

Bolstering up the existence of $P_s(2080)$

Breno Agatão^{1,*}, A. Vertel Nieto^{1,†}, K. P. Khemchandani^{2,‡}, A. Martínez Torres^{1,§} and Seung-il Nam^{3,4,||}

¹*Universidade de Sao Paulo, Instituto de Fisica, Postal Code 05389-970, Sao Paulo, Brazil*

²*Universidade Federal de Sao Paulo, Postal Code 01302-907, Sao Paulo, Brazil*

³*Department of Physics, Pukyong National University (PKNU), Busan 48513, Korea*

⁴*Asia Pacific Center for Theoretical Physics (APCTP), Pohang 37673, Korea*



(Received 3 February 2025; accepted 21 May 2025; published 16 June 2025)

We present a detailed study of the partial decay widths of a spin-parity resonance $J^P = 3/2^- N^*$ with a mass of $\simeq 2070$ MeV obtained from the coupled channel s -wave vector-baryon ρN , ωN , ϕN , $K^* \Lambda$, and $K^* \Sigma$ dynamics. This state, which couples strongly to the $K^* \Sigma$ channel, corresponds to a nucleon with a hidden strange quark content, in analogy to the P_c states discovered by the LHCb Collaboration, and we denote it as $P_s(2080)$. A state with such a nature can decay to vector-baryon, pseudoscalar-baryon, and pseudoscalar-baryon resonance channels, involving triangular loops in the latter two cases. As we will show, the partial decay widths to pseudoscalar-baryon resonance channels, like $\pi N^*(1535)$, $\pi N^*(1650)$, $K \Lambda(1405)$, are comparable to those related to ground state baryons in the final state, like πN , ηN , $K \Lambda$. In this way, reactions involving such lighter baryon resonances in the final state can be used as an alternative source of information on the properties of an N^* with hidden strangeness.

DOI: [10.1103/v61h-vddp](https://doi.org/10.1103/v61h-vddp)

I. INTRODUCTION

The discovery of the P_c pentaquarks by the LHCb Collaboration [1–3] has undoubtedly proven the existence of exotic baryons whose properties cannot be understood in terms of three quarks. Their nature and quantum numbers, however, are still unclear, and different spin-parity assignments and inner structures, like pentaquarks or meson-baryon molecular type of hadrons, have been proposed for describing the P_c states [4–15].

The P_c states, being observed in the J/ψ - p invariant mass distribution of the process $\Lambda_b^0 \rightarrow J/\psi p K^-$, correspond to nucleon resonances with hidden charm and one could wonder if there may exist in Nature their hidden strange partners. If the P_c states would be generated from the meson-baryon dynamics, $P_c^+(4450)$ seems to be described as a spin-parity $J^P = 3/2^-$, isospin $I = 1/2$ baryon obtained mainly from the interaction of $\bar{D}^*$ and Σ_c in the s -wave, and whose nominal mass is $\simeq 8$ MeV

below the threshold of the latter channel. One of the relevant contributions in the description of the $\bar{D}^*$ and Σ_c interaction consists of exchanging a vector meson, like ρ , ω , in the t -channel [16]. In such a case, the quark $\bar{c}$ in $\bar{D}^*$ and the quark c in Σ_c act as spectators, as shown in Fig. 1. If the quarks $\bar{c}$ and c are now replaced by the quarks $\bar{s}$ and s , respectively, the $\bar{D}^* \Sigma_c$ system would become $K^* \Sigma$, interacting via vector meson exchange in the t -channel, with the quarks $\bar{s}$ and s continuing being spectators as well. Since in both cases, the heavy quarks in the respective systems behave as spectators, assuming the relevant dynamics needed to form states in such systems to be the t -channel exchange of vector mesons, the formation of an isospin $1/2$ state, with $J^P = 3/2^-$ and a mass of $\simeq 2077$ MeV, i.e., $\simeq 8$ MeV below the $K^* \Sigma$ threshold, in analogy with $\bar{D}^* \Sigma_c$, seems almost compelling.

After the discovery of the P_c states, several authors have investigated the existence of the hidden strange partners of the former. For instance, in Ref. [17], the $3/2^-$ nucleon resonances $N^*(1875)$ and $N^*(2120)$ were interpreted as hadronic molecular states, generated from the coupled channel interactions $\Sigma^* K$ and ΣK^* considering a boson exchange potential model to solve the Bethe-Salpeter equation. In Ref. [18], by assuming that $N^*(1875)$ and $N^*(2120)$ are indeed s -wave $K \Sigma^*$ and $K^* \Sigma$ states, and by fixing the mass of these states to be, respectively, 1875 and 2080 MeV, the coupling constants of the former resonances to the latter states were determined by considering the Weinberg compositeness condition [19]. Using the obtained coupling constants and effective Lagrangians to describe the

*Contact author: bgarcia@if.usp.br

†Contact author: anieto@if.usp.br

‡Contact author: kanchan.khemchandani@unifesp.br

§Contact author: amartine@if.usp.br

||Contact author: sinam@pknu.ac.kr

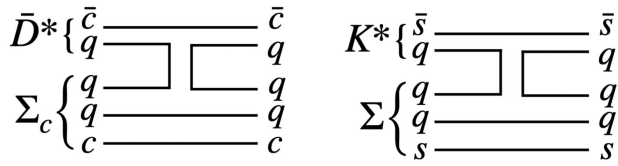


FIG. 1. Left: vector exchange in the t -channel for the process $\bar{D}^* \Sigma_c \rightarrow \bar{D}^* \Sigma_c$. Right: same mechanism but for $K^* \Sigma \rightarrow K^* \Sigma$. In both cases, the heavy quarks behave as spectators.

vertices, the partial decay widths of the mentioned N^* resonances to final states formed by vector-baryon, pseudoscalar-baryon and $K\Lambda(1405)$, $K\Lambda(1520)$ were determined by considering a pion exchange in a triangular loop.

The production of hidden strangeness nucleon resonances with a mass of ~ 2000 – 2100 MeV has been theoretically studied [20–25] in the past and related to some of the bumplike structures observed in the experimental data in the same energy region [26,27], in different processes involving final states such as ϕN , $K\Sigma$. More recently, within the context of the existence of hidden strangeness partners of the P_c states, the presence of $P_s(2080)$ has been investigated in data on processes such as $\gamma p \rightarrow \phi p$, $K^+ p \rightarrow K^+ \phi p$, $\pi^- p \rightarrow \phi n$ [28–31]. In some of these studies, the existence of such states, the mass, width, and quantum numbers are assumed using the analogy with the $\bar{D}^* \Sigma_c$ interaction, and the coupling constants needed to evaluate the corresponding cross sections are determined either via the Weinberg compositeness condition, considering the P_s states to be bound states of two hadrons whose threshold is close to the assumed mass, or from fits to the data [29–31].

Before continuing with further discussions, we should mention that no N^* resonance with a mass of 2080 MeV and quantum numbers $J^P = 3/2^-$ is listed in the most recent version of the Review of Particle Physics (PDG) [32]: before the 2012 version of the PDG, any evidence for N^* resonances with $J^P = 3/2^-$ and mass above 1800 MeV were collected under the label of $N^*(2080)$. It is worth recalling that there existed a proposal for replacing $N^*(2080)$ by $D_{13}(1895)$ and identifying it as a missing nucleon resonance [33,34], on the basis of the SAPHIR experimental data on $\gamma p \rightarrow K^+ p$. In a different work, it was proposed that $N^*(2080)$ could be related to $N^*(2200)5/2^-$ [35]. In a more recent work [36], it was shown that the contribution of $N^*(2080)$ is small but not negligible, and that $P_{13}(1900)$ seems to be more present in the process. In fact, $N^*(2080)$ seems to remain consistently absent in the latest analysis of data on the photoproduction of $K\Lambda/K\Sigma$ [37–39]. This could imply that $N^*(2080)$ couples weakly to $K\Lambda/K\Sigma$ states, and other alternative processes could be more useful to determine the fate of this state. It is one of the purposes of this paper to provide useful alternatives, which are much needed given that the state has been removed from the PDG listings: in the latest

volume of PDG, two $J^P = 3/2^-$ states, a three-star $N^*(1875)$ and a two-star $N^*(2120)$, are cataloged. Further, a closer look at the papers listed in the PDG in these entries shows a large uncertainty (~ 100 MeV) in these states' mass and width values.

Despite the absence of $N^*(2080)$ in the latest Review of Particle Physics, theoretical evidence for its existence and its nature as a $3/2^-$ $K^* \Sigma$ quasibound state was reported in Ref. [40], long before the discovery of the P_c states by the LHCb Collaboration. In Refs. [40,41], the coupled channel $K^* \Sigma$, $K^* \Lambda$, ϕN , ωN , and ρN vector-baryon dynamics was studied by using effective Lagrangians based on the hidden local symmetry [42], considering t -, s -, u -channel exchange contributions as well as a contact interaction whose origin lies in the nature of the Lagrangian considered. The amplitudes were projected on the s -wave and further on the spin $1/2$ and $3/2$ bases. As a consequence of the aforementioned dynamics, the generation of several $J^P = 1/2^-$ and $3/2^-$ N^* and Δ resonances were found, and, in particular, for the case of $J^P = 3/2^-$ and isospin $1/2$, a pole in the second Riemann sheet with a mass of $\simeq 2071$ MeV and a width¹ of $\simeq 60$ – 70 MeV was obtained, with the state having a large coupling to the $K^* \Sigma$ channel.

Denoting the former state as $N^*(2080)$, given its large coupling to $K^* \Sigma$ and the proximity of its mass to the threshold of this channel, such a state can be considered as a nucleon resonance with hidden strangeness. In analogy to the notation for the P_c states, we could use the nomenclature $P_s(2080)$ to represent the state, where the letter P refers to the five quark (pentaquark) content (four quarks and an antiquark) and the subscript s to the presence of a $s\bar{s}$ pair in the inner structure of the state.

It is worth mentioning that the generation of a nucleon resonance with hidden strangeness content, from vector-baryon dynamics, was also investigated in Ref. [43]. In this former work, considering t -channel exchange contributions, spin-degenerate amplitudes were obtained, which led to the finding of two N^* resonances, both with a mass of 1977 MeV, and a width of 106 MeV, but different spin-parity quantum numbers (one having $J^P = 1/2^-$ and another having $J^P = 3/2^-$).

The study of Ref. [43] was revisited in Ref. [20], where the cross sections for $\gamma p \rightarrow K^0 \Sigma^+$, $\gamma n \rightarrow K^0 \Sigma^0$ were determined and the role of the production of N^* resonances with hidden strangeness near the $K^* \Lambda$ and $K^* \Sigma$ thresholds was studied. By readjusting the model parameters used in Ref. [43] to regularize the vector-baryon loops entering the Bethe-Salpeter equation, the pole at $M - i\Gamma/2 = 1977 - i53$ MeV was shifted to $\simeq 2035 - i63$ MeV, providing an interpretation to the bump observed in the cross section of $\gamma p \rightarrow K^0 \Sigma^+$ at energies around 2000 MeV (which is close to the $K^* \Lambda$ threshold). In this way, according

¹There is a typo in the original work, in which the full width obtained for the state is referred to as the half-width of the state.

to the authors of Ref. [20], there should be two N^* resonances with hidden strangeness at $\approx 2035 - i63$ MeV, one with $J^P = 1/2^-$ and other with $J^P = 3/2^-$. An enhancement in this energy region has been reported in Ref. [44].

The findings of Refs. [40,41] are different to those of Ref. [43]. In Refs. [40,41] N^* resonances with different masses for $J^P = 1/2^-$ and $J^P = 3/2^-$ were obtained in the energy region of ~ 1900 – 2100 MeV. In $J^P = 1/2^-$ two overlapping poles were found at $1801 - i96$ and $1912 - i54$ MeV, which produce one peak on the real axis and were related to $N^*(1895)$. Such a nature of $N^*(1895)$ was found to be useful in describing the cross sections for $\gamma p \rightarrow K^+ \Lambda(1405)$ [45]. The $J^P = 3/2^-$ state, obtained at $\approx 2071 - i35$ MeV, was related to the $J^P = 3/2^-$ $N^*(2080)$ appearing in the previous version of the PDG.

To summarize this discussion, we can say that there seems to be progress gathering evidence for the existence of a $3/2^-$ state with mass around 2080 MeV in recent times. Some works assume such a possibility and search for the signals of a hidden strange partner of $P_c(4457)$ in the experimental data, and in some works a simplified model is used to determine meson-baryon scattering amplitudes. The experimental data too are still scarce to draw clear conclusions. Here, we benefit from the work of Ref. [40] which, using a more complete framework, predicted the existence of $N^*(2080)$, and study its decay to channels like πN , ηN , $K\Lambda$. We also explore decay channels involving baryon resonances, such as $\pi N^*(1535)$, $\pi N^*(1650)$, $\eta N^*(1535)$, $K\Lambda(1405)$, which could serve as alternative processes to search for a P_s -state, i.e., a nonstrange partner of the $\bar{D}^* \Sigma$ quasibound state.

II. CALCULATION OF THE PARTIAL DECAY WIDTHS

We start the discussions by showing in Fig. 2 different decay mechanisms for the $P_s(2080)$ found in Ref. [40]. Since the former state is obtained from the s -wave vector-octet baryon (VB) coupled channel dynamics with $J^P = 3/2^-$, we can have a direct decay mode of $P_s(2080)$ to the VB channels considered for its generation: $K^* \Sigma$, $K^* \Lambda$, ϕN , ωN , and ρN . In this case, the amplitude describing such a process can be written as

$$-it_{P_s \rightarrow V_i B_i} = i g_{P_s \rightarrow V_i B_i} \bar{u}_{B_i}(P-k) \epsilon_{V_i}^\mu(k) u_{P_s \mu}(P), \quad (1)$$

where $g_{P_s \rightarrow V_i B_i}$ represents the coupling constant of $P_s(2080)$ to a VB channel i constituted by a vector V_i and a baryon B_i , $\epsilon_{V_i}^\mu$ is the polarization vector associated with the vector meson V_i , $u_{P_s \mu}$ is a Rarita-Schwinger spinor [46] related to P_s , and P^μ , k^μ represent the four-momenta of P_s and of the meson in the final state, respectively. To simplify the notation, the dependence of the spinors on the

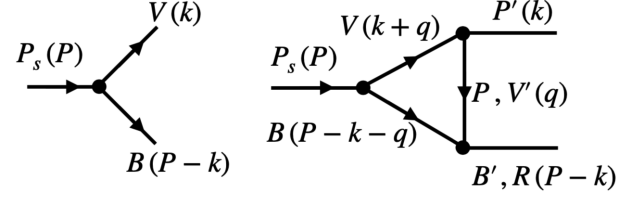


FIG. 2. Decay mechanisms for $P_s(2080)$ [$J^P = 3/2^-$] to VB (left) and to PB/PR (right) channels, where R represents either $N^*(1535)$, $N^*(1650)$, or $\Lambda(1405)$, which are $J^P = 1/2^-$ states. The four-momenta assignment for each particle is shown between brackets.

spin projection of the corresponding particle has been omitted in Eq. (1). The Dirac and Rarita-Schwinger spinors related, respectively, to particles of four-momenta Q , masses m and M and spin projections α and β , are normalized such that [22,29]

$$\begin{aligned} \sum_{\alpha=-1/2}^{1/2} u(Q, \alpha) \bar{u}(Q, \alpha) &= \frac{(\not{Q} + m\mathbb{I})}{2m}, \\ \sum_{\beta=-3/2}^{3/2} u_\mu(Q, \beta) \bar{u}_\nu(Q, \beta) &= \frac{\not{Q} + M\mathbb{I}}{2M} P_{\mu\nu}, \end{aligned} \quad (2)$$

where

$$P_{\mu\nu} = -g_{\mu\nu} \mathbb{I} + \frac{1}{3} \gamma_\mu \gamma_\nu + \frac{2}{3} \frac{Q_\mu Q_\nu}{M^2} \mathbb{I} + \frac{\gamma_\mu Q_\nu - \gamma_\nu Q_\mu}{3M},$$

with $\mathbb{I}$ being the identity matrix.

