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ESTIMATION OF A STRUCTURAL LINEAR REGRESSION MODEL WITH A KNOWN RELIABILITY RATIO

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Abstract

In this paper, we consider the estimation of the slope parameter β of a simple structural linear regression model when the reliability ratio (Fuller, 1987) is considered to be known. By making use of an orthogonal transformation of the unknown parameters, the maximum likelihood estimator of β and its asymptotic distribution are derived. Likelihood ratio statistics based on the profile and conditional profile likelihoods are proposed. An exact marginal posterior distribution of β, which is shown to be a t-distribution is obtained. Results of a small Monte Carlo study are also reported.

1. Introduction

The classical structural simple linear regression model is defined by the equations

$$Y_k = y_k + e_k, \quad X_k = x_k + u_k, \quad y_k = \beta x_k \tag{1}$$

where e_k, u_k and x_k are independent and normally distributed with means zero and variances σ_e^2, σ_u^2 and σ_x^2, respectively, that is, $e_k \sim N(0, \sigma_e^2)$, $u_k \sim N(0, \sigma_u^2)$ and $x_k \sim N(\mu_x, \sigma_x^2)$, $k = 1, \ldots, n$. Extensive bibliographies on the structural and functional simple regression models are given in Kendall and Stuart (1961) and Fuller (1987). The main idea behind the equations (1) is that $x_1, \ldots, x_n$ are not observed directly and the estimation has to be based on $(X_1, Y_1), \ldots, (X_n, Y_n)$. Let $\mathbf{X} = (X_1, \ldots, X_n)'$ and $\mathbf{Y} = (Y_1, \ldots, Y_n)'$. Examples of practical situations where the x_k are not observed directly are provided in Fuller (1987). An interesting situation is the case where x_k is the amount of nitrogen in the soil and y_k is the yield of a certain cereal. Values for X_k in this case can only be determined by laboratory analysis and are only estimates of the true x_k values.

From the previous assumptions, it follows that the joint distribution of (X_k, Y_k) is bivariate normal, that is,

$$\begin{pmatrix} Y_k \\ X_k \end{pmatrix} \sim N_2\left(\begin{pmatrix} \beta\mu_x \\ \mu_x \end{pmatrix}; \begin{pmatrix} \beta^2\sigma_x^2 + \sigma_e^2 & \beta\sigma_x^2 \\ \beta\sigma_x^2 & \sigma_x^2 + \sigma_u^2 \end{pmatrix} \right). \tag{2}$$

Bayesian estimation of β when the ratio $\lambda = \sigma_e^2/\sigma_u^2$ is known and $\mu_x = 0$ is considered in Lindley and El-Sayad (1968). Likelihood based inferences are considered in Fuller (1987) and Wong (1989). In the present paper, we assume that the reliability ratio,

$$k_x = \frac{\sigma_x^2}{\sigma_x^2 + \sigma_u^2}, \tag{3}$$

is known. As pointed out by Fuller (1987), there are great number of situations, particularly in psychology, sociology and survey sampling where k_x is so well estimated that it may be treated as being known. Table 1.1.1 in Fuller (1987) describes values of k_x for several variables. For example, measurement error is about 15% of the observed variation for income. By considering k_x as known, we may write

$$\sigma_u^2 = k\sigma_x^2,$$

where $k = (1 - k_x)/k_x$. Hence, the covariance matrix in (2) may be written as

$$\begin{pmatrix} \beta^2\sigma_x^2 + \sigma_\epsilon^2 & \beta\sigma_x^2 \\ \beta\sigma_x^2 & (k+1)\sigma_x^2 \end{pmatrix}. \tag{4}$$

In Section 2, we consider an orthogonal parametrization (Cox and Reid, 1987) of the unknown parameters which simplifies considerably the task of deriving the maximum likelihood estimators and approximate confidence intervals for β. Likelihood ratio statistics based on the profile and on the conditional profile likelihood (Cox and Reid, 1987) are also considered. Additionally, we perform a small Monte Carlo study in order to illustrate the performance of the confidence intervals based on the asymptotic distribution of the maximum likelihood estimator of β and also on the profile and conditional profile likelihoods. A conditional model which yields an exact confidence interval for β is proposed in Section 3. In Section 4 we consider the noninformative Jeffrey's priors for the unknown orthogonal parameters and show that the marginal posterior distribution of β is a t–distribution centered about the maximum likelihood estimator. Thus, exact inference for β can also be based on this posterior distribution.

