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Short communication

Moment arms and musculotendon lengths estimation for a three-dimensional lower-limb model[☆]

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Abstract

This paper presents a set of polynomial expressions that can be used as regression equations to estimate length and three-dimensional moment arms of 43 lower-limb musculotendon actuators. These equations allow one to find, at a low computational cost, the musculotendon geometric parameters required for numerical simulation of large musculoskeletal models. Nominal values for these biomechanical parameters were established using a public-domain musculoskeletal model of the lower limb (IEEE Trans. Biomed. Eng. 37 (1990) 757). To fit these nominal values, regression equations with different levels of complexity were generated, based on the number of generalized coordinates of the joints spanned by each musculotendon actuator. Least squares fitting was used to identify regression equation coefficients. The goodness of the fit and confidence intervals were assessed, and the best fitting equations selected.

Keywords: Musculoskeletal modeling; Muscle geometry; Muscle moment arm; Lower limb

1. Introduction

The development of musculoskeletal modeling software has provided new opportunities to understand and numerically simulate musculoskeletal systems. Nevertheless, several applications require explicit analytical expressions for biomechanical parameters. This is the case for open-loop muscle excitations determined using optimal control techniques, where the biomechanical model dynamic equations must be known explicitly to achieve the optimal control solution (Kaplan and Heegaard, 2001). Engineering control design techniques are often based on linear models, or piecewise linearized models with variable control gains. To obtain the state-space matrices, usually the Jacobian of the non-linear state-space equations has to be calculated. If these state-space non-linear equations are differentiable throughout the domain of the state variables, the Jacobian can be found more easily. This is an important reason to look for differentiable regression equations.

This paper presents a set of equations, as well as the methodology used to find and evaluate them, which can be used to estimate the lengths and moment arms of 43 musculotendon actuators in the lower limb. Nominal values of the lengths and moment arms were obtained from a public-domain biomechanical model[1] (Delp, 1990; Delp et al., 1990; Delp and Loan, 1995) built to run in the SIMM Software for Interactive Musculoskeletal Modeling (MusculoGraphics Inc., 1997).

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[1] Available to download at International Society of Biomechanics (ISB) site: http://isb.ri.ccf.org/data/delp/.

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2. Description of the biomechanical model

The three-dimensional kinematic model of the lower limb described by Delp et al. (1990) was used as the basis for the regression equations. SIMM allows estimation of the desired parameters as functions of generalized coordinates (GCs) that specify joint kinematics. The kinematic behavior of each joint of Delp's model depends on an assumed number of GCs (one, two, three or four).

The origins of the coordinate systems that define the GCs are described in Table 1. The definition of the GC names and the range of motion (ROM) in degrees for each GC is shown in Table 2. The coordinate systems are oriented as follows. Once the inertial coordinate system (Pelvis) is chosen, the subsequent coordinate systems are orderly defined through three translations plus three rotations, taking into account "kinematic functions" of the GCs (see details in Delp, 1990). In the anatomic position, the X-axis points anterior, the Y-axis points superior and the Z-axis laterally. However, the axes that define the ankle angle, the subtalar angle and the metatarsus–phalangeal angle are not orthogonal, as defined by Inman (1976). The coordinate systems that define each GC and the respective axes of rotation are shown in Table 3. The specific GCs for each musculotendon actuator can be found in the *Web Supplementary Material*, Section 1.

Every musculotendon actuator crosses one or two joints, and its lengths and moment arms depend on the GCs of the corresponding joints. For all the analyzed mono or biarticular muscles, the maximum number of overall GCs was four, which corresponds to the biarticular muscles that cross hip and knee. Hence, four families of feasible fitting equations were proposed, with one–four GCs dependency.

