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SPECTRAL ESTIMATION FOR TIME SERIES WITH AMPLITUDE MODULATED OBSERVATIONS: A REVIEW

by

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SPECTRAL ESTIMATION FOR TIME SERIES WITH AMPLITUDE MODULATED OBSERVATIONS: A REVIEW.

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1. Introduction.

Suppose that $\{X(n), n=0,\pm 1,\ldots\}$ is a strictly stationary sequence and $\{a(n), n=0,\pm 1,\ldots\}$ is a sequence (deterministic or stochastic) independent of $X(n)$. Then the observable process

$$Y(n) = a(n)X(n), \quad n=0,\pm 1,\ldots \tag{1.1}$$

is called an *amplitude modulated version of* $X(n)$ and $a(n)$ is the *amplitude modulated function*.

An important special case of (1.1) appears when we have missing observations. In this case (Parzen, 1962),

$$a(n) = \begin{cases} 1, & \text{if } X(n) \text{ is observed at time } n \\ 0, & \text{if } X(n) \text{ is missing at time } n. \end{cases}$$

Another possibility is that the 0's and 1's are independent realizations of Bernoulli's trials, which would imply an amplitude modulated function that is random and asymptotically stationary (see section 2) and would result that missed values of

X(n) occur with a probability p, independently one of the others. This situation was considered by Scheinok (1965) and Hinich and Weber (1981).

Bloomfield (1970) considered the case where the observations are missing according to certain covariance structure. A variation of the proposed scheme is the one in which the probability of a missing value varies periodically, with known period, as considered by Thrall (1980).

Another possibility is to have sistematically missing values, represented by a deterministic sequence a(n), formed of repetitions of the same sequence of 0's and 1's. This is known as the scheme of regular sampling, considered by Jones (1962), Parzen (1963), Alekseev and Savitskii (1973) and others. For further details, see Dunsmuir (1981, 1983).

But (1.1) can be used to represent more general versions of amplitude modulated time series, since a(n) can assume any real value or it can be random. Parzen (1962) considers the situation where $\{X(t),\ t \geq 0\}$ is a stationary time series, with a continuous covariance function and $\{a(n),\ n \geq 0\}$ is bounded, deterministic, asymptotically stationary sequence. As an example, a(n) may be a finite sum of harmonics.

Dunsmuir and Robinson (1981a) considered several sets of assumptions on the sequences {a(n)} and {X(n)} in such a way that {Y(n)} is asymptotically stationary.

Further references on the analysis of time series with unequally spaced observations are Dunsmuir and Robinson (1981b) and Shapiro and Silverman (1960).

2. Estimation of the Covariance Function.

Given observations $\{Y(n), a(n), n=1,\ldots,N\}$ of the sequences $\{Y(n)\}$ and $\{a(n)\}$, Parzen (1962) defines an estimator for the autocovariance function $\gamma_X(\ell)$ of $X(n)$, given by

$$\hat{\gamma}_X(\ell) = \frac{\hat{\gamma}_Y(\ell)}{C_a(\ell)}, \quad C_a(\ell) \neq 0, \tag{2.1}$$

where

$$\hat{\gamma}_Y(\ell) = \frac{1}{N-\ell} \sum_{i=1}^{N-\ell} Y(n)Y(n+\ell), \quad 0 \leq \ell \leq N-1 , \tag{2.2}$$

and

$$C_a(\ell) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N-\ell} a(n)a(n+\ell), \quad 0 \leq \ell \leq N-1 ,$$

assuming that $E\{X(n)\} = 0$. In this same article, it is shown that $\hat{\gamma}_X(\ell)$ is a consistent estimator of $\gamma_X(\ell)$ if $X(n)$ is Gaussian and ergodic.

