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Some properties of the automorphisms of a Bernstein algebra

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SOME PROPERTIES OF THE AUTOMORPHISMS OF A BERNSTEIN ALGEBRA

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ABSTRACT. In this short note we show that if A is a nuclear Bernstein algebra then the group of automorphisms of $M(A)$, its multiplication algebra, has a proper subgroup isomorphic to Aut$A$.

1. INTRODUCTION

Some of the tools for the study of a Bernstein algebra are its multiplication algebra and also its group of automorphisms. In this note we stablish another connection between these two structures, among others which are well known in genetic algebra theory.

Let F be a field of characteristic not 2. A *baric algebra* (A,ω) over F is formed by an algebra A over F and a nonzero homomorphism $\omega\colon A \to F$. A commutative baric algebra (A,ω) is *Bernstein* if $(x^2)^2 = \omega(x)^2x^2$, for all $x \in A$. Bernstein algebras have idempotents e such that $\omega(e) = 1$ and for each of them, the Peirce decomposition of A is

$$A = Fe \oplus U \oplus V = Fe \oplus N,$$

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with $U=\{x\in\ker\omega: 2ex=x\}$, $V=\{x\in\ker\omega: ex=0\}$ and $N=\ker\omega$. These subspaces U and V satisfy the relations

$$U^2\subseteq V,\quad UV\subseteq U,\quad V^2\subseteq U,$$

and if $u\in U$ and $v\in V$ the following identities hold:

$$u^3=(u^2)^2=u(uv)=(uv)^2=uv^2=0.$$

The subspace $L=\mathrm{ann}_U U=\{u\in U: uU=0\}$ is an ideal of A, independent of the choice of the idempotent $e\in A$. The quotient algebra A/L is a Jordan algebra. We say that A is *nuclear* if $A^2=A$ and in this case, we have $V=U^2$. For more information, see [5].

Given a Bernstein algebra A, its multiplication algebra, denoted by $M(A)$, is the subalgebra of $\mathrm{End}A$ generated by $\{L_x: x\in A\}$, where L_x is the linear operator of A defined by $L_x(a)=xa$, for all $x,a\in A$. If $e\in A$ is an idempotent then $2L_e^2-L_e\in M(A)$ is also an idempotent and $M(A)=F(2L_e^2-L_e)\oplus(N:A)$, where $(N:A)=\{\sigma\in M(A):\sigma(A)\subseteq N\}$. We can decompose the ideal $(N:A)$ as $(N:A)=\widetilde{U}\oplus\widetilde{V}$ with $\widetilde{U}=\{\sigma\in(N:A):\sigma(2L_e^2-L_e)=\sigma\}=\{\sigma\in(N:A):\sigma(N)=0\}$ and $\widetilde{V}=\{\sigma\in(N:A):\sigma(2L_e^2-L_e)=0\}=\{\sigma\in(N:A):\sigma(e)=0\}$. The study of these multiplication algebras began in [1], where the authors establish some connections between A and $M(A)$. In [2] and [3], other properties of $M(A)$ are presented.

For each $x\in U\oplus U^2$ we define the linear operator $\psi_x\in\mathrm{End}A$ by $\psi_x(e)=x$ and $\psi_x(N)=0$. Then $\psi_x\in M(A)$ and $\widetilde{U}=\{\psi_x: x\in U\oplus U^2\}$. As particular cases, we have $\psi_u=2L_eL_u+2L_uL_e-L_u$ and $\psi_{ux}=2L_{ux}L_e+L_uL_x+L_xL_u$, for all $u,x\in U$.

Remark If $I\subseteq N$ is an ideal of A and $(I:A)=\{\sigma\in M(A):\sigma(A)\subseteq I\}$ then $M(A/I)\cong M(A)/(I:A)$. We observe that all the elements of $M(A)$ have their images in A^2. So if we take an ideal I of A such that $I\cap A^2=0$ then $(I:A)=0$ and in this case $M(A/I)\cong M(A)$. Ideals of A satisfying $I\cap A^2=0$ are contained in $\mathrm{ann}A$, the annihilator of A. Therefore we can assume that $\mathrm{ann}A\subseteq U^2$.