Table 1
Location of coordinate systems

Reference system	Location
Pelvis	Midpoint of the line that connects the two ASIS
Femur	Center of femoral head
Tibia	Midpoint between femoral epicondyles with knee in anatomic position
Patella	Most distal point of patella apex
Talus	Midpoint between lateral and medial maleoli
Calcaneus	Most distal and inferior point of calcaneus posterior surface
Toes	Base of the second metatarsus

3. Regression equations and least-squares fitting

The problem of finding the musculotendon length and moment arms fitting equations, as functions of joint angles GCs, may be formulated as determining the coefficients a_i ($i = 1, \ldots, n$, n is a natural number) in a generic equation of the form:

$$\begin{aligned} F(Q_1, Q_2, Q_3, Q_4) = a_1 &+ a_2 f_1(Q_1, Q_2, Q_3, Q_4) \\ &+ a_3 f_2(Q_1, Q_2, Q_3, Q_4) \\ &+ \cdots + a_n f_{n-1}(Q_1, Q_2, Q_3, Q_4), \end{aligned} \quad (1)$$

where the function F may represent a musculotendon length (L_{mt}) or the moment arms r_i ($i = 1, 2, 3, 4$), in relation to the GCs (Q_1, Q_2, Q_3, Q_4). The equation above is formulated for the case of four GCs dependency, but the structure is similar for any number of coordinates. The $f_i (i = 1, \ldots, n-1)$ are generic nonlinear functions, and may be chosen according to the shape of the surface to be fitted.

If k samples of data are available (i.e. the values of the GCs and the corresponding value of F), the *Definition Matrix A* (Press et al., 1992) may be constructed for this problem, as well as the $(n \times 1)$ vector $\mathbf{a}$ of unknown coefficients and the $(k \times 1)$ vector $\mathbf{b}$ of values of F:

$$A = \begin{bmatrix} 1 & f_1(1) & f_2(1) & \cdots & f_{n-1}(1) \\ 1 & f_1(2) & f_2(2) & \cdots & f_{n-1}(2) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & f_1(k) & f_2(k) & \cdots & f_{n-1}(k) \end{bmatrix}, \quad (2)$$

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \cdots & a_n \end{bmatrix}^{\mathrm{T}}, \quad (3)$$

$$\mathbf{b} = \begin{bmatrix} F(1) & F(2) & \cdots & F(k) \end{bmatrix}^{\mathrm{T}}. \quad (4)$$

Table 2
Generalized coordinates and their range of motion

Generalized coordinate		Minimum (deg)	Maximum (deg)
Hip flexion	HF	− 10 extension	95 flexion
Hip adduction	HA	−50 abduction	15 adduction
Hip rotation	HR	−20 external rotation	20 internal rotation
Knee angle	KA	−120 flexion	0 (full extension)
Ankle angle	AA	−30 plantar flexion	30 dorsiflexion
Subtalar angle	SA	−20 eversion	20 inversion
Metatarsus—phalangeal angle	MA	−30 flexion	30 extension

Table 3
Orientation of the generalized coordinates

Generalized coordinate	First coordinate system	Second coordinate system	Axis of rotation between the first and the second coordinate systems
HF	Pelvis	Femur	Z
HA	Pelvis	Femur	X
HR	Pelvis	Femur	Y
KA	Femur	Tibia	Z
AA	Tibia	Talus	Z
SA	Talus	Calcaneus	X
MP	Talus	Toes	X

Table 4
Selected muscle groups according to dependence on the same GCs[a,b]

Groups	Number of generalized coordinates	Muscles
1	3: (HF, HA, HR)	gmed1, gmed2, gmed3, gmin1, gmin2, gmin3, addlong, tfl, adbrev, amag1, amag2, amag3, pect, gmax1, gmax2, gmax3, iliacus, psoas, quadfem, gem, peri
2	4: (HF, HA, HR, KA)	semimem, semiten, bifemlh, sar, gra, rf
3	1: (KA)	bifemsh, vasmed, vasint, vaslat, ligpat
4	3: (KA, AA, SA)	medgas, latgas
5	2: (AA, SA)	tibipost, tibiant, perbrev, perlong, pertert, sol
6	3: (AA, SA, MA)	flexdig, flexhal, extdig, exthal