In the case that $\{a(n)\}$ is a stochastic sequence, an alternative estimator is given by (Dunsmuir, 1983)

$$\tilde{\gamma}_X(\ell) = \frac{\hat{\gamma}_Y(\ell)}{v(\ell)}, \tag{2.4}$$

where $v(\ell) = E\{C_a(\ell)\}$, if $v(\ell)$ is known. If $v(\ell)$ is unknown, we can fit a model to it and estimate its parameters using $\{a(n)\}$. Then,

$$\bar{\gamma}_X(\ell) = \frac{\hat{\gamma}_Y(\ell)}{\bar{v}(\ell)}, \tag{2.5}$$

where $\bar{v}(\ell)$ is the estimate of $v(\ell)$.

Dunsmuir and Robsinson (1981) show, under certain assumptions, that the estimators (2.1), (2.4) and (2.5) are strongly consistent. An important assumption is that the sequence $\{a(n)\}$ be *asymptotically stationary* (Parzen, 1963), in the sense that

$$\lim_{N\to\infty} \sum_{n=1}^{N} a(n)/N = \mu \quad \text{a.s.}, \tag{2.6}$$

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N-\ell} a(n)a(n+\ell) = v_a(\ell) \quad \text{a.s.} \tag{2.7}$$

3. Estimation of the Spectrum.

Under the assumtpion that $\sum_\ell |\gamma_X(\ell)| < \infty$, the spectral density function (simply, the spectrum) of $\{X(n)\}$ is defined by

$$f_X(\lambda) = (2\pi)^{-1} \sum_{\ell=-\infty}^{\infty} \gamma_X(\ell)e^{-i\lambda\ell}, \quad -\pi \le \lambda \le \pi. \tag{3.1}$$

The question here is how to estimate $f_X(\lambda)$, given observations of the modulated process $\{Y(n)\}$. Several authors dealt with the problem, and most of them considered the case where the modulated sequence $\{a(n)\}$ is formed by zeros and ones, indicating that there are missing values of $\{X(n)\}$, the missing scheme being deterministic or stochastic.

From now on we shall assume that $\{X(n)\}$ is strictly stationary, with $f_X(\lambda)$ given by (3.1) and continuous.

Denoting by $\gamma_X^*(\ell)$ any of the above considered estimators of $\gamma_X(\ell)$, we can consider the usual estimators of $f_X(\lambda)$, namely:

(i) smoothed covariance estimator (SCE),

$$\hat{f}_X(\lambda) = \frac{1}{2\pi} \sum_{j=-N}^{N} w_N(j)\gamma_X^*(j)e^{-i\lambda j}, \tag{3.1}$$

$-\pi \leq \lambda \leq \pi$;

(ii) smoothed periodogram estimator (SPE),

$$\hat{f}_X(\lambda) = \int_0^{2\pi} W_N(\lambda-\alpha)I_X^{(N)}(\alpha)d\alpha$$

$$= \frac{2\pi}{N} \sum_{s=1}^{N-1} W_N(\lambda - \frac{2\pi s}{N})I_X^{(N)}(\frac{2\pi s}{N}), \tag{3.2}$$

where $w_N(j)$ is a sequence of weights with Fourier transforms $W_N(\lambda)$ and $I_X^{(N)}(\lambda)$ is the *periodogram* of the N observations of $\{X(n)\}$, given by

$$I_X^{(N)}(\lambda) = \frac{1}{2\pi N} \left| \sum_{n=0}^{N-1} X(n)e^{-i\lambda n}\right|^2. \tag{3.3}$$

We shall give in the sequel a summary of the estimators of $f_X(\lambda)$, considered in the literature, in different situations, and see that sometimes they are different from (3.1) or (3.2). We shall consider initially the case of series with missing values.

3.1. Modulated sequence 0-1 Deterministic.

3.1.1. *Regular and periodic sampling*

The situation where

$$a(n) = \begin{cases} 1, & n = t_1, \ldots, t_A, t_1+k, \ldots, t_A+k, \text{ etc} \\ 0, & \text{otherwise} \end{cases} \qquad (3.4)$$

with t_j, k integers, was considered by several authors, including Jones (1962), Parzen (1963), Alekseev and Savitski (1973), Neave (1970), and it is called *periodic sampling*. The special case where $t_{j+1} = t_j+1$ and $k = A+B$ is called *regular sampling*.