2. The Orthogonal Transformation

It follows from (2) and (3) that the log of the likelihood function of the unknown parameters β, σ_x^2 and σ_ϵ^2 is proportional to

$$\begin{aligned} -\frac{n}{2}log(D) - \frac{1}{2D}\{(\beta^2\sigma_x^2 + \sigma_\epsilon^2)\sum_{i=1}^{n}(X_i - \mu_x)^2 \\ - 2\sigma_x^2\sum_{i=1}^{n}(X_i - \mu_x)(Y_i - \beta\mu_x) + \sigma_x^2(k+1)\sum_{i=1}^{n}(y_i - \beta\mu_x)^2\}, \end{aligned} \tag{5}$$

where

$$D = k\beta^2\sigma_x^4 + (k+1)\sigma_x^2\sigma_\epsilon^2.$$

The main interest in the present paper is to make inferences about the slope parameter β, considering the other unknown parameters as nuisance parameters. It is not an easy task to find the maximum likelihood estimators of the unknown parameters by using the likelihood (5). The likelihood equations are quite complicated to solve. Moreover, the asymptotic distribution of the maximum likelihood estimators are also hard to deal with since the 3x3 Fisher information matrix is not diagonal. Bayesian inference about the parameter β has to perfomed numerically or by means of approximations. One can use, for example, the Laplace method for integrals. As pointed out by Cox and Reid (1987), inference about β is typically simplified by an orthogonal parametrization, where β is orthogonal to parameters λ_0 e λ_1. After computing the elements of the Fisher information matrix needed for finding the orthogonal parametrization (β, λ_0, λ_1); we arrive at the following differential equations:

$$(1+k^2)\sigma_x^4 \frac{d\sigma_\epsilon^2}{d\beta} + k(k+1)\beta^2\sigma_x^4 \frac{d\sigma_x^2}{d\beta} = -2k(k+1)\beta\sigma_x^6$$

and

$$k(k+1)\beta^2\sigma_x^4 \frac{d\sigma_\epsilon^2}{d\beta} + [2k\beta^2 D + \sigma_\epsilon^4(k+1)^2]\frac{d\sigma_x^2}{d\beta} = -2\beta^3 k^2 \sigma_x^6.$$

One solution to this set of differential equations gives the one to one transformation

(6)
$$\begin{cases} \lambda_0 = (k+1)\sigma_\epsilon^2 + k\lambda_1\beta^2, \\ \lambda_1 = \sigma_x^2. \end{cases}$$

Therefore, it follows from (5) that the log of the likelihood function in the new parametrization is proportional to

(7)
$$-\frac{n}{2}log(\lambda_0\lambda_1) - \frac{1}{2\lambda_0\lambda_1}\{\frac{(\lambda_0+\lambda_1\beta^2)}{k+1}\sum_{i=1}^n (X_i-\mu)^2$$
$$-2\beta\lambda_1\sum_{i=1}^n (X_i-\mu_x)(Y_i-\beta\mu_x) + \lambda_1(k+1)\sum_{i=1}^n (Y_i-\beta\mu_x)^2\}.$$

After some algebraic manipulations, we arrive at the following maximum likelihood estimators:

(8)
$$\hat{\beta} = \frac{\sum_{i=1}^n (X_i-\mu_x)Y_i + \mu_x(k+1)\sum_{i=1}^n Y_i}{\frac{1}{k+1}S_x^2 + 2\mu_x\sum_{i=1}^n (X_i-\mu_x) + n(k+1)\mu_x^2}$$
$$= \frac{\sum_{i=1}^n (X_i-\mu_x)Y_i + \frac{\mu_x}{k_x}\sum_{i=1}^n Y_i}{k_x(S_x^2 + 2\frac{\mu_x}{k_x}\sum_{i=1}^n (X_i-\mu_x) + n(\frac{\mu_x}{k_x})^2)},$$