[a] The abbreviated muscle names are: gmed: *gluteus medius* (compartments, e.g. gmed1: 1—anterior, 2—middle, 3—posterior); gmin (compartments: 1—anterior, 2—middle, 3—posterior): *gluteus minimus*; semimem: *semimembranosus*; semiten: *semitendinosus*; bifemlh: *biceps femoris caput longus*; bifemsh: *biceps femoris caput brevis*; sar: *sartorius*; addlong: *adductor longus*; addbrev: *adductor brevis*; amag (compartments: 1—superior, 2—middle, 3—inferior): *adductor magnus*; tfl: *tensor fasciae latae*; pect: *pectineus*; gra: *gracilis*; gmax (compartments: 1—superior, 2—middle, 3—inferior): *gluteus maximus*; quadfem: *quadratus femoris*; gem: *gemelli*; peri: *periformis*; rf: *rectus femoris*; vasmed: *vastus medialis*; vasint: *vastus intermedius*; vaslat: *vastus lateralis*; medgas: *gastrocnemius* (medial head); latgas: *gastrocnemius* (lateral head); sol: *soleus*; tibipost: *tibialis posterior*; flexdig: *flexor digitorum longus*; flexhal: *flexor hallucis longus*; tibiant: *tibialis anterior*; perbrev: *peroneus brevis*; perlong: *peroneus longus*; pertert: *peroneus tertius*; extdig: *extensor digitorum longus*; exthal: *extensor hallucis longus*.

[b] See Section 1 of *Web Supplementary Material* to know the function of each muscle in relation to the generalized coordinates.

Using the least-squares normal equation:

$$\mathbf{a} = (A^{\mathrm{T}}A)^{+}A^{\mathrm{T}}\mathbf{b}, \tag{5}$$

the coefficients a_i can be estimated. In the above equation, $(\cdot)^{+}$ represents a pseudo-inverse matrix.

Each musculotendon actuator was placed in one of six groups, depending on the GCs spanned (see Table 4) and several fitting equations, with different levels of complexity (and of computational cost), were evaluated. Table 5 lists the candidate equations for 1–4 GCs. In using these equations for the specific muscles, note that sequence r_1, r_2, r_3 or Q_1, Q_2, Q_3 and Q_4 refers to the GCs *in the same order* as they appear in Table 4. For example, the muscle *semimembranosus* depends on HF, HA, HR and KA (Table 4). Thus, r_1 is the HF moment arm, r_2 is the HA, r_3 is the HR and r_4 is the KA moment arm. Q_1, Q_2, Q_3 and Q_4 are, for this particular muscle, the HF, HA, HR and KA angles, respectively.

4. Generation of input and output files

A set of Matlab® routines was written for the automatic generation of the input files[2] to SIMM. Initially, a set of vectors containing the GCs was established, sampling the full ROM in 20 points. If a muscle's geometrical properties depended on two (or more) GCs, all combinations of the GCs were sampled. If one GC was used, 20 points were generated, if two, 400 points and for three coordinates, a sequence of 8000 points was created. When four GCs were used, the same procedure was adopted, but the sampling was made using only 15 points per GC, to decrease the size of the files and the computational cost. In this case, $15^4 = 50,625$ points were created. These data, describing the lengths and moment arms of the musculotendon actuators, were used to fit the equations proposed in Table 5 using Eqs. (1)–(5).

[2] All files, original data and routines are available upon request from the authors.

Table 5
Proposed fitting equations for L_{mt} and r depending on the number of GCs