When considering the process $V_i B_i \rightarrow P_s \rightarrow V_j B_j$ in the s -wave, the amplitude in Eq. (1) gives rise to the following isospin 1/2, s -wave, and spin 3/2 projected amplitude $T_{ij}^{S=3/2}$, i.e., $I(J^P) = 1/2(3/2^-)$, in the nonrelativistic limit,

$$T_{ij}^{S=3/2}(\sqrt{s}) = \frac{g_{P_s \rightarrow V_i B_i} g_{P_s \rightarrow V_j B_j}}{\sqrt{s} - m_{P_s} + i\Gamma_{P_s}/2}, \quad (3)$$

with m_{P_s} (Γ_{P_s}) being the mass (width) of $P_s(2080)$ and $\sqrt{s}$ representing the center-of-mass energy of the system. Equation (3) shows that the coupling constants $g_{P_s \rightarrow V_i B_i}$ needed in Eq. (1) can be directly obtained from the residue of the t -matrix describing the $V_i B_i \rightarrow V_i B_i$ interaction in which P_s is dynamically generated. These coupling constants were determined in Ref. [40] from the analytical continuation of the t -matrix in the second Riemann sheet. Alternatively, it is possible to calculate the mentioned coupling constants by using the t -matrix determined in Ref. [40] on the real-energy plane. In this case, from Eq. (3), we can calculate the coupling constants $g_{P_s \rightarrow V_i B_i}$ as

$$g_{P_s \rightarrow V_i B_i} = \sqrt{i \frac{\Gamma_{P_s}}{2} T_{ii}^{S=3/2}(m_{P_s})}. \quad (4)$$

For a channel $j \neq i$, the couplings $g_{P_s \rightarrow V_j B_j}$ are obtained from the ratio between $T_{ij}^{S=3/2}$ and $T_{ii}^{S=3/2}$ at $\sqrt{s} = m_{P_s}$, i.e.,

$$g_{P_s \rightarrow V_j B_j} = g_{P_s \rightarrow V_i B_i} \frac{T_{ij}^{S=3/2}(m_{P_s})}{T_{ii}^{S=3/2}(m_{P_s})}. \quad (5)$$

In this way, all the relative phases between the couplings for $i \neq j$ are all related to the same channel i . This latter procedure of calculating the coupling constants is more convenient when considering the finite width of the ρ - and K^* -mesons through a convolution of the loop functions while solving the Bethe-Salpeter equation for the coupled channel system. Here, we follow the latter approach and provide the obtained coupling constants in Table I. As can be seen in the table, the coupling of $P_s(2080)$ to the $K^*\Sigma$ channel, whose nominal threshold (2085 MeV) is the closest to the mass of P_s , is the largest, as implicitly assumed in Refs. [29,30] when considering the Weinberg compositeness condition to determine the coupling constant of $P_s(2080)$ to $K^*\Sigma$ by considering P_s to be a $K^*\Sigma$ bound state. However, this does not necessarily mean that

TABLE I. Coupling constants (dimensionless) of $P_s(2080)$ to the vector-baryon channels, in the isospin 1/2 basis, considered for its generation.

Channel	ρN	ωN	ϕN
Coupling	$-0.231 - i0.284$	$-0.175 + i0.038$	$0.285 + i0.01$
Channel	$K^*\Lambda$	$K^*\Sigma$	
Coupling	$0.112 + i0.553$	$2.313 - i0.856$	

the other coupled channels listed in Table I will have no relevant contributions to the partial decay widths of P_s , especially when considering the triangular loop mechanisms shown in Fig. 2, where the interference effects between different coupled channels can play a relevant role obtaining the partial decay widths.

With the coupling constants listed in Table I, and considering the rest frame of the decaying particle, the amplitudes in Eq. (1) can be evaluated and the partial decay width of $P_s(2080)$ to a $V_i B_i$ channel can be determined from

$$\Gamma_{P_s \rightarrow V_i B_i}(m_{P_s}, m_{V_i}, m_{B_i}) = \frac{m_{B_i}}{m_{P_s}} \frac{|\mathbf{p}_i|}{(2\pi) 2s_{P_s} + 1} \sum_{\text{pol}} |t_{P_s \rightarrow V_i B_i}|^2 \Theta(m_{P_s} - m_{V_i} - m_{B_i}), \quad (6)$$

where $|\mathbf{p}_i|$ is the modulus of the center-of-mass linear momentum of the particles in the final state, s_{P_s} is the spin of $P_s(2080)$, $\Theta(\dots)$ is the Heaviside Θ -function, and the symbol $\sum_{\text{pol}}$ represents summing over the polarizations of the particles in the initial and final states. The finite width of P_s can be incorporated by considering a convolution of the expression in Eq. (6) with the corresponding spectral function for P_s :

$$\Gamma_{P_s \rightarrow V_i B_i} = \frac{1}{N_{P_s}} \int_{m_{P_s}-2\Gamma_{P_s}}^{m_{P_s}+2\Gamma_{P_s}} d\tilde{m}_{P_s} \rho_{P_s}(\tilde{m}_{P_s}) \times \Gamma_{P_s \rightarrow V_i B_i}(\tilde{m}_{P_s}, m_{V_i}, m_{B_i}), \quad (7)$$

where

$$\rho_{P_s}(\tilde{m}_{P_s}) = -\frac{1}{\pi} \text{Im} \left(\frac{1}{\tilde{m}_{P_s} - m_{P_s} + i\Gamma_{P_s}/2} \right), \quad (8)$$

and N_{P_s} is the normalization of the spectral function of Eq. (8) when considering $\tilde{m}_{P_s} \in [m_{P_s} - 2\Gamma_{P_s}, m_{P_s} + 2\Gamma_{P_s}]$,

$$N_{P_s} = \int_{m_{P_s}-2\Gamma_{P_s}}^{m_{P_s}+2\Gamma_{P_s}} d\tilde{m}_{P_s} \rho_{P_s}(\tilde{m}_{P_s}). \quad (9)$$

Note that the effect of the finite width of the vector mesons in the final state is already present in the coupling constants listed in Table I.

In the case of P_s decaying to pseudoscalar-baryon ($P'_i B_i$) or pseudoscalar-baryon resonance ($P'_i R_i$, with R_i having $J^P = 1/2^-$) channels, the decay mechanism proceeds via triangular loops, as shown in Fig. 2. Now we can have contributions from the exchange of pseudoscalars (P) or vector mesons (V') between the vectors (V) and baryons (B) produced in the primary vertex. For instance, we can have channels in the final state like πN , ηN , $K\Lambda$, $K\Sigma$, $\eta' N$, $K\Lambda(1405)$, $\pi N^*(1535)$, $\eta N^*(1535)$, or $\pi N^*(1650)$, and intermediate states in the triangular loop like $K^*\Sigma\pi$, $K^*\Sigma\eta$, $K^*\Sigma\eta'$, $K\Lambda\pi$, $\rho N\bar{K}$, $\omega N\bar{K}$, $\phi N\bar{K}$, $K^*\Sigma\omega$, etc. Thus, to evaluate the contribution to the partial decay widths of P_s from the diagrams represented in Fig. 2, we need amplitudes describing the vector-pseudoscalar-pseudoscalar (VPP), vector-vector-pseudoscalar (VVP), pseudoscalar-baryon-baryon (PBB), and vector-baryon-baryon (VBB) vertices. These latter contributions are determined from effective Lagrangians based on the chiral and hidden local symmetries [42,47,48], with

$$\begin{aligned}
\mathcal{L}_{\text{VPP}} &= -ig\langle V^\mu [P, \partial_\mu P] \rangle, \\
\mathcal{L}_{\text{VVP}} &= \frac{G}{\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \langle \partial_\mu V_\nu \partial_\alpha V_\beta P \rangle, \\
\mathcal{L}_{\text{PBB}} &= -\frac{D+F}{\sqrt{2}f_\pi} \langle \bar{B} \gamma^\mu \gamma_5 \partial_\mu P B \rangle - \frac{D-F}{\sqrt{2}f_\pi} \langle \bar{B} \gamma^\mu \gamma_5 B \partial_\mu P \rangle, \\
\mathcal{L}_{\text{VBB}} &= g[\langle \bar{B} \gamma_\mu [V^\mu, B] \rangle + \langle \bar{B} \gamma_\mu B \rangle \langle V^\mu \rangle], \quad (10)
\end{aligned}$$

where $g = m_V/(2f_\pi)$, $m_V \simeq m_\rho$, $G = 3g^2/(4\pi^2 f_\pi)$, $D \simeq 0.80$, $F \simeq 0.46$, $f_\pi \simeq 93$ MeV, $u = e^{iP/(\sqrt{2}f_\pi)}$, $u_\mu = iu^\dagger \partial_\mu U u^\dagger$, $U = u^2$, P , B , and V^μ are matrices whose elements are, respectively, the pseudoscalar, baryon, and vector fields from the octet,

$$\begin{aligned}
P &= \begin{pmatrix} \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{\eta}{\sqrt{3}} + \frac{\eta'}{\sqrt{6}} - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & -\frac{\eta}{\sqrt{3}} + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}, \\
B &= \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}, \\
V_\mu &= \begin{pmatrix} \frac{\omega+\rho^0}{\sqrt{2}} & \rho^+ & K^{*+} \\ \rho^- & \frac{\omega-\rho^0}{\sqrt{2}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_\mu, \quad (11)
\end{aligned}$$

and the symbol $\langle \dots \rangle$ indicates SU(3) trace. Here ideal η - η' mixing, i.e., a mixing angle of $\beta \simeq -19.43^\circ$ ($\sin \beta = -1/3$) has been assumed when writing the elements of the matrix P . A value of β in the range $\simeq -15^\circ$ to -22° is compatible with the experimental data [49–51], and such uncertainty will be considered in the calculation of the partial decay widths. The expression of P in terms of a general mixing angle β can be found in Appendix B.

In the case of a $J^P = 1/2^-$ baryon resonance in the final state, we consider the amplitudes [45]

$$\begin{aligned}
-it_{PB \rightarrow R} &= ig_{R \rightarrow PB} \bar{u}_R(p) u_B(P-k-q), \\
-it_{V'B \rightarrow R} &= -\frac{g_{R \rightarrow V'B}}{\sqrt{3}} \epsilon_{V'}^\mu(q) \bar{u}_R(p) \gamma_\mu \gamma_5 u_B(P-k-q), \quad (12)
\end{aligned}$$

with $g_{R \rightarrow PB(V'B)}$ being the coupling constant of the resonance R to the PB and VB channels considered for its generation and u_R being the Dirac-spinor related to the $J^P = 1/2^-$ baryon in the final state. The factor $1/\sqrt{3}$ in Eq. (12) has its origin in the fact that the $g_{R \rightarrow V'B}$ coupling in Refs. [41,52,53] are determined by parametrizing the meson-baryon t -matrices as Breit-Wigner amplitudes while Eq. (12) provides a spin dependent expression (see Ref. [45]

for more details). Here we consider the low-lying Λ and N^* resonances for which phase space is available for decaying, i.e., $\Lambda(1405)$, $N^*(1535)$, and $N^*(1650)$, and use the coupling constants determined in Refs. [41,52,53], where PB and VB channels were treated as coupled channels when solving the Bethe-Salpeter equations and the couplings constants were determined from the residues of the corresponding T -matrix in the complex energy plane.

In the case of the process $P_s \rightarrow P'B'$ shown in Fig. 2, using the previous amplitudes and the effective Lagrangians in Eq. (10), we get the following contribution for a particular vector-baryon-pseudoscalar (VBP) channel in the triangular loop shown in Fig. 2:

$$\begin{aligned}
-it_{P_s \rightarrow P'B'}^{\text{VBP}} &= gC_{PB \rightarrow B'} C_{V \rightarrow P'P} g_{P_s \rightarrow VB} \bar{u}_{B'}(p) \gamma_5 \\
&\times \left[(2p^\nu + (m_B + m_{B'})\gamma^\nu)(-I_{\nu\mu}^{(1)} + k_\mu I_\nu^{(2)}) \right. \\
&\left. + I_\mu^{(3)} - k_\mu I^{(4)} \right] u_{P_s}^\mu(P), \quad (13)
\end{aligned}$$

where $C_{PB \rightarrow B'}$ and $C_{V \rightarrow P'P}$ are coefficients obtained from the effective Lagrangians of Eq. (10) and

$$\begin{aligned}
I_{\nu\mu}^{(1)} &= \left(1 + \frac{k^2}{m_V^2}\right) \mathbb{I}_{\nu\mu}^{(1)} - \frac{1}{m_V^2} \mathbb{I}_{\nu\mu}^{(2)}, \\
I_\nu^{(2)} &= \left(1 - \frac{k^2}{m_V^2}\right) \mathbb{I}_\nu^{(3)} + \frac{1}{m_V^2} \mathbb{I}_\nu^{(4)}, \\
I_\mu^{(3)} &= \left(1 + \frac{k^2}{m_V^2}\right) \mathbb{I}_\mu^{(4)} - \frac{1}{m_V^2} \mathbb{I}_\mu^{(5)}, \\
I^{(4)} &= \left(1 - \frac{k^2}{m_V^2}\right) \mathbb{I}_\nu^{(6)} + \frac{1}{m_V^2} \mathbb{I}^{(7)}, \quad (14)
\end{aligned}$$

with

$$\begin{aligned}
\mathbb{I}_{\nu\mu}^{(1)} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q_\nu q_\mu}{\mathbb{D}}; & \mathbb{I}_{\nu\mu}^{(2)} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^2 q_\nu q_\mu}{\mathbb{D}}; \\
\mathbb{I}_\nu^{(3)} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q_\nu}{\mathbb{D}}; & \mathbb{I}_\nu^{(4)} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^2 q_\nu}{\mathbb{D}}; \\
\mathbb{I}_\mu^{(5)} &= \int \frac{d^4 q}{(2\pi)^4} \frac{q^4 q_\mu}{\mathbb{D}}; & \mathbb{I}^{(6)} &= g^{\nu\mu} \mathbb{I}_{\nu\mu}^{(1)}; \\
\mathbb{I}^{(7)} &= g^{\nu\mu} \mathbb{I}_{\nu\mu}^{(2)}; \\
\mathbb{D} &= [(P-k-q)^2 - m_B^2 + i\epsilon][(k+q)^2 - m_V^2 + i\epsilon] \\
&\times [q^2 - m_P^2 + i\epsilon]. \quad (15)
\end{aligned}$$

It should be noted that the expressions in Eqs. (12)–(15) depend on the particular channel considered in the final and intermediate states, but to simplify the notation we omit writing such a dependence.

The integrals in Eq. (15) can be written as combinations of the four-momenta P_α and k_β by using Lorentz covariance. For example, after integrating in d^4q , $\mathbb{I}_{\mu\nu}^{(1)}$ must be a symmetric tensor of order 2 depending on the four-momenta P and k . Thus, we can write

$$\mathbb{I}_{\nu\mu}^{(1)} = a_1^{(1)} g_{\nu\mu} + a_2^{(1)} P_\nu P_\mu + a_3^{(1)} k_\nu k_\mu + a_4^{(1)} (P_\nu k_\mu + P_\mu k_\nu), \quad (16)$$

where $a_j^{(1)}$, $j = 1, 2, \dots, 4$ are the coefficients of the combinations, and which need to be determined. Similar arguments can be used for the other tensor integrals. Details on the calculation of the $a_j^{(i)}$, $i = 1, 2, \dots, 5$, coefficients (which depend on the final and intermediate states) can be found in Appendix A. The main steps to follow are to use the Passarino-Veltman decomposition of tensor integrals [54], then determine the dq^0 integration analytically by using Cauchy's theorem and the d^3q integration numerically, by using a cutoff or form factors to regularize it. We have varied the cutoff in the range 600–850 MeV and considered three types of form factors at the vertices (Gaussian, Lorentz, and a Heaviside Θ -function), and estimated uncertainties in the results.

In terms of the $a_j^{(i)}$ coefficients, the amplitude for the process $P_s \rightarrow P'B'$, considering the different VPB intermediate states, thus, exchanging pseudoscalars in the triangular loop of Fig. 2, can be written as

$$\begin{aligned} -it_{P_s \rightarrow P'B'}^{\text{pseudo}} &= -i \sum_{\text{VBP}} t_{P_s \rightarrow P'B'}^{\text{VBP}} \\ &= g \bar{u}_{B'}(p) \gamma_5 t_\mu^{(A)}(A, \tilde{A}) u_{P_s}^\mu(P), \end{aligned} \quad (17)$$

where

$$\begin{aligned} t_\mu^{(A)}(A, \tilde{A}) &= \sum_{k=1}^5 [2A_k P^\nu + (\tilde{A}_k + m_{B'} A_k) \gamma^\nu] T_{\nu\mu}^{(k)} \\ &\quad + A_6 P_\mu + A_7 k_\mu. \end{aligned} \quad (18)$$

In Eq. (18), $T_{\nu\mu}^{(k)}$ represents the k th element of $T_{\nu\mu}$, with

$$T_{\nu\mu} = \{g_{\nu\mu}, P_\nu P_\mu, k_\nu k_\mu, P_\nu k_\mu, P_\mu k_\nu\}, \quad (19)$$

and A_i and $\tilde{A}_i$ are coefficients given by

$$\begin{aligned} A_i &= \sum_{\text{VBP}} C_{PB \rightarrow B'} C_{V \rightarrow P'P} g_{P_s \rightarrow VB} A_i^{\text{VBP}}, \\ \tilde{A}_i &= \sum_{\text{VBP}} C_{PB \rightarrow B'} C_{V \rightarrow P'P} g_{P_s \rightarrow VB} m_B A_i^{\text{VBP}}, \end{aligned} \quad (20)$$

with $i = 1, 2, \dots, 7$. The A_i^{VBP} coefficients appearing in Eq. (20) depend on the four-momenta of the initial and final particles as well as of $a_j^{(i)}$, and their definition can be found in Appendix A.