Lemma 2.4 of [4] states that if A and B are algebras over F and $\varphi: A\to B$ is an epimorphism then there exists an epimorphism $\widehat{\varphi}: M(A)\to M(B)$ such that

$\widehat{\varphi}(L_a) = L_{\varphi(a)}$ for all $a \in A$. In particular if φ is an automorphism of A then $\widehat{\varphi}$ is an automorphism of $M(A)$. This defines the mapping $\Lambda\colon \mathrm{Aut}A \longrightarrow \mathrm{Aut}M(A)$ such that $\varphi \longmapsto \widehat{\varphi}$. This mapping Λ is a homomorphism of groups. In fact, for all $x \in A$ and $\varphi_1, \varphi_2 \in \mathrm{Aut}A$ we have on the generators L_x of $M(A)$:

$$\Lambda(\varphi_1\varphi_2)(L_x) = \widehat{\varphi_1\varphi_2}(L_x) = L_{\varphi_1\varphi_2(x)} = \widehat{\varphi_1}L_{\varphi_2(x)} = \widehat{\varphi_1}\widehat{\varphi_2}(L_x) = \Lambda(\varphi_1)\Lambda(\varphi_2)(L_x).$$

Moreover $\Lambda(\mathrm{Id}_A) = \mathrm{Id}_{M(A)}$ and this proves that Λ is a homomorphism. We determine now the kernel of Λ. Let $\varphi \in \mathrm{Aut}A$ such that $\widehat{\varphi} = \mathrm{Id}_{M(A)}$. Then we have $\widehat{\varphi}(L_x) = L_{\varphi(x)} = L_x$ for all $x \in A$, that is $\varphi \in \ker\Lambda$ if and only if $\varphi(x) - x \in \mathrm{ann}A$ for all $x \in A$. Since $\varphi \in \mathrm{Aut}A$, $\varphi(e)$ is an idempotent of A. Therefore there exists $u_0 \in U$ such that $\varphi(e) = e + u_0 + u_0^2$ and from $\varphi(e) - e \in \mathrm{ann}A$ we have $u_0 + u_0^2 \in \mathrm{ann}A \subseteq V$, that is $u_0 = 0$ and this implies $u_0^2 = 0$ and $\varphi(e) = e$. For elements $u \in U$ we have $u = 2eu$. So $\varphi(u) = 2\varphi(e)\varphi(u) = 2e\varphi(u)$ and so $\varphi(u) \in U$. Using the relation $\varphi(u) - u \in \mathrm{ann}A$ we obtain $\varphi(u) = u$. Given $u_1, u_2 \in U$ we have $\varphi(u_1u_2) = \varphi(u_1)\varphi(u_2) = u_1u_2$. Consequently

$$\varphi(x) = x, \text{ for all } x \in A^2. \tag{1}$$

Decomposing $V = U^2 \oplus W$, for each $w \in W$ we have $\varphi(w) - w = z_w \in \mathrm{ann}A$. Thus

$$\varphi(w) = w + z_w, \text{ with } z_w \in \mathrm{ann}A. \tag{2}$$

We prove now that a linear mapping of A verifying the above two conditions (1) and (2) is an automorphism of A. We may assume $\mathrm{ann}A \subseteq U^2$ by the previous Remark. Let $A = Fe \oplus U \oplus U^2 \oplus W$ be a Bernstein algebra. If $\{w_1, \ldots, w_t\}$ is a basis of W and $\{z_1, \ldots, z_t\} \subseteq \mathrm{ann}A$ then $\sigma\colon A \to A$ defined by $\sigma(a) = a$ for all $a \in A^2$ and $\sigma(w_i) = w_i + z_i$ for $i = 1, \ldots, t$ is an automorphism of A. In fact, σ is linear and since $\mathrm{ann}A \subseteq U^2$, σ is bijective because $w_i = \sigma(w_i - z_i)$, for each i. Given $x = a + w$ and $y = b + w'$ in A with $a, b \in A^2$ and $w, w' \in W$ we have $\sigma(xy) = xy$ because $xy \in A^2$. As $\sigma(w) = w + z_w$ and $\sigma(w') = w' + z_{w'}$, where $z_w, z_{w'} \in \mathrm{ann}A$ we obtain $\sigma(x)\sigma(y) = (x + z_w)(y + z_{w'}) = xy = \sigma(xy)$. If we denote the above defined operator σ by $\sigma_{(z_1,\ldots,z_t)}$ we have $\sigma_{(z_1,\ldots,z_t)} \in \ker\Lambda$, for each t-uple $(z_1, \ldots, z_t)$ of elements in $\mathrm{ann}A$. Thus if we fix a basis $\{w_1, \ldots, w_t\}$ of W, $\ker\Lambda = \{\sigma_{(z_1,\ldots,z_t)} : z_1, \ldots, z_t \in \mathrm{ann}A\}$. We obtain, as particular cases, that Λ is injective in two situations:

Proposition 1. *If A is nuclear or* $\mathrm{ann}A = 0$ *then* $\mathrm{Aut}M(A)$ *has a subgroup isomorphic to* $\mathrm{Aut}A$. ■

The above proposition shows that in many cases Λ is injective. On the other side we will prove that Λ is not onto and so $M(A)$ has more automorphisms than A. We construct now a new class of automorphisms of $M(A)$ to ilustrate this fact.

