditions on weaker initial-data norms. From a technical viewpoint, since the Morrey norms prevent the common use of energy-type and integration by parts arguments, we need to obtain some estimates for the localizations of the heat semigroup, the commutator, and the volume-preserving map in our setting.

## On the Cauchy problem for nonlinear Schrödinger equations with nonlinearities of Power-Type

**Edison Fausto Cuba Huamani**, Lucas C. F. Ferreira  
Unicamp

The aim of this talk is to examine the Cauchy problem associated with the nonlinear Schrödinger equations in homogeneous Besov-Lorentz spaces  $\dot{B}_{p,\infty}^{s,\infty}$  with  $s > 0$ . Obtain estimates for the nonlinear term in this type of spaces is the main challenge in establishing our results. However, we can solve this difficulty by using composition estimates to show the existence and uniqueness of global solutions for the nonlinear Schrödinger equation with nonlinearities of power-type.

## Numerical mathematics for computation of local and nonlocal nonlinear transport models in fluid mechanics

**Eduardo Abreu**  
University of Campinas

Local and nonlocal nonlinear hyperbolic-transport models are relevant on the mathematical foundations of modeling in fluid mechanics anchored on the basic and applied sciences. For instance, such models appear in a close connection with several dynamics and models in fluids (e.g., the surface quasi-geostrophic equation, vortex sheet and formation of vortex streets) and also in porous media flow and related equilibrium and nonequilibrium problems (e.g., nonlinear transport of fluids in porous medium as well as in shallow water equations with variable topography and discontinuous initial data). In this work, we will discuss how new insights on the improved concepts no-flow curves and no-flow surfaces in a Lagrangian-Eulerian (forward) formulation via numerical analysis linked to local and non-local nonlinear hyperbolic-transport models are key ingredients to address their inherent properties: nonlinearity, wave-breaking phenomena (shocks) and unique weak-entropy solution for both local and nonlocal (scalar) models. Indeed, we also undertake a numerical-analytical study of an initial value problem associated with 1D doubly (nonlocal) fractional conservation and its interplay between the Hilbert transform and the Riesz potential. We will also present advances in the design of a new class of semi-discrete positive Lagrangian-Eulerian schemes for solving multidimensional systems of (local) hyperbolic-transport problems supported by rigorous analysis.

## Algebraic decay in critical Sobolev spaces for 4D energy-critical nonlinear heat equation

**Gabriela Planas**, Leonardo Kosloff, César Niche  
Universidade Estadual de Campinas

We address the decay of solutions to the four-dimensional energy-critical nonlinear heat equation in the critical space  $\dot{H}^1$ . Recently, it was proven that the  $\dot{H}^1$  norm of solutions goes to zero when time goes to infinity, but no decay rates were established. By means of the Fourier Splitting Method and using properties arising from the scale invariance, we obtain an algebraic upper bound for the decay rate of solutions.

## Dirichlet Problem for Degenerate Fractional Parabolic Hyperbolic Equations

**Gerardo Jonatan Huaroto Cardenas**, Wladimir Neves  
UFAL- Universidade Federal de Alagoas

We are concerned in this paper with the degenerate fractional diffusion advection equations posed in bounded domains. Due to a suitable formulation, we show the existence of weak entropy solutions for measurable and bounded initial and Dirichlet boundary data. Moreover, we prove a L1-type contraction property for weak entropy solutions obtained via parabolic perturbation. This is a weak selection principle which means that the weak entropy solutions are stable in this class.

It is a joint work with Wladimir Neves da UFRJ, email: wladimir@im.ufrj.br.

## Well-posedness of the equations governing the motion of a fluid-filled elastic solid

**Giusy Mazzone**  
Queen's University

I will present some results concerning the existence of solutions to the equations governing the motion of an elastic solid with an interior cavity completely filled by a viscous incompressible fluid. The equations of motion are the Navier equations of linear elasticity for the solid, and the Navier-Stokes equations for the fluid. Continuity of stresses and velocities are imposed at the fluid-solid interface, and a zero-traction condition is imposed at the other free boundary of the solid. After demonstrating the existence of local (in time) strong solutions to the governing equations, we will discuss current challenges related to the existence of global solutions and their stability.

## On energy balance in 2D incompressible ideal fluid flow

**Helena J. Nussenzveig Lopes**  
Universidade Federal do Rio de Janeiro

In this talk I will discuss energy balance for weak solutions of the 2D incompressible Euler equations, with periodic boundary conditions, which are limits of vanishing viscosity, i.e. physically realizable weak solutions. I will show that, under relatively weak regularity assumptions on the external force, energy balance holds if and only if the Navier-Stokes solutions converge strongly in  $L_t^2 L_x^2$ . I will also discuss the largest space of initial data which implies this strong convergence. This is joint work with Fabian Jin (ETH-Zurich), Samuel Lanthaler (CalTech), Milton C Lopes Filho (UFRJ).

## **Lagrangian Structure of Vlasov-Maxwell system.**

**Henrique Borrin de Souza**, Marcelo Martins Santos  
UNICAMP

In this talk, we explore the existence of a suitable notion of flow for the Vlasov-Maxwell system, and equivalence between Lagrangian solutions, i.e., the transport of the initial condition by its flow, distributional solutions, and renormalized solutions. Moreover, we discuss the existence of renormalized solutions and physically suitable conditions for global well-posedness of such flow, namely, boundedness of initial energy. This is a joint work with my PhD advisor prof. Marcelo Santos, extending results provided by Ambrosio, Colombo, and Figalli for Vlasov-Poisson equation and Borrin-Marcon for relativistic Vlasov equations.

## **Asymptotic and numerical investigations on viscoelastic fluid flows**

**Irineu L. Palhares Junior**, Jonathan D. Evans, Cassio M. Oishi  
UNESP

Viscoelastic flow with singularities is a challenge in the area of computational fluid dynamics, since the presence of singularities can result in spurious solutions or simulation breakdown. Furthermore, for such flows, analytical solutions are not known. However, through the use of asymptotic analysis it is possible to obtain some analytical progress in such flows. In this sense, this study determines the asymptotic behavior of the stress tensor which describes the viscoelastic fluids. In particular, we have investigated the stick-slip flow which is a benchmark problem for studying the effect of singularities. In addition, boundary layer equations were determined in the stick and slip boundary layer regions. Our investigations included comparison between the asymptotic and numerical solutions in a transient fluid flow considering different viscoelastic models. The results of this study aim to bring more understanding on the behavior of stresses close to the singularity, as well as guide the construction of stable and accurate numerical methods.

## **A rescaled approach for the 3D-Boussinesq system in critical Fourier-Besov spaces**

**Leithold L. Aurazo-Alvarez**  
University of Campinas

It is proved a global well-posedness result for a tridimensional rescaled Boussinesq system, with positive full viscosity and diffusivity parameters, in the framework of critical Fourier-Besov spaces, which allow homogeneous functions with negative degree. The rescaled approach implies to rescale both the velocity, dividing by a positive parameter, and the temperature, dividing by the square of the same parameter, and study the obtained system. This rescaled approach permits to deal with the right hand linear term for the Boussinesq system, in order to apply a fixed point lemma, and to know a qualitative behaviour of the system, according to relations between both the parameters and the initial velocity and temperature; for instance, it is possible to consider, for small enough viscosity and large diffusivity, a large enough critical Fourier-Besov norm for the initial temperature and it is also possible to consider, for small enough diffusivity and large viscosity, a large enough critical Fourier-Besov norm for both the velocity and the temperature.

## **Stochastic traveling waves**

**Lynnyngs Kelly Arruda**

Federal University of São Carlos

In this talk we discuss the existence of some weak stochastic soliton-like solutions for some stochastic partial differential equations.

## A family of systems including the Herschel-Bulkley fluid equations

**Marcelo Santos**, Anderson L. A. de Araujo, Nikolai V. Chemetov  
UNICAMP

To a family of systems with a “power law stress tensor” which includes models for visco-plastic fluids (more generally, Bingham fluid equations/Herschel-Bulkley type equations), we use some basic facts on Convex Analysis to show that the tensor is the sub-differential of a convex function. Consequently, it defines a monotone operator. The convexity and monotonicity help to prove the existence of a solution to the system of equations. This is a joint work with Anderson L. A. de Araujo and Nikolai V. Chemetov.

## Lower bounds on the radius of analyticity for the Navier-Stokes and Kuramoto-Sivashinsky systems

**Milton da C. Lopes**,  
Universidade Federal do Rio de Janeiro

In 2011, Z. Lei and F. Lin presented a proof of existence of a global mild solution of the three-dimensional Navier-Stokes equations in scale-critical function spaces based on the Wiener algebra, for sufficiently small initial data in full space. In 2015, H. Bae modified the technique used by Lei and Lin to prove lower bounds on the radius of analyticity of the solution obtained. Recently, Ambrose, Lopes Filho and Nussenzweig Lopes adapted Bae’s argument to periodic flows, which changes the radius estimate a little. The same authors have adapted these ideas to revisiting the 2004 existence result by M. Cannone and G. Karch in spaces of pseudo-measures, extending it to a larger space of initial data, and to the scalar Kuramoto-Sivashinsky equation, where we improve known existence results and obtain new estimates on the radius of analyticity of the solution, both in spaces based on pseudo-measures and on the Wiener algebra. The purpose of this talk is to survey these results and illustrate the technique involved.

## The Rigid Body Motion in Cosserat’s Fluid With Navier’s Slip Boundary Conditions

**Nikolai V. Chemetov**,  
USP - Universidade de São Paulo

The aim of the talk is to give a brief presentation of novel results related to the body-fluid interaction problem. The motion is described by a system of coupled differential equations: Newton’s second law and Navier Stokes type equations. We shall discuss the global solvability result of weak solution of the problem, when the slippage is allowed at the boundaries of the rigid body and of the bounded domain, occupied by the fluid. This result completely resolves a famous non-contact paradox between the rigid body and the domain boundary.

## On the self-similar blowup for the generalized SQG equation

**Ricardo Martins M. Guimaraes**, Anne Bronzi, Cecília Mondaini  
UNICAMP

In this work, we propose to study the locally self-similar solutions of the generalized equation (SQG). We proved that under an  $L^q$  growth assumption on the self-similar profile and its gradient, we identify appropriate

ranges of the self-similar parameter where the profile is either identically zero (no blowup) or its  $L^p$  asymptotic behavior can be characterized, for suitable  $p$ . This is a joint work with A. Bronzi (Unicamp) and C. Mondaini (Drexel).

## Asymptotic description of the long linear water wave propagation over the non-uniform bottom with fast-oscillating parts

Sergey Sergeev  
PUC-Rio

We pose the Cauchy-Poisson problem for the potential of the velocity vector field in a basin with non-uniform bottom and we assume that there may be fast-oscillating areas of this bottom. The initial condition is chosen in the form of a localized function. We are interested in the description of the propagation of the long waves on the free surface of the basin for the far distances. In such formulation the problem has several natural small parameters: localization of the initial condition, the parameter of the oscillations of the bottom and inverse value to the distance of wave propagation.

We reduce the problem for the potential to the two-dimensional Cauchy problem on the surface and the resulting governing equation has the pseudo-differential form. For the long waves we can provide a semi-classical asymptotic description of the solution while the localization parameter tends to zero. The resulting asymptotic solution can be written explicitly and presented in terms of Airy functions. This explicit representation allows us to provide the fast simulations for the wave propagation and its analysis on the initial parameters of the problem.