In the case of regular sampling, Jones (1962) used the estimator (3.1) with $\gamma_X^*(j)$ given by (2.1) and the $C_a(\ell)$ replaced by

$$C_a(\ell) = \begin{cases} \frac{A+B}{A-\ell}, & 0 \le \ell \le B \\ \frac{A+B}{A-B}, & B \le \ell \le A \\ \frac{A+B}{\ell-B}, & A \le \ell \le A+B \end{cases}$$

The proposed estimator is asymptotically unbiased and consistent if $\mu_X = 0$. Assuming that the process is Gaussian, the author derives an expression for the asymptotic variance and an upper bound for it.

Alekseev and Savitski (1973) consider regular sampling of a zero mean Gaussian process, having spectral density satisfying certain regularity conditions. The estimator proposed, for frequencies $\lambda \neq \pm k\pi/(A+B)$, is given by (3.1), with $w_N(j)\gamma_N^*(j)$ replaced by $w_N^*(j)\hat{\gamma}_Y(j)$, where $w_N^*(j)$ is a weight function which involves a set of contants $\{c_o,\ldots,c_{A+B-1}\}$; these are determined in such a way that the estimator be asymptotically unbiased, with a bias of given order. Moreover, an upper bound for the variance of the estimator is found, which vanishes as $N \to \infty$. The results hold for $A > B$.

3.1.2 *Scarce Sampling*

This is the case where the sampling interval is shortened at some point t^* during the period of observation. Assume that we have a sample where the m initial data (first segment) are read at intervals of r time units and the next pr data (second segment) are read at the unit time interval (r and p are integers). Thus, $N = (m+p)r$. Assume also that $\lambda = \lim p/m$, $N \to \infty$.

Here,

$$a(n) = \begin{cases} 1, & \text{if } r \text{ divides } n \\ 0, & \text{otherwise.} \end{cases}$$

Neave (1970) proposes two estimators for this situation:

(a) Estimator given by (3.1), with $\gamma_X^*(\ell)$ replaced by (2.1) and

$$C_a(\ell) = \begin{cases} \dfrac{1+\lambda r}{(1+\lambda)r}, & \text{if } r \text{ divides } \ell \\ \lambda/(1+\lambda), & \text{otherwise,} \end{cases}$$

called simple estimator by the author and which possess some disadvantages: provides negative values in many cases, the estimate at frequency $2\Pi/r$ is poorly correlated with values at neighbouring frequencies, and $E\{\hat{\gamma}_X(\ell)\} = MR_X(\ell)$, where $M = M(T)$ is not a steadly decreasing function.

(b) An alias-improved estimator, given by

$$\hat{f}_B(\lambda) = f^*(\lambda) \cdot \frac{\hat{f}_2^{[r]}(\lambda)}{\hat{f}_1^{[r]}(\lambda)}, \qquad (3.5)$$

where $f^*(\lambda)$ is the classical covariance estimator, using the second segment of the sample, $\hat{f}_1^{[r]}(\lambda)$ is the covariance estimator formed by using a sub-sample of the second segment, $X(t^*), X(t^*+r),\ldots,X(N)$, and $\hat{f}_2^{[r]}(\lambda)$ is the estimator formed using the sub-sample $X(1), X(1+r),\ldots,X(N)$.

Neave proves that $\hat{f}_B(\lambda)$ is asymptotically unbiased, gives an asymptotic expression for both estimators and concludes that, asymptotically, there are not big differences between the estimators. But through simulations with moderate sample sizes it is shown that the estimator (3.5) is superior to the simple estimator.

3.1.3. *Missing observations without a fixed pattern.*

Jones (1971) studied the general case

$$a(n) = \begin{cases} 1, & \text{if } X(n) \text{ is observed} \\ 0, & \text{otherwise} \end{cases},$$

where X(n) is a zero mean, weakly stationary complex process.

The proposed estimator is similar to that given by (3.1), with $w_N(j) = 0$, $|j| > m$, and $\gamma_X^*(j) = \frac{(N-j)}{N} \cdot \frac{\hat{\gamma}_Y(j)}{N^{-1} \sum_{n=1}^{N-j} a(n)a(n+j)}$; m is the *truncation point* of the sequence of weights.

