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# VISCOELASTIC AND CONTINUUM IN-PLANE DAMAGE MODELS FOR FIBROUS COMPOSITE MATERIALS: A REVIEW

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**Abstract.** *Fibrous composite materials are of generalized use nowadays, although inherently having a variety of failure modes which must be considered for design; as several of these failure modes may be related to damage, there has been increasing efforts to model the latter. Moreover, as many fibrous composites have polymer matrices which exhibit time-dependent effects at least in manufacture, and given the growing interest of modeling biomaterials, many viscoelastic models have been proposed, some of which considering also damage phenomena and possible coupling effects. Thus, the purpose of this work is to present a review of viscoelastic and in-plane continuum damage models for composite materials, along with a discussion of some models which incorporate both effects.*

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## 1. INTRODUCTION

Fibrous composite materials with polymer matrices are currently of generalized use both in the aeronautical and automobile industry, as stressed in several works [1,2,3,4]. Nevertheless, this type of material is subject to multiple failure modes, shown, for example, in [3,4], which must be assessed and taken into account for design purposes.

Several of the aforementioned failure mechanisms may be considered as related to damage: according to [5], the brittle behavior of composite polymer matrices may be described as the irreversible nucleation and evolution of microcracks and voids, whose main effect is degrading the material mechanical stiffnesses, without production of significant permanent deformation. Nevertheless, as pointed out in [6], there is experimental evidence of ductile behavior, and, hence, of permanent deformation, when this class of material is subjected to shear: this phenomenon, in contrast to the brittle material behavior when in transverse tension, is related to damage on the fiber-matrix interface, which combines to the damage on the matrix itself.

On the other hand, over the past decades, there has been increasing interest in modeling composite manufacturing processes, such as thermoforming [7,8,9]: in this process, the matrix has not yet been cured, exhibiting, thus, pronounced time-dependent stress-strain (viscoelastic) behavior, which is also often temperature-dependent; all of these effects must be taken into account for the simulations to be performed with sufficient accuracy. In addition, the progressive interest in thermoplastic matrices [10], and even fibers [11], coupled with the growing use of toughened resins [12], has brought a necessity of taking their intrinsic time-dependent behavior into account for design and manufacturing purposes; moreover, these effects are also present in biological tissues [13], being increasingly more studied nowadays.

This work, thus, presents a review concerning computational models proposed for considering viscoelastic and damage phenomena in fibrous composite materials of polymer matrix, along with some proposed models which consider both effects and their coupling.

## 2. VISCOELASTIC MODELS

Most viscoelastic models regard the material as being a set of connected networks, which may be equivalently of the Maxwell type (spring and dashpot elements in series, with the networks themselves laid in parallel), or of the Kelvin-Voigt type (spring and dashpot elements in parallel, with the networks themselves in series), with the former represented in Fig. 1.

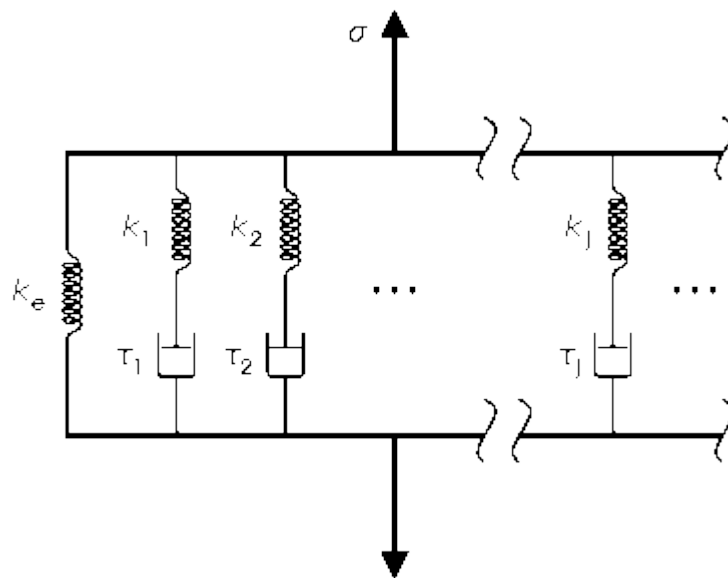


Figure 1 – Solid model with Maxwell networks [14]