## On the 2D inviscid Boussinesq equations with critical regularity and dispersive effects in homogeneous Besov spaces

Vladimir Angulo, Lucas Catão de Freitas Ferreira, Leonardo Kosloff  
Universidad Nacional de Colombia

We study the long-time solvability for 2D inviscid Boussinesq equations

$$\begin{cases} \partial_t u + (u \cdot \nabla)u + \nabla p = \kappa \theta e_2, \\ \partial_t \theta + (u \cdot \nabla)\theta = 0, \\ \operatorname{div} u = 0, \\ u(x, 0) = u_0(x), \theta(x, 0) = \theta_0(x), \end{cases} \quad (10)$$

where  $(x, t) \in \mathbb{R}^2 \times (0, \infty)$ ,  $u$  is the fluid velocity,  $\theta$  denotes the temperature (or the density in geophysical flows),  $p$  stands for the pressure, and the constant  $\kappa$  is a parameter associated with the strength of gravity. The unit vector  $e_2 = (0, 1)$  indicates the positive vertical direction.

We wish to consider the initial temperature close to a nontrivial balance, namely  $\theta_0(x) = \rho_0(x) - \kappa x_2$ , and then look for solutions with the temperature in the form  $\theta(t, x) = \rho(t, x) - \kappa x_2$ . Moreover, applying the “curl” to the first equation in (10) and recalling that  $\omega = \operatorname{curl}(u) = \nabla^\perp \cdot u$ , we will work with the following new system

$$\begin{cases} \partial_t \omega + (u \cdot \nabla)\omega = \kappa \partial_1 \rho, \\ \partial_t \rho + (u \cdot \nabla)\rho = \kappa u_2, \\ u = \nabla^\perp (-\Delta)^{-1} \omega, \\ \omega(x, 0) = \omega_0(x), \rho(x, 0) = \rho_0(x), \quad \text{in } \mathbb{R}^2. \end{cases} \quad (11)$$

In fact, we show the local solvability in Besov spaces uniformly with respect to a physical parameter  $\kappa$  associated with the strength of gravity. After, employing a blow-up criterion and Strichartz-type estimates, the long-time solvability is obtained for large  $\kappa$  and regardless of the size of initial data. Our results provide a larger class of initial data as well as cover borderline regularity for the system.

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## The Buckley-Leverett system. Revisited.

Wladimir Neves

Universidade Federal do Rio de Janeiro

In this talk, we discuss about the important issue of existence of solutions for the Buckley-Leverett system, that is to say, we consider the following system

$$\begin{cases} \partial_t u + \operatorname{div}(g(u)\mathbf{v}) = 0, \\ h(u)\mathbf{v} = -\nabla p, \\ \operatorname{div}(\mathbf{v}) = 0. \end{cases} \quad (12)$$

The first equation is a conservation law that expresses the mass balance for the density  $u$  of a fluid evolving according to a nonlinear continuity equation. The second equation expresses the Darcy's law, that is an empirical law which describes the dynamics of the velocity field  $\mathbf{v}$  of the flow in relation with the scalar function  $p$ , usually called pressure, exerted on it. The third and last equation is the incompressibility condition for the velocity field of the flow.

This is a joint work with André de Oliveira Gomes.

## Interfaces for Shock Capturing Scheme and Boundary Conditions

Zamurat A. Adegboye, Reula O. Oscar, Helen O. Edogbanya

Institute of Mathematical and Physical Sciences, IMSP-UAC, Dangbo, Benin

We formulate Adomian Decomposition Method Implementation technique for Solving Partial Differential Equations to obtain analytic solutions in a rapidly convergent series to some class of higher order partial differential equation. After that we try them in our Force-Free codes to generate jets of astrophysical interest around rotating black holes. This scheme is optimal for it respects the accuracy of the internal method and introduces minimal dissipation. We also investigate if there is any connection between the work and Interfaces for Shock Capturing Scheme, in particular whether the method can be used as a truncated approximation scheme and study their respective accuracy, stability in the complex plane and possible TVD properties. We are able to find conditions for passing the information from one of these grids to the neighboring one so that perturbations continue traveling without distortion.

## PSEUDO DIFFERENTIAL OPERATORS

**Organizer:** Man Wah Wong (York University Toronto, Canada)

### Pseudo-differential operator associated with index Whittaker transform

**Akhilesh Prasad**, S. K. Verma, Jeetendrasingh Maan  
Indian Institute of Technology (ISM)

Some estimates of the kernel of index Whittaker transform and Young's type convolution inequality in a family of weighted Lebesgue spaces are discussed. An iterated convolution is defined and obtained its estimate. Two test function spaces are introduced and discussed the continuity of the index Whittaker transform, differential operator on these spaces. Moreover, pseudo-differential operator associated with index Whittaker transform is defined and studied its continuity between the given function spaces. Some applications of the index Whittaker transform in pseudo-differential equation are discussed.

### Global hypoellipticity for a class of systems of periodic P.D.O.

**Fernando de Avila Silva**  
UFPR - Universidade Federal do Paraná

We present an investigation on the global hypoellipticity problem for a class of systems of pseudo-differential operators on the torus. The approach consists in establishing conditions on the matrix symbol of the system such that it can be transformed into a suitable triangular form involving a nilpotent upper triangular matrix. Hence, global hypoellipticity is studied by analyzing the behavior of the eigenvalues and their averages.

### Orlicz modulation spaces and pseudo-differential operators

**Joachim Toft**  
Linnaeus University

A convex function  $\Phi$  from  $[0, \infty]$  to  $[0, \infty]$  with properties

$$\Phi(0) = 0, \quad \lim_{t \rightarrow \infty} \Phi(t) = \Phi(\infty) = \infty,$$

is called a Young function. For any Young function  $\Phi$ , the Orlicz space  $L^\Phi$  is a Banach space, and consists of all measurable functions  $f$  such that  $\Phi(t \cdot |f|) \in L^1$  for some  $t > 0$ . By choosing  $\Phi$  in suitable ways we gain the definition of any (Banach) Lebesgue space  $L^p$ , as well as sums of such spaces like  $L^p + L^q$ ,  $p, q \in [1, \infty]$ . In particular, the family of Orlicz spaces contain any Lebesgue space.

The Orlicz modulation space  $M^\Phi$  is obtained by imposing  $L^\Phi$  norm conditions of the short-time Fourier transforms of the involved functions and distributions. In the same way we may discuss Orlicz modulation spaces  $M^{\Phi, \Psi}$  of mixed normed types. Again, by choosing the Young functions  $\Phi$  and  $\Psi$  in suitable ways,  $M^{\Phi, \Psi}$  becomes the classical Feichtinger's modulation space  $M^{p, q}$ . In the talk we explain some basic properties and give some examples on interesting Orlicz spaces and Orlicz

modulation spaces. We also explain some classical results on pseudo-differential operators acting on modulation spaces, and give examples on how such results can be extended to the framework of Orlicz modulation spaces.

*The talk is based on joint works with A. Gumber, N. Rana, S. Öztop and R. Üster.*

## **$M$ -Ellipticity of Fredholm Pseudo-Differential Operators on $L^p(\mathbb{R}^n)$ and GRarding's Inequality**

**Lalit Mohan**, Aparajita Dasgupta  
Indian Institute of Technology Delhi

In this talk, we study  $M$ -elliptic pseudo-differential operators on  $\mathbb{R}^n$ . First, we will discuss the  $M$ -ellipticity of Fredholm pseudo-differential operators associated with weighted symbols on  $L^p(\mathbb{R}^n)$ ,  $1 < p < \infty$ . Then, we explore the GRarding's inequality for SG  $M$ -elliptic class of pseudo-differential operators. Finally, we present some applications of the result above to show the existence and uniqueness of parabolic type IVP with weighted symbols.

## **$L^p_\alpha(\mathbb{R}_+^{n+1})$ - Boundedness of Pseudo-differential Operators involving the Weinstein Transform**

**Mohd Sartaj**, S.K. Upadhyay  
Indian Institute of Technology (BHU)

In this paper, an  $L^p_\alpha(\mathbb{R}_+^{n+1})$ -boundedness of pseudo-differential operators associated with class of symbol  $S^0$  are proven by utilizing the theory of the Weinstein transform. Using the aforesaid theory various properties and boundedness results on  $L^p_\alpha(\mathbb{R}_+^{n+1})$  - type Sobolev spaces are given.

## **Adams and Trudinger-Moser embeddings on weighted Sobolev spaces and applications**

**Raoní Cabral Ponciano**, João Marcos Bezerra do Ó, Guozhen Lu  
Universidade Federal da Paraíba

In this talk, we will explore sharp Sobolev embeddings and Adams-type inequalities on a class of Sobolev spaces with potential weights, without assuming boundary conditions. We will focus on the borderline Sobolev embedding into the exponential class, highlighting its sharp constant. By applying these concepts, we will demonstrate the existence of nontrivial solutions for elliptic equations with nonlinearities including both polynomial and exponential growths.

Additionally, we will investigate embeddings on a class of Sobolev spaces with potential weights on unbounded domains. Our findings will reveal embeddings into weighted Lebesgue spaces, specifically  $L^q_\theta$ , using radial power weights. Based on these embeddings, we will investigate the existence and non-existence of maximizers for Trudinger-Moser type inequalities. Furthermore, we will present a technique to enhance the maximal integrability by selectively removing necessary terms from the exponential series while maintaining the continuity of the embedding.

## Holomorphic One-Parameter Semigroups of Bounded Linear Operators Generated by Strongly $M$ -Elliptic Pseudo-Differential Operators on Euclidean Spaces

Yao Dong Gao, M W Wong  
York University

For a  $M$ -elliptic pseudo-differential operator on  $\mathbf{R}^n$ , we give a sufficient condition on its symbol to ensure that it is the infinitesimal generator of a holomorphic one-parameter semigroup of bounded linear operators on  $L^p(\mathbf{R}^n)$ ,  $1 < p < \infty$ .

## QUATERNIONIC AND CLIFFORD ANALYSIS

**Organizer:** Swanhild Bernstein (TU Bergakademie Freiberg, Germany) & Uwe Kähler (University of Aveiro, Portugal) & Irene Sabadini (Politecnico di Milano, Italy) & Franciscus Sommen (Ghent University, Belgium)

### Nuclear operators and the Grothendieck-Lidskii formula in quaternionic spaces

**Alberto Debernardi Pinos**, Paula Cerejeiras, Fabrizio Colombo, Uwe Kähler, Irene Sabadini  
Universitat Autònoma de Barcelon

In this talk we discuss the Grothendieck-Lidskii formula in quaternionic Hilbert spaces, along with the particularities of the problem, which include, among others, the definition of an appropriate trace that differs from the usual one.

We will also discuss  $r$ -nuclear operators in quaternionic Banach (or, more generally, locally convex) spaces  $X$ , i.e., operators of the form

$$\sum_{k=1}^{\infty} r_k(x_k \otimes x'_k), \quad x_k \in X, x'_k \in X',$$

where  $\{r_k\} \in \ell^r$ , and the associated Grothendieck-Lidskii's formula for the case  $r \leq 2/3$ . In order to establish these results, the use of the aforementioned traces (and their invariance with respect to the basis choices) is essential.

This is a joint work with P. Cerejeiras, F. Colombo, U. Kähler, and I. Sabadini.