(9)
$$\hat{\lambda}_0 = \frac{k_x\hat{\beta}^2 S_x^2 - 2\hat{\beta}\sum_{i=1}^n (X_i-\mu_x)(Y_i-\mu_x\hat{\beta}) + \frac{1}{k_x}\sum_{i=1}^n (Y_i-\mu_x\hat{\beta})^2}{n}$$

and

$$\hat{\lambda}_1 = \frac{k_x S_x^2}{n},$$

where

$$S_x^2 = \sum_{i=1}^n (X_i-\mu_x)^2.$$

In the particular case where $\mu_x = 0$, it follows from (8) that

$$\hat{\beta} = \frac{\sum_{i=1}^{n} X_i Y_i}{k_x \sum_{i=1}^{n} X_i^2}. \tag{10}$$

Fuller (1987) suggested, in a more general situation, an estimator similar to (10), based on unbiasedness grounds. It can be easily verified that $\hat{\beta}$ is a consistent estimator of β. Moreover, after some algebraic manipulations, it follows that the per observation Fisher information matrix in the new parametrization is given by

$$\begin{pmatrix} \frac{1}{2\lambda_0^2} & 0 & 0 \\ 0 & \frac{1}{2\lambda_1^2} & 0 \\ 0 & 0 & \frac{\lambda_1 + (\frac{\mu_x}{k_x})^2}{\lambda_0} \end{pmatrix}. \tag{11}$$

Thus, an approximate $(1-\alpha)$ level confidence interval for β is given by

$$\left(\hat{\beta} - z_\alpha \sqrt{\frac{\hat{\lambda}_0}{n(\hat{\lambda}_1 + (\frac{\mu_x}{k_x})^2)}}; \hat{\beta} + z_\alpha \sqrt{\frac{\hat{\lambda}_0}{n(\hat{\lambda}_1 + (\frac{\mu_x}{k_x})^2)}}\right), \tag{12}$$

where z_α denotes the upper $\alpha/2$ point for the standard normal distribution. Furthermore, it follows that the profile likelihood, $l_p(\beta)$, of the parameter β is

$$l_p(\beta) \propto -\frac{n}{2} log(\hat{\lambda}_0(\beta)), \tag{13}$$

where

$$\hat{\lambda}_0(\beta) = \frac{k_x \beta^2 S_x^2 - 2\beta \sum_{i=1}^{n}(X_i - \mu_x)(Y_i - \beta\mu_x) + \frac{1}{k_x}\sum_{i=1}^{n}(Y_i - \beta\mu_x)^2}{n}. \tag{14}$$

For testing the hypothesis $\beta = \beta_0$, we may use the likelihood ratio statistic, which is based on the profile likelihood above and is given by

$$W = 2\{l_p(\hat{\beta}) - l_p(\beta_o)\}, \tag{15}$$

where $l_p(\hat{\beta})$ and $l_p(\beta_o)$ are the profile likelihood (13) computed at the maximum likelihood estimator $\hat{\beta}$ and at β_o, respectively. Thus, a $1-\alpha$ level confidence interval for β can be obtained from $W < \chi_1^2(\alpha)$, where $\chi_1^2(\alpha)$ denotes the upper α level probability point for a chi-squared variate with one degree of freedom.

The conditional profile likelihood function (Cox and Reid, 1987) of the parameter β, $l_c(\beta)$, is

$$l_c(\beta) \propto -\frac{(n-2)}{2} log(\hat{\lambda}_0(\beta)), \tag{16}$$

where $\hat{\lambda}_0(\beta)$ is given in (14). Thus, we can obtain confidence intervals for β by using the conditional likelihood ratio statistic

$$W_c = 2\{l_c(\hat{\beta}) - l_c(\beta_o)\},$$

as considered with the statistic W, given in (15).