One of the main issues in modeling composites is their anisotropy: fiber and matrix are often made of different materials, and, hence, have distinct viscoelastic properties. A way to cope with this characteristic is the decomposition of the stress tensor, as proposed in [15], initially proposed for plasticity: the stress is divided into stress parallel to the fibers, volumetric-related stress, and extra stress, with the latter being orthogonal to the former two. This decomposition is employed in [16], considering that only the extra stress tensor suffers viscoelastic effects: in other words, the volumetric behavior and the behavior parallel to the fibers were considered as linear elastic; on the other hand, [17] employs this decomposition at finite strains, in order to build a final viscoelastic response by summing the contributions of the components (fiber and matrix), also considering the volumetric-related behavior as purely elastic.

When working in finite strains (visco-hyperelasticity), an important foundation was laid in [18,19] for finite-element implementation: as viscoelasticity is time-dependent, numerical algorithms must be capable to perform the time integration in an objective (frame-independent) manner with finite increments. The solution proposed in these models is to work in the material frame of reference, through use of the Green-Lagrange strain, and the second Piola-Kirchhoff stress, which are trivially

objective, before switching to the Cauchy stress for momentum balance calculations; in addition, these models allow nonlinear viscoelasticity to be coupled with any hyperelastic law. This material approach was followed by [20] and [13], specifically for composites: whereas [18,19] already decompose the second Piola-Kirchhoff stress into its volumetric and isochoric parts (regarding the volumetric behavior as purely elastic), the latter models extend this further: in [20], there is also no viscoelasticity at the fiber direction, while in [13] the material may possess two viscoelastic families of fibers, thus allowing simulation of biological tissues.

Alternatively, there have been other approaches in order to deal with large deformations and anisotropy: in the context of thermoforming, [8] proposes an anisotropic viscoelastic model for finite strains, performing time integration at the rotated configuration, and taking temperature into account by time-temperature superposition; [9], on the other hand, avoids implementation of a new model through mesh and material superposition in finite-element analyses: the final behavior was attained by superposing an isotropic visco-hyperelastic model with an orthotropic elastic one. In addition, another way of addressing large rotations is proposed in [17]: the constitutive law is integrated after the stress tensor has been rotated to a fixed reference system.

Lastly, some of the assessed models allowed viscoplasticity, that is, possibility of permanent (non-recoverable) time-dependent strains. In [10], both phenomena, along with the plastic surface, are written as functions of the five invariants for fiber-reinforced materials with a single family of fibers [15], in order to make the constitutive law independent of the fiber orientation. On the other hand, [12] proposes linear and nonlinear viscoelastic work potentials, generalized in [21] for viscoplastic behavior; in both works, the potentials employ internal variables based on more general microstructural phenomena, making it possible to take into account other phenomena, such as temperature changes, humidity, and chemical degradation, all represented by timescale changes, as well as damage, which will be treated in more detail at the following section.

### 3. CONTINUUM IN-PLANE DAMAGE MODELS

Several damage models work with Continuum Damage Mechanics: material behaviors representing the macroscopic effects of microscopic damage on the composite properties (their reduction [5]), employing homogenization [22]; hence, damage is considered as a reduction in the material effective area, giving rise to effective stresses [22]. Anisotropy is again of concern, because the distinct components become damaged in different ways: according to [5], there is experimental evidence that transverse and shear stresses do not influence fiber degradation, and that matrix shear damage does not depend on the sign of shear stresses. In addition [5], matrix microcracks do not propagate in transverse compression, but do not close elastically when the material is unloaded; that is, neither transverse compression nor unloading change the material damaged state.

Given these evidences, the model proposed in [5] considers three damage variables for an individual ply: one for the fiber-dominated direction, and the other for the matrix-dominated properties (in transverse normal stress, and in shear). The authors assume four different modes for ultimate failure, and, also, mention the fact that the composite shear behavior is nonlinear but attribute this behavior solely to damage. This model was also used in [23], for matrix degradation, with its onset governed by the criterion of Puck and Schürmann.