### On fractional d-bar derivatives

**Arran Fernandez**, Cihan Guder, Walaa Yasin  
Eastern Mediterranean University

The d-bar derivative is one of the fundamental operators of complex analysis, quaternion analysis, and Clifford analysis. As fractional derivatives are becoming popular nowadays in various branches of mathematics and sciences, it is natural to think about fractional versions of the d-bar derivative. Previous work in this direction has not taken into account the idea of fractional powers, leading to natural properties such as a semigroup law and composition relations analogous to those of the classical fractional calculus. This talk will present a new approach for obtaining fractional d-bar derivatives with natural properties.

### First Order Differential Operators Associated to $\tilde{\Delta} = \text{div}(B\nabla)$ Operator in Clifford Analysis

**Carmen Judith Vanegas**, Franklin Vargas Jiménez, Eusebio Ariza  
Yachay Tech University

Let  $\mathcal{F}$  be a given differential operator acting with respect to spacelike variables, then a function space  $\mathcal{B}$  is called an associated space to  $\mathcal{F}$  if  $\mathcal{F}$  applies  $\mathcal{B}$  into itself. In this research work, a

characterization of all the first-order differential operators with coefficients in the Clifford algebra  $CL_n$  that are associated to the space of solution functions of the partial differential equation  $\tilde{\Delta}u = \text{div}(B\nabla)u = 0$  has been determined. To make this possible the equation  $\tilde{\Delta}u = 0$  and some special solutions of such equation were used. The results found imply that the technique of associated spaces allows to solve certain initial value problems.

## On slice regular functions and fiber bundle theory

J. Oscar González-Cervantes,

Several topological methods have been used to study the hypercomplex analysis. Particularly, the fiber bundle theory has recently been used to interpret some phenomena in the quaternionic right-linear space of slice regular functions. The main idea of this talk is to explain the following advances in the theory of slice regular functions obtained from the fiber bundle theory:

1. The Splitting Lemma and the Representation Formula intrinsically determine a fiber bundle over the space of quaternionic slice regular functions.
2. Quaternionic slice regular functions define on the total space on specific sphere bundles that generalize the usual concept of slice regular functions on axially symmetric s-domains
3. An interpretation of the behavior of the zero sets of some quaternionic slice regular polynomials in terms of the theory of fiber bundles.
4. A concept of slice regular function in several quaternionic variables can be established from the fiber bundle theory.

## Fock spaces and superoscillations

Kamal Diki, Daniel Alpay, Fabrizio Colombo, Irene Sabadini, Daniele C. Struppa  
Chapman University

In this talk we use techniques in Fock spaces theory and compute how the Segal-Bargmann transform acts on special wave functions obtained by multiplying superoscillating sequences with normalized Hermite functions. It turns out that these special wave functions can be constructed also by computing the approximating sequence of the normalized Hermite functions. First, we start by treating the case when a superoscillating sequence is multiplied by the Gaussian function. Then, we extend these calculations to the case of normalized Hermite functions leading to interesting relations with Weyl operators. In particular, we show that the Segal-Bargmann transform maps superoscillating sequences onto a superposition of coherent states. As a consequence, we obtain two new integral Bargmann-type representations of superoscillating sequences. We also present some results relating superoscillation functions with Weyl operators and Fourier transform.

## White and Grey noise analysis in the case of generalised Grassmann algebras

Paula Cerejeiras  
Universidade de Aveiro

One approach to the study of stochastic calculus with white noise is the creation of sequence of Gelfand triples where each space is a reproducing kernel Hilbert space linked to the Fock space

via the Hermite transform. We shall extend this approach in the context of a supersymmetry with  $Z_3$ -graded Grassmannian algebras by establishing a ternary Fock space and the construction of the corresponding strong algebra of stochastic distributions.

## Clifford B-Splines in Higher Dimensions

**Peter Massopust**, Jeffrey Hogan  
Technical University of Munich

We introduce a generalization of the classical concept of univariate B-spline to higher dimensions using a Clifford analytic setting. This generalization employs a definition in the Fourier domain. Several properties of these new higher-dimensional B-splines are derived and a construction of these functions in the time domain is presented. This is joint work with J. Hogan (University of Newcastle, Australia).

## On the Jackson-Stechin Theorems in Clifford algebras

**Radouan Daher**, Othman Tyr  
University Hassan II

In this talk, we look at problems in the theory of approximation of functions in real Clifford algebras. We prove analogues of direct and inverse approximation theorems in terms of best approximations of functions with bounded spectrum and the moduli of smoothness of all orders constructed by the generalized Steklov operator.

## Szegő-Radon transform in hypermonogenic setting and complex harmonic setting

**Ren Hu**  
Tianjin Sino-German University of Applied Sciences

In this talk, we study the Szegő-Radon transform in the hypermonogenic setting and the complex harmonic setting. Firstly, we introduce the hypermonogenic function which is the null-solution of the modified Dirac operator. Then we present the Szegő-Radon transform in this setting and show its explicit expression. Compare to the hypermonogenic case, we introduce the complex harmonic setting and consider the corresponding Szegő-Radon transform. We present its explicit formula and construct its dual transform and inversion.

## Spectral theory for quaternionic non-self-adjoint operators

**Uwe Kähler**  
Universidade de Aveiro

One of the principal problems in studying spectral theory for quaternionic or Clifford-algebra-valued operators lies in the fact that due to the noncommutativity many methods from classic spectral theory are not working anymore in this setting. For instance, even in the simplest case of finite rank operators there are different notions of a left and right spectrum. Hereby, the notion of a left spectrum has little practical use while the notion of a right spectrum is based on a nonlinear eigenvalue problem. In the present talk we will recall the notion of  $S$ -spectrum as a natural way to consider a spectrum in a noncommutative setting and use it to study quaternionic non-selfadjoint operators. To this end we will

discuss quaternionic Volterra operators and triangular representation of quaternionic operators similar to the classic approaches by Gohberg, Krein, Livsic, Brodskii and de Branges. Hereby we introduce spectral integral representations with respect to quaternionic chains and discuss the concept of P-triangular operators in the quaternionic setting. This will allow us to study the localization of spectra of non-selfadjoint quaternionic operators.

## **Quaternionic Kolosov-Muskhelishvili formulae for three dimensional transversely anisotropic elasticity**

**Yuri Grigor'ev**, Andrei Yakovlev  
North-Eastern Federal University

The quaternionic functions method is an analytical tool used in elasticity theory. For isotropic elasticity, there are known a few variants of three-dimensional analogues of the Kolosov-Muskhelishvili formulae. In this case, a general solution of the Lamé equation for the spatial theory of elasticity is expressed in terms of two regular quaternionic or monogenic Clifford functions. For the anisotropic elasticity, a close approach exists when equations of an equilibrium are factorized by means of matrix algebra. In our early works quaternionic factorization of equations of the transversely isotropic theory of elasticity is proposed. The model of the transversely isotropic elasticity is described by five elastic constants, and for example, can be used in the mechanics of rocks in permafrost conditions. In this report we present the new quaternionic Kolosov-Muskhelishvili formulae for the transversely isotropic elasticity when displacement vector is expressed by means of quasi-regular quaternionic functions.

The work is supported by Russian Science Foundation, grant N 20-31-90065.

## RECENT PROGRESS IN EVOLUTION EQUATIONS

**Organizer:** Marcello D'Abbico (University of Bari, Italy) & Michael Reissig (TU Bergakademie Freiberg, Germany)

### Functional dissipativity of second order differential operators with complex coefficients

**Alberto Cialdea**, Vladimir Maz'ya  
University of Basilicata

It is well known that the concept of dissipativity of an operator  $E$  is strictly related to the contractivity of the generated semigroup and to the maximum principle. In this talk I present some recent results obtained with Vladimir Maz'ya concerning a new concept of dissipativity. We say that the second order differential operator  $E = \nabla(A\nabla)$  ( $A$  being a matrix with complex valued  $L^\infty$  entries) is dissipative with respect to a given positive function  $\varphi : \mathbb{R}^+ \rightarrow \mathbb{R}^+$  if

$$\operatorname{Re} \int_{\Omega} \langle A\nabla u, \nabla(\varphi(|u|)u) \rangle dx \geq 0$$

for any complex valued  $u \in \dot{H}^1(\Omega)$  such that  $\varphi(|u|)u \in \dot{H}^1(\Omega)$ ,  $\Omega$  being a domain in  $\mathbb{R}^N$ . Here  $\langle \cdot, \cdot \rangle$  denotes the usual complex scalar product.

The functional dissipativity is an extension of the concept of  $L^p$ -dissipativity, which is obtained taking  $\varphi(t) = t^{p-2}$  ( $1 < p < \infty$ ). We remark that the class of operators whose principal part is strictly  $L^p$ -dissipative coincides with the class of the so called  $p$ -elliptic operators, which was recently considered by several authors.

A motivation for the study of the more general functional dissipativity comes from the decrease of the Luxemburg norm in a proper Orlicz space of solutions of the Cauchy–Dirichlet problem  $u' = Eu$ ,  $u(0) = u_0$ .

We give necessary and sufficient conditions for the functional dissipativity not only for scalar operators but also for some systems.

### Sharp decay assumptions in the Cauchy problem for $p$ -evolution equations in Gevrey classes

**Alessia Ascanelli**, Alexandre Arias Junior, Marco Cappiello  
University of Ferrara

We deal with the class of  $p$ -evolution equations with real characteristics. This is a very wide class of pdes containing, for instance, the Schrödinger equation and KDV equation. Necessary and/or sufficient conditions for well-posedness of the associated Cauchy problem in Sobolev classes in the case of  $(t, x)$ -depending coefficients have been widely investigated, while there are few results concerning the Gevrey setting. In this talk we prove the sharpness of some decay assumptions on the coefficients of the subprincipal part of the equation leading to a well-posed in Gevrey classes Cauchy problem.

## Some remarks on the Cauchy Problem for $p$ -evolution equations in Gelfand-Shilov spaces

Alexandre Arias Junior  
University of Torino

We consider the Cauchy problem

$$\begin{cases} P(t, x, D_t, D_x)u(t, x) = f(t, x), & (t, x) \in [0, T] \times \mathbb{R}, \\ u(0, x) = g(x), & x \in \mathbb{R}, \end{cases} \quad (13)$$

where

$$P(t, x, D_t, D_x) = D_t + a_p(t)D_x^p + \sum_{j=0}^{p-1} a_j(t, x)D_x^j,$$

$p = 2, 3$ ,  $a_p(t)$  is a continuous real-valued function which never vanishes and  $a_j(t, x)$  are continuous with respect to time and Gevrey regular with respect to space variable  $x$ .

In this talk, we shall discuss a sufficient decay condition at  $|x| \rightarrow \infty$  on the coefficients  $a_j(t, x)$  to obtain well-posedness for the Cauchy problem (13) in Gelfand-Shilov spaces.