Structure (number of generalized coordinates)	Eq. no.	Proposed fitting equations
4	1	$L_{mt}, r_1, r_2, r_3, r_4(Q_1, Q_2, Q_3, Q_4) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_4 + a_6Q_1^2 + a_7Q_2^2 + a_8Q_3^2 + a_9Q_4^2 + a_{10}Q_1^3 + a_{11}Q_2^3 + a_{12}Q_3^3 + a_{13}Q_4^3$
	2	$L_{mt}, r_1, r_2, r_3, r_4(Q_1, Q_2, Q_3, Q_4) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_4 + a_6Q_1Q_2 + a_7Q_1Q_3 + a_8Q_1Q_4 + a_9Q_2Q_3 + a_{10}Q_2Q_4 + a_{11}Q_3Q_4$
	3	$L_{mt}, r_1, r_2, r_3, r_4(Q_1, Q_2, Q_3, Q_4) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_4$
	4	$L_{mt}, r_1, r_2, r_3, r_4(Q_1, Q_2, Q_3, Q_4) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_4 + a_6Q_1^2 + a_7Q_2^2 + a_8Q_3^2 + a_9Q_4^2 + a_{10}Q_1Q_2Q_3Q_4$
3	1	$L_{mt}, r_1, r_2, r_3(Q_1, Q_2, Q_3) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_1^2 + a_6Q_2^2 + a_7Q_3^2 + a_8Q_1^3 + a_9Q_2^3 + a_{10}Q_3^3$
	2	$L_{mt}, r_1, r_2, r_3(Q_1, Q_2, Q_3) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_1Q_2 + a_6Q_1Q_3 + a_7Q_2Q_3$
	3	$L_{mt}, r_1, r_2, r_3(Q_1, Q_2, Q_3) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3$
	4	$L_{mt}, r_1, r_2, r_3(Q_1, Q_2, Q_3) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_3 + a_5Q_1^2 + a_6Q_2^2 + a_7Q_3^2 + a_8Q_1Q_2Q_3$
2	1	$L_{mt}, r_1, r_2(Q_1, Q_2) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_1^2 + a_5Q_2^2 + a_6Q_1^3 + a_7Q_2^3$
	2	$L_{mt}, r_1, r_2(Q_1, Q_2) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_1Q_2$
	3	$L_{mt}, r_1, r_2(Q_1, Q_2) = a_1 + a_2Q_1 + a_3Q_2$
	4	$L_{mt}, r_1, r_2(Q_1, Q_2) = a_1 + a_2Q_1 + a_3Q_2 + a_4Q_1^2 + a_5Q_2^2 + a_6Q_1Q_2$
1	1	$L_{mt}, r_1(Q_1) = a_1 + a_2Q_1 + a_3Q_1^2 + a_4Q_1^3$
	2	$L_{mt}, r_1(Q_1) = a_1 + a_2Q_1 + a_3Q_1^2$
	3	$L_{mt}, r_1(Q_1) = a_1 + a_2Q_1$

5. Results

The regression coefficients **a**, which were found for the fitting equations proposed in Table 5 are shown in Section 2 of the *Web Supplementary Material*, as well as the mean fitting errors (MFE) and standard deviation of the errors for all the equations. The ankle dorsiflexion moment arm of the *tibialis anterior* is shown to demonstrate the accuracy of the methodology described in this paper. This actuator depends on two GCs (AA and SA), and the variation in moment arm with joint angles can be easily visualized in a 3D plot (Fig. 1A). The distance (error) between the surface generated from the model and the surface generated from the regression equation is shown in Fig. 1B. A reasonably good agreement between the two surfaces was obtained, with a maximum error of 0.15 cm. As usual in polynomial fitting, the largest errors are found close to the borders, which correspond to large joint angular displacements.

6. Error analysis

As a measure of the accuracy for the fit, a *Mean Fitting Error* (MFE) and a *standard deviation* (σ) of the error were calculated for the fitted data (see Eqs. (6) and (7)). In these equations, $\hat{X}$ is the SIMM generated value for L_{mt} or r, X is the same parameter evaluated with the fitting equation and N is the number of sample points ($N = 20$, 400, 8000 or 50,625, for 1, 2, 3 or 4 GCs). (MFE) and (σ) give an estimate of the mean error that can be expected, and the error dispersion, respectively, when the fitting equations are used to approximate the original parameters from Delp's model. In *Web Supplementary Material*, Section 3, the recommended equations for each parameter and musculotendon actuator are listed according to the minimum fitting error and standard deviation achieved. In general, the equation that produced the smallest fitting error was different for different muscles, although some trends were observed among some muscles with similar biomechanical functions. For example, in gmed, vasti and ilipsoas the same best fitting equations are defined for all the actuators.

$$\mathrm{MFE} = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{X}_i - X_i\right|, \tag{6}$$

$$\sigma = \left(\frac{1}{N-1}\sum_{i=1}^{N}\left(\left|\hat{X}_i - X_i\right| - \mathrm{MFE}\right)^2\right)^{0.5} \tag{7}$$

The reader is encouraged to examine the MFE of the equations to determine if they are within the tolerances of his/her particular application. For example, at neutral position (AA and SA null), the ankle angle moment arm of *tibialis anterior* is approximately 0.042 m = 4.2 cm, applying the fourth equation of Structure 2 in Table 5, or observing Fig. 1B. The MFE associated with the 4th equation (according to Section 2 of *Web Supplementary Material*) is 3.1590