Independently of the previous works, [24] proposed a model with also three damage variables (fiber, transverse tension, and transverse shear), regarding the fiber-dominated direction as brittle linear elastic in tension, but nonlinear elastic in compression, and considering anisotropic plasticity. This model has been extended in [6], taking fiber-matrix interfacial stresses into account for damage

calculations, in [25], which considers the influence of the out-of-plane stresses in the in-plane damage, and also the isotropic growth of delamination-related damage (followed by degradation of the out-of-plane material properties), and also in [26], which does not consider plasticity, but instead proposes a new criterion for the onset of damage, new nonlinear laws both for longitudinal and transverse compression, and explicit dependence of the damage variables on the fiber orientation.

Moreover, there have been models considering progressive, not brittle fiber damage: the model proposed in [27] is a triaxial damage model, with exponential fiber damage, progressive matrix degradation whose onset is governed by the Hashin criterion, along with cubic in-plane shear, and a quadratic criterion for initiation of interlaminar failure. On the other hand, [28] independently proposed progressive fiber damage based on the critical energy release rate, with independent damage variables for tension and compression; an appropriate critical energy release rate was also employed in a mixed-mode matrix damage model, governed by equivalent strains, coupled to a cubic elastoplastic model for in-plane shear, and to a mixed-mode delamination model.

#### **4. MODELS COUPLING DAMAGE AND VISCOELASTICITY**

Some authors have proposed coupled models, which consider both damage and viscoelasticity. A first example is the model proposed in [18], although damage growth is isotropic, as it was initially meant for polymers. In turn, [29] proposed two sets of internal variables: one related to microdamage, and the other to transverse cracking, with the possibility of rate-dependent damage growth; viscoelasticity is considered by employing equivalent displacements, in order to reduce the viscoelastic problem to an equivalent elastic one. This solution out of the time domain was also employed in [30], aiming not for characterizing damage evolution, but for assessing anisotropic viscoelastic effects in already damaged laminates, considering the influence of the adjacent layers: for this purpose, damage was to be directly measured, and then homogenized in the model, without using hypotheses regarding equal strains or equal strain energies for both the damaged and the undamaged composites. Another feature of this model is the possibility of a layer to lose its orthotropy due to damage.

Lastly, another model coupling viscoelasticity and damage has been presented in [31]: the authors use the effective stress, calculated through a proper linear transformation of the stress tensor (built in a way that the latter would remain symmetric), in order to yield equivalent damaged material properties; damage is strain-driven in an exponential fashion, and also proposed in rate form with evolving damage surfaces and isotropic “hardening” (expansion of the damage surface). Viscoelasticity, in turn, was formulated as a series of nonlinear Kelvin chains, through the use of strain-related internal variables, unlike that of models, such as the one proposed in [19], which employs stress-related internal variables.

#### **5. CONCLUSIONS**

This work presented some trends in composite material modeling, specifically in the development of viscoelastic and continuum in-plane damage models for fibrous composite materials. Anisotropy was a major concern, as distinct components show distinct time-dependent behaviors and distinct damaging mechanisms, thus commonly used approaches in order to deal with this material characteristic were stress tensor decompositions and model superpositions for viscoelastic modeling, along with use of several damage variables with experiment-based decoupling, in the case of damage modeling.

Finite strains (visco-hyperelasticity) were often allowed in viscoelastic models, given the increasing interest in modeling of biological tissues and composite manufacture through thermoforming, for example; this phenomenon, thus, needed to be addressed by performing stress integration in an objective (frame-independent) way. In addition, many damage models and some viscoelastic models considered permanent deformations, in the form of plasticity or viscoplasticity, as another source of material nonlinearity, with some damage models even employing nonlinear elastic laws for some material directions.

For models considering both viscoelastic and damage phenomena, they tended to be calculated through separate formulations, although there were concerns about rate-dependent damage propagation [29], and coupling of both phenomena, with the existence of damage influencing viscoelastic material parameters, and vice-versa [30]. This was considered in the model proposed in this latter reference, which, along with [21], also considered possible orthotropy losses due to damage.

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