## A weakly coupled system of noneffectively damped evolution equations

Marcello D'Abbicco, Antonio Lagioia  
University of Bari

In this talk, we consider the critical curve for the system

$$\begin{cases} \partial_t^2 U + (-\Delta)^\sigma U + D\partial_t U = F(U), & t > 0, x \in \mathbb{R}^n, \\ (U, U_t)(0, x) = (0, \Phi(x)) & x \in \mathbb{R}^n, \end{cases}$$

where  $\sigma > 1$ ,  $U = (u_1, u_2)^T$ ,  $\Phi = (\varphi_1, \varphi_2)^T$ , the term

$$DU = \begin{pmatrix} a_1(-\Delta)^{\frac{\theta_1}{2}} & 0 \\ 0 & a_2(-\Delta)^{\frac{\theta_2}{2}} \end{pmatrix}, \quad (14)$$

with  $\theta_1, \theta_2 \in [\sigma, 2\sigma]$ ,  $a_1, a_2 \geq 0$ , represents a noneffective damping term, and the coupling term is semilinear, in particular, we may take

$$F(U) = \begin{pmatrix} |u_2|^{p_2} \\ |u_1|^{p_1} \end{pmatrix}. \quad (15)$$

To obtain the result, we employ the  $L^p - L^q$  estimates for the corresponding linear equation obtained in [1].

## References

1. M. D'Abbicco, M.R. Ebert,  $L^p - L^q$  estimates for a parameter-dependent multiplier with oscillatory and diffusive components, *J. Math. Anal. Appl.* **504** (2021), 1, 125393.
2. M. D'Abbicco, A. Lagioia, *Critical curves for a weakly coupled system of  $\sigma$ -evolution equations with non-effective damping*, preprint.

## Global solvability for semi-discrete Kirchhoff equation

Fumihiko Hirose  
Yamaguchi University

We consider the following initial value problem of semi-discrete Kirchhoff equation, which is a discretized model of the Cauchy problem of nonlinear wave equation of Kirchhoff type:

$$\begin{cases} u''(t)[n] - \Phi(\|D^+u(t)\|^2)D^+D^-u(t)[n] = 0, & (t, n) \in (0, \infty) \times \mathbb{Z}, \\ u(0)[n] = u_0[n], \quad u'(0)[n] = u_1[n], & n \in \mathbb{Z}, \end{cases} \quad (16)$$

where  $\Phi \in C^1([0, \infty))$  satisfying  $\inf_{\eta \geq 0} \{\Phi(\eta)\} > 0$ ,  $D^+$  and  $D^-$  are difference operators for  $f = \{f[n]\}_{n \in \mathbb{Z}}$  defined by

$$D^\pm f[n] := \pm (f[n \pm 1] - f[n]),$$

and  $\|f\| := \|f\|_{l^2(\mathbb{Z})} := (\sum_{n \in \mathbb{Z}} |f[n]|^2)^{1/2}$ . For the solution  $u(t) = \{u(t)[n]\}_{n \in \mathbb{Z}}$  of (16), the total energy  $E(t; u)$  is defined by

$$E(t; u) := \frac{1}{2} \left( \|u'(t)\|^2 + \int_0^{\|D^+u(t)\|^2} \Phi(\eta) d\eta \right).$$

Then we have the following theorem.

**Theorem 0.0.1.** *If  $E(0; u) < \infty$ , then (16) has a unique global solution such that  $E(t; u) = E(0; u)$  for any  $t \geq 0$ .*

## Global Wellposedness for a Class of Weakly Hyperbolic Cauchy Problems on $\mathbb{R}^d$

Giovanni Girardi, Sandro Coriasco, N. Uday Kiran  
Università Politecnica delle Marche

We consider a class of second order weakly hyperbolic equations, with coefficients that are unbounded in the space variable ( $x \in \mathbb{R}^d$ ) and have an oscillatory behavior in the time variable. We develop a generalized parameter-dependent symbolic calculus, in order to construct a suitable parametrix for the Cauchy problem, employing theories of (generalized) Fourier integral operators and pseudodifferential operators. Through the properties of the obtained parametrix we prove some results about the local (in time) wellposedness of our problem.

## Visco-elastic damped wave models with time-dependent coefficient

Halit Sevki Aslan, Michael Reissig  
University of São Paulo

We study the following Cauchy problem for the linear visco-elastic damped wave equation with a time-dependent coefficient  $g = g(t)$ :

$$\begin{cases} u_{tt} - \Delta u + g(t)(-\Delta)u_t = 0, & (t, x) \in (0, \infty) \times \mathbb{R}^n, \\ u(0, x) = u_0(x), \quad u_t(0, x) = u_1(x), & x \in \mathbb{R}^n. \end{cases} \quad (17)$$

Here  $g = g(t)$  is a time-dependent coefficient satisfying some appropriate conditions like positivity, continuity, monotonicity, and some behavior of derivatives. The role played by the visco-elastic damping in (17) varies with the choices of the coefficient  $g = g(t)$  and different approaches are required to study its influence on the asymptotic profile of the solution as  $t \rightarrow \infty$ . Our main goal is to study decay rates of energies of higher order for solutions to the Cauchy problem (17) with a general time-dependent coefficient in the visco-elastic damping. These estimates rely on the structural properties of representations of solutions. Our approach is based on asymptotic representations combined with an extended phase space analysis under some assumptions, mostly adapted from the treatment of problems in WKB-analysis.

We will divide our considerations into the following cases of the time-dependent coefficients:

- Models with increasing time-dependent coefficient  $g = g(t)$ .
- Models with integrable and decreasing time-dependent coefficient  $g = g(t)$ .
- Models with non-integrable and decreasing time-dependent coefficient  $g = g(t)$ .
- Models with non-integrable and slowly increasing time-dependent coefficient  $g = g(t)$ .

## Decay estimates for space-time fractional equations with structural damping and nonlinear memory

Jorge Marques  
University of Coimbra

In this talk we study a wide class of space-time fractional equations with structural damping and nonlinear memory:

$$\partial_t^{2\alpha} u(t, x) + \mu(-\Delta)^{\frac{\sigma}{2}} \partial_t^\alpha u(t, x) + (-\Delta)^\sigma u(t, x) = I_{0+}^{1-\alpha} f(t, x), \quad t \geq 0, \quad x \in \mathbf{R}^n$$

where the parameters satisfying  $0 < \alpha \leq 1$ ,  $\mu > 0$  and  $\sigma > 1$ . Here, we take the so-called Caputo-Djrbashian derivative with respect to time  $t$ ,  $\partial_t^\beta$  of order  $\beta$  with  $\beta = 2\alpha$ ,  $\alpha$ , the fractional Laplacian in the space variable  $x$ ,  $(-\Delta)^\gamma$  with  $\gamma = \frac{\sigma}{2}, \sigma$ , and the Riemann-Liouville integral  $I_{0+}^{1-\alpha} f(t, x)$ .

We first obtain solution representations for two Cauchy type problems expressed in terms of Mittag-Leffler's functions by using a Duhamel type formula with the aid of Laplace (in time) and Fourier (in space) transforms. Taking into account that the multipliers are radially symmetric functions, the convolution kernels are represented through the Hankel transform. Then, our purpose is to establish decay estimates of the derivatives of its solutions for initial data with  $L^p$  and Sobolev regularity. We highlight that the strategy to be employed relies on harmonic analysis tools.

We also provide applications employing Strichartz's estimates to prove existence results for mild solutions in particular cases, in which the inhomogeneous term is given by  $f(t, x) = |u|^k$ , for  $k > 1$  or  $f$  belongs to a time weighted Banach space.

This is a joint work with Nelson Faustino (University of Aveiro, Portugal).

## Exponential stability for viscoelastic waves with delay effects

To Fu Ma  
University of Brasília

This talk is concerned with viscoelastic waves featuring small delays on the velocity. We show there exists a class of admissible memory kernels that is necessary and sufficient for exponential stability.

## Stability for partially damped models in curved beams

**Marcio A. Jorge da Silva**  
Universidade Estadual de Londrina

Curved beams are widely used in various engineering applications, and understanding their stability behavior is essential from a mathematical and applied point of view. The presence of damping in these systems can significantly affect their dynamic response and stability characteristics. The results contribute to the theoretical understanding of partially damped systems and provide a foundation for future research on more complex structural dynamics problems involving viscoelastic and thermoelastic curved beams. In particular, a partially damped thermoelastic curved beam system will be approached in the present talk.

## The Cauchy problem for KdV-type equations in projective Gevrey spaces

**Marco Cappiello**, Alexandre Arias Junior, Alessia Ascanelli  
University of Turin

We discuss the well-posedness of the Cauchy problem for a class of third order quasilinear evolution equations in a suitable class of Gevrey-Sobolev spaces. The approach we use is a combination of classical techniques coming from the theory of linear  $p$ -evolution equations with the Nash-Moser inversion theorem in tame Fréchet spaces.

## Global solutions to the Kirchhoff equation with spectral gap data in the energy space

**Marina Ghisi**, Massimo Gobbino  
University of Pisa

We prove that the classical hyperbolic Kirchhoff equation admits global-in-time solutions for some classes of initial data in the energy space. We also show that there are enough such solutions so that every initial datum in the energy space is the sum of two initial data for which a global-in-time solution exists.

The proof relies on the notion of spectral gap data, namely initial data whose components vanish for large intervals of frequencies. We do not pass through the linearized equation, because it is not well-posed at this low level of regularity.

## A road map to the blow-up for a Kirchhoff equation

**Massimo Gobbino**, Marina Ghisi  
University of Pisa

It is well-known that the classical hyperbolic Kirchhoff equation admits infinitely many simple modes, namely time-periodic solutions with only one Fourier component in the space variables.

We assume that, for a suitable choice of the nonlinearity, there exists a heteroclinic connection between two simple modes with different frequencies. Under this assumption, we cook up a forced Kirchhoff equation that admits a solution that blows-up in finite time, despite the regularity and boundedness of the forcing term.

The forcing term can be chosen with the maximal regularity that prevents the application of the classical global existence results in analytic and quasi-analytic classes.

## Mathematician on three continents - on the occasion of Karen Yagdjian's 70th birthday

**Michael Reissig**

Technical University Bergakademie Freiberg

Karen Yagdjian from University of Texas - Rio Grande Valley had his 70'th birthday two years ago. In honour of this occasion the talk is reminiscent of his mathematical life on three different continents. Some interesting aspects round off the presentation.

## Asymptotic profile of solutions to wave equations with critical log-damping

**Ruy Coimbra Charao**, Ryo Ikehata

Universidade Federal de Santa Catarina

We consider wave equations with a special kind of fractional damping of log-type. We study the Cauchy problem for this model in  $R^n$  and we obtain an asymptotic profile and optimal estimates of solutions when  $t$  goes to infinity in the  $L^2$ -norm sense. An important result of this work is that under effective damping, in the case of  $n = 1$  that quantity blows up in infinite time, and in the case of  $n = 2$  the quantity never decays and never blows up. This last phenomenon seems to be new in the community. The log-damping term depends on a parameter  $\mu > 0$ . In particular, in the case when  $\mu > 2$  from the profile point of view the so called double diffusion structure is captured explicitly. This structure has been first discovered by D'Abbicco-Ebert to the fractionally small-parameter ( $0 < \theta < 1/2$ ) damped wave equation, and in Piske-Charao-Ikehata to the wave equation with a log-type damping. A double diffusion structure for the case  $\theta = 1/2$  and  $\mu > 2$  seems not to be clearly mentioned so far.