If μ_x is unknown, we may replace it by its consistent estimator $\bar{X} = \sum_{i=1}^{n} X_i/n$. If n is large or moderate, resulting inferences based on the above procedures (with μ_x replaced by $\bar{X}$) are close to the case where μ_x is known. This fact is illustrated in the simulation study that follows.

Tables 1 and 2 below present the results of a Monte Carlo study based on 500 simulated samples generated according to model (1) with σ_u^2 =1.0, β =1.0 and 4.0, k_x =3/4, 1/2 and 1/3, and μ_x=0 and 2.0. We took α =0.10. Table 1 shows that the approximate confidence interval for β, given in (12), is better for μ_x =0 than for μ_x =2.0. Furthermore, in this last case the coverage frequencies are below 80% for k_x=1/2 and 1/3. As expected, the results are better for n =50. The profile likehood based limits, I_2 and I_3 perform quite well; both present coverage probabilities above 90% for μ_x =2.0. The limits based on the conditional likelihood ratio statistic (I_3) have the same performance in all the entries of the table for μ_x =0. The coverage probabilities for unknown μ_x and n =50 are quite similar.

Table 1. Relative frequencies (x1000) where the confidence intervals (I_1, I_2 and I_3) covers $\beta = 1.0$

	β = 1						β = 4					
	k_x=3/4		k_x=1/2		k_x=1/3		k_x=3/4		k_x=1/2		k_x=1/3	
	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2
n=10												
I1	838	796	860	728	848	600	848	756	822	700	836	600
I2	856	942	872	986	870	1000	876	924	862	992	864	998
I3	898	968	908	994	906	1000	912	954	900	996	914	1000
n=50												
I1	902	848	892	754	878	640	906	870	894	748	902	600
I2	902	950	902	998	882	1000	912	966	900	996	902	1000
I3	908	954	912	998	890	1000	918	970	906	996	908	1000
	μ_x unknown											
n=50												
I1	886	866	856	716	840	568	902	836	852	622	748	448
I2	892	972	904	984	918	1000	906	956	914	986	918	998
I3	902	974	910	988	922	1000	916	966	918	988	924	998

(I_1: given by (12); I_2: based on (13); I_3 based on (16))

Table 2 presents in its first entry the mean values of $\hat{\beta}$ for n =10 and n =50. Note the closeness of these values to β, which is better achieved for μ_x known and n =50, as expected. The second entry corresponds to the coefficient of variation of $\hat{\beta}$, which is always lower for β=4.0. Further, when n =50, its values for known μ_x are close to the case where μ_x (unknown) is estimated by $\bar{X}$. The last entry of the table presents the mean size of the 500 simulated intervals I_1, as given in (12). The largest intervals were always obtained for μ_x=0, with k_x=1/2,1/3 and β =4.0. This behaviour was expected, since the variance of $\hat{\beta}$, given by the Fisher information matrix, is directly proportional to β and inversely proportional to μ_x.

Table 2. Mean value of $\hat{\beta}$, coefficient of variation of $\hat{\beta}$ and mean size of I_1, based on 500 simulated samples

	β = 1						β = 4					
	k_x=3/4		k_x=1/2		k_x=1/3		k_x=3/4		k_x=1/2		k_x=1/3	
	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2	μ_x=0	μ_x=2
n=10	0.999	0.989	1.032	0.997	1.035	1.008	4.001	3.980	3.999	3.995	4.031	4.051
	(30%)	(18%)	(57%)	(18%)	(98%)	(18%)	(22%)	(13%)	(40%)	(11%)	(54%)	(10%)
	0.903	0.456	1.811	0.398	2.954	0.316	2.496	1.246	4.275	0.979	6.326	0.694
n=50	0.995	1.004	0.990	0.997	0.996	0.995	4.003	3.990	3.964	4.006	3.999	4.001
	(13%)	(7%)	(25%)	(8%)	(41%)	(8%)	(8%)	(5%)	(16%)	(5%)	(22%)	(5%)
	0.407	0.220	0.801	0.191	1.310	0.151	1.102	0.600	1.957	0.472	2.856	0.330
	μ_x unknown											
n=50	0.996	1.003	0.971	1.008	0.909	1.009	3.967	4.001	3.850	4.025	3.575	4.030
	(12%)	(8%)	(26%)	(9%)	(42%)	(10%)	(8%)	(5%)	(17%)	(7%)	(26%)	(7%)
	0.398	0.225	0.760	0.196	1.120	0.153	1.099	0.599	1.876	0.479	2.430	0.333