In the case of exchanging a vector (V') between the vector and baryon produced in the primary vertex of the diagram in Fig. 2, we find the following amplitude describing the process when considering contributions from the different intermediate VBV' channels:

$$\begin{aligned} -it_{P_s \rightarrow P'B'}^{\text{vector}} &= -i \sum_{\text{VBV}'} t_{P_s \rightarrow P'B'}^{\text{VBV}'} \\ &= -\frac{gG}{\sqrt{2}} \bar{u}_{B'}(p) t_\nu^{(B)}(\mathbf{B}) u_{P_s}^{\nu'}(P), \end{aligned} \quad (21)$$

where

$$\begin{aligned} t_\nu^{(B)}(\mathbf{B}) &= \epsilon_{\mu'\nu'\alpha\beta} k^{\mu'} \left[(B_1 g^{\sigma\alpha'} + B_2 P^\sigma P^{\alpha'} \right. \\ &\quad \left. + B_3 P^{\alpha'} k^\sigma) \gamma_\sigma \gamma^{\beta'} + (B_4 - m_{B'} B_5) P^{\alpha'} \gamma^{\beta'} \right], \\ B_i &= \sum_{\text{VBV}'} g_{P_s \rightarrow VB} C_{V'B \rightarrow B'} C_{V \rightarrow V'P'} B_i^{\text{VV}'} B, \end{aligned} \quad (22)$$

with $i = 1, 2, \dots, 5$. The $B_i^{\text{VV}'} B$ coefficients appearing in Eq. (22) are defined in Appendix A. They depend on coefficients, $b_j^{(i)}$, which can be obtained from the expressions for $a_j^{(i)}$ replacing $\omega_P(\mathbf{q})$ by $\omega_{V'}(\mathbf{q}) = \sqrt{\mathbf{q}^2 + m_{V'}^2}$. The $C_{V'B \rightarrow B'}$, and $C_{V \rightarrow V'P'}$ in Eq. (22) are coefficients obtained from the Lagrangians in Eq. (10), and their values can be found in Appendix B.

Using Eqs. (17) and (21), the sum over the polarizations of the initial and final states for

$$|t_{P_s \rightarrow P'B'}|^2 = |t_{P_s \rightarrow P'B'}^{\text{pseudo}} + t_{P_s \rightarrow P'B'}^{\text{vector}}|^2 \quad (23)$$

can be calculated, obtaining

$$\begin{aligned} \sum_{\text{pol}} |t_{P_s \rightarrow P'B'}|^2 &= \sum_{\text{pol}} |t_{P_s \rightarrow P'B'}^{\text{pseudo}}|^2 + \sum_{\text{pol}} |t_{P_s \rightarrow P'B'}^{\text{vector}}|^2 \\ &\quad + 2\text{Re} \left\{ \sum_{\text{pol}} t_{P_s \rightarrow P'B'}^{\text{pseudo}} (t_{P_s \rightarrow P'B'}^{\text{vector}})^\dagger \right\}, \end{aligned} \quad (24)$$

where

$$\begin{aligned}
\sum_{\text{pol}} |t_{P_s \rightarrow P' B'}^{\text{pseudo}}|^2 &= \frac{|g|^2}{4m_{B'} m_{P_s}} \text{Tr} \left[(\not{p} - m_{B'}) t_\mu^{(A)}(A, \tilde{A}) (\not{p} + m_{P_s}) P^{\mu\sigma} t_\sigma^{(A)}(A^*, \tilde{A}^*) \right] \\
&= \frac{|g|^2}{4m_{B'} m_{P_s}} \sum_{l=0}^5 F^{(l)}(P \cdot k)^l, \\
\sum_{\text{pol}} |t_{P_s \rightarrow P' B'}^{\text{vector}}|^2 &= \frac{|g|^2 |G|^2}{8m_{B'} m_{P_s}} \text{Tr} \left[(\not{p} + m_{B'}) t_{\nu'}^{(B)}(B) (\not{p} + m_{P_s}) P^{\nu'\sigma'} \tilde{t}_{\sigma'}^{(B)}(B^*) \right] \\
&= \frac{|g|^2 |G|^2}{8m_{B'} m_{P_s}} \sum_{l=0}^4 H^{(l)}(P \cdot k)^l, \sum_{\text{pol}} t_{P_s \rightarrow P' B'}^{\text{pseudo}} \left(t_{P_s \rightarrow P' B'}^{\text{vector}} \right)^\dagger \\
&= -\frac{|g|^2 G}{4\sqrt{2} m_{B'} m_{P_s}} \text{Tr} \left[(\not{p} + m_{B'}) \gamma_5 t_\mu^{(A)}(A, \tilde{A}) (\not{p} + m_{P_s}) P^{\mu\nu'} \tilde{t}_{\nu'}^{(B)}(B^*) \right] \\
&= -i \frac{|g|^2 G}{4\sqrt{2} m_{B'} m_{P_s}} \sum_{l=0}^4 J^{(l)}(P \cdot k)^l, \tag{25}
\end{aligned}$$

with

$$\begin{aligned}
\tilde{t}_\sigma^{(B)}(B^*) &= \epsilon_{\mu'\sigma\alpha'\beta'} k^{\mu'} \left[-(B_1^* g^{\sigma\alpha'} + B_2^* P^{\sigma'} P^{\alpha'} \right. \\
&\quad \left. + B_3^* P^{\alpha'} k^{\sigma'} \gamma_\sigma \gamma^{\beta'} + 2B_3^* P^{\alpha'} k^{\beta'} \right. \\
&\quad \left. + (B_4^* - m_{B'} B_5^*) P^{\alpha'} \gamma^{\beta'} \right]. \tag{26}
\end{aligned}$$

As can be seen from the preceding equations, the traces present in Eq. (25) can be written as an expansion of powers of $P \cdot k$, with $F^{(l)}$, $H^{(l)}$ and $J^{(l)}$ being the coefficients for such expansions, and their values are given in Appendix C.

Next, we can also have a resonance, like $\Lambda(1405)$, $N^*(1535)$, and $N^*(1650)$, in the final state of a decaying P_s since the former resonances have sizable couplings to PB [55–60] and VB channels [41,52,53,61,62]. We have considered final states formed by a pseudoscalar and one of these resonances, which we denote as R . As shown in Fig. 2, the decay mechanism of $P_s \rightarrow P'R$ proceeds via triangular loops as well. In this case, the amplitude describing such a process, involving the exchange of pseudoscalar mesons between the hadrons of the primary vertex, can be written as

$$t_{P_s \rightarrow P'R}^{\text{pseudo}} = \sum_{\text{VBP}} t_{P_s \rightarrow P'R}^{\text{VBP}} = g \bar{u}_R t_\mu^{(C)}(C) u_{P_s}^\mu(P), \tag{27}$$

where

$$t_\mu^{(C)}(C) = \sum_{k=1}^7 C_k U_\mu^{(k)}, \tag{28}$$

with $U_\mu^{(k)}$ being the k th element of

$$U_\mu = \{\gamma^\nu g_{\nu\mu}, \not{P} P_\mu, \not{k} k_\mu, \not{P} k_\mu, \not{P} k_\nu, P_\mu, k_\mu\}. \tag{29}$$

In Eq. (28), C_k are coefficients given by

$$C_i = \sum_{\text{VBP}} g_{R \rightarrow PB} g_{P_s \rightarrow VB} C_{V \rightarrow P' P} C_i^{\text{VBP}}, \tag{30}$$

with $i = 1, 2, \dots, 7$, and the definition of C_i^{VBP} , which depend on $a_j^{(i)}$, and the four-momenta of the particles in the initial, intermediate, and final states, can be found in Appendix A.

In the case of exchanging a vector meson between the particles produced in the primary vertex of the triangular loop for the reaction $P_s \rightarrow P'R$, the amplitude can be written, once the contribution from different VBV' channels is included, as

$$t_{P_s \rightarrow P'R}^{\text{vector}} = \frac{1}{\sqrt{6}} G \bar{u}_R(p) t_{\nu'}^{(D)}(D) u_{P_s}^{\nu'}(P), \tag{31}$$

where

$$\begin{aligned}
t_{\nu'}^{(D)}(D) &= \epsilon_{\mu'\nu'\alpha'\beta'} k^{\mu'} \gamma_5 [(D_1 g^{\sigma\alpha'} + D_2 P^{\sigma} P^{\alpha'} \\
&\quad + D_3 P^{\alpha'} k^{\sigma}) \gamma^{\beta'} \gamma_\sigma - D_4 P^{\alpha'} \gamma^{\beta'}], \tag{32}
\end{aligned}$$

with

$$D_i = \sum_{\text{VBV}'} g_{R \rightarrow V'B} g_{P_s \rightarrow VB} C_{V \rightarrow V' P'} D_i^{\text{VBV}'}, \tag{33}$$

where $i = 1, 2, \dots, 4$. We refer the reader to Appendix A for the definition of the coefficients $D_i^{\text{VBV}'}$. We should mention at this point that the coupling constants $g_{R \rightarrow PB}$ and $g_{R \rightarrow V'B}$ can be found, for instance, in Refs. [41,53]. There, $\eta - \eta'$ mixing was not considered, but the coupling constants of R

to the channels ηB ($\eta' B$) can be estimated by multiplying those obtained in Refs. [41,53] by $\cos\beta$ ($\sin\beta$), where the latter factor is the coefficient multiplying the octet component in the wave function of η (η') in terms of the singlet and octet of SU(3) [43].

Considering the amplitudes of Eqs. (27) and (31), we can determine the sum over the polarizations of

$$|t_{P_s \rightarrow P'R}|^2 = |t_{P_s \rightarrow P'R}^{\text{pseudo}} + t_{P_s \rightarrow P'R}^{\text{vector}}|^2, \quad (34)$$

finding

$$\sum_{\text{pol}} |t_{P_s \rightarrow P'R}|^2 = \sum_{\text{pol}} |t_{P_s \rightarrow P'R}^{\text{pseudo}}|^2 + \sum_{\text{pol}} |t_{P_s \rightarrow P'R}^{\text{vector}}|^2 + 2\text{Re}\left\{\sum_{\text{pol}} t_{P_s \rightarrow P'R}^{\text{pseudo}} (t_{P_s \rightarrow P'R}^{\text{vector}})^\dagger\right\}, \quad (35)$$

where

$$\begin{aligned} \sum_{\text{pol}} |t_{P_s \rightarrow P'R}^{\text{pseudo}}|^2 &= \frac{|g|^2}{4m_R m_{P_s}} \text{Tr}\left\{(\not{P} - \not{k} + m_R) t_\mu^{(C)}(C) (\not{P} + m_{P_s}) P^{\mu\sigma} t_\sigma^{(C)}(C^*)\right\} = \frac{|g|^2}{4m_R m_{P_s}} \sum_{l=0}^4 L^{(l)}(P \cdot k)^l, \\ \sum_{\text{pol}} |t_{P_s \rightarrow P'R}^{\text{vector}}|^2 &= \frac{|G|^2}{24m_R m_{P_s}} \text{Tr}\left\{(\not{P} + m_R) t_\nu^{(D)}(D) (\not{P} + m_{P_s}) P^{\nu\bar{\nu}} t_{\bar{\nu}}^{(D)}(D^*)\right\} = \frac{|G|^2}{24m_R m_{P_s}} \sum_{l=0}^4 M^{(l)}(P \cdot k)^l, \\ \sum_{\text{pol}} t_{P_s \rightarrow P'R}^{\text{pseudo}} (t_{P_s \rightarrow P'R}^{\text{vector}})^\dagger &= \frac{gG}{4\sqrt{2}m_R m_{P_s}} \text{Tr}\left\{(\not{P} + m_R) t_\mu^{(C)}(C) (\not{P} + m_{P_s}) P^{\mu\nu'} t_{\nu'}^{(D)}(D^*)\right\} \\ &= i \frac{gG}{4\sqrt{2}m_R m_{P_s}} \sum_{l=0}^4 N^{(l)}(P \cdot k)^l, \end{aligned} \quad (36)$$

with the coefficients $L^{(l)}$, $M^{(l)}$, and $N^{(l)}$ listed in the Appendix C.

With the above amplitudes, the partial decay width of $P_s \rightarrow P'_i B'_i$, or $P_s \rightarrow P'_i R_i$, can be determined from Eq. (6) replacing $\sum_{\text{pol}} |t_{P_s \rightarrow V_i B_i}|^2$ by either $\sum_{\text{pol}} |t_{P_s \rightarrow P'_i B'_i}|^2$ or $\sum_{\text{pol}} |t_{P_s \rightarrow P'_i R_i}|^2$, and m_{V_i} by $m_{P'_i}$, m_{B_i} by m_{R_i} . The unstable character of the vector mesons in the intermediate states has been taken into account replacing $\omega_{V(V')} - i\epsilon$, with $\omega_{V(V')}$ representing their energies, by $\omega_{V(V')} - i\Gamma_{V(V')}/2$, with $\Gamma_{V(V')}$ being their widths. In the case of having a resonance in the final state, its unstable character has been

implemented by convoluting Eq. (6) with the corresponding spectral function for the resonance, i.e.,

$$\begin{aligned} \Gamma_{P_s \rightarrow P'_i R_i} &= \frac{1}{N_{P_s} N_{R_i}} \int_{m_{P_s}-2\Gamma_{P_s}}^{m_{P_s}+2\Gamma_{P_s}} d\tilde{m}_{P_s} \rho_{P_s}(\tilde{m}_{P_s}) \\ &\times \int_{m_{R_i}-2\Gamma_{R_i}}^{m_{R_i}+2\Gamma_{R_i}} d\tilde{m}_{R_i} \rho_{R_i}(\tilde{m}_{R_i}) \Gamma_{P_s \rightarrow P'_i R_i}(\tilde{m}_{P_s}, m_{P'_i}, \tilde{m}_{R_i}), \end{aligned}$$

TABLE II. Partial decay widths (in MeV) of $P_s(2080)$ to final states formed by a pseudoscalar and an octet baryon and a pseudoscalar and $\Lambda(1405)/N^*(1535)/N^*(1650)$. We present the results obtained by considering the triangular loop mechanism of Fig. 2, including only the exchange of pseudoscalars between the vector and baryon produced from the primary vertex (P exch.) and considering the exchange of vector mesons too (P + V exch.). Here $\Lambda_1(1405)$ [$\Lambda_2(1405)$] represents the lower (higher) pole related to $\Lambda(1405)$ [53,56,57].

Channel	Width		Channel	Width	
	P exchange	P + V exchange		P exchange	P + V exchange
$\pi^+ n$	0.77 ± 0.21	0.95 ± 0.27	$K^+ \Lambda_2(1405)$	5.05 ± 0.76	5.10 ± 0.77
$\pi^0 p$	0.38 ± 0.11	0.47 ± 0.13	$\pi^+ N^{*0}(1535)$	1.18 ± 0.28	1.18 ± 0.28
ηp	0.87 ± 0.23	0.77 ± 0.21	$\pi^0 N^{*+}(1535)$	0.59 ± 0.14	0.59 ± 0.14
$K^+ \Lambda$	3.83 ± 0.84	3.74 ± 0.82	$\eta N^{*0}(1535)$	0.33 ± 0.05	0.34 ± 0.06
$K^+ \Sigma^0$	1.56 ± 0.31	1.45 ± 0.29	$\pi^+ N^{*0}(1650)$	0.34 ± 0.03	0.26 ± 0.02
$K^0 \Sigma^+$	3.11 ± 0.62	2.90 ± 0.57	$\pi^0 N^{*+}(1650)$	0.17 ± 0.01	0.13 ± 0.01
$\eta' p$	0.014 ± 0.005	0.07 ± 0.02			
$K^+ \Lambda_1(1405)$	16.97 ± 2.67	17.16 ± 2.71			

with

$$\rho_{R_i}(\tilde{m}_{R_i}) = -\frac{1}{\pi} \text{Im} \left(\frac{1}{\tilde{m}_{R_i} - m_{R_i} + i\Gamma_{R_i}/2} \right),$$

$$N_{R_i} = \int_{m_{R_i}-2\Gamma_{R_i}}^{m_{R_i}+2\Gamma_{R_i}} d\tilde{m}_{R_i} \rho_{R_i}(\tilde{m}_{R_i}). \quad (37)$$

III. RESULTS

In Table II we show the partial decay widths obtained for the processes $P_s(2080) \rightarrow P'B'$, $P'R$ without considering the convolution over the widths of the P_s and of the resonance R in the final state. The central values obtained represent the average values resulting from consideration of different form factors at the vertices, different cutoffs in those form factors as well as different $\eta - \eta'$ mixing angles, while the uncertainty shown in the results of Table II corresponds to the standard deviation obtained. As can be seen, the contribution to the partial decay widths of diagrams in which a vector is exchanged between the vector and baryon produced at the primary vertex of the diagram in Fig. 2 is small, except for the case of the $\eta'p$ final state.