## Test function method in KdV hierarchy, some confirmations, some news

**Sandra Lucente**

Università degli Studi di Bari Aldo Moro

The KdV equation is the most famous quasilinear evolution equation in fluid dynamics. KdV equation has an infinite amount of conserved quantities; it presents a class of infinite soliton solutions; an infinite sequence of partial differential equations can be obtained from KdV by means of Lax pairs called KdV hierarchy. In addition, by means of derivation and different algebraic modification one can extend it from 1 dimension to higher dimension finding many equations that appears in mathematical physics. For example one has KP, BLMP, YTSF, CBS equations. They inherits from KdV a quasilinear evolution structure:  $L(\partial_t, \nabla)u + Q(u, \nabla u) = 0$  with linear  $L$  and  $Q$  in divergence form. The presence of solitons is then assured. We imagine to perturb one of these equations with a semilinear forcing

term. We gain  $Lu + Q(u, \nabla u) = N(u, \nabla u)$  with polynomial growth for  $N$ . The interaction between  $L, Q, N$  destroy the symmetry of the equations and for suitable  $N$  one can prove non-existence of solutions. The test function method is the first idea for proving this, but due to the peculiar form of  $L$  and  $Q$  something new appears in initial data condition.

## Stochastic and deterministic evolution PDEs with polynomially bounded coefficients

**Sandro Coriasco**

Università degli Studi di Torino

I will give a survey about existing results and future challenges concerning evolution PDEs with variable coefficients admitting polynomial bounds, of hyperbolic and Schrödinger types. These classes of PDEs have been studied by various authors along the recent decades, both in the deterministic case, as well as, in more recent times, in the stochastic case. I will focus on hyperbolic and Schrödinger type PDEs on  $\mathbb{R}^d$  in the above class. If time allows, I will also briefly discuss the extension of the analysis to analogous problems on asymptotically Euclidean manifolds. The talk is based on joint works with A. Ascanelli, A. Abdeljawad, and A. Süß, with K. Johansson and Joachim Toft, and with S. Pilipović and D. Selešić.

## The move from Fujita type exponent to a shift of it for a class of semilinear evolution equations with scale-invariant time-dependent damping

**Wanderley Nunes do Nascimento**, Marcelo Rempel Ebert, Jorge Marques

Universidade Federal do Rio Grande do Sul

In this lecture, we derive suitable optimal  $L^p - L^q$  decay estimates,  $1 \leq p \leq 2 \leq q \leq \infty$ , for the solutions to the  $\sigma$ -evolution equation,  $\sigma > 1$ , with scale-invariant time-dependent damping and power nonlinearity  $|u|^p$ ,

$$u_{tt} + (-\Delta)^\sigma u + \frac{\mu}{1+t} u_t = |u|^p, \quad t \geq 0, \quad x \in \mathbf{R}^n,$$

where  $\mu > 0$ ,  $p > 1$ . The critical exponent  $p = p_c$  for the global (in time) existence of small data solutions to the Cauchy problem is related to the long time behavior of solutions, which changes accordingly  $\mu \in (0, 1)$  or  $\mu > 1$ . Under the assumption of small initial data in  $L^m(\mathbf{R}^n) \cap L^2(\mathbf{R}^n)$ ,  $m = 1, 2$ , we find the critical exponent at low space dimension  $n$  with respect to  $\sigma$ , namely,

$$p_c = \max \{ \bar{p}(\gamma_m), \bar{p}(\gamma_m + \mu - 1) \}, \quad \gamma_m \doteq \frac{n}{m\sigma}, \quad \mu > 1 - \gamma_m,$$

where  $\bar{p}(\gamma) \doteq 1 + \frac{2}{\gamma}$  is the well known Fujita exponent. Hence,  $p_c = \bar{p}(\gamma_m)$  if  $\mu > 1$ , whereas  $p_c = \bar{p}(\gamma_m + \mu - 1)$  is a shift of Fujita type exponent if  $\mu \in (0, 1)$ .

## Lifespan estimates for semilinear damped wave equation in a 2D exterior domain

**Yuta Wakasugi**, Masahiro Ikeda, Koichi Taniguchi, Motohiro Sobajima

Hiroshima University

We study the initial-boundary value problem of the semilinear damped wave equation with the critical nonlinearity in the exterior of the ball in two dimensions. We prove that for radially symmetric initial data, the lifespan of the solution is estimated from both above and below by  $\exp(\exp(C\varepsilon^{-1}))$ , where  $\varepsilon$  is the size of the initial data. This shows that the lifespan is longer than that of the whole space case in which the single exponential-type estimate holds.

## STOCHASTIC PROCESSES

**Organizer:** Benedetta Ferrario (University of Pavia, Italy) & Rafael Andres Rosales Mitrowsky (USP, Brazil) Fernanda Cipriano (Universidade Nova de Lisboa, Portugal) & Fernando Pigeard de Almeida Prado (USP, Brazil)

### On the asymptotic behaviour of the elephant random walk

**Cristian Favio Coletti (UFABC)**, Lucas Roberto de Lima (UFABC), Denis Araujo Luiz (UFABC), Gunter Schütz (Institute of Complex Systems II; Jülich, Germany), Renato Gava (UFSCAR)  
UFABC - Centro de Matemática, Computação e Cognição

In this talk we consider the so-called elephant random walk (ERW) which is a non-Markovian discrete-time random walk on  $\mathbb{Z}$  (the set of integer numbers) with unbounded memory which exhibits a phase transition from diffusive to superdiffusive behaviour. We show that a law of large numbers and a central limit theorem holds. Remarkably the central limit theorem applies not only to the diffusive regime but also to the phase transition point which is superdiffusive. Inside the superdiffusive regime the ERW converges to a non-degenerate random variable which is not normal. We also show that the ERW is almost surely well approximated by a Brownian motion. As a by-product of our result we get the law of iterated logarithm and the central limit theorem for the ERW. If time allows we are going to discuss a model with reinforcement. This is joint work with Günter Schutz, Renato Gava and Lucas R. de Lima.

### Invariant measures for stochastic parabolic-hyperbolic equations in the space of almost periodic functions

**Daniel Marroquin**

Universidade Federal do Rio de Janeiro

We study the well-posedness and the long-time behavior of almost periodic solutions to stochastic degenerate parabolic-hyperbolic equations in any space dimension, under the assumption of Lipschitz continuity of the flux and viscosity functions and a non-degeneracy condition. We show the existence and uniqueness of an invariant measure in a separable subspace of the space of Besicovitch almost periodic functions. This is a joint work with Hermano Frid and Claudia Espitia

### A random walk with memory perturbed by a dynamical system

**Denis Araujo Luiz**, Cristian F. Coletti, Lucas R. De Lima, Renato J. Gava

Universidade Federal do ABC

A random walk with unbounded memory is introduced as a mixture of the Elephant Random Walk and the Dynamic Random Walk which we call the Dynamic Elephant Random Walk (DERW). We prove a strong law of large numbers for the DERW and, in a particular case, we provide an explicit expression for its speed. Also, we give sufficient conditions for the Central Limit Theorem and the Law of the Iterated Logarithm to hold.

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001. Research also supported by grants #2017/10555-0, #2019/19056-2, #2017/10555-0 and #2018/04764-9, São Paulo Research Foundation (FAPESP).

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### Price model with common noise

**Diogo Gomes**, Julian Gutierrez, Yuri Ashrafyan, Tigran Bakaryan  
King Abdullah University of Science and Technology

We consider the mean-field game price formation model introduced by Gomes and Saúde. In this MFG model, agents trade a commodity whose supply can be deterministic or stochastic. Agents maximize profit, taking into account current and future prices. The balance between supply and demand determines the price. We introduce a potential function that converts the MFG into a convex variational problem. This variational formulation is particularly suitable for machine learning approaches. Here, we use a recurrent neural network to solve this problem. In the last section of the paper, we compare our results with known analytical solutions.

### Smoothness of densities for path-dependent SDEs under Hörmander's condition

**Evelina Shamarova**, Alberto Ohashi, Francesco Russo  
Universidade Federal da Paraíba

We establish the existence of smooth densities for solutions to a broad class of path-dependent SDEs under a Hörmander-type condition. The classical scheme based on the reduced Malliavin matrix turns out to be unavailable in the path-dependent context. We approach the problem by lifting the given  $n$ -dimensional path-dependent SDE into a suitable  $L_p$ -type Banach space in such a way that the lifted Banach-space-valued equation becomes a state-dependent reformulation of the original SDE. We then formulate Hörmander's bracket condition in  $\mathbb{R}^n$  for non-anticipative SDE coefficients defining the Lie brackets in terms of vertical derivatives in the sense of the functional Itô calculus. Our pathway to the main result engages an interplay between the analysis of SDEs in Banach spaces, Malliavin calculus, and rough path techniques.

### A boundary control problem for stochastic 2D-Navier-Stokes equations

**Fernanda Cipriano**, Nikolai Chemetov  
NOVA University Lisbon

In this talk, we discuss a stochastic velocity tracking problem for the 2D-Navier-Stokes equations perturbed by a multiplicative Gaussian noise. From physical point of view, the control acts through a boundary injection/suction device with uncertainty, modelled by non-homogeneous Navier-slip boundary conditions. We show the existence and uniqueness of solution to the state equation and prove the existence of an optimal solution to the control problem. In addition, the first-order necessary optimization conditions are analysed.

## Numerical schemes for various stochastic models in Hydrodynamic

**Hakima Bessaih**

Florida International University

We will introduce space/time numerical schemes for some models in Hydrodynamic, including the 2d stochastic Navier-Stokes and Boussinesq equations. We will also discuss various rates of convergences in probability and in mean square. This is a joint work joint with Annie Millet.

## Random walks driven by interacting particle systems in one dimension

**Marcelo Hilario**

Universidade Federal de Minas Gerais

In this talk, we review some recent developments in the study of random walks in dynamic random environments via multiscale renormalization techniques. We consider the case when the environment is given by interaction particle systems such as the simple symmetric exclusion process and the zero-range process. Our main results are laws of large numbers for the displacements of the walk and, in some ballistic cases, central limits theorems.

## No blow-up by nonlinear Itô noise for Euler equations

**Marco Bagnara**, Mario Maurelli, Fanhui Xu

Scuola Normale Superiore di Pisa

We consider the 2D and 3D stochastic Euler equations. It is well-known that (under suitable assumptions on the noise) regular solutions exist locally in time. We show, by means of the Lyapunov function method and a Galerkin approximation, that the choice of a suitable non-linear multiplicative Itô noise provides a regularizing effect. Namely, we establish that with full probability the regular solutions are global in time. The presentation is based on a joint work with Mario Maurelli and Fanhui Xu.

The research is supported by the European Commission Project NoisyFluid, n. 101053472.

## Ergodicity of infinite particle systems and applications

**Mikhail Neklyudov**

UFAM

I am going to discuss algebraic structure which naturally appears in the study of ergodicity of certain degenerate infinite dimensional sub-elliptic generators. I will show different applications of the structure (which is extension of Jordan-Schwinger map) to algebra (i.e. construction of explicit representations of finite and infinite dimensional algebras) and analysis (i.e. construction of new quantisation) and present some open problems and open directions of research. (based on joint works with B. Zegarlinski, J. Inglis, W. Bock, V. Futorny).

## Stochastic n-point D-bifurcations of stochastic Lévy flows and their complexity on finite spaces

**Paulo Ruffino**, P. H. da Costa, Michael Högele

University of Campinas

Brownian flows of diffeomorphisms are known to be rigid in the sense that any  $\omega$ -wise invariant measure of the flow is uniquely determined by the usual invariant measure of the respective 1 and 2-point motion.

For general Markovian systems this turns out to be false. In order to quantify this defect we introduce the notion of a stochastic  $n$ -point bifurcation which provides new information about the random dynamics.

We construct several classes of examples already over finite spaces including the minimal example where this phenomenon occurs.