3. A Conditional Model

By using the orthogonal transformation (6), it follows that the joint distribution of (Y_i, X_i) is bivariate normal with mean vector and covariance matrix given, respectively, by

$$\begin{pmatrix} \beta\mu_x \\ \mu_x \end{pmatrix} \text{ and } \begin{pmatrix} \frac{\lambda_0+\lambda_1\beta^2}{k+1} & \beta\lambda_1 \\ \beta\lambda_1 & (k+1)\lambda_1 \end{pmatrix}.$$

Then, the conditional distribution of Y_i given X_i is

$$N(\beta Z_i; k_x\lambda_0),$$

where

$$Z_i = \mu_x + k_x(X_i - \mu_x),$$

$i = 1, \ldots, n$. After some algebraic manipulations it follows that the likelihood estimators of β and of λ_0 (given **X**) are given by

$$\hat{\beta} = \frac{\sum_{i=1}^{n} Z_i Y_i}{\sum_{i=1}^{n} Z_i^2} \text{ and } \hat{\lambda}_0 = \frac{1}{nk_x}\sum_{i=1}^{n}(Y_i - \hat{\beta} Z_i)^2. \tag{17}$$

Since

$$Var[\hat{\beta}|\mathbf{X}] = \frac{k_x \lambda_0}{\sum_{i=1}^{n} Z_i^2},$$

it follows that an exact 1-α level conditional confidence interval for β is given by

$$(\hat{\beta} - t_\alpha \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{\beta} Z_i)^2}{(n-1)\sum_{i=1}^{n} Z_i^2}}; \hat{\beta} + t_\alpha \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{\beta} Z_i)^2}{(n-1)\sum_{i=1}^{n} Z_i^2}}), \tag{18}$$

where t_α is the upper $\alpha/2$ point for the standard t distribution. A confidence interval similar to (18) was derived by Rodrigues and Cordani (1989).

4. A Bayesian Analysis

As will be seen below, the orthogonal tranformation (6) simplifies considerably the task of finding the exact marginal posterior distribution of the parameter β. We assign a uniform improper prior for β. Given β, we assign the Jeffrey's noninformative prior for (λ_0, λ_1). From the Fisher information matrix (11), the Jeffreys' noninformative prior for (λ_0, λ_1) is

$$p(\lambda_o, \lambda_1|\beta) \propto \frac{1}{\lambda_0 \lambda_1}.$$

Therefore, the joint posterior density of $(\beta, \lambda_0, \lambda_1)$ is proportional to

$$\begin{aligned}(\frac{1}{\lambda_0\lambda_1})^{\frac{n+2}{2}} exp\{-\frac{1}{2\lambda_1(k+1)}\sum_{i=1}^{n}(X_i-\mu_x)^2\} exp\{-\frac{1}{2\lambda_0}[\frac{\beta^2}{k+1}\sum_{i=1}^{n}(X_i-\mu_x)^2 \\ -2\beta\sum_{i=1}^{n}(X_i-\mu_x)(Y_i-\beta\mu_x)+(k+1)\sum_{i=1}^{n}(Y_i-\beta\mu_x)^2]\}.\end{aligned} \tag{19}$$