It is interesting to notice that the partial decay width of $P_s(2080)$ to pseudoscalar-baryon channels with hidden strangeness, like $K\Lambda$ ($\Gamma_{K\Lambda} = 3.74 \pm 0.82$ MeV) and $K\Sigma$ ($\Gamma_{K\Sigma} = \Gamma_{K^+\Sigma^0} + \Gamma_{K^0\Sigma^+} = 4.35 \pm 0.86$), is larger than the partial decay width to a channel like πN ($\Gamma_{\pi N} = \Gamma_{\pi^+n} + \Gamma_{\pi^0p} = 1.42 \pm 0.40$ MeV). This result suggests that considering reactions in which $P_s(2080)$ is produced and decays to a final state like $K\Sigma$ and $K\Lambda$ can be more relevant than those involving πN for identifying the generation of $P_s(2080)$. But even more interesting is the fact that the partial decay width of $P_s(2080)$ to a final state formed by $\pi N^*(1535)$, for which $\Gamma_{\pi N^*(1535)} = \Gamma_{\pi^+N^0(1535)} + \Gamma_{\pi^0N^{*+}(1535)} = 1.77 \pm 0.42$ MeV, is also comparable to the previous partial decay widths. There are several studies suggesting that $N^*(1535)$ has a sizable hidden strangeness $K\Sigma$ component in its wave function [41,58,59], producing a partial decay width of $P_s(2080)$ which is similar to that of πN , even if there is more phase space available for the latter channel.

As can be seen in Table II, the decay of $P_s(2080)$ to $K\Lambda_1(1405)$ produces the largest contribution of the final states considered. Here we denote as $\Lambda_1(1405)$ and $\Lambda_2(1405)$ to the lower and upper mass poles, respectively, obtained in Refs. [53,56,57], where a double pole structure is suggested for $\Lambda(1405)$, with the lower (upper) pole having a mass ~ 1380 (1426) MeV and a larger coupling to the $\pi\Sigma$ ($\bar{K}N$) channel. In this way, reactions with a final state like $K\pi\Sigma$, where $\pi\Sigma$ has its origin in the decay of $\Lambda(1405)$, can be very relevant to extract information on the properties of $P_s(2080)$.

TABLE III. Partial decay widths (in MeV) of $P_s(2080)$ to the vector-baryon channels (in the isospin 1/2 basis) used for its generation in Refs. [41,52].

Channel	Width
ρN	5.66
ωN	1.33
ϕN	1.92
$K^*\Lambda$	6.64
$K^*\Sigma$	49.97

Considering all the partial decay widths listed in Table II, we obtain a width of $\sim 35 \pm 6$ MeV, which is to be added to the width of ~ 60 – 70 MeV obtained from vector-baryon channels in Ref. [41]. In this way, within our model, summing up the partial decay widths to all channels, the total width for $P_s(2080)$ turns out to be around 100 MeV. Thus, using as an estimation for the total width of $P_s(2080)$ a value of ~ 100 MeV, we determine the partial decay widths of P_s to the vector-baryon channels investigated in Ref. [41]. We can also estimate the effect of convoluting the partial decay widths of $P_s(2080)$ to $P'B'$ and $P'R$ with the spectral function related to P_s and, for the $P'R$ channels, we can incorporate the finite width of the resonances R in the calculation of the partial decay widths of $P_s(2080)$. The results obtained are similar to those found without implementing such effects, with the exception that when varying the masses of P_s and R , the channel $\eta N^{*+}(1650)$ would be open for decay, finding a very small partial decay width (~ 0.005 MeV).

In Table III we list the partial decay widths of $P_s(2080)$ to the vector-baryon channels considered in Refs. [41,52]. As can be seen, the largest contribution to the width comes from the $K^*\Sigma$ channel, to which P_s couples more strongly, and whose nominal threshold is slightly above the mass of P_s , thus, the convolution here plays a relevant role for the calculation of the decay widths.

It should also be mentioned that the consideration of all the VB channels listed in Table III is necessary when determining the partial decay widths of $P_s \rightarrow P'B'$, $P'R$ via the triangular loop mechanism shown in Fig. 2. For instance, considering only the primary vertex $P_s \rightarrow K^*\Sigma$, in view that the coupling constant of P_s to $K^*\Sigma$ is the largest, drastically reduces the partial decay widths found. For example, to mention a few cases, the partial decay width to πN would be ~ 26 times smaller, to $K\Sigma$ it will be a factor of ~ 2 smaller, and to ηN about 3 times smaller.

IV. PROCESSES USEFUL FOR STUDYING THE PROPERTIES OF P_s

Our study shows that the P_s state couples strongly to $K^*\Sigma$ and $K\Lambda(1405)$ channels. These findings imply that processes with $K\pi\Sigma$ final states would be the most

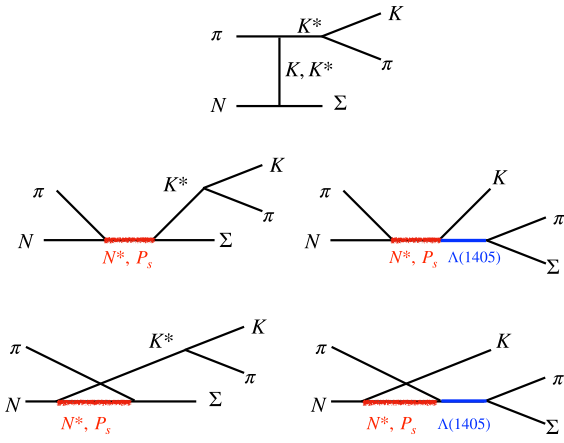


FIG. 3. Different diagrams contributing to the process $\pi^- p \rightarrow K\pi\Sigma$.

appropriate ones to study the properties of P_s . We find it useful to provide concrete examples of processes that are going to be studied, those on which data are already available, and those that can be studied in the existing experimental facilities.

The first encouraging example is a proposal to investigate the $\pi^- p \rightarrow K\pi\Sigma$ process, which is currently being prepared to be submitted for measurements at the J-PARC (Japan Proton Accelerator Complex) facility [63]. Given our findings, besides a K/K^* exchange in the t -channel, an exchange of the P_s state in $\pi^- p \rightarrow K\pi\Sigma$ would proceed through the s - and u -channel diagrams shown in Fig. 3. The results obtained in our present work can be used to describe the vertices $P_s \rightarrow \pi N, K^*\Sigma, K\Lambda(1405)$ in a theoretical analysis on the process. For a complete study, one must include exchange of other possible N^* s following Ref. [45], though the strong couplings of P_s to $K^*\Sigma$ and $K\Lambda(1405)$ found in our present work indicate that the exchange of P_s must dominate. In fact, the results of this work can also be used to study the excitation of P_s in the $\gamma N \rightarrow K\Lambda(1405)$ and such a consideration can improve the description of the experimental data studied in Ref. [45].

With the recent progress in the analysis of data on weak processes, another source of useful information on the properties of P_s can be decays of heavy hadrons like Λ_c . In fact, an attempt to describe the data on $\Lambda_c^+ \rightarrow \pi^0 p \phi$ has

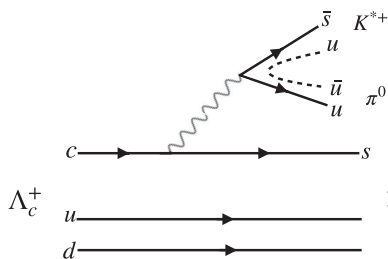


FIG. 4. The weak decay $\Lambda_c^+ \rightarrow \pi^0 K^{*+} \Sigma^0$, where K^{*+} and Σ^0 can interact and produce $P_s(2080)$, which can decay to $K^*\Sigma$.

been made in Ref. [64], and it was shown that interesting effects like triangular singularities can be seen in the data, though the statistics are poor at this moment. Findings of our work can be used to further analyze $\Lambda_c^+ \rightarrow \pi^0 p \phi$. However, even though the phase space would be small, data on $\Lambda_c^+ \rightarrow \pi^0 K^{*+} \Sigma^0$, if measured, could show a much better signal of the P_s state in the $K^{*+} \Sigma^0$ invariant mass spectrum [or in that of $K\pi\Sigma$, where the kaon and the pion would come from the decay of $K^*(892)$]. A possible mechanism for such a weak decay of Λ_c is shown in Fig. 4. We hope that data on such a process can be obtained in the near future. Yet another possibility could be the process $\psi(3686) \rightarrow \bar{p} K^{*+} \Sigma^0$, which is similar to $\psi(3686) \rightarrow \bar{p} K^{*+} \Lambda$ studied by the BESIII Collaboration [65]. The invariant mass spectrum of the $K^{*+} \Lambda$ system shown in Ref. [65] does not show any clear evidence of the presence of the P_s state, which can be understood from the weak coupling of P_s to $K^* \Lambda$ found in our work. However, a similar process, with Σ present in the final state, instead of Λ , should be more useful to find a clear signal of P_s . We encourage experimental groups to determine data on such a process.

V. CONCLUSIONS

The interest in studying the existence of N^* resonances with hidden strangeness and masses around 2000 MeV has grown since the discovery of the P_c states by the LHCb Collaboration. Understanding the existence of lighter partners of these P_c states with hidden strange content is part of the program of several experimental collaborations. However, detecting such states can be challenging due to the existence of several N^* resonances in the same energy region. For this reason, studying the decay properties of these states and proposing nonstandard final states, where the hidden strange quark content of the state could play a major role, is important for a better understanding of the properties of these states. In this work, we have focused our attention on the $J^P = 3/2^-$ $P_s(2080)$ state generated from vector-baryon dynamics in Refs. [41,52] and show that its partial decay widths to channels like $K\Sigma, K\Lambda, \pi N^*(1535)$ are as big as that to πN , with the decay to $K\Lambda(1405)$ giving a larger contribution. In this way, considering final states like $K\pi\Sigma$ could be relevant to understanding the properties of $P_s(2080)$. We have provided a list of processes which can produce $K\pi\Sigma$ in the final state. We hope that our work encourages experimental investigation of such processes.

ACKNOWLEDGMENTS

This study was partly financed by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior—Brasil (CAPES)—Finance Code 001. The partial support from other Brazilian agencies is also gratefully acknowledged: We thank CNPq (K. P. K.: Grants No. 407437/2023-1 and No. 306461/2023-4; A. M. T.: Grant No. 304510/2023-8),

FAPESP (K. P. K.: Grant No. 2022/08347-9; A. M. T.: Grant No. 2023/01182-7). The work of S.-i. N. is supported by a grant from the National Research Foundation of Korea (NRF), funded by the Korean government (MSIT) (NRF-2022K2A9A1A06091761).

DATA AVAILABILITY

The data supporting this study's findings are available within the article.

APPENDIX A: EVALUATION OF THE TENSOR INTEGRALS APPEARING IN THE FORMALISM

To calculate the integrals in Eq. (15), we use the Passarino-Veltman decomposition of tensor integrals [54]: Let us consider, for example, the tensor integral $I_{\nu\mu}^{(1)}$. As a consequence of the Lorentz covariance, we see from Eq. (15) that $I_{\nu\mu}^{(1)}$ can be written as a linear combination of the metric tensor $g_{\nu\mu}$ and combinations of the four-momenta P and k forming a symmetric tensor of rank 2 under the interchange of μ and ν , i.e.,

$$I_{\nu\mu}^{(1)} = a_1^{(1)} g_{\nu\mu} + a_2^{(1)} P_\nu P_\mu + a_3^{(1)} k_\nu k_\mu + a_4^{(1)} (P_\nu k_\mu + P_\mu k_\nu). \quad (\text{A1})$$

The coefficients $a_i^{(1)}$, $i = 1, 2, \dots, 4$, in Eq. (A1) can be determined by contracting the latter equation by the different Lorentz structures present on it, i.e., $g^{\nu\mu}$, $P^\nu P^\mu$, $k^\nu k^\mu$, and $P^\nu k^\mu$. In this way, we can form a system of four coupled equations that allow us to express the coefficients $a_i^{(1)}$ in terms of the integrals

$$\begin{aligned} \mathbb{G}\mathbb{I}^{(1)} &\equiv g^{\nu\mu} I_{\nu\mu}^{(1)} = \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{q^2}{\mathbb{D}}, \\ \mathbb{P}\mathbb{P}\mathbb{I}^{(1)} &\equiv P^\nu P^\mu I_{\nu\mu}^{(1)} = \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{(P \cdot q)^2}{\mathbb{D}}, \\ \mathbb{K}\mathbb{K}\mathbb{I}^{(1)} &\equiv k^\nu k^\mu I_{\nu\mu}^{(1)} = \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{(k \cdot q)^2}{\mathbb{D}}, \\ \mathbb{P}\mathbb{K}\mathbb{I}^{(1)} &\equiv P^\nu k^\mu I_{\nu\mu}^{(1)} = \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{(P \cdot q)(k \cdot q)}{\mathbb{D}}, \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} a_1^{(1)} &= -\frac{1}{2[(P \cdot k)^2 - P^2 k^2]} \left[\mathbb{G}\mathbb{I}^{(1)} \{-(P \cdot k)^2 + P^2 k^2\} + 2\mathbb{P}\mathbb{K}\mathbb{I}^{(1)}(P \cdot k) - \mathbb{P}\mathbb{P}\mathbb{I}^{(1)} k^2 - \mathbb{K}\mathbb{K}\mathbb{I}^{(1)} P^2 \right], \\ a_2^{(1)} &= -\frac{1}{2[(P \cdot k)^2 - P^2 k^2]^2} \left[\mathbb{G}\mathbb{I}^{(1)} k^2 \{-(P \cdot k)^2 + P^2 k^2\} \right. \\ &\quad \left. - \mathbb{K}\mathbb{K}\mathbb{I}^{(1)} \{2(P \cdot k)^2 + P^2 k^2\} + 6\mathbb{P}\mathbb{K}\mathbb{I}^{(1)} k^2 (P \cdot k) - 3\mathbb{P}\mathbb{P}\mathbb{I}^{(1)} k^4 \right], \\ a_3^{(1)} &= -\frac{1}{2[(P \cdot k)^2 - P^2 k^2]^2} \left[\mathbb{G}\mathbb{I}^{(1)} P^2 \{-(P \cdot k)^2 + P^2 k^2\} \right. \\ &\quad \left. - \mathbb{P}\mathbb{P}\mathbb{I}^{(1)} \{2(P \cdot k)^2 + P^2 k^2\} + 6\mathbb{P}\mathbb{K}\mathbb{I}^{(1)} P^2 (P \cdot k) - 3\mathbb{K}\mathbb{K}\mathbb{I}^{(1)} P^4 \right], \\ a_4^{(1)} &= -\frac{1}{2[(P \cdot k)^2 - P^2 k^2]^2} \left[\mathbb{G}\mathbb{I}^{(1)} (P \cdot k) \{(P \cdot k)^2 - P^2 k^2\} \right. \\ &\quad \left. - 2\mathbb{P}\mathbb{K}\mathbb{I}^{(1)} \{2(P \cdot k)^2 + P^2 k^2\} + 3\mathbb{K}\mathbb{K}\mathbb{I}^{(1)} P^2 (P \cdot k) + 3\mathbb{P}\mathbb{P}\mathbb{I}^{(1)} k^2 (P \cdot k) \right]. \end{aligned} \quad (\text{A3})$$

Next, to determine the integrals in Eq. (A2), and find, in this way, the value of $a_i^{(1)}$, we first perform the dq^0 integration by considering Cauchy's theorem, finding

$$\mathcal{I}_n = \int_{-\infty}^{+\infty} \frac{dq^0}{2\pi} \frac{(q^0)^n}{\mathbb{D}} = -i \frac{N_n}{\mathcal{D}}, \quad (\text{A4})$$

where

$$\begin{aligned} \mathcal{D} &= 2\omega_B \omega_V \omega_P (P^0 + \omega_B + \omega_V)(k^0 + \omega_V + \omega_P) \\ &\quad \times [P^0 - k^0 - \omega_B - \omega_P + i\epsilon][k^0 - P^0 - \omega_B - \omega_P + i\epsilon] \\ &\quad \times [P^0 - \omega_B - \omega_V + i\epsilon][k^0 - \omega_V - \omega_P + i\epsilon], \end{aligned} \quad (\text{A5})$$

with

$$\begin{aligned}
\omega_B &= \sqrt{(\mathbf{P} - \mathbf{k} - \mathbf{q})^2 + m_B^2}, \\
\omega_V &= \sqrt{(\mathbf{k} + \mathbf{q})^2 + m_V^2}, \\
\omega_P &= \sqrt{\mathbf{q}^2 + m_P^2}.
\end{aligned} \tag{A6}$$