Let $A = Fe \oplus U \oplus V$ be a Bernstein algebra and $M(A) = F(2L_e^2 - L_e) \oplus \widetilde{U} \oplus \widetilde{V}$ its multiplication algebra. We recall that $\widetilde{U} = \{\psi_x : x \in U \oplus U^2\}$. For each $x_0 \in U \oplus U^2$ we define $\Gamma_{x_0} \colon M(A) \longrightarrow M(A)$ by the rules $2L_e^2 - L_e \longmapsto 2L_e^2 - L_e + \psi_{x_0}$, $\psi_x \longmapsto \psi_x$ and $\theta \longmapsto \theta - \theta\psi_{x_0}$, for all $\psi_x \in \widetilde{U}$ and $\theta \in \widetilde{V}$. Clearly Γ_{x_0} is linear and bijective. Moreover $\Gamma_{x_0}(\sigma\tau) = \Gamma_{x_0}(\sigma)\Gamma_{x_0}(\tau)$ for all $\sigma, \tau \in M(A)$. In fact, decomposing $\sigma = \alpha(2L_e^2 - L_e) + \psi_x + \theta_1$ and $\tau = \beta(2L_e^2 - L_e) + \psi_y + \theta_2$ with $\psi_x, \psi_y \in \widetilde{U}$ and $\theta_1, \theta_2 \in \widetilde{V}$ we have $\sigma\tau = \alpha\beta(2L_e^2 - L_e) + \beta\psi_x + \theta_1\psi_y + \theta_1\theta_2$. Then $\Gamma_{x_0}(\sigma\tau) = \alpha\beta(2L_e^2 - L_e) + \alpha\beta\psi_{x_0} + \beta\psi_x + \theta_1\psi_y + \theta_1\theta_2 - \theta_1\theta_2\psi_{x_0}$ and $\Gamma_{x_0}(\sigma)\Gamma_{x_0}(\tau) = \Big(\alpha(2L_e^2 - L_e) + \alpha\psi_{x_0} + \psi_x - \theta_1\psi_{x_0} + \theta_1\Big)\Big(\beta(2L_e^2 - L_e) + \beta\psi_{x_0} + \psi_y - \theta_2\psi_{x_0} + \theta_2\Big) = \alpha\beta(2L_e^2 - L_e) + \alpha\beta\psi_{x_0} + \beta\psi_x - \beta\theta_1\psi_{x_0} + \beta\theta_1\psi_{x_0} + \theta_1\psi_y - \theta_1\theta_2\psi_{x_0} + \theta_1\theta_2 = \Gamma_{x_0}(\sigma\tau)$. Therefore Γ_{x_0} is an automorphism of $M(A)$. This defines the application $\Gamma \colon U \oplus U^2 \longrightarrow \mathrm{Aut}M(A)$ by $x \longmapsto \Gamma_x$. Note that Γ is a homomorphism from the additive group $U \oplus U^2$ to the group $\mathrm{Aut}M(A)$: the image of $0 \in U \oplus U^2$ is the identity operator in $M(A)$ and given $x_0, y_0 \in U \oplus U^2$ we have $\Gamma_{x_0+y_0} = \Gamma_{x_0}\Gamma_{y_0}$ because $\Gamma_{x_0}\Gamma_{y_0}(2L_e^2 - L_e) = \Gamma_{x_0}(2L_e^2 - L_e) + \psi_{x_0} + \psi_{y_0} = 2L_e^2 - L_e + \psi_{x_0+y_0} = \Gamma_{x_0+y_0}(2L_e^2 - L_e)$; $\Gamma_{x_0}\Gamma_{y_0}(\psi_x) = \psi_x = \Gamma_{x_0+y_0}(\psi_x)$ for all $\psi_x \in \widetilde{U}$ and $\Gamma_{x_0}\Gamma_{y_0}(\theta) = \Gamma_{x_0}(\theta - \theta\psi_{y_0}) = \theta - \theta\psi_{x_0} - \theta\psi_{y_0} = \theta - \theta\psi_{x_0+y_0} = \Gamma_{x_0+y_0}(\theta)$ for all $\theta \in \widetilde{V}$. Moreover Γ is injective: if $\Gamma_x = \mathrm{Id}_M(A)$ then, in particular $2L_e^2 - L_e = 2L_e^2 - L_e + \psi_x$, that is $\psi_x = 0$ and consequently $x = 0$.