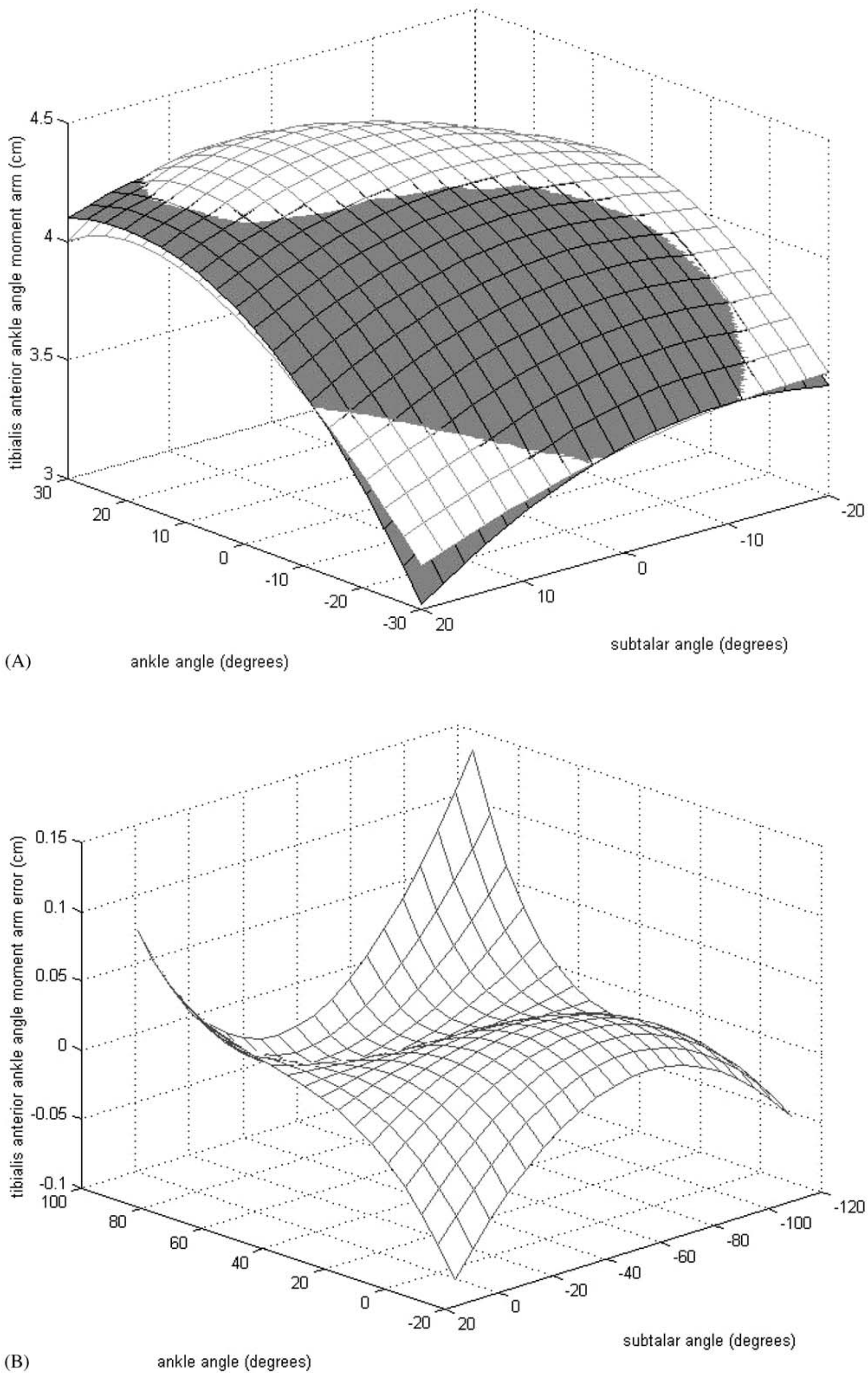

Fig. 1. (A) Ankle angle moment arm of *tibialis anterior*. White: SIMM generated surface; Gray: interpolated surface. (B) Space distribution of ankle angle moment arm fitting error in *tibialis anterior*.