The numerators N_n in Eq. (A4) for the cases concerned in Eq. (A2) are

$$\begin{aligned}
N_0 &= 2k^0 P^0 \omega_B \omega_P - (k^0)^2 \omega_P \omega_{B+V} + \omega_{P+V} [-(P^0)^2 \omega_B + \omega_{B+P} \omega_{B+V} \omega_{B+V+P}], \\
N_1 &= \omega_P \left(-k^0 (P^0)^2 \omega_B - (k^0)^3 \omega_{B+V} + (k^0)^2 P^0 (2\omega_B + \omega_V) - P^0 \omega_V \omega_{P+V} (2\omega_B + \omega_{P+V}) \right. \\
&\quad \left. + k^0 \omega_{B+V} [\omega_B^2 + \omega_{P+V}^2 + \omega_B (2\omega_P + \omega_V)] \right), \\
N_2 &= \omega_P \left((k^0)^2 \omega_{B+V} [(P^0 - k^0)^2 - \omega_{B+P}^2 - 2\omega_P \omega_V - \omega_V^2] + \omega_V \omega_{P+V} [\omega_B \omega_{B+P} \omega_{B+V} - (P^0)^2 \omega_{B+P+V}] \right. \\
&\quad \left. + 2k^0 P^0 \omega_V [\omega_{P+V}^2 + \omega_B (2\omega_P + \omega_V)] \right), \\
N_3 &= \omega_P \left(-(k^0)^5 \omega_{B+V} + (k^0)^4 P^0 (2\omega_B + 3\omega_V) + (k^0)^2 P^0 \omega_V [(P^0)^2 - \omega_B^2 - 3\omega_{P+V}^2 - 2\omega_B \right. \\
&\quad \times (3\omega_P + \omega_V)] + (k^0)^3 [-(P^0)^2 (\omega_B + 3\omega_V) + \omega_{B+V} (\omega_{B+P}^2 + 2\omega_P \omega_V + \omega_V^2)] + P^0 \omega_V \omega_{P+V} \\
&\quad \times [-(P^0)^2 \omega_{P+V} + \omega_B (2\omega_P \omega_V + \omega_B \omega_{P+V})] + k^0 \omega_V [(P^0)^2 (3\omega_{P+V}^2 + \omega_B (2\omega_P + \omega_V)) \\
&\quad \left. - \omega_B \omega_{B+V} (\omega_B (2\omega_P + \omega_V) + \omega_P (3\omega_P + 2\omega_V))] \right), \\
N_4 &= \omega_P \left((k^0)^6 \omega_{B+V} - 2(k^0)^5 P^0 (\omega_B + 2\omega_V) + 4(k^0)^3 P^0 \omega_V [-(P^0)^2 + \omega_{B+P}^2 + (\omega_B + 2\omega_P) \omega_V + \omega_V^2] \right. \\
&\quad + (k^0)^4 [(P^0)^2 (\omega_B + 6\omega_V) - \omega_{B+V} (\omega_{B+P}^2 + (\omega_B + 2\omega_P) \omega_V + \omega_V^2)] - \omega_V \omega_{P+V} [(P^0)^4 \omega_{P+V} \\
&\quad + \omega_B \omega_{B+P} \omega_{B+V} (\omega_P \omega_V + \omega_B \omega_{P+V}) - (P^0)^2 \omega_B (\omega_P^2 + 3\omega_P \omega_V + \omega_V^2 + 2\omega_B \omega_{P+V})] + 2k^0 P^0 \omega_V \\
&\quad \times [2(P^0)^2 \omega_{P+V}^2 - \omega_B (2\omega_B \omega_{P+V}^2 + \omega_V (2\omega_P + \omega_V)^2)] + (k^0)^2 \omega_V [(P^0)^4 - 2(P^0)^2 (\omega_B^2 + 3\omega_{P+V}^2 + \omega_B (2\omega_P + \omega_V)) \\
&\quad \left. + \omega_B \omega_{B+V} (\omega_B^2 + 6\omega_P^2 + 4\omega_P \omega_V + \omega_V^2 + \omega_B (4\omega_P + \omega_V))] \right), \\
N_5 &= \omega_P \left(-(k^0)^7 \omega_{B+V} + (k^0)^6 P^0 (2\omega_B + 5\omega_V) - \omega_V \omega_{P+V} [(P^0 + \omega_B) (P^0)^4 \omega_{P+V} + \omega_B \omega_{B+P}^2 \omega_{B+V}^2 \omega_{P+V} \right. \\
&\quad - 2(P^0)^3 \omega_B \omega_{P+V} \omega_{B+P+V} + P^0 \omega_B (\omega_B^2 \omega_P (\omega_B + 2\omega_P) + \omega_B (\omega_B + 2\omega_P)^2 \omega_V + 2\omega_{B+P}^2 \omega_V^2) - (P^0)^2 \omega_B \omega_{P+V} \\
&\quad \times (2\omega_B^2 + \omega_P^2 + \omega_V^2 + 2\omega_B \omega_{P+V})] - (k^0)^4 \omega_V [-10(P^0)^3 - (P^0)^2 \omega_B + \omega_B \omega_{B+V}^2 + 5P^0 \\
&\quad \times (2\omega_B^2 + 2\omega_B \omega_{P+V} + \omega_{P+V}^2)] - (k^0)^3 \omega_V [5(P^0)^4 + 2(P^0)^3 \omega_B - 2P^0 \omega_B \omega_{B+V}^2 - 2(P^0)^2 (4\omega_B^2 + 4\omega_B \omega_{P+V} \\
&\quad + 5\omega_{P+V}^2) + \omega_B \omega_{B+V} (3\omega_B^2 + 10\omega_P^2 + 8\omega_P \omega_V + 3\omega_V^2 + \omega_B (8\omega_P + \omega_V))] + (k^0)^5 [-(P^0)^2 (\omega_B + 10\omega_V) \\
&\quad + \omega_{B+V} (\omega_B^2 + \omega_{P+V}^2 + \omega_B (2\omega_P + 3\omega_V))] \\
&\quad + k^0 \omega_V [5(P^0)^4 \omega_{P+V}^2 + 2(P^0)^3 \omega_B \omega_{P+V}^2 - 2P^0 \omega_B \omega_{B+V}^2 \omega_{P+V}^2 - (P^0)^2 \omega_B (4\omega_P^3 + 18\omega_P^2 \omega_V \\
&\quad + 20\omega_P \omega_V^2 + 7\omega_V^3 + 8\omega_B \omega_{P+V}^2) + \omega_B \omega_{B+V} (3\omega_B^2 \omega_{P+V}^2 + \omega_P^2 \omega_V (4\omega_P + 3\omega_V) + \omega_B \omega_P \\
&\quad \times (\omega_P + 2\omega_V) (4\omega_P + 3\omega_V))] + (k^0)^2 \omega_V [(P^0)^5 + (P^0)^4 \omega_B + \omega_B \omega_{B+V}^2 (\omega_{B+P}^2 + \omega_{P+V}^2) \\
&\quad - 2(P^0)^2 \omega_B (\omega_B^2 + \omega_P^2 + \omega_P \omega_V + \omega_V^2 + \omega_B \omega_{P+V}) - 2(P^0)^3 (\omega_B^2 + \omega_B \omega_{P+V} + 5\omega_{P+V}^2) + P^0 \omega_B \\
&\quad \left. \times (\omega_B^3 + 2\omega_B^2 \omega_{P+V} + 2\omega_V (10\omega_P^2 + 11\omega_P \omega_V + 4\omega_V^2) + 2\omega_B (5\omega_P^2 + 12\omega_P \omega_V + 5\omega_V^2))] \right), \tag{A7}
\end{aligned}$$

where

$$\begin{aligned}
\omega_{B+V} &= \omega_B + \omega_V, \\
\omega_{B+P} &= \omega_B + \omega_P, \\
\omega_{P+V} &= \omega_P + \omega_V, \\
\omega_{B+P+V} &= \omega_B + \omega_P + \omega_V.
\end{aligned} \tag{A8}$$

Here we consider the rest frame of the decaying particle to determine the partial decay widths of $P_s(2080)$, thus, $\mathbf{P} = \mathbf{0}$, $P^0 = \sqrt{s} = m_{P_s}$, and

$$k^0 = \frac{s + m_{P'}^2 - m_{B'(R)}^2}{2\sqrt{s}}, \tag{A9}$$

with $m_{P'}$, $m_{B'(R)}$ being, respectively, the masses of the pseudoscalar and baryon (resonance) in the final state.

Using Eq. (A4), we can write the integrals in Eq. (A2) as

$$\begin{aligned}
\mathbb{G}\mathbb{I}^{(1)} &= \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} [\mathcal{I}_2 - \mathcal{I}_0 q^2], \\
\mathbb{P}\mathbb{P}\mathbb{I}^{(1)} &= \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} (P^0)^2 \mathcal{I}_2, \\
\mathbb{K}\mathbb{K}\mathbb{I}^{(1)} &= \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} [(k^0)^2 \mathcal{I}_2 - 2k^0(\mathbf{k} \cdot \mathbf{q}) \mathcal{I}_1 + (\mathbf{k} \cdot \mathbf{q})^2 \mathcal{I}_0], \\
\mathbb{P}\mathbb{K}\mathbb{I}^{(1)} &= \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} [P^0 k^0 \mathcal{I}_2 - P^0(\mathbf{k} \cdot \mathbf{q}) \mathcal{I}_1],
\end{aligned} \tag{A10}$$

and the $d^3 q$ integral is given by

$$\int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} = \int_0^\infty d|q| \int_{-1}^1 d\cos\theta |q|^2 \frac{1}{(2\pi)^2}. \tag{A11}$$

The $d|q|$ integral is regularized with form factors. We consider either Gaussian (F_G), Lorentzian (F_L), or Heaviside (F_H) Θ -function form factors at each vertex, i.e.,

$$\begin{aligned}
F_G(\mathbf{q}) &= e^{-|\mathbf{q}|^2/(2\Lambda^2)}, \\
F_L(\mathbf{q}) &= \frac{\Lambda^2}{\Lambda^2 + |\mathbf{q}|^2}, \\
F_H(\mathbf{q}) &= \Theta(\Lambda - |\mathbf{q}|),
\end{aligned} \tag{A12}$$

where $\Lambda \sim 600\text{--}900$ MeV. To compare results obtained with different form factors, we consider the normalization

$$\int_0^\infty d|q| F_H^2(\mathbf{q}) = \int_0^\infty d|q| F_G^2(\mathbf{q}) = \int_0^\infty d|q| F_L^2(\mathbf{q}), \tag{A13}$$

which implies a different value of Λ for each type of form factor. When considering final states involving a resonance, the cutoff Λ used is for the modulus of the center of mass momentum of the particles forming the resonance, thus, a boost needs to be performed from the rest frame of the decaying particle to the rest frame of the resonance in the final state.

Similarly, we can write

$$\begin{aligned}
\mathbb{I}_{\nu\mu}^{(2)} &= a_1^{(2)} g_{\nu\mu} + a_2^{(2)} P_\nu P_\mu + a_3^{(2)} k_\nu k_\mu \\
&\quad + a_4^{(2)} (P_\nu k_\mu + P_\mu k_\nu),
\end{aligned} \tag{A14}$$

and

$$\mathbb{I}_\nu^{(i)} = a_1^{(i)} P_\nu + a_2^{(i)} k_\nu, \tag{A15}$$

with $i = 3, 4, 5$, where the coefficients $a_i^{(2)}$ can be obtained from Eq. (A3) by changing $\mathbb{G}\mathbb{I}^{(1)}$, $\mathbb{P}\mathbb{P}\mathbb{I}^{(1)}$, $\mathbb{K}\mathbb{K}\mathbb{I}^{(1)}$, and $\mathbb{P}\mathbb{K}\mathbb{I}^{(1)}$ to

$$\begin{aligned}
\mathbb{G}\mathbb{I}^{(2)} &= \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{q^4}{\mathbb{D}} = \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} (\mathcal{I}_4 - 2\mathbf{q}^2 \mathcal{I}_2 + \mathbf{q}^4 \mathcal{I}_0), \\
\mathbb{P}\mathbb{P}\mathbb{I}^{(2)} &= \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{q^2 (P \cdot q)^2}{\mathbb{D}} = \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} (P^0)^2 (\mathcal{I}_4 - \mathbf{q}^2 \mathcal{I}_2), \\
\mathbb{K}\mathbb{K}\mathbb{I}^{(2)} &= \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{q^2 (\mathbf{k} \cdot \mathbf{q})^2}{\mathbb{D}} = \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} [(k^0)^2 \mathcal{I}_4 - 2k^0(\mathbf{k} \cdot \mathbf{q}) \mathcal{I}_3 + ((\mathbf{k} \cdot \mathbf{q})^2 - (k^0)^2 \mathbf{q}^2) \mathcal{I}_2 + 2k^0(\mathbf{k} \cdot \mathbf{q}) \mathbf{q}^2 \mathcal{I}_1 - \mathbf{q}^2 (\mathbf{k} \cdot \mathbf{q})^2 \mathcal{I}_0], \\
\mathbb{P}\mathbb{K}\mathbb{I}^{(2)} &= \int_{-\infty}^{+\infty} \frac{d^4 q}{(2\pi)^4} \frac{q^2 (P \cdot q) (\mathbf{k} \cdot \mathbf{q})}{\mathbb{D}} = \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} P^0 [k^0 \mathcal{I}_4 - \mathbf{k} \cdot \mathbf{q} \mathcal{I}_3 - k^0 \mathbf{q}^2 \mathcal{I}_2 + \mathbf{q}^2 (\mathbf{k} \cdot \mathbf{q}) \mathcal{I}_1],
\end{aligned}$$

and

$$\begin{aligned}
a_1^{(i)} &= -\frac{k^2 \mathbb{P} \mathbb{I}^{(i)} - (P \cdot k) \mathbb{K} \mathbb{I}^{(i)}}{(P \cdot k)^2 - k^2 P^2}, \\
a_2^{(i)} &= \frac{(P \cdot k) \mathbb{P} \mathbb{I}^{(i)} - P^2 \mathbb{K} \mathbb{I}^{(i)}}{(P \cdot k)^2 - k^2 P^2},
\end{aligned} \quad (\text{A16})$$

with $i = 3, 4, 5$.

Once we have determined the coefficients $a_j^{(i)}$, we define the following combinations appearing in Eq. (20):

$$\begin{aligned}
A_1^{\text{VBP}} &= -\left[\left(1 + \frac{k^2}{m_V^2}\right) a_1^{(1)} - \frac{1}{m_V^2} a_1^{(2)} \right], \\
A_2^{\text{VBP}} &= -\left[\left(1 + \frac{k^2}{m_V^2}\right) a_2^{(1)} - \frac{1}{m_V^2} a_2^{(2)} \right], \\
A_3^{\text{VBP}} &= -a_3^{(1)} + a_2^{(3)} - \frac{k^2}{m_V^2} (a_3^{(1)} + a_2^{(3)}) + \frac{1}{m_V^2} (a_3^{(2)} + a_2^{(4)}), \\
A_4^{\text{VBP}} &= -a_4^{(1)} + a_1^{(3)} - \frac{k^2}{m_V^2} (a_4^{(1)} + a_1^{(3)}) + \frac{1}{m_V^2} (a_4^{(2)} + a_1^{(4)}), \\
A_5^{\text{VBP}} &= -\left[\left(1 + \frac{k^2}{m_V^2}\right) a_4^{(1)} - \frac{1}{m_V^2} a_4^{(2)} \right],
\end{aligned} \quad (\text{A17})$$

$$\begin{aligned}
A_6^{\text{VBP}} &= \left(1 + \frac{k^2}{m_V^2}\right) a_1^{(4)} - \frac{1}{m_V^2} a_1^{(5)}, \\
A_7^{\text{VBP}} &= a_2^{(4)} - 4a_1^{(1)} - a_2^{(1)} P^2 - a_3^{(1)} k^2 - 2a_4^{(1)} (P \cdot k) \\
&\quad + \frac{k^2}{m_V^2} (a_2^{(4)} + 4a_1^{(1)} + a_2^{(1)} P^2 + a_3^{(1)} k^2 \\
&\quad + 2a_4^{(1)} (P \cdot k)) - \frac{1}{m_V^2} (a_2^{(5)} + 4a_1^{(2)} \\
&\quad + a_2^{(2)} P^2 + a_3^{(2)} k^2 + 2a_4^{(2)} (P \cdot k)).
\end{aligned} \quad (\text{A18})$$

Next, when dealing with a particular vector-vector-baryon intermediate state, $VV'B$, in the triangular loop, we define the coefficients

$$\begin{aligned}
B_1^{VV'B} &= b_1^{(1)}; & B_2^{VV'B} &= b_2^{(1)}; & B_3^{VV'B} &= b_4^{(1)}; \\
B_4^{VV'B} &= b_1^{(3)} m_B; & B_5^{VV'B} &= b_1^{(3)},
\end{aligned} \quad (\text{A19})$$

which appear in Eq. (22). Here, the coefficients $b_j^{(i)}$ are analogous to $a_j^{(i)}$ but replacing ω_P by $\omega_{V'}$.