In [3] the authors study properties of isomorphisms of multiplication algebras of Bernstein algebras. Some of them are used to prove the following result.

Proposition 2. *For each Bernstein algebra $A = Fe \oplus U \oplus V$, the group of automorphisms of $M(A)$ has a normal abelian subgroup isomorphic to the additive group $U \oplus U^2$.*

PROOF: As explained above, it remains only to show that $\Gamma(U\oplus U^2)$ is normal in $\mathrm{Aut}M(A)$. Let $\Gamma_{x_0}\in\Gamma(U\oplus U^2)$ and $\Phi\in\mathrm{Aut}M(A)$. Proposition 6 of [3] states that $\Phi(\widetilde{U})=\widetilde{U}$. Then there exists $y_0\in U\oplus U^2$ such that $\Phi(\psi_{y_0})=\psi_{x_0}$. We prove that $\Phi^{-1}\Gamma_{x_0}\Phi=\Gamma_{y_0}$. By [3, Prop.5], we have $\Phi(2L_e^2-L_e)=2L_e^2-L_e+\psi_y$ for some $y\in U\oplus U^2$. So $(\Phi^{-1}\Gamma_{x_0}\Phi)(2L_e^2-L_e)=\Phi^{-1}\Gamma_{x_0}(2L_e^2-L_e+\psi_y)=\Phi^{-1}(2L_e^2-L_e+\psi_{x_0}+\psi_y)=2L_e^2-L_e+\psi_{y_0}=\Gamma_{y_0}(2L_e^2-L_e)$. For $\psi_x\in\widetilde{U}$ we have $\Phi(\psi_x)\in\widetilde{U}$. Thus $\Gamma_{x_0}(\Phi(\psi_x))=\Phi(\psi_x)$, that is $(\Phi^{-1}\Gamma_{x_0}\Phi)(\psi_x)=\Phi^{-1}\Phi(\psi_x)=\psi_x=\Gamma_{y_0}(\psi_x)$. If $\theta\in\widetilde{V}$ and $\Phi(\theta)=\psi_z+\theta'$ then $(\Phi^{-1}\Gamma_{x_0}\Phi)(\theta)=\Phi^{-1}\Gamma_{x_0}(\psi_z+\theta')=\Phi^{-1}(\psi_z+\theta'-\theta'\psi_{x_0})=\Phi^{-1}(\psi_z+\theta'-\psi_z\psi_{x_0}-\theta'\psi_{x_0})=\Phi^{-1}(\psi_z+\theta'-(\psi_z+\theta')\psi_{x_0})=\theta-\theta\psi_{y_0}=\Gamma_{y_0}(\theta)$. Therefore $\Phi^{-1}\Gamma_{x_0}\Phi=\Gamma_{y_0}\in\Gamma(U\oplus U^2)$ and consequently $\Gamma(U\oplus U^2)$ is normal in $\mathrm{Aut}M(A)$. ■

We compare the subgroups $\Lambda(\mathrm{Aut}A)$ and $\Gamma(U\oplus U^2)$ of $\mathrm{Aut}M(A)$. Recall that $L=\{u\in U : uU=0\}$

Proposition 3. *For any Bernstein algebra $A=Fe\oplus U\oplus V$, the set $\Lambda(\mathrm{Aut}A)\cap\Gamma(U\oplus U^2)$ is a subgroup contained in $\Gamma(L)$. Moreover if A is nuclear then $\Lambda(\mathrm{Aut}A)\cap\Gamma(U\oplus U^2)=\Gamma(L)$.*