## Interacting vertex reinforced random walks on complete sub-graphs

**Rafael A. Rosales**

Universidade de São Paulo

We introduce a model of interacting vertex reinforced random walks that covers a large variety of interactions including cooperative and competitive dynamics. We will describe some results concerning the almost sure asymptotic behaviour of the vertex occupation measures by each walk. This work is the result of a collaboration with Fernando P.A. Prado (Universidade de São Paulo, Brasil).

## CLT and almost sure CLT for dependent Bernoulli random variables

**Renato Gava**

Universidade Federal de São Carlos

We consider a sequence of correlated Bernoulli variables whose probability of success of the current trial depends conditionally on the previous trials as a linear function of the sample mean. We establish the central limit theorem and the almost sure central limit theorem for the model and discuss their relationship to other models.

## Large deviations for return times

**Sandro Gallo**, Miguel Abadi, Vitor Amorim, Jean-René Chazottes  
UFSCar

For a discrete time stochastic process we denote by  $R_n$  the *return time of order  $n$* , first time the process repeats its first  $n$  outputs. The well-known Ornstein-Weiss theorem states that, for ergodic processes with positive Shannon entropy  $h$ ,  $\frac{\log R_n}{n}$  converges a.s. to  $h$ . A central limit theorem is also known for this quantity. We will present some recent results about its large deviations.

## Stochastic Transport Equations. Recent results

**Wladimir Neves**

Universidade Federal do Rio de Janeiro

In this talk, we discuss about the important issue of uniqueness of solutions for the linear stochastic transport equations with rough coefficients, that is to say, we consider the following Cauchy problem

$$\begin{cases} \partial_t u + b \cdot \nabla u + \frac{dB_t}{dt} \cdot \nabla u = 0, \\ u|_{t=0} = u_0, \end{cases} \quad (18)$$

where the unknown  $u$  is a real value function,  $u_0 : \mathbb{R}^d \rightarrow \mathbb{R}$  is a given initial data,  $b = b(t, x) : [0, T] \times \mathbb{R}^d \rightarrow \mathbb{R}^d$  is the drift vector field,  $B_t = (B_t^1, \dots, B_t^d)$  is a standard Brownian motion in  $\mathbb{R}^d$ , and  $T > 0$  is any real fixed number.

This is a joint work with Christian Olivera.

## TRENDS IN NONLINEAR DISPERSIVE EQUATIONS

**Organizer:** Ademir Pastor (UNICAMP, Brazil) & Felipe Linares (IMPA, Brazil) & Luiz Gustavo Farah (UFMG, Brazil) & Mahendra Panthee (UNICAMP, Brazil)

### Controllability of a model system for strong interaction between internal solitary waves

**Ademir Fernando Pazoto**, Fernando Gallego, Jon Asier Barcena-Petisco, Roberto de Almeida Capistrano-Filho, Sergio Guerrero  
Universidade Federal do Rio de Janeiro

In this talk, we present local controllability results for a nonlinear coupled system of two Korteweg-de Vries equations posed on a bounded interval. The model was derived by Gear and Grimshaw to describe the interactions of two dimensional, long, internal gravity waves propagation in a stratified fluid. We address both, the internal and the boundary control problems. Our approach consists mainly in proving the controllability of the linearized system, which is done by using a classical duality approach. A fixed point argument and a local inversion theorem are then applied to get the results for the nonlinear system.

### Global well posedness for inhomogeneous nonlinear Schrödinger equation with combined power-type nonlinearities

**Andressa Gomes**, Mykael Cardoso  
Universidade Federal do Piauí

In this talk we consider the inhomogeneous nonlinear Schrödinger equation (INLS) with combined power-type nonlinearities. More precisely, we study the case when a nonlinearity is  $\dot{H}_x^1$ -critical and the other nonlinearity is subcritical. We develop a energy-critical stability result for the INLS equation with one critical-nonlinearity to prove well-posedness in the energy space for the INLS equation with two power-type nonlinearities. For this purpose, we treat the energy-subcritical nonlinearity as a perturbation to the energy-critical INLS, which is globally wellposed. This is a joint work with Prof. Mykael Cardoso (UFPI).

### Scattering for the non-radial inhomogeneous Schrodinger-type equations

**Carlos Manuel Guzman Jimenez**  
Federal Fluminense University

In this talk, we show scattering for some inhomogeneous Schrödinger-type equations, in the non-radial case. Precisely, we consider de Inhomogeneous Schrodinger and Hartree equations. First, we consider the inhomogenous NLS equation in the different cases, giving more attention to the case of critical energy. In the sequel, we study the non-radial energy critical Hartree model in  $3D$ . To this end, we used the robust technique of Kenig Merle combined with new ingredients to address the non-radial setting. The inhomogeneous models present some new challenges arising from the broken translation symmetry. Here, we overcome this. Finally, we compare the results with the homogeneous models and discuss some open problems.

## Local well-posedness for the gKdV equation on the background of a bounded function

**José Palacios**  
University of Toronto

In this talk we will prove the local well-posedness for the generalized Korteweg-de Vries equation in  $H^s(\mathbf{R})$ ,  $s > 1/2$ , under general assumptions on the nonlinearity  $f(x)$ , on the background of an  $L_{t,x}^\infty$ -function  $\Psi(t, x)$ , with  $\Psi(t, x)$  satisfying some suitable conditions. As a consequence of our estimates, we will also obtain the unconditional uniqueness of the solution in  $H^s(\mathbf{R})$ . This result not only gives us a framework to solve the gKdV equation around a Kink, for example, but also around a periodic solution, that is, to consider localized non-periodic perturbations of a periodic solution. We also prove global existence in the energy space  $H^1(\mathbf{R})$ , in the case where the nonlinearity satisfies that  $|f''(x)| \lesssim 1$ .

## Properties of the Support of Solutions of a Class of Nonlinear Evolution Equations

**José Manuel Jiménez**, Eddy Bustamante  
Universidad Nacional de Colombia Sede Medellín

In this work we consider equations of the form

$$\partial_t u + P(\partial_x)u + G(u, \partial_x u, \dots, \partial_x^l u) = 0,$$

where  $P$  is any polynomial without constant term, and  $G$  is any polynomial without constant or linear terms. We prove that if  $u$  is a sufficiently smooth solution of the equation, such that  $\sup u(0), \sup u(T) \subset (-\infty, B]$  for some  $B > 0$ , then there exists  $R_0 > 0$  such that  $\sup u(t) \subset (-\infty, R_0]$  for every  $t \in [0, T]$ . Then, as an example of the application of this result, we employ it to show unique continuation properties for some nonlinear dispersive models

## Long-time behavior of inhomogeneous Schrödinger equations with inverse-square potentials

**Luccas Campos**  
UFMG

We consider the initial value problem for the inhomogeneous nonlinear Schrödinger equation with inverse-square potential:

$$\begin{cases} i\partial_t u + \Delta u - \frac{a}{|x|^2}u + \frac{1}{|x|^b}|u|^{2\sigma}u = 0, \\ u|_{t=0} = u_0. \end{cases} \quad (\text{INLS}_a)$$

This model can be interpreted as an extension to the nonlinear Schrödinger equation. For instance, when  $a = b = 0$  we have the classical nonlinear Schrödinger equation, extensively studied over the last decades. The case  $b = 0$  and  $a \neq 0$  is known as the NLS equation with inverse square potential, denoted by  $\text{NLS}_a$ , which has also been attracting attention in recent years. The  $\text{NLS}_a$  equation appears in several physical settings, such as in quantum field equations or black hole solutions of the Einstein's equations. Alternatively, if  $a = 0$  and  $b \neq 0$  we have the inhomogeneous NLS equation, denoted by  $\text{INLS}$ , which also has received substantial attention recently. It can model, for example, nonlinear optical systems with spatially dependent interactions. In particular, it can be thought of as

modeling inhomogeneities in the medium in which the wave propagates. The INLS<sub>a</sub> can thus be seen as a general model for various physical contexts.

In this talk, we discuss the recent developments in the theory for the INLS<sub>a</sub> equation, such as local and global well-posedness, together with scattering and blow up results. The techniques used involve, for example, Sobolev spaces adapted to the linear part of the equation, and either the Kenig-Merle's concentration-compactness or the Dodson-Murphy's virial-Morawetz approach for scattering, and the Merle-Raph el approach for the blow-up rate. The technical parte here is that, unlike several perturbed Schr odinger equations, we deal with singularities in both the linear and the nonlinear part, which also preserve the scaling-invariance of the equation.

## Blow-up solutions for the inhomogeneous nonlinear Schr odinger equation

**Mykael Cardoso**

Universidade Federal do Piau  - UFPI

In this talk, we consider the inhomogeneous nonlinear Schr odinger (INLS) equation

$$i\partial_t u + \Delta u + |x|^{-b}|u|^{2\sigma}u = 0, \quad \text{in } \mathbb{R}^N$$

where  $N \geq 3$ ,  $0 < b < 2$  and  $\frac{2-b}{N} < \sigma < \frac{2-n}{N-2}$ . This is an interesting extension of the classical nonlinear Schr odinger equation ( $b = 0$ ). Due to the local well-posedness theory for the INLS equation in the Sobolev space  $H^1$ , given  $u_0 \in H^1$ , there exists a time  $T > 0$  and a unique corresponding solution  $u \in C([0, T]; H^1)$  to the INLS equation such that  $u(0) = u_0$ . Considering  $T^* > 0$  as the maximal time of existence for such a solution, we say that the solution is global if  $T^* = \infty$ , and that the solution blows up in finite time if  $T^* < \infty$ . Our objective is to present some recent results about the existence of blow-up solutions for the  $L^2$ -critical INLS equation, as well as the behavior of the critical norm of blow-up solutions when the time is close enough to its maximal time of existence. This is a joint work with Luiz Gustavo Farah (UFMG) and Luccas Campos (UFMG).

## Coupled NLS equations with double power nonlinearities

**Nataliia Goloshchapova**

USP - Universidade de S o Paulo

We study a system of coupled NLS equations with double power nonlinearities (classical and point). The associated energy is defined by

$$\begin{aligned} E(u, v) &= \|u'\|_2^2 + \|v'\|_2^2 - 2(G_\delta(u, v) + G(u, v)), \\ G_\delta(u, v) &= \frac{1}{p_1} |u(0)|^{p_1} + \frac{1}{r_1} |v(0)|^{r_1} + \frac{1}{q_1} |u(0)v(0)|^{q_1}, \\ G(u, v) &= \frac{1}{p_2} \|u\|_{p_2}^{p_2} + \frac{1}{r_2} \|v\|_{r_2}^{r_2} + \frac{1}{q_2} \|uv\|_{q_2}^{q_2}, \end{aligned}$$

where  $p_j, q_j, r_j > 2, j = 1, 2$ , and  $p_1, r_1, 2q_1 \leq 4, p_2, r_2, 2q_2 \leq 6$ . The interest for this nonlinear model is relatively recent (and related to the applications of the standard NLS equation). It was introduced in the 1990s to describe phenomena mostly connected with solid state and condensed matter physics.