In case of Eq. (30), we have the following coefficients:

$$\begin{aligned}
C_l^{\text{VBP}} &= -A_l^{\text{VBP}}, \quad l = 1, 2, \dots, 5 \\
C_6^{\text{VBP}} &= (m_R + m_B) \left[-\left(1 + \frac{k^2}{m_V^2}\right) a_1^{(3)} + \frac{a_1^{(4)}}{m_V^2} \right], \\
C_7^{\text{VBP}} &= (m_R + m_B) \left[\mathbb{I}^8 - a_2^{(3)} - \frac{k^2}{m_V^2} (a_2^{(3)} + \mathbb{I}^{(8)}) \right. \\
&\quad \left. + \frac{1}{m_V^2} (a_2^{(4)} + 4a_1^{(1)} + a_2^{(1)} P^2 + a_3^{(1)} k^2 + 2a_4^{(1)} P \cdot k) \right],
\end{aligned} \quad (\text{A20})$$

where the integral $\mathbb{I}^{(8)}$ is given by

$$\mathbb{I}^{(8)} = \int_{-\infty}^{+\infty} \frac{d^3 q}{(2\pi)^3} \mathcal{I}_0. \quad (\text{A21})$$

To determine Eq. (33), we need the coefficients $D_i^{\text{VBP}'}$, which are given by

$$\begin{aligned}
D_1^{\text{VBP}'} &= b_1^{(1)}; & D_2^{\text{VBP}'} &= b_2^{(1)}, \\
D_3^{\text{VBP}'} &= b_4^{(1)}; & D_4^{\text{VBP}'} &= (m_R + m_B) b_1^{(3)},
\end{aligned} \quad (\text{A22})$$

where $m_B(m_R)$ represents the mass of the baryon (resonance) in the intermediate (final) state.

TABLE IV. Coefficients $C_{V \rightarrow PP'}$, $C_{BP \rightarrow B'}$, $C_{V \rightarrow V' P'}$, and $C_{BV' \rightarrow B'}$ for the final state $K^0 \Sigma^+$. To simplify the notation, we define $C_\beta = \cos \beta$ and $S_\beta = \sin \beta$.

VBP	$C_{V \rightarrow PP'}$	$C_{BP \rightarrow B'}$
$K^{*0} \Sigma^+ \pi^0$	$-1/\sqrt{2}$	$-F/f_\pi$
$K^{*0} \Sigma^+ \eta$	$\sqrt{3}/2 C_\beta$	$(-DC_\beta + \sqrt{2} DS_\beta)/(\sqrt{3} f_\pi)$
$K^{*0} \Sigma^+ \eta'$	$\sqrt{3}/2 S_\beta$	$-D(\sqrt{2} C_\beta + S_\beta)/(\sqrt{3} f_\pi)$
$K^{*0} \Sigma^0 \pi^+$	1	F/f_π
$K^{*0} \Lambda \pi^+$	1	$-D/(\sqrt{3} f_\pi)$
$\rho^0 p \bar{K}^0$	$1/\sqrt{2}$	$(-D + F)/(\sqrt{2} f_\pi)$
$\omega p \bar{K}^0$	$-1/\sqrt{2}$	$(-D + F)/(\sqrt{2} f_\pi)$
$\phi p \bar{K}^0$	1	$(-D + F)/(\sqrt{2} f_\pi)$
VBV'	$C_{V \rightarrow V' P'}$	$C_{BV' \rightarrow B'}$
$K^{*0} \Sigma^+ \rho^0$	$-1/\sqrt{2}$	$\sqrt{2}$
$K^{*0} \Sigma^+ \omega$	$1/\sqrt{2}$	$\sqrt{2}$
$K^{*0} \Sigma^+ \phi$	1	1
$K^{*0} \Sigma^0 \rho^+$	1	$-\sqrt{2}$
$K^{*0} \Lambda \rho^+$	1	0
$\rho^0 p \bar{K}^{*0}$	$-1/\sqrt{2}$	-1
$\omega p \bar{K}^{*0}$	$1/\sqrt{2}$	-1
$\phi p \bar{K}^{*0}$	1	-1

APPENDIX B: COEFFICIENTS $C_{PB \rightarrow B'}, \dots, C_{V \rightarrow V'P'}$

In Tables IV–VIII, we provide the coefficients $C_{PB \rightarrow B'}$, $C_{V \rightarrow P'P}$, $C_{V'B \rightarrow B'}$, $C_{V \rightarrow V'P'}$ needed to evaluate the amplitudes associated with the triangular diagrams shown in Fig. 2 for the different final and intermediate states. In the case of the coefficients related to vector-pseudoscalar-pseudoscalar, baryon-baryon-pseudoscalar, vector-vector-

pseudoscalar vertices, we have considered an $\eta - \eta'$ mixing angle in the range $\beta \simeq -15^\circ$ to -22° [49–51] instead of assuming ideal mixing, which corresponds to an angle β with $\sin \beta = -1/3$, i.e., $\beta \simeq -19.43^\circ$.

It should be noted that for a general mixing angle β , the matrix P related to the pseudoscalar fields in Eq. (11) reads as [66]

$$\mathbb{P} = \begin{pmatrix} A(\beta)\eta + B(\beta)\eta' + \frac{\pi^0}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & A(\beta)\eta + B(\beta)\eta' - \frac{\pi^0}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & C(\beta)\eta + D(\beta)\eta' \end{pmatrix}, \quad (\text{B1})$$

where

TABLE V. Same as in Table IV but for the $K^+\Sigma^0$, $K^+\Lambda$ and $K^+\Lambda(1405)$ final states. In the case of $P_S \rightarrow K^+\Lambda(1405)$, $C_{BP \rightarrow B'} = g_{\Lambda^* \rightarrow PB}$, $C_{BV' \rightarrow B'} = g_{\Lambda^* \rightarrow V'B}$, i.e., the coupling constants of the resonance to the PB and VB channels. Here, by $\Lambda(1405)$, we refer to any of the two poles obtained in Refs. [52,57].

VBP	$C_{V \rightarrow PP'}$	$K^+\Sigma^0/K^+\Lambda/K^+\Lambda(1405)$	$C_{BP \rightarrow B'}$	
			$K^+\Sigma^0$	$K^+\Lambda$
$K^{*0}\Sigma^+\pi^-$		1	F/f_π	$-D/(\sqrt{3}f_\pi)$
$K^{*+}\Sigma^0\pi^0$		$1/\sqrt{2}$	0	$-D/(\sqrt{3}f_\pi)$
$K^{*+}\Sigma^0\eta$		$\sqrt{3/2}C_\beta$	$(-DC_\beta + \sqrt{2}DS_\beta)/(\sqrt{3}f_\pi)$	0
$K^{*+}\Sigma^0\eta'$		$\sqrt{3/2}S_\beta$	$-D(\sqrt{2}C_\beta + S_\beta)/(\sqrt{3}f_\pi)$	0
$K^{*+}\Lambda\pi^0$		$1/\sqrt{2}$	$-D/(\sqrt{3}f_\pi)$	0
$K^{*+}\Lambda\eta$		$\sqrt{3/2}C_\beta$	0	$D(C_\beta + \sqrt{2}S_\beta)/(\sqrt{3}f_\pi)$
$K^{*+}\Lambda\eta'$		$\sqrt{3/2}S_\beta$	0	$D(-\sqrt{2}C_\beta + S_\beta)/(\sqrt{3}f_\pi)$
$\rho^0 p K^-$		$-1/\sqrt{2}$	$(-D + F)/(2f_\pi)$	$(D + 3F)/(2\sqrt{3}f_\pi)$
$\rho^+ n \bar{K}^0$		-1	$(D - F)/(2f_\pi)$	$(D + 3F)/(2\sqrt{3}f_\pi)$
$\omega p K^-$		$-1/\sqrt{2}$	$(-D + F)/(2f_\pi)$	$(D + 3F)/(2\sqrt{3}f_\pi)$
$\phi p K^-$		1	$(-D + F)/(2f_\pi)$	$(D + 3F)/(2\sqrt{3}f_\pi)$

VBV'	$C_{V \rightarrow V'P'}$	$K^+\Sigma^0/K^+\Lambda/K^+\Lambda(1405)$	$C_{BV' \rightarrow B'}$	
			$K^+\Sigma^0$	$K^+\Lambda$
$K^{*0}\Sigma^+\rho^-$		1	$-\sqrt{2}$	0
$K^{*+}\Sigma^0\rho^0$		$1/\sqrt{2}$	0	0
$K^{*+}\Sigma^0\omega$		$1/\sqrt{2}$	$\sqrt{2}$	0
$K^{*+}\Sigma^0\phi$		1	1	0
$K^{*+}\Lambda\rho^0$		$1/\sqrt{2}$	0	0
$K^{*+}\Lambda\omega$		$1/\sqrt{2}$	0	$\sqrt{2}$
$K^{*+}\Lambda\phi$		1	0	1
$\rho^0 p K^{*-}$		$1/\sqrt{2}$	$-1/\sqrt{2}$	$-\sqrt{3/2}$
$\rho^+ n \bar{K}^{*0}$		1	$1/\sqrt{2}$	$-\sqrt{3/2}$
$\omega p K^{*-}$		$1/\sqrt{2}$	$-1/\sqrt{2}$	$-\sqrt{3/2}$
$\phi p K^{*-}$		1	$-1/\sqrt{2}$	$-\sqrt{3/2}$

TABLE VI. Same as in Table IV but for the $\pi^0 p$ and $\pi^0 N^{*+}$ final states. Here N^{*+} represents either $N^{*+}(1535)$ or $N^{*+}(1650)$. In the case of $P_s \rightarrow \pi^0 N^{*+}$, $C_{BP \rightarrow B'} = g_{N^{*+} \rightarrow PB}$, $C_{BV' \rightarrow B'} = g_{N^{*+} \rightarrow V'B}$, i.e., the coupling constants of the resonance to the PB and VB channels.

VBP	$C_{V \rightarrow PP'}$ $\pi^0 p / \pi^0 N^{*+}$	$C_{BP \rightarrow B'}$ $\pi^0 p$
$K^{*0} \Sigma^+ K^0$	$1/\sqrt{2}$	$(-D + F)/(\sqrt{2}f_\pi)$
$K^{*+} \Sigma^0 K^+$	$-1/\sqrt{2}$	$(-D + F)/(2f_\pi)$
$K^{*+} \Lambda K^+$	$-1/\sqrt{2}$	$(D + 3F)/(2\sqrt{3}f_\pi)$
$\rho^0 p \pi^0$	0	$-(D + F)/(2f_\pi)$
$\rho^0 p \eta$	0	$(C_\beta(D - 3F) + 2\sqrt{2}DS_\beta)/(\sqrt{2}f_\pi)$
$\rho^0 p \eta'$	0	$(-2\sqrt{2}DC_\beta + (D - 3F)S_\beta)/(\sqrt{2}f_\pi)$
$\rho^+ n \pi^+$	$-\sqrt{2}$	$-(D + F)/(\sqrt{2}f_\pi)$
$\omega p \pi^0$	0	$-(D + F)/(2f_\pi)$
$\omega p \eta$	0	$(C_\beta(D - 3F) + 2\sqrt{2}DS_\beta)/(\sqrt{2}f_\pi)$
$\omega p \eta'$	0	$(-2\sqrt{2}DC_\beta + (D - 3F)S_\beta)/(\sqrt{2}f_\pi)$
$\phi p \pi^0$	0	$-(D + F)/(2f_\pi)$
$\phi p \eta$	0	$(C_\beta(D - 3F) + 2\sqrt{2}DS_\beta)/(\sqrt{2}f_\pi)$
$\phi p \eta'$	0	$(-2\sqrt{2}DC_\beta + (D - 3F)S_\beta)/(\sqrt{2}f_\pi)$
VBV'	$C_{V \rightarrow V'P'}$ $\pi^0 p / \pi^0 N^{*+}$	$C_{BV' \rightarrow B'}$ $\pi^0 p$
$K^{*0} \Sigma^+ K^{*0}$	$-1/\sqrt{2}$	-1
$K^{*+} \Sigma^0 K^{*+}$	$1/\sqrt{2}$	$-1/\sqrt{2}$
$K^{*+} \Lambda K^{*+}$	$1/\sqrt{2}$	$-\sqrt{3}/2$
$\rho^0 p \rho^0$	0	$1/\sqrt{2}$
$\rho^0 p \omega$	$\sqrt{2}$	$3/\sqrt{2}$
$\rho^0 p \phi$	0	0
$\rho^+ n \rho^+$	0	1
$\omega p \rho^0$	$\sqrt{2}$	$1/\sqrt{2}$
$\omega p \omega$	0	$3/\sqrt{2}$
$\omega p \phi$	0	0
$\phi p \rho^0$	0	$1/\sqrt{2}$
$\phi p \omega$	0	$3/\sqrt{2}$
$\phi p \phi$	0	0

TABLE VII. Same as in Table IV but for the $\pi^+ n$ and $\pi^+ N^{*0}$ final states. Here N^{*0} represents either $N^{*0}(1535)$ or $N^{*0}(1650)$. In the case of $P_s \rightarrow \pi^+ N^{*0}$, $C_{BP \rightarrow B'} = g_{N^{*0} \rightarrow PB}$, $C_{BV' \rightarrow B'} = g_{N^{*0} \rightarrow V'B}$, i.e., the coupling constants of the resonance to the PB and VB channels.

VBP	$C_{V \rightarrow PP'}$ $\pi^+ n / \pi^+ N^{*0}$	$C_{BP \rightarrow B'}$ $\pi^+ n$
$K^{*0} \Sigma^0 K^0$	-1	$(D - F)/(2f_\pi)$
$K^{*+} \Lambda K^0$	-1	$(D + 3F)/(2\sqrt{3}f_\pi)$
$\rho^0 p \pi^-$	$-\sqrt{2}$	$-(D + F)/(\sqrt{2}f_\pi)$
$\rho^+ n \pi^0$	$\sqrt{2}$	$(D + F)/(2f_\pi)$
$\rho^+ n \eta$	0	$(C_\beta(D - 3F) + 2\sqrt{2}DS_\beta)/(\sqrt{2}f_\pi)$
$\rho^+ n \eta'$	0	$(-2\sqrt{2}DC_\beta + (D - 3F)S_\beta)/(\sqrt{2}f_\pi)$
$\omega p \pi^-$	0	$-(D + F)/(\sqrt{2}f_\pi)$
$\phi p \pi^-$	0	$-(D + F)/(\sqrt{2}f_\pi)$
VBV'	$C_{V \rightarrow V'P'}$ $\pi^+ n / \pi^+ N^{*0}$	$C_{BV' \rightarrow B'}$ $\pi^+ n$
$K^{*0} \Sigma^0 K^{*0}$	1	$1/\sqrt{2}$
$K^{*+} \Lambda K^{*0}$	1	$-\sqrt{3}/2$
$\rho^0 p \rho^-$	0	1
$\rho^+ n \rho^0$	0	$-1/\sqrt{2}$
$\rho^+ n \omega$	$\sqrt{2}$	$3/\sqrt{2}$
$\rho^+ n \phi$	0	0
$\omega p \rho^-$	$\sqrt{2}$	1
$\phi p \rho^-$	0	1

APPENDIX C: COEFFICIENTS

$$\mathbf{F}^{(l)}, \mathbf{H}^{(l)}, \mathbf{J}^{(l)}, \dots, \mathbf{N}^{(l)}$$

The coefficients $F^{(l)}$, $H^{(l)}$, $J^{(l)}$, $L^{(l)}$, $M^{(l)}$, and $N^{(l)}$ appearing in Eqs. (25) and (36) can be written in terms of the coefficients A_i , B_i , C_i , and D_i as follows:

$$\begin{aligned}
 A(\beta) &= -\frac{\sin \beta}{\sqrt{3}} + \frac{\cos \beta}{\sqrt{6}}, \\
 B(\beta) &= \frac{\sin \beta}{\sqrt{6}} + \frac{\cos \beta}{\sqrt{3}}, \\
 C(\beta) &= -\frac{\sin \beta}{\sqrt{3}} - \sqrt{\frac{2}{3}} \cos \beta, \\
 D(\beta) &= -\sqrt{\frac{2}{3}} \sin \beta + \frac{\cos \beta}{\sqrt{3}}.
 \end{aligned} \tag{B2}$$

$$\begin{aligned}
 F^{(l)} &= \mathbf{c}_F^{(l)} \cdot \mathbf{v}_F; & H^{(l)} &= \mathbf{c}_H^{(l)} \cdot \mathbf{v}_B; & J^{(l)} &= \mathbf{c}_J^{(l)} \cdot \mathbf{v}_J; \\
 L^{(l)} &= \mathbf{c}_L^{(l)} \cdot \mathbf{v}_L; & M^{(l)} &= \mathbf{c}_M^{(l)} \cdot \mathbf{v}_M; & N^{(l)} &= \mathbf{c}_N^{(l)} \cdot \mathbf{v}_N,
 \end{aligned} \tag{C1}$$

where $\mathbf{v}_F$, $\mathbf{v}_B$, ..., $\mathbf{v}_N$ are vectors whose elements are

TABLE VIII. Same as in Table IV but for the ηp , $\eta' p$, and ηN^{*+} final states. In the case of $P_S \rightarrow \eta N^{*+}$, $C_{BP \rightarrow B'} = g_{N^{*+} \rightarrow PB}$, $C_{BV' \rightarrow B'} = g_{N^{*+} \rightarrow V'B}$, i.e., the coupling constants of the resonance to the PB and VB channels. Here N^{*+} refers to either $N^{*+}(1535)$ or $N^{*+}(1650)$.