E-4 m, and $\sigma = 2.5389$ E-4 m. It corresponds to a MFE of 0.75% of the ankle moment arm at this position. According to the Central Limit Theorem of estimation, with 95% of probability in a hypothetically normal distribution, the maximum expected MEF should be given by

$$\mathrm{MEF_{MAX}} = \mathrm{MEF} + 1.96\sigma. \quad (8)$$

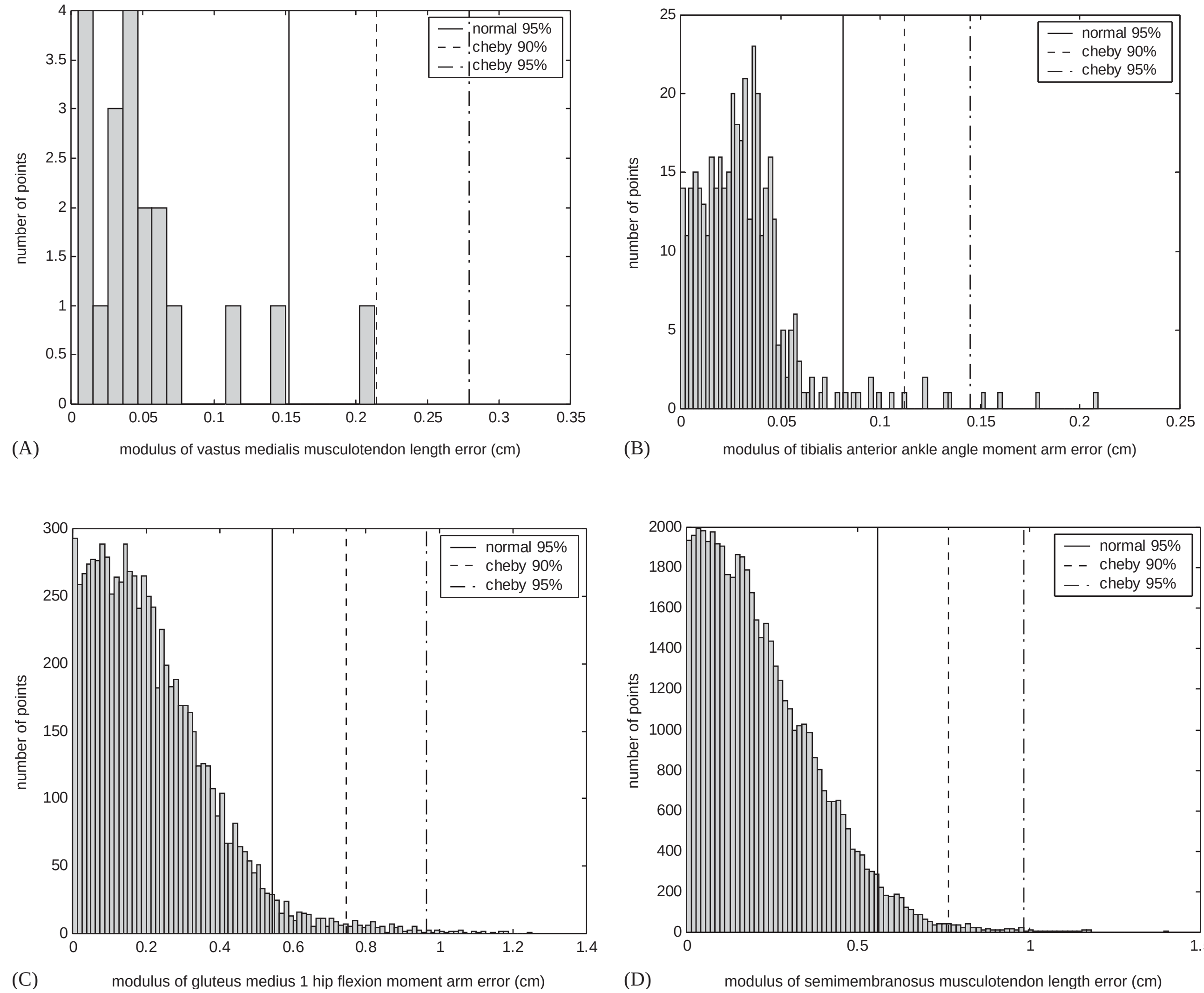

Fig. 2. (A) Histogram of the musculotendon length MFE for *vastus medialis* (1 GC, 20 points). (B) Histogram of the ankle angle moment arm MFE for *tibialis anterior* (2 GC, 400 points). (C) Histogram of the hip flexion moment arm MFE for *gluteus medius* 1 (3 GCs, 8000 points). (D) Histogram of the musculotendon length MFE for *semimembranosus* (4 GCs, 50625 points).