We seek for *the minimizers* of the energy under a fixed masses constraint. One of the main peculiarities of the model is that the *ground state*  $(\phi, \psi)$  belongs to the following nonlinear set

$$\mathcal{D} = \left\{ \begin{array}{l} (\phi, \psi) : \phi, \psi \in H^1(\mathbb{R}) \cap H^2(\mathbb{R} \setminus \{0\}) \\ \phi'(0+) - \phi'(0-) = - \left[ |\phi(0)|^{p_1-2} + |\psi(0)|^{q_1} |\phi(0)|^{q_1-2} \right] \phi(0) \\ \psi'(0+) - \psi'(0-) = - \left[ |\psi(0)|^{r_1-2} + |\phi(0)|^{q_1} |\psi(0)|^{q_1-2} \right] \psi(0) \end{array} \right\},$$

that is, on the contrary to the standard NLS models, it does not belong to the domain of some self-adjoint operator.

Our main tools are *the classical concentration-compactness principle* and the recent research [F. Boni, S. Dovetta. *Prescribed mass ground states for a doubly nonlinear Schrödinger equation in dimension one*, J. Math. Anal. Appl., 496(1): 2021] on a single NLS equation with analogous double power nonlinearity.

We prove the existence of ground states at every mass when power parameters are  $L^2$ -subcritical ( $p_1, r_1, 2q_1 < 4$  and  $p_2, r_2, 2q_2 < 6$ ). On the other hand, we show that when one or several power parameters are  $L^2$ -critical, the existence of the ground state strongly depends on the prescribed masses.

## Decay of the radius of spatial analyticity for the modified KdV equation

**Renata O. Figueira**, Mahendra Panthee  
University of Campinas

We shall consider the Cauchy problem on the line for the modified KdV equation in the defocusing case, where the initial data belongs in a class of analytic functions on the line that can be extended holomorphically in a symmetric strip around  $x$ -axis.

From the result about local well-posedness in these classes of functions, we guarantee the analytic regularity of the solutions in space variable without shrinking the width of the strip for short times, which means that the uniform radius of spatial analyticity remains the same till some lifespan  $0 < T_0 \leq 1$ . This talk is devoted to discuss the evolution of the radius of spatial analyticity  $\sigma(t)$  when the local solution extends globally in time and prove that for any time  $T \geq T_0$  it is bounded from below by  $cT^{-\frac{4}{3}}$ .

This work is in collaboration with Mahendra Panthee and partially supported by FAPESP.

## WAVELET THEORY AND ITS RELATED TOPICS

**Organizer:** Keiko Fujita ( University of Toyama, Japan) & Akira Morimoto (Osaka Kyoiku Toyama, Japan)

### Feature extraction from mixed speech data using wavelet analysis

**Akira Morimoto**, Ryuichi Ashino, Takeshi Mandai  
Osaka Kyoiku University

There are two mixtures of speech sounds with multiple scaling along the time axis. Extract features from each mixture, using wavelet analysis. By comparing the extracted feature points, match each speech sound contained in two mixtures. We discuss methods for estimating parameters such as scaling, sound volume, and time delay.

### On statistical operators formalism in coherent states Hilbert spaces

**Isiaka Aremua**, Mahouton Norbert Hounkonnou, Komi Sodoga  
Université de Lomé

This work gives value to the importance of the density operator expansion in coherent states quantum Hilbert spaces and the related mathematical and physical aspects. First, we deal with Hilbert-Schmidt operators Hilbert space in the study of coherent states construction using the Glauber-Sudarshan (GS)- $P$ -representation. The definition of the density operator on quantum states in Hilbert spaces and some of its features relevant to thermodynamics and information-theoretical entropy calculations have been provided. As an application, the physical model describing an electron in a magnetic field has been studied. First, the exotic Landau problem in the noncommutative plane has been investigated, and the related coherent states have been constructed. In addition, the density operator formalism has been applied to the motion of an electron in the noncommutative  $xy$  plane, subject to a constant magnetic field background coupled with a harmonic potential. Also, the quantum model, for which modular structures emerging for two underlying von Neumann algebras have been provided, has been examined. The diagonal  $P$ -representation of the density operator in the constructed coherent states Hilbert space basis has been established, with the  $Q$ -Husimi distribution and the Wehrl entropy determined. Next, a matrix formulation of the Landau problem has been given with matrix operators acting on an abstract Hilbert space defined. Main statistical properties have been investigated in the basis of the multi matrix vector coherent states built in the corresponding Hilbert space. Besides, the matrix formulation of the density operator representation is also achieved in two-component vector coherent state basis representation for a supersymmetric harmonic oscillator. The diagonal representation of the density operator has been investigated and discussed in the supersymmetric vector coherent states basis. A link with quantum information has been also given via an integral representation of a qubit using the supersymmetric vector coherent states resolution of the identity.

### On characterization of the Gabor wavelet transform of analytic functionals

**Keiko Fujita**

University of Toyama

The wavelet transformation is usually applied for the square integrable functions. Since the Gabor function is an exponential type, we can apply the Gabor wavelet transformation to the analytic functionals. As the Gabor function is a Gaussian type, the Gabor wavelet transformation is similar to the windowed Fourier transformation. That is, the Gabor wavelet transformation is closely related to the Fourier transformation. Therefore our previous results on Fourier transformation are useful when we consider the Gabor transformation.

In this talk, we will review our previous results and will consider the Gabor wavelet transform of analytic functional based on our previous results.

## **Coupled Fractional Fourier Transform: Convolution and Correlation Theorems and Uncertainty Principle**

**Ryuichi Ashino**, Mawardi Bahri  
Osaka Kyoiku University

The coupled fractional Fourier transform is an extended version of the fractional Fourier transform. In the present work, we investigate some basic properties of the generalized transformation. We also propose convolution and correlation theorems and explore an uncertainty principle for the coupled fractional Fourier transform.

## **The $p$ -adic wavelet expansion of the locally constant test function and its Fourier transform**

**Toshio Suzuki**, Munehiro Kobayashi  
Tokyo University of Science

$p$ -adic analysis is a field of number theory that explores how  $p$ -adic numbers, a type of numbers that behave differently from ordinary real numbers, can be used to define functions.

This study investigates the Fourier transform of functions that take complex values on  $p$ -adic fields.

When we deal with functions on  $p$ -adic fields, we use the parameter of constancy instead of the interval fineness to approximate them by step functions, which is related to the discrete Fourier transform.

In this research, we have derived a formula to compute the discrete Fourier transform of complex-valued functions on  $p$ -adic fields.

We also explain how the  $p$ -adic wavelet and the Fourier transform are linked together.

## **Fractional neural network interpolation operator of irregular grid points**

**Uday Singh**, Manju Sharma  
Indian Institute of Technology Roorkee

The problem of data interpolation and approximation of function of several variables is of central interest in general theory of neural networks, for instance, for applications concerning the training of NNs. The interpolation and approximation of function by neural network operator for irregular grid of data points is frequently used in the applications of neural networks such as image and signal processing.

In this paper, we prove interpolation and approximation for continuous function defined on a box-domain of  $\mathbb{R}^d$  by neural network operators for multivariate data for irregular grid points. We will measure the rate of approximation in terms of the modulus of continuity of the functions being approximated. We will define this neural network interpolation operator by using the linear combinations of sigmoidal functions from new class of sigmoidal functions and by using the fractional mean value of function instead of its sample values. We prove approximation and interpolation results for irregular grid points.

We specify the type of irregularity of grid points we are considering. We denote  $\mathcal{R} := [a_1, b_1] \times \dots \times [a_d, b_d] \subset \mathbb{R}^d$  be a box domain in  $\mathbb{R}^d$ . We consider the  $n + 1$  nodes  $a_i = z_0^i < z_1^i < \dots < z_{n-1}^i < z_n^i = b_i$  for  $i = 1, \dots, d$ . Corresponding to  $n$  suppose that there exist  $0 < d_1 \leq d_2$ , with  $d_2 < 2d_1$  such that,

$$d_1 \leq z_{k_i+1}^i - z_{k_i}^i \leq d_2, \text{ for every } k_i = 0, \dots, n-1; i = 1, \dots, d.$$

We denote by  $\mathbf{z}_{\mathbf{k}} := (z_{k_1}^1, \dots, z_{k_d}^d)$ , where  $\mathbf{k} := (k_1, \dots, k_d)$ .

More precisely, we prove the following results.

**Theorem 1.** Let  $f : \mathcal{R} \rightarrow \mathbb{R}$ ,  $\mathcal{R} \subset \mathbb{R}^d$ , be a bounded function.

$$T_n^m(f, \mathbf{z}_{\mathbf{k}}) = f(\mathbf{z}_{\mathbf{k}}),$$

for  $\mathbf{z}_{\mathbf{k}} := (z_{k_1}^1, \dots, z_{k_d}^d)$ , where  $\mathbf{k} := (k_1, \dots, k_d)$ , with  $k_i = 0, 1, \dots, n$ , and  $i = 1, \dots, d$ .

$$\|T_n^m(f, \cdot) - f(\cdot)\|_{\infty} \leq 2\omega(f, \|\mathbf{h}\|_2),$$

where  $\omega$  denotes the modulus of continuity of objective function and  $\mathbf{h} = (h_1, h_2, \dots, h_d)$ , where  $h_i = \frac{b_i - a_i}{n}$  for all  $i = 1, 2, \dots, d$ .

**Keywords:** Neural network operators, Interpolation, Sigmoidal function, modulus of continuity, irregularity.

**AMS 2010 Subject Classification:** 41A30, 41A35, 41A65.

**Category:** Analysis, Approximation theory, Neural network interpolation operator.

## A class of quaternionic Fourier orthonormal bases

Yun-Zhang Li, Xiao-Li Zhang  
Beijing University of Technology

The quaternion algebra is an extension of the complex number field. Recently, quaternionic Fourier analysis has received increasing attention due to its applications in signal analysis and image processing. In this talk, we will give a class of quaternionic Fourier orthonormal bases of exponential type.

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Chapter

Programme



## Schedule Overview

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
8:00-8:30	Registration				
8:30-9:00	Opening*				Thematic Sessions
9:00-10:00	Loukas Grafakos*	Zdzislaw Brzezniak*	Danylo Radchenko*	Irena Lasiiecka*	
10:00-10:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break	Coffee Break
10:30-12:10	Thematic Sessions	Thematic Sessions	Hubert Lacoïn*	Thematic Sessions	Carlos Pérez*
			Photo		Closing*
12:10-14:00	Lunch	Lunch	Lunch	Lunch	Lunch
14:00-15:00	Anna Mazzucato*	Gustavo Ponce*	Recital de Piano*	Monica Musso*	
15:00-15:30	Coffee Break	Coffee Break	Coffee Break	Coffee Break	
15:30-18:20	Thematic Sessions	Thematic Sessions	Thematic Sessions	Thematic Sessions	
20:00			Conference Dinner		

(\*) Auditorium of the Faculty of Law

- Session 1 – Applications of Dynamical Systems Theory in Biology
- Session 2 – Complex Geometry
- Session 3 – Complex Variables and Potential Theory
- Session 4 – Evolution Equations and Dynamical Systems
- Session 5 – Function Spaces and their Applications to Nonlinear Evolutional Equations
- Session 6 – Harmonic Analysis and Partial Differential Equations
- Session 7 – Harmonic Analysis and Related Topics
- Session 8 – Integral Transforms and Reproducing Kernels
- Session 9 – Partial Differential Equations on Curved Spacetimes
- Session 10 – PDEs in Fluid Mechanics
- Session 11 – Pseudo Differential Operators
- Session 12 – Quaternionic and Clifford Analysis
- Session 13 – Recent Progress in Evolution Equations
- Session 14 – Stochastic Processes
- Session 15 – Trends in Nonlinear dispersive Equations
- Session 16 – Wavelet Theory and its Related Topics