VBP	$C_{V \rightarrow PP'}$		$C_{BP \rightarrow B'} \eta p / \eta' p$
	$\eta p / \eta N^{*+}$	$\eta' p$	
$K^{*0} \Sigma^+ K^0$	$-\sqrt{3/2} C_\beta$	$-\sqrt{3/2} S_\beta$	$(-D + F) / (\sqrt{2} f_\pi)$
$K^{*+} \Sigma^0 K^+$	$-\sqrt{3/2} C_\beta$	$-\sqrt{3/2} S_\beta$	$(-D + F) / (2 f_\pi)$
$K^{*+} \Lambda K^+$	$-\sqrt{3/2} C_\beta$	$-\sqrt{3/2} S_\beta$	$(D + 3F) / (2\sqrt{3} f_\pi)$
$\rho^0 p \pi^0$	0	0	$-(D + F) / (2 f_\pi)$
$\rho^0 p \eta$	0	0	$(C_\beta(D - 3F) + 2\sqrt{2} D S_\beta) / (2\sqrt{3} f_\pi)$
$\rho^0 p \eta'$	0	0	$(-2\sqrt{2} D C_\beta + (D - 3F) S_\beta) / (2\sqrt{3} f_\pi)$
$\rho^+ n \pi^+$	0	0	$-(D + F) / (\sqrt{2} f_\pi)$
$\omega p \pi^0$	0	0	$-(D + F) / (2 f_\pi)$
$\omega p \eta$	0	0	$(C_\beta(D - 3F) + 2\sqrt{2} D S_\beta) / (2\sqrt{3} f_\pi)$
$\omega p \eta'$	0	0	$(-2\sqrt{2} D C_\beta + (D - 3F) S_\beta) / (2\sqrt{3} f_\pi)$
$\phi p \pi^0$	0	0	$-(D + F) / (2 f_\pi)$
$\phi p \eta$	0	0	$(C_\beta(D - 3F) + 2\sqrt{2} D S_\beta) / (2\sqrt{3} f_\pi)$
$\phi p \eta'$	0	0	$(-2\sqrt{2} D C_\beta + (D - 3F) S_\beta) / (2\sqrt{3} f_\pi)$

VBV'	$C_{V \rightarrow V' P'}$		$C_{BV' \rightarrow B'} \eta p / \eta' p$
	$\eta p / \eta N^{*+}$	$\eta' p$	
$K^{*0} \Sigma^+ K^{*0}$	$-(C_\beta + 2\sqrt{2} S_\beta) / \sqrt{6}$	$(2\sqrt{2} C_\beta - S_\beta) / \sqrt{6}$	-1
$K^{*+} \Sigma^0 K^{*+}$	$-(C_\beta + 2\sqrt{2} S_\beta) / \sqrt{6}$	$(2\sqrt{2} C_\beta - S_\beta) / \sqrt{6}$	$-1/\sqrt{2}$
$K^{*+} \Lambda K^{*+}$	$-(C_\beta + 2\sqrt{2} S_\beta) / \sqrt{6}$	$(2\sqrt{2} C_\beta - S_\beta) / \sqrt{6}$	$-\sqrt{3/2}$
$\rho^0 p \rho^0$	$(\sqrt{2} C_\beta - 2S_\beta) / \sqrt{3}$	$(2C_\beta + \sqrt{2} S_\beta) / \sqrt{3}$	$1/\sqrt{2}$
$\rho^0 p \omega$	0	0	$3/\sqrt{2}$
$\rho^0 p \phi$	0	0	0
$\rho^+ n \rho^+$	$(\sqrt{2} C_\beta - 2S_\beta) / \sqrt{3}$	$(2C_\beta + \sqrt{2} S_\beta) / \sqrt{3}$	1
$\omega p \rho^0$	0	0	$1/\sqrt{2}$
$\omega p \omega$	$(\sqrt{2} C_\beta - 2S_\beta) / \sqrt{3}$	$(2C_\beta + \sqrt{2} S_\beta) / \sqrt{3}$	$3/\sqrt{2}$
$\omega p \phi$	0	0	0
$\phi p \rho^0$	0	0	$1/\sqrt{2}$
$\phi p \omega$	0	0	$3/\sqrt{2}$
$\phi p \phi$	$-2(\sqrt{2} C_\beta + S_\beta) / \sqrt{3}$	$2(C_\beta - \sqrt{2} S_\beta) / \sqrt{3}$	0

$$\mathbf{v}_F = \{|A_1|^2, 2\text{Re}(A_1 A_3^*), 2\text{Re}(A_1 A_4^*), 2\text{Re}(A_1 A_7^*), 2\text{Re}(A_1 \tilde{A}_3^*), 2\text{Re}(A_1 \tilde{A}_4^*), |A_3|^2, 2\text{Re}(A_3 A_4^*), \\ 2\text{Re}(A_3 A_7^*), 2\text{Re}(A_3 \tilde{A}_3^*), 2\text{Re}(A_3 \tilde{A}_4^*), |A_4|^2, 2\text{Re}(A_4 A_7^*), 2\text{Re}(A_4 \tilde{A}_3^*), 2\text{Re}(A_4 \tilde{A}_4^*), |A_7|^2, \\ 2\text{Re}(A_7 \tilde{A}_3^*), 2\text{Re}(A_7 \tilde{A}_4^*), |\tilde{A}_3|^2, 2\text{Re}(\tilde{A}_3 \tilde{A}_4^*), |\tilde{A}_4|^2\},$$

$$\mathbf{v}_H = \{|B_1|^2, 2\text{Re}(B_1 B_2^*), 2\text{Re}(B_1 B_3^*), 2\text{Re}(B_1 B_4^*), 2\text{Re}(B_1 B_5^*), |B_2|^2, 2\text{Re}(B_2 B_3^*), 2\text{Re}(B_2 B_4^*), \\ 2\text{Re}(B_2 B_5^*), |B_3|^2, 2\text{Re}(B_3 B_4^*), 2\text{Re}(B_3 B_5^*), |B_4|^2, 2\text{Re}(B_4 B_5^*), |B_5|^2\},$$

$$\mathbf{v}_J = \{A_1 B_1^*, A_1 B_2^*, A_1 B_3^*, A_1 B_4^*, A_1 B_5^*, A_3 B_1^*, A_3 B_2^*, A_3 B_3^*, A_3 B_4^*, A_3 B_5^*, A_4 B_1^*, A_4 B_2^*, A_4 B_3^*, A_4 B_4^*, A_4 B_5^*, A_7 B_1^*, A_7 B_2^*, \\ A_7 B_3^*, A_7 B_4^*, A_7 B_5^*, \tilde{A}_3 B_1^*, \tilde{A}_3 B_2^*, \tilde{A}_3 B_3^*, \tilde{A}_3 B_4^*, \tilde{A}_3 B_5^*, \tilde{A}_4 B_1^*, \tilde{A}_4 B_2^*, \tilde{A}_4 B_3^*, \tilde{A}_4 B_4^*, \tilde{A}_4 B_5^*\},$$

$$\mathbf{v}_L = \{|C_3|^2, 2\text{Re}(C_3 C_4^*), 2\text{Re}(C_3 C_7^*), |C_4|^2, 2\text{Re}(C_4 C_7^*), |C_7|^2\},$$

$$\mathbf{v}_M = \{|D_1|^2, 2\text{Re}(D_1 D_2^*), 2\text{Re}(D_1 D_3^*), 2\text{Re}(D_1 D_4^*), |D_2|^2, 2\text{Re}(D_2 D_3^*), 2\text{Re}(D_2 D_4^*), |D_3|^2, 2\text{Re}(D_3 D_4^*), |D_4|^2\},$$

$$\mathbf{v}_N = \{C_3 D_1^*, C_3 D_2^*, C_3 D_3^*, C_3 D_4^*, C_4 D_1^*, C_4 D_2^*, C_4 D_3^*, C_4 D_4^*, C_7 D_1^*, C_7 D_2^*, C_7 D_3^*, C_7 D_4^*\}.$$

In the following, we give some of the elements of the vectors $\mathbf{c}_F^{(l)}$, $\mathbf{c}_H^{(l)}$, ..., $\mathbf{c}_N^{(l)}$:

$$\begin{aligned}
\mathbf{c}_F^{(0)} &= \left\{ -\frac{32}{3} m_{P_s} (m_{P_s} - m_{B'}) m_{P'}^2, \frac{16}{3} (m_{B'} - 2m_{P_s}) m_{P_s} m_{P'}^4, -\frac{16}{3} m_{P_s}^2 (m_{B'}^2 + m_{P_s} m_{B'} - 2m_{P_s}^2) m_{P'}^2, \dots \right\}, \\
\mathbf{c}_F^{(1)} &= \left\{ \frac{32}{3} m_{P'}^2, -\frac{16}{3} m_{P'}^2 [m_{B'}^2 + m_{P_s} m_{B'} - 2(m_{P_s}^2 + m_{P'}^2)], \frac{16}{3} (m_{B'} - 4m_{P_s}) m_{P_s} m_{P'}^2, \dots \right\}, \\
\mathbf{c}_F^{(2)} &= \left\{ -\frac{32(m_{B'} - m_{P_s})}{3m_{P_s}}, -\frac{16m_{B'} m_{P'}^2}{3m_{P_s}}, \frac{16}{3} (m_{B'}^2 + m_{P_s} m_{B'} - 2m_{P_s}^2 + 2m_{P'}^2), \dots \right\}, \\
\mathbf{c}_F^{(3)} &= \left\{ -\frac{32}{3m_{P_s}^2}, \frac{16[m_{B'}^2 + m_{P_s} m_{B'} - 2(m_{P_s}^2 + m_{P'}^2)]}{3m_{P_s}^2}, -\frac{16(m_{B'} - 4m_{P_s})}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_F^{(4)} &= \left\{ 0, \frac{32}{3m_{P_s}^2}, -\frac{32}{3m_{P_s}^2}, \dots \right\}, \\
\mathbf{c}_F^{(5)} &= \left\{ 0, 0, 0, 0, 0, 0, -\frac{32}{3m_{P_s}^2}, \frac{32}{3m_{P_s}^2}, \dots \right\}, \\
\mathbf{c}_H^{(0)} &= \left\{ \frac{32}{3} m_{P_s} m_{P'}^2 (m_{B'} - m_{P_s}), \frac{16}{3} m_{P_s}^3 m_{P'}^2 (m_{B'} - m_{P_s}), \frac{16}{3} m_{P_s}^2 m_{P'}^4, \dots \right\}, \\
\mathbf{c}_H^{(1)} &= \left\{ \frac{32m_{P'}^2}{3}, \frac{16m_{P_s}^2 m_{P'}^2}{3}, \frac{16m_{P_s} m_{P'}^2 (m_{B'} - m_{P_s})}{3}, \dots \right\}, \\
\mathbf{c}_H^{(2)} &= \left\{ -\frac{32(m_{B'} - m_{P_s})}{3m_{P_s}}, -\frac{16m_{P_s} (m_{B'} - m_{P_s})}{3}, -\frac{16m_{P'}^2}{3}, \dots \right\}, \\
\mathbf{c}_H^{(3)} &= \left\{ -\frac{32}{3m_{P_s}^2}, -\frac{16}{3}, -\frac{16(m_{B'} - m_{P_s})}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_H^{(4)} &= \left\{ 0, 0, 0, 0, 0, 0, 0, 0, \frac{16}{3}, \dots \right\}, \\
\mathbf{c}_J^{(0)} &= \left\{ -\frac{32m_{P_s} m_{P'}^2 (m_{P_s} - m_{B'})}{3}, \frac{16}{3} m_{P_s}^3 m_{P'}^2 (m_{B'} - m_{P_s}), \frac{16}{3} m_{P_s}^2 m_{P'}^4, \dots \right\}, \\
\mathbf{c}_J^{(1)} &= \left\{ \frac{32m_{P'}^2}{3}, \frac{16m_{P_s}^2 m_{P'}^2}{3}, -\frac{16}{3} m_{P_s} m_{P'}^2 (m_{P_s} - m_{B'}), \dots \right\}, \\
\mathbf{c}_J^{(2)} &= \left\{ -\frac{32(m_{B'} - m_{P_s})}{3m_{P_s}}, -\frac{16}{3} m_{P_s} (m_{B'} - m_{P_s}), -\frac{16}{3} m_{P'}^2, \dots \right\}, \\
\mathbf{c}_J^{(3)} &= \left\{ -\frac{32}{3m_{P_s}^2}, -\frac{16}{3}, -\frac{16(m_{B'} - m_{P_s})}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_J^{(4)} &= \left\{ 0, 0, 0, 0, 0, \frac{32}{3m_{P_s}^2}, \frac{16}{3}, \dots \right\}, \\
\mathbf{c}_L^{(0)} &= \left\{ \frac{8m_{P_s} m_{P'}^4 (m_{P_s} - m_R)}{3}, \frac{8m_{P_s}^2 m_{P'}^4}{3}, \frac{8m_{P_s} m_{P'}^4}{3}, \dots \right\},
\end{aligned}$$

$$\begin{aligned}
\mathbf{c}_L^{(1)} &= \left\{ \frac{8m_{P'}^4}{3}, -\frac{8m_{P_s}m_{P'}^2(m_{P_s}+m_R)}{3}, -\frac{8}{3}m_{P'}^2(m_{P_s}+m_R), \dots \right\}, \\
\mathbf{c}_L^{(2)} &= \left\{ \frac{8}{3}\left(\frac{m_R}{m_{P_s}}-3\right)m_{P'}^2, -\frac{8m_{P'}^2}{3}, -\frac{8m_{P'}^2}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_L^{(3)} &= \left\{ -\frac{8m_{P'}^2}{3m_{P_s}^2}, \frac{8(m_{P_s}+m_R)}{3m_{P_s}}, \frac{8(m_{P_s}+m_R)}{3m_{P_s}^2}, \dots \right\}, \\
\mathbf{c}_L^{(4)} &= \left\{ \frac{16}{3m_{P_s}^2}, 0, 0, \dots \right\}, \\
\mathbf{c}_M^{(0)} &= \left\{ -\frac{32m_{P_s}(m_{P_s}+m_R)m_{P'}^2}{3}, -\frac{16m_{P_s}^3m_{P'}^2(m_{P_s}+m_R)}{3}, \frac{16m_{P_s}^2m_{P'}^4}{3}, \dots \right\}, \\
\mathbf{c}_M^{(1)} &= \left\{ \frac{32m_{P'}^2}{3}, \frac{16m_{P_s}^2m_{P'}^2}{3}, -\frac{16m_{P_s}m_{P'}^2(m_{P_s}+m_R)}{3}, \dots \right\}, \\
\mathbf{c}_M^{(2)} &= \left\{ \frac{32(m_{P_s}+m_R)}{3m_{P_s}}, \frac{16m_{P_s}(m_{P_s}+m_R)}{3}, -\frac{16m_{P'}^2}{3}, \dots \right\}, \\
\mathbf{c}_M^{(3)} &= \left\{ -\frac{32}{3m_{P_s}^2}, -\frac{16}{3}, \frac{16(m_{P_s}+m_R)}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_M^{(4)} &= \left\{ 0, 0, 0, 0, 0, 0, \frac{16}{3}, \dots \right\}, \\
\mathbf{c}_N^{(0)} &= \left\{ \frac{16m_{P'}^4m_{P_s}}{3}, \frac{8m_{P'}^4m_{P_s}^3}{3}, \frac{8m_{P'}^4m_{P_s}^2(m_{P_s}-m_R)}{3}, \dots \right\}, \\
\mathbf{c}_N^{(1)} &= \left\{ -\frac{16m_{P'}^2(m_{P_s}+m_R)}{3}, -\frac{8m_{P'}^2m_{P_s}^2(m_{P_s}+m_R)}{3}, \frac{8m_{P'}^4m_{P_s}}{3}, \dots \right\}, \\
\mathbf{c}_N^{(2)} &= \left\{ -\frac{16m_{P'}^2}{3m_{P_s}}, -\frac{8m_{P'}^2m_{P_s}}{3}, -\frac{8m_{P'}^2(3m_{P_s}-m_R)}{3}, \dots \right\}, \\
\mathbf{c}_N^{(3)} &= \left\{ \frac{16(m_{P_s}+m_R)}{3m_{P_s}^2}, \frac{8(m_{P_s}+m_R)}{3}, -\frac{8m_{P'}^2}{3m_{P_s}}, \dots \right\}, \\
\mathbf{c}_N^{(4)} &= \left\{ 0, 0, \frac{16}{3m_{P_s}}, 0, \dots \right\}.
\end{aligned} \tag{C2}$$