The value of this maximum error is 8.1352E−4 m, corresponding to 1.94% of the ankle angle moment arm value.

The estimation of the confidence interval of the error, as suggested above, assumes a random error with a normal distribution. In fact, the error distribution is generally not random, as can be observed in Fig. 1B.

A more conservative upper bound of the error CI can be estimated using Chebyshev's Theorem, that can be applied to any probability distribution where the mean and the standard deviation is known (Fisher and van Belle, 1993; Johnson, 1994):

Chebyshev's Theorem: "If a probability distribution has mean μ and standard deviation σ, the probability of getting a value X which deviates from μ by at least $k\sigma$ is at most $1/k^2$":

$$P(|X-\mu| > k\sigma) \leqslant \frac{1}{k^2}. \quad (9)$$

If $P = 0.05$ (95% of probability of the sample to be inside the CI), Eq. (9) gives $k = 4.47$. In the case of $P = 0.1$ (90% of probability), $k = 3.16$. The two CIs defined by this approach are

$$\mathrm{MEF}_{\mathrm{MAX}} = \mathrm{MEF} + 3.16\sigma \ (90\%), \quad (10)$$

$$\mathrm{MEF}_{\mathrm{MAX}} = \mathrm{MEF} + 4.47\sigma \ (95\%).$$

The number of errors that has fallen inside the CI defined by Eqs. (8) and (10) was counted and normalized for all the muscles and proposed fitting equations (Section 4 of *Web Supplementary Material*). Examples of histograms of the MFE, as well as the CIs are shown for 1, 2, 3 and 4 GCs in Figs. 2A–D.

7. Discussion

A set of regression equations to estimate musculotendon lengths and moment arms of 43 lower-limb musculotendon actuators, in three dimensions, has been presented. Several equations have been proposed for each group of musculotendon actuators, with different levels of complexity and computational cost. It is worth noting that, for muscles whose geometries depend on more than one GC, the regression equations presented here still reproduce the interdependencies between the coordinates in the values of L_{mt} and r. This property is a consequence of the way the fitting process was performed, over a discrete mesh in space, considering all the possible combinations of joint angles on which each muscle depends. MFEs and standard deviations were evaluated for all equations and, as a consequence, the most appropriate equations can be chosen for a particular application, taking into account the tradeoff between analytical complexity and desired accuracy. Although the error distributions are neither normal or random, the CI estimated for a normal distribution is a good approximation. (Section 4 of *Web Supplementary Material*). More conservative CI bounds can be defined by using Chebyshev's theorem, but due to the uncertainties associated with the anatomic data themselves, the authors believe that the normal CI is appropriate for most applications.

The choice of a particular regression equation for a specific application depends on the muscle that is being analyzed, as well as on the numerical requirements of the application. For example, when a linearized musculoskeletal model is used for control design a linear regression equation (Eq. (3) in Table 5, for all structures) should be employed. On the other hand, if there are no strong numerical cost limitations, the authors recommend the use of the equations stated in the *Web Supplementary Material*, Section 3. Most of the recommended equations give the smallest MFE, or a slightly greater error than the minimum, but with a smaller σ.

None of the proposed equations are themselves expensive from the numerical point of view. However, if the equations will be used as a part of a more complex biomechanical model that is required to be analytically differentiable (e.g. the calculus of an analytic Jacobian), or will be integrated several times in some iterative numerical process, the choice of more complex fitting equations may increase significantly the computational costs. This study suggests that no one fitting equation type can be declared as the most appropriate for all muscles and parameters in all applications; hence, this work offers some guidelines to choose them.

The lower-limb musculoskeletal model used as a basis for parameter estimation (Delp, 1990; Delp et al., 1990) was developed from several surveys of anatomic data (Brand et al., 1982), representing mean values of muscle coordinates, but retaining plausible anatomical and anthropometric dimensions. In this paper, a methodology to obtain fitting surfaces with the aid of SIMM has been described. Thus, more accurate models of the lower limb, with subject-specific individual parameters, as well as musculoskeletal systems other than the lower limb, can be fitted by similar polynomial expressions.

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