PROOF: Given $u,x\in U$ we have $\psi_u=2L_eL_u+2L_uL_e-L_u$ and $\psi_{ux}=2L_{ux}L_e+L_uL_x+L_xL_u$. We also remind that if $\varphi\in\mathrm{Aut}A$ and $\varphi(e)=f$ then $\varphi(U_e)=U_f$ and $\varphi(V_e)=V_f$. So we have $\widehat{\varphi}(\psi_x)=2L_{\varphi(e)}L_{\varphi(u)}+2L_{\varphi(u)}L_{\varphi(e)}-L_{\varphi(u)}=\psi_{\varphi(u)}$ and $\widehat{\varphi}(\psi_{ux})=2L_{\varphi(u)\varphi(x)}L_{\varphi(e)}+L_{\varphi(u)}L_{\varphi(x)}+L_{\varphi(x)}L_{\varphi(u)}=\psi_{\varphi(u)\varphi(x)}=\psi_{\varphi(ux)}$. Let $\Gamma_{x_0}\in\Lambda(\mathrm{Aut}A)\cap\Gamma(U\oplus U^2)$. Then there exists $\varphi\in\mathrm{Aut}A$ such that $\Gamma_{x_0}=\widehat{\varphi}$. As $\Gamma_{x_0}(\psi_x)=\psi_x$ for all $x\in U\oplus U^2$ we have $\psi_{\varphi(x)}=\widehat{\varphi}(\psi_x)=\psi_x$, that is $\varphi(x)=x$ for all $x\in U\oplus U^2$. If $\varphi(e)=e_0=e+u_0+u_0^2$ then $U=\varphi(U)=U_{e_0}=\{u+2u_0u : u\in U\}$. Therefore $2u_0u=0$ for all $u\in U$, that is $u_0\in L$. If we evaluate $\widehat{\varphi}$ in $2L_e^2-L_e$ we obtain $\widehat{\varphi}(2L_e^2-L_e)=2L_{e_0}^2-L_{e_0}=2L_e^2-L_e+\psi_{u_0}$. On the other hand $\Gamma_{x_0}(2L_e^2-L_e)=2L_e^2-L_e+\psi_{x_0}$. So $x_0=u_0\in L$ and $\Lambda(\mathrm{Aut}A)\cap\Gamma(U\oplus U^2)\subseteq\Gamma(L)$. Suppose now that A is nuclear. Let us prove that each Γ_{u_0} with $u_0\in L$ is an element of $\Lambda(\mathrm{Aut}A)$. Let $u_0\in L$ and $\varphi_{u_0}: A\to A$ defined by $\varphi_{u_0}(e)=e+u_0$, $\varphi_{u_0}(u)=u$ and $\varphi_{u_0}(v)=v$ for all $u\in U$ and $v\in V$. Then φ_{u_0} is linear and bijective and for $x=\alpha e+u_1+v_1$, $y=\beta e+u_2+v_2$, we have $xy=(\alpha e+u_1+v_1)(\beta e+u_2+v_2)=\alpha\beta e+\frac{1}{2}(\alpha u_2+\beta u_1)+u_1v_2+u_2v_1+v_1v_2+u_1u_2$. Then $\varphi_{u_0}(xy)=xy+\alpha\beta u_0$ and since A is nuclear, elements in L are in the annihilator of N. Therefore $\varphi_{u_0}(x)\varphi_{u_0}(y) = (x+\alpha u_0)(y+\beta u_0) = xy+\frac{1}{2}\alpha\beta u_0 + \frac{1}{2}\alpha\beta u_0 = xy+\alpha\beta u_0 = \varphi_{u_0}(xy)$. Thus φ_{u_0} is an automorphism. Let us show that $\Lambda(\varphi_{u_0}) = \Gamma_{u_0}$. Again using that elements in L are in ann$N$, we have $L_{u_0} = \frac{1}{2}\psi_{u_0}$ and $L_x\psi_{u_0} = 0$ for all $x \in N$. Then, $\Gamma_{u_0}(L_e) = \Gamma_{u_0}(2L_e^2 - L_e + \frac{1}{2}(4L_e - 4L_e^2)) = 2L_e^2 - L_e + \psi_{u_0} + \frac{1}{2}(4L_e - 4L_e^2) - \frac{1}{2}\psi_{u_0} = L_e + \frac{1}{2}\psi_{u_0} = L_e + L_{u_0} = \widehat{\varphi_{u_0}}(L_e)$. For $u \in U$, $\Gamma_{u_0}(L_u) = \Gamma_{u_0}(\frac{1}{2}\psi_u + L_u - \frac{1}{2}\psi_u) = \frac{1}{2}\psi_u + L_u - \frac{1}{2}\psi_u - (L_u - \frac{1}{2}\psi_u)\psi_{u_0} = L_u = \widehat{\varphi_{u_0}}(L_u)$ and if $v \in V$, $\Gamma_{u_0}(L_v) = L_v - L_v\psi_{u_0} = L_v = \widehat{\varphi_{u_0}}(L_v)$. Thus $\Gamma_{u_0} = \Lambda(\varphi_{u_0})$ and consequently $\Lambda(\mathrm{Aut}A) \cap \Gamma(U \oplus U^2) = \Gamma(L)$. ■

When A is nuclear we have proved that Aut$M(A)$ has a subgroup isomorphic to Aut$A$ (which is the image of the application Λ) and each automorphism of type Γ_x with $x \in U \oplus U^2$ and $x \notin L$ is not in this subgroup. Then the application Λ is not onto Aut$M(A)$.

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