-
- [1] Roel Aaij *et al.*, Observation of $J/\psi p$ resonances consistent with pentaquark states in $\Lambda_b^0 \rightarrow J/\psi K^- p$ decays, *Phys. Rev. Lett.* **115**, 072001 (2015).
- [2] Roel Aaij *et al.*, Model-independent evidence for $J/\psi p$ contributions to $\Lambda_b^0 \rightarrow J/\psi p K^-$ decays, *Phys. Rev. Lett.* **117**, 082002 (2016).
- [3] Roel Aaij *et al.*, Observation of a narrow pentaquark state, $P_c(4312)^+$, and of two-peak structure of the $P_c(4450)^+$, *Phys. Rev. Lett.* **122**, 222001 (2019).
- [4] L. Maiani, A.D. Polosa, and V. Riquer, The new pentaquarks in the diquark model, *Phys. Lett. B* **749**, 289 (2015).
- [5] L. Roca, J. Nieves, and E. Oset, LHCb pentaquark as a $\bar{D}^*\Sigma_c - \bar{D}^*\Sigma_c^*$ molecular state, *Phys. Rev. D* **92**, 094003 (2015).
- [6] Feng-Kun Guo, Ulf-G. Meißner, Wei Wang, and Zhi Yang, How to reveal the exotic nature of the $P_c(4450)$, *Phys. Rev. D* **92**, 071502 (2015).

- [7] Xiao-Hai Liu, Qian Wang, and Qiang Zhao, Understanding the newly observed heavy pentaquark candidates, *Phys. Lett. B* **757**, 231 (2016).
- [8] Xin-Zhen Weng, Xiao-Lin Chen, Wei-Zhen Deng, and Shi-Lin Zhu, Hidden-charm pentaquarks and P_c states, *Phys. Rev. D* **100**, 016014 (2019).
- [9] Ming-Zhu Liu, Ya-Wen Pan, Fang-Zheng Peng, Mario Sánchez Sánchez, Li-Sheng Geng, Atsushi Hosaka, and Manuel Pavon Valderrama, Emergence of a complete heavy-quark spin symmetry multiplet: Seven molecular pentaquarks in light of the latest LHCb analysis, *Phys. Rev. Lett.* **122**, 242001 (2019).
- [10] T. J. Burns and E. S. Swanson, Molecular interpretation of the $P_c(4440)$ and $P_c(4457)$ states, *Phys. Rev. D* **100**, 114033 (2019).
- [11] Jun He, Study of $P_c(4457)$, $P_c(4440)$, and $P_c(4312)$ in a quasipotential Bethe-Salpeter equation approach, *Eur. Phys. J. C* **79**, 393 (2019).
- [12] Cheng-Jian Xiao, Yin Huang, Yu-Bing Dong, Li-Sheng Geng, and Dian-Yong Chen, Exploring the molecular scenario of $P_c(4312)$, $P_c(4440)$, and $P_c(4457)$, *Phys. Rev. D* **100**, 014022 (2019).
- [13] Zhi-Hui Guo and J. A. Oller, Anatomy of the newly observed hidden-charm pentaquark states: $P_c(4312)$, $P_c(4440)$ and $P_c(4457)$, *Phys. Lett. B* **793**, 144 (2019).
- [14] C. Fernández-Ramírez, A. Pilloni, M. Albaladejo, A. Jackura, V. Mathieu, M. Mikhasenko, J. A. Silva-Castro, and A. P. Szczepaniak, Interpretation of the LHCb $P_c(4312)^+$ signal, *Phys. Rev. Lett.* **123**, 092001 (2019).
- [15] Samson Clymton, Hyun-Chul Kim, and Terry Mart, Production mechanism of the hidden charm pentaquark states P_{cc} , *Phys. Rev. D* **110**, 094014 (2024).
- [16] Jia-Jun Wu, R. Molina, E. Oset, and B. S. Zou, Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV, *Phys. Rev. Lett.* **105**, 232001 (2010).
- [17] Jun He, Nucleon resonances $N(1875)$ and $N(2100)$ as strange partners of LHCb pentaquarks, *Phys. Rev. D* **95**, 074031 (2017).
- [18] Yong-Hui Lin, Chao-Wei Shen, and Bing-Song Zou, Decay behavior of the strange and beauty partners of P_c hadronic molecules, *Nucl. Phys. A* **980**, 21 (2018).
- [19] Steven Weinberg, Evidence that the deuteron is not an elementary particle, *Phys. Rev.* **137**, B672 (1965).
- [20] A. Ramos and E. Oset, The role of vector-baryon channels and resonances in the $\gamma p \rightarrow K^0 \Sigma^+$ and $\gamma n \rightarrow K^0 \Sigma^0$ reactions near the $K^* \Lambda$ threshold, *Phys. Lett. B* **727**, 287 (2013).
- [21] Alvin Kiswandhi, Ju-Jun Xie, and Shin Nan Yang, Is the nonmonotonic behavior in the cross section of phi photoproduction near threshold a signature of a resonance?, *Phys. Lett. B* **691**, 214 (2010).
- [22] Ju-Jun Xie and J. Nieves, The role of the $N^*(2080)$ resonance in the $\bar{\gamma} p \rightarrow K^+ \Lambda(1520)$ reaction, *Phys. Rev. C* **82**, 045205 (2010).
- [23] Ju-Jun Xie and Bo-Chao Liu, Role of the $N^*(2080)$ in $pp \rightarrow p K^+ \Lambda(1520)$ and $\pi^- p \rightarrow K^0 \Lambda(1520)$ reactions, *Phys. Rev. C* **87**, 045210 (2013).
- [24] Alvin Kiswandhi, Shin Nan Yang, and Yu Bing Dong, Near-threshold incoherent ϕ photoproduction on the deuteron: Searching for traces of a resonance, *Phys. Rev. C* **94**, 015202 (2016).
- [25] Haiyan Gao, Hongxia Huang, Tianbo Liu, Jialun Ping, Fan Wang, and Zhiwen Zhao, Search for a hidden strange baryon-meson bound state from ϕ production in a nuclear medium, *Phys. Rev. C* **95**, 055202 (2017).
- [26] T. Mibe *et al.*, Diffractive phi-meson photoproduction on proton near threshold, *Phys. Rev. Lett.* **95**, 182001 (2005).
- [27] B. Dey, C. A. Meyer, M. Bellis, and M. Williams, Data analysis techniques, differential cross sections, and spin density matrix elements for the reaction $\gamma p \rightarrow \phi p$, *Phys. Rev. C* **89**, 055208 (2014); **90**, 019901(A) (2014).
- [28] Seung-Il Nam, Studies on the $K^* \Sigma$ bound-state via $K^+ p \rightarrow K^+ \phi p$, *Phys. Rev. D* **103**, 054040 (2021).
- [29] Shu-Ming Wu, Fei Wang, and Bing-Song Zou, Strange molecular partners of P_c states in the $\gamma p \rightarrow \phi p$ reaction, *Phys. Rev. C* **108**, 045201 (2023).
- [30] Xiao-Yun Wang, Hui-Fang Zhou, and Xiang Liu, Prospects for detecting the hidden-strange pentaquarklike state $N^*(2080)$ in the $\pi^- p \rightarrow \phi n$ reaction, *Phys. Rev. D* **110**, 014026 (2024).
- [31] Di Ben, Ai-Chao Wang, Fei Huang, and Bing-Song Zou, Effects of $N(2080)3/2^-$ and $N(2270)3/2^-$ molecules on $K^* \Sigma$ photoproduction, *Phys. Rev. C* **108**, 065201 (2023).
- [32] S. Navas *et al.*, Review of particle physics, *Phys. Rev. D* **110**, 030001 (2024).
- [33] T. Mart and C. Bennhold, Evidence for a missing nucleon resonance in kaon photoproduction, *Phys. Rev. C* **61**, 012201 (2000).
- [34] C. Bennhold, H. Haberzettl, and T. Mart, A new resonance in K^+ lambda electroproduction: The $D(13)(1895)$ and its electromagnetic form-factors, in *Proceedings of the 2nd ICTP International Conference on Perspectives in Hadronic Physics* (World Scientific, Trieste, Italy, 1999), 9, pp. 328–337.
- [35] A. V. Anisovich, R. Beck, E. Klempt, V. A. Nikonov, A. V. Sarantsev, and U. Thoma, Properties of baryon resonances from a multichannel partial wave analysis, *Eur. Phys. J. A* **48**, 15 (2012).
- [36] T. Mart and M. J. Kholili, Origin of the second peak in the cross section of the $K^+ \Lambda$ photoproduction, *Phys. Rev. C* **86**, 022201 (2012).
- [37] T. Mart, Electromagnetic production of kaon in all isospin channels: Summary of the progress and application, *Few Body Syst.* **62**, 31 (2021).
- [38] T. Mart, S. Clymton, and A. J. Arifi, Nucleon resonances with spin 3/2 and 5/2 in the isobar model for kaon photoproduction, *Phys. Rev. D* **92**, 094019 (2015).
- [39] S. Clymton and T. Mart, Extracting the pole and Breit-Wigner properties of nucleon and Δ resonances from the $\gamma N \rightarrow K \Sigma$ photoproduction, *Phys. Rev. D* **104**, 056015 (2021).
- [40] K. P. Khemchandani, H. Kaneko, H. Nagahiro, and A. Hosaka, Vector meson-Baryon dynamics and generation of resonances, *Phys. Rev. D* **83**, 114041 (2011).
- [41] K. P. Khemchandani, A. Martinez Torres, H. Nagahiro, and A. Hosaka, Role of vector and pseudoscalar mesons in understanding $1/2^- N^*$ and Δ resonances, *Phys. Rev. D* **88**, 114016 (2013).
- [42] Masako Bando, Taichiro Kugo, and Koichi Yamawaki, Nonlinear realization and hidden local symmetries, *Phys. Rep.* **164**, 217 (1988).

- [43] E. Oset and A. Ramos, Dynamically generated resonances from the vector octet-baryon octet interaction, *Eur. Phys. J. A* **44**, 445 (2010).
- [44] Thomas Jude *et al.*, The BGOOD experiment at ELSA, exotic structures in the light quark sector?, *EPJ Web Conf.* **291**, 01004 (2024).
- [45] Sang-Ho Kim, K. P. Khemchandani, A. Martínez Torres, Seung-il Nam, and Atsushi Hosaka, Photoproduction of Λ^* and Σ^* resonances with $J^P = 1/2^-$ off the proton, *Phys. Rev. D* **103**, 114017 (2021).
- [46] William Rarita and Julian Schwinger, On a theory of particles with half integral spin, *Phys. Rev.* **60**, 61 (1941).
- [47] G. Ecker, Chiral perturbation theory, *Prog. Part. Nucl. Phys.* **35**, 1 (1995).
- [48] V. Bernard, Norbert Kaiser, and Ulf-G. Meissner, Chiral dynamics in nucleons and nuclei, *Int. J. Mod. Phys. E* **04**, 193 (1995).
- [49] Frederick J. Gilman and Russel Kauffman, The η — η' mixing angle, *Phys. Rev. D* **36**, 2761 (1987); **37**, 3348 (E) (1988).
- [50] R. Akhoury and J. M. Frere, η , η' mixing and anomalies, *Phys. Lett. B* **220**, 258 (1989).
- [51] A. Bramon, R. Escribano, and M. D. Scadron, Mixing of η — η' mesons in $J\psi$ decays into a vector and a pseudoscalar meson, *Phys. Lett. B* **403**, 339 (1997).
- [52] K. P. Khemchandani, A. Martínez Torres, H. Kaneko, H. Nagahiro, and A. Hosaka, Coupling vector and pseudoscalar mesons to study baryon resonances, *Phys. Rev. D* **84**, 094018 (2011).
- [53] K. P. Khemchandani, A. Martínez Torres, and J. A. Oller, Hyperon resonances coupled to pseudoscalar- and vector-baryon channels, *Phys. Rev. C* **100**, 015208 (2019).
- [54] G. Passarino and M. J. G. Veltman, One loop corrections for e^+e^- annihilation into $\mu^+\mu^-$ in the Weinberg model, *Nucl. Phys. B* **160**, 151 (1979).
- [55] Norbert Kaiser, P. B. Siegel, and W. Weise, Chiral dynamics and the low-energy kaon—nucleon interaction, *Nucl. Phys. A* **594**, 325 (1995).
- [56] E. Oset and A. Ramos, Nonperturbative chiral approach to S wave anti-K N interactions, *Nucl. Phys. A* **635**, 99 (1998).
- [57] D. Jido, J. A. Oller, E. Oset, A. Ramos, and U. G. Meissner, Chiral dynamics of the two $\Lambda(1405)$ states, *Nucl. Phys. A* **725**, 181 (2003).
- [58] T. Inoue, E. Oset, and M. J. Vicente Vacas, Chiral unitary approach to S wave meson baryon scattering in the strangeness $S = 0$ sector, *Phys. Rev. C* **65**, 035204 (2002).
- [59] J. Nieves and E. Ruiz Arriola, The $S_{11} - N(1535)$ and $-N(1650)$ resonances in meson baryon unitarized coupled channel chiral perturbation theory, *Phys. Rev. D* **64**, 116008 (2001).
- [60] Peter C. Bruns, Maxim Mai, and Ulf G. Meissner, Chiral dynamics of the $S_{11}(1535)$ and $S_{11}(1650)$ resonances revisited, *Phys. Lett. B* **697**, 254 (2011).
- [61] E. J. Garzon and E. Oset, Effects of pseudoscalar-baryon channels in the dynamically generated vector-baryon resonances, *Eur. Phys. J. A* **48**, 5 (2012).
- [62] E. J. Garzon and E. Oset, Mixing of pseudoscalar-baryon and vector-baryon in the $J^P = 1/2^-$ sector and the $N^*(1535)$ and $N^*(1650)$ resonances, *Phys. Rev. C* **91**, 025201 (2015).
- [63] Takatsugu Ishikawa (private communication).
- [64] Ju-Jun Xie, Li-Sheng Geng, and Feng-Kun Guo, Role of $N^*(1535)$ in the $\Lambda_c^+ \rightarrow K^0 \eta p$ decay and the possible ϕp state in the $\Lambda_c^+ \rightarrow \pi^0 \phi p$ decay, *EPJ Web Conf.* **241**, 02010 (2020).
- [65] M. Ablikim *et al.*, Study of the decays $\psi(3686) \rightarrow \gamma \chi_{cJ} \rightarrow \gamma \bar{p} K^{*+} \Lambda + \text{c.c.}$ and $\psi(3686) \rightarrow \bar{p} K^{*+} \Lambda + \text{c.c.}$, *Phys. Rev. D* **100**, 052010 (2019).
- [66] Brenda B. Malabarba, K. P. Khemchandani, and A. Martínez Torres, $\phi(2170)$ decaying to $\phi\eta$ and $\phi\eta'$, *Phys. Rev. D* **108**, 036010 (2023).