Integrating (19) over (λ_0, λ_1), we arrive at the marginal posterior density of β, which is

$$\begin{aligned}p(\beta|\mathbf{X},\mathbf{Y}) \propto \{&\frac{\beta^2}{k+1}\sum_{i=1}^{n}(X_i-\mu_x)^2 \\ &-2\beta\sum_{i=1}^{n}(X_i-\mu_x)(Y_i-\beta\mu_x)+(k+1)\sum_{i=1}^{n}(Y_i-\beta\mu_x)^2\}^{-\frac{n}{2}}.\end{aligned} \tag{20}$$

After some rearrangements, we can write the posterior density (20) as

$$p(\beta|\mathbf{X},\mathbf{Y}) \propto \{1 + \frac{(\beta-\hat{\beta})^2}{(n-1)\Sigma}\}^{-\frac{n}{2}}, \tag{21}$$

where

$$\Sigma = \frac{\sum_{i=1}^n Y_i^2 - \frac{(\sum_{i=1}^n (X_i-\mu_x)Y_i + \frac{\mu_x}{k_x}\sum_{i=1}^n Y_i)^2}{\sum_{i=1}^n (X_i-\mu_x)^2 + 2\frac{\mu_x}{k_x}\sum_{i=1}^n (X_i-\mu_x) + n(\frac{\mu_x}{k_x})^2}}{(n-1)k_x^2(\sum_{i=1}^n (X_i-\mu_x)^2 + 2\frac{\mu_x}{k_x}\sum_{i=1}^n (X_i-\mu_x) + n(\frac{\mu_x}{k_x})^2)}.$$

Thus, according to (21), it follows that the posterior distribution of β is a t–distribution with $n-1$ degrees of freedom, centered at the maximum likelihood estimator $\hat{\beta}$ and with variance given by

$$Var[\beta|\mathbf{Y},\mathbf{X}] = \frac{(n-1)}{(n-3)}\Sigma.$$

In the particular case where $\mu_x = 0$, we have

$$\Sigma = \frac{\{\sum_{i=1}^n X_i^2 \sum_{i=1}^n Y_i^2 - (\sum_{i=1}^n X_i Y_i)^2\}}{(n-1)k_x^2(\sum_{i=1}^n X_i^2)^2}.$$

Bayesian inference for β may also be obtained by using the conditional model of Section 3. By considering the noninformative prior

$$p(\beta,\lambda_0) \propto \frac{1}{\lambda_0},$$

it can be shown that the posterior marginal distribution of β is again a t–distribution with $n-1$ degrees of freedom and centered at $\hat{\beta}$ given in (17). The posterior variance is given by

$$Var[\beta|\mathbf{X},\mathbf{Y}] = \frac{\sum_{i=1}^n (Y_i - \hat{\beta}Z_i)^2}{(n-3)\sum_{i=1}^n Z_i^2},$$

where $\hat{\lambda}_0$ is given in (17).

5. Final Discussion

The main object of the present paper is to estimate the regression parameter β under the structural linear regression model (2) considering the reliability ratio k_x and the mean μ_x as known parameters. If μ_x is unknown, it was sugested to replace it by the sample mean $\bar{X}$, which is a consistente estimator of μ_x. For large or moderate n, inferences about β when μ_x is known and when it is replaced by $\bar{X}$ are typically close. But, the method of obtaining an orthogonal parametrization in which β is orthogonal to the remaining parameters can be extended to include μ_x. It can be shown after some algebraic manipulations that to have an orthogonal parametrization with μ_x, σ_u^2 and σ_e^2 as unknown parameters, we need λ_0 and λ_1 as given by (6) and also λ_2 given by the equation

$$\mu_x = \frac{\lambda_2\sqrt{1+k}}{\sqrt{\lambda_0 + \beta^2 k^2 \lambda_1}}.$$

Then β is orthogonal to $(\lambda_0, \lambda_1, \lambda_2)$. Unfortunately, in this new parametrization, the likelihood equations are complicated to solve and we haven't yet found explicit solutions for the maximum likelihood estimators. Thus, the problem considered in this paper (with μ_x unknown) is a situation where an explicit solution is possible for the differential equations, but, explicit solutions for the likelihood equations are yet to be found.

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