## Thematic Session

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
<b>Session 1 – Applications of Dynamical Systems Theory in Biology (Building B4 - Room 16 )</b>					
8:30-10:00					
Chair					
10:30-11:00				G. Röst	
11:05-11:35				B. Pires	
11:40-12:10				Á. Garab	
Chair					
15:30-16:00				A. Corli	
16:05-16:35				T. Lindström	
16:40-17:10					
17:15-17:45					
17:50-18:20					
<b>Session 2 - Complex Geometry (Building B4 - Room 15 )</b>					
8:30-10:00					
Chair					
10:30-11:00	A. Rojas	P. Rodriguez		G. Giorgadze	
11:05-11:35	S. Reyes-Carocca	Á. Castañeda		C. Goretti	
11:40-12:10	G. Bresciani	F. Martin		F. Neumann	
Chair					
15:30-16:00		A. Herrero		A. Mandini	
16:05-16:35	J. González - Cervantes*	A. Wilson		P. Dalakov	
16:40-17:10	C. Kiselman*	J. Bernal		U. Bruzzo	
17:15-17:45		G. Comaschi			
<b>Session 3 - Complex Variables and Potential Theory (Building B1 - Room 600-B)</b>					
8:30-10:00					
Chair		N. Manjavidze		G. Giorgobiani	
10:30-11:00		G. Giorgobiani		N. Manjavidze	
11:05-11:35		C. Green		K. Ospanov	
11:40-12:10		F. Lanzara		L. Tovar	
Chair		Lanza de Cristoforis	F. Lanzara	L. Tovar	
15:30-16:00		I. Denega*	B. Klishchuk*	V. Shpakivskiy*	
16:05-16:35		O. Dovhopiatyi*	Lanza de Cristoforis	M. Stefanchuk*	
16:40-17:10		A. Golberg*	S. Plaksa*	Y. Zabolotnyi*	
17:15-17:45		S. Gryshchuk*	E. Sevost'yanov*		

(\*) remote lecture



## Thematic Session

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
<b>Session 4 - Evolution Equations and Dynamical Systems (Building B4 - Room 14 )</b>					
8:30-10:00					
Chair					
10:30-11:00	A. Carvalho	A. Verri			
11:05-11:35	J. Simsen	M. Pereira			
11:40-12:10	L. Pires	J. Nakasato			
Chair					
15:30-16:00	E. Bonotto	P. Lopes			
16:05-16:35	J. Hwang	Soon-Yeong Chung			
16:40-17:10	R. Araujo	E. Moreira			
17:15-17:45	Salah-Eddine Rebiai	M. Marrocos			
17:50-18:20					
<b>Session 5—Function Spaces and their Applications to Nonlinear Evolutional Equations (Building B4 - Room 13)</b>					
8:30-10:00					
Chair					
10:30-11:00		R. Muramatsu			
11:05-11:35		L. Wu			
11:40-12:10		S. Kinoshita			
Chair					
15:30-16:00	B. Liu	L. Zhao			
16:05-16:35	N. Kishimoto	T. Kato			
16:40-17:10	Li Ze	S. Juriaans			
17:15-17:45	C. de Andrade				
17:50-18:20					



## Thematic Session

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
<b>Session 6 – Harmonic Analysis and Partial Differential Equations (Building B4 - Room 26)</b>					
Chair					
8:30-9:00					
9:00-9:30					<b>M. Ospanov</b>
9:30-10:00					<b>A. Kassymov</b>
Chair					
10:30-1:00		<b>D. Suragan</b>		<b>R. Kandy</b>	
11:05-11:35		<b>T. Nurlybekuly</b>		<b>M. Raikhan</b>	
11:40-12:10		<b>D. Sanchez</b>		<b>K. Gonzalez</b>	
Chair					
15:30-16:00		<b>D. Garrisi</b>	<b>A. Kirilov</b>	<b>P. Dattori da Silva</b>	
16:05-16:35		<b>S. Mondal</b>	<b>A. Kowacz</b>	<b>N. Yessirkegenov</b>	
16:40-17:10		<b>A. Tushir</b>	<b>A. Almeida</b>	<b>I. Beschastnyi</b>	
17:15-17:45			<b>P. Agarwal</b>	<b>W. Moraes</b>	
17:50-18:20					
<b>Session 7 – Harmonic Analysis and Related Topics (Building B4 - Room 25)</b>					
8:30-10:00					
Chair	<b>T. Picon</b>	<b>L. Oliveira</b>		<b>Conde Alonso</b>	
10:30-11:00	<b>G. Dafni</b>	<b>G. Hoepfner</b>		<b>L. Castro</b>	
11:05-11:35	<b>L. Moonens</b>	<b>M. Cejas</b>		<b>A. Silva</b>	
11:40-12:10	<b>L. Oliveira</b>	<b>J. Alonso</b>		<b>R. Guerra</b>	
Chair	<b>G. Hoepfner</b>	<b>L. Moonens</b>			
15:30-16:00	<b>T. Jordão</b>	<b>E. Q. Herrera</b>			
16:05-16:35	<b>C. Vasconcelos</b>	<b>C. Mosquera</b>			
16:40-17:10	<b>A. Ramos</b>	<b>El M. Loualid</b>			
17:15-17:45	<b>R. Daher</b>	<b>P. Takemura</b>			
17:50-18:20		<b>V. Biliatto</b>			
<b>Session 8 – Integral Transforms and Reproducing Kernels (Building B4 - Room 21)</b>					
8:30-9:00					<b>P. Cerejeiras</b>
9:00-9:30					<b>J. Toft</b>
9:30-10:00					<b>Z. Mouayn</b>
Chair					
10:30-11:00				<b>D. Cho</b>	
11:05-11:35				<b>Y. Kraid</b>	
11:40-12:10					



## Thematic Session

	Monday 17	Tuesday 18	Wednesday19	Thursday20	Friday 21
<b>Session 9 – Partial Differential Equations on Curved Spacetimes (Building B1 - Room 500 )</b>					
Chair					
8:30-9:00					N. Kita
9:00-9:30					J. Marques
9:30-10:00					A. Galstian
Chair					
10:30-11:00				M. G. Reissig	
11:05-11:35				J. Krieger	
11:40-12:10				A. Palmieri	
Chair					
15:30-16:00			M. G. Reissig (joint with 13)	K. Yagdjian	
16:05-16:35			M. Nakamura (joint with 13)	M. Ohta	
16:40-17:10			G. Girardi (joint with 13)	T. Wada	
17:15-17:45			R. Charão (joint with 13)		
<b>Session 10 – PDEs in Fluid Mechanics (Building B4 – Room 11 )</b>					
8:30-10:00					
Chair	H. Lopes	A. Mazzucato		G. Planas	
10:30-11:00	G. Planas	M. Lopes		H. Lopes	
11:05-11:35	L. Arruda	H. de Souza		J. Acevedo	
11:40-12:10	G. Cardenas	B. Calsavara		G. Mazzone	
Chair	A. Bronzi	M. Santos	G. Mazzone	M. Lopes	
15:30-16:00	M. Santos	A. Bronzi	W. Neves	E. Abreu	
16:05-16:35	I. Palhares Jr.	Z. Adegboye	S. Sergeev	N. Chemetov	
16:40-17:10	D. Machado	L. Alvarez	R. Guimarães		
17:15-17:45	C. Perusato	E. Huamani	V. Castilho		
17:50-18:20					



## Thematic Session

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
<b>Session 11 – Pseudo Differential Operators (Building B4 - Room 22)</b>					
8:30-10:00					
Chair					
15:30-16:00	J. Toft	R. Ponciano			
16:05-16:35	Y. Gao	M. Sartaj			
16:40-17:10	L. Mohan	A. Prasad			
17:15-17:45	F. Avila Silva				
<b>Session 12 – Quaternionic and Clifford Analysis (Building B4 - Room 23)</b>					
8:30-10:00					
Chair					
10:30-11:00	P. Massopust	A. Debernardi			
11:05-11:35	P. Cerejeiras	Y. Grigor'ev			
11:40-12:10	A. Fernandez	R. Hu			
Chair					
15:30-16:00	K. Diki	C. Vanegas			
16:05-16:35	H. Monaim	J. González-Cervantes*			
16:40-17:10	B. De Zayas	R. Daher			
17:15-17:45	D. Cedeño	U. Kähler			
17:50-18:20	W. Barrera				
<b>Session 13 – Recent Progress in Evolution Equations (Building B1 – Room 502)</b>					
Chair					
8:30-9:00					M. D'Abbicco
9:00-9:30					W. Nunes
9:30-10:00					F. Hirosawa
Chair					
10:30-11:00				A. Cialdea	
11:05-11:35				A. Ascanelli	
11:40-12:10				M. Cappiello	
Chair					
15:30-16:00		Ma To Fu	M. G. Reissig (joint with 9)	M. Gobbino	
16:05-16:35		S. Coriasco	M. Nakamura(joint with 9)	M. Ghisi	
16:40-17:10		M. da Silva	G. Girardi (joint with 9)	J. Marques	
17:15-17:45		Y. Wakasugi	R. Charão (joint with 9)	H. Aslan	
17:50-18:20		S. Lucente		A. Arias Jr.	

(\*) remote lecture



## Thematic Session

	Monday 17	Tuesday 18	Wednesday 19	Thursday 20	Friday 21
<b>Session 14 – Stochastic Processes (Building B4 - Room 24)</b>					
8:30-10:00					
Chair					
10:30-11:00	<b>D. Gomes</b>	<b>H. Bessaih</b>			
11:05-11:35	<b>R. Rosa</b>	<b>W. Neves</b>		<b>M. Hilario</b>	
11:40-12:10	<b>M. Neklyudov</b>	<b>D. Marroquin</b>		<b>S. Gallo</b>	
Chair					
15:30-16:00	<b>P. Ruffino</b>	<b>J. Lora</b>		<b>C. Coletti</b>	
16:05-16:35	<b>R. Gava</b>	<b>E. Shamarova</b>		<b>D. Luiz</b>	
16:40-17:10	<b>Y. Tahraoui</b>	<b>M. Bagnara</b>		<b>R. Rosales</b>	
17:15-17:45		<b>F. Cipriano</b>			
17:50-18:20					
<b>Session 15 – Trends in Nonlinear Dispersive Equations (Building B4 - Room 21)</b>					
8:30-10:00					
10:30-11:00					
11:05-11:35					
11:40-12:10					
Chair					
15:30-16:00		<b>A. Pazoto</b>	<b>J. Jiménez</b>	<b>N. Goloshchapova</b>	
16:05-16:35		<b>R. Figueira</b>	<b>A. Gomes</b>	<b>M. Cardoso</b>	
16:40-17:10		<b>C. Jimenez</b>	<b>L. Campos</b>	<b>J. Palacios</b>	
17:15-17:45					
17:50-18:20					
<b>Session 16 – Wavelet Theory and its Related Topics (Building B4 - Room 22)</b>					
8:30-10:00					
Chair					
10:30-11:00	<b>A. Morimoto*</b>	<b>R. Ashino*</b>		<b>T. Suzuki*</b>	
11:05-11:35	<b>U. Singh</b>	<b>Yun-Zhang Li*</b>		<b>I. Aremua*</b>	
11:40-12:10				<b>K. Fujita</b>	
15:30-16:00					
16:05-16:35					
16:40-17:10					
17:15-17:45					
17:50-18:20					

(\*) remote lecture

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Chapter

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