The estimator is asymptotically unbiased and its variance is computed under the assumption that X(n) is Gaussian. The expression for the variance of the estimator is the same as that given by Parzen (1963). The author also calculates the "white noise variance", which is the variance for $f_X(\lambda)$ assumed constant. Also, a(n) is generalized to any known real function of n.

3.2. Modulated Sequence 0-1 Stochastic.

Several types of modulated sequences 0-1 stochastic were considered in the literature. Scheinok (1965), Bloomfield (1970) and Thrall (1980) are among those who proposed estimates for $f_X(\lambda)$ in some situations.

3.2.1. *Random Sequence of Independent Bernoulli's.*

Scheinok (1965) considered the case of

$$a(n) = \begin{cases} 1, & \text{if } X(n) \text{ is observed} \\ 0, & \text{otherwise} \end{cases}$$

and $P\{a(n) = 1\} = p$, independent of n, $0 < p < 1$.

The estimator of $f_X(\lambda)$ is of the form (3.2), with the periodogram replaced by

$$I'_X(\lambda) = \frac{1}{2\pi N} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{Y(j)Y(k)}{E\{a(j)a(k)\}} e^{-i(j-k)\lambda},$$

called modified normalized periodogram. This estimator is assymptotically unbiased and has a bias bounded by $K_N . \log N/N$, where K_N is a function of the true spectrum and of the spectral window. Moreover, under the hypothesis that X(n) is Gaussian, the exact and asymptotic variance of the estimator are found, the latter being of order $(N.B_N)^{-1}$.

3.2.2. *Correlated Random Sequence.*

Bloomfield (1970) considered the same situation as in 3.2.1. but with

$P\{a(n) = 1\} = p$, for all n,

$E\{a(n)a(n+r)\} = p.u_r$, $u_r > 0$,

$E\{a(n)a(n+q)a(n+r)a(n+s)\} = p^2 . v_{q,r,s}$, $q,r,s = 0, \pm 1, \ldots$

The author makes other assumptions on a(n) and also on

X(n). The proposed estimator of $f_X(\lambda)$ is of the form (3.1), with $\gamma_X^*(j)$ replaced by

$$\gamma_X^*(j) = \frac{N-j}{N}\,\hat{\gamma}_X(j)/E\{a(n)a(n+j)\} \qquad (3.6)$$

and $\hat{\gamma}_Y(j)$ given by (2.2).

Under some additional conditions, the estimator has the following properties:

(a) It is asymptotically unbiased.

(b) It is weakly consistent.

(c) If $E\{a(n)a(n+j)\}$ in (3.6) is replaced by $\frac{N\,C_a(\ell)}{N-\ell}$ (or $pu/2$, if this value is too small, u being the lower bound for u_r), then the new estimator will have the same asymptotic mean and variance as (3.6).

3.2.3. *Periodic Random Sequence.*

Thrall (1980) considered the case where $\{X(n)\}$ is strictly stationary and mixing and a(n) as in 3.2.1., independent with $P\{a(n)=1\} = \rho(n \bmod S)$. In the example given in the paper, $S=7$, and $\rho(s)$, $s=0,\ldots,S-1$, correspond to the probabilities of observing the process of interest in each day of the week, that is, the probability of the process being observed depends only on the day of the week.

To construct the estimator, the process

$$\tau(t) = \frac{Y(t)}{\rho(t)} = \frac{Y(t)}{E\{a(t)\}}$$

is considered and then an estimator of type (3.2) is used, bias-corrected, given by

$$\hat{f}_X^T(\lambda) = \frac{2\pi}{N} \sum_{t \neq 0 (\mathrm{mod}\ R)} W_N(\lambda - \frac{2\pi t}{N}) I_N^{\tau}(\frac{2\pi t}{N}) - $$

$$- (2\pi S)^{-1} \sum_{s=0}^{S-1} \frac{1-\rho(s)}{\rho(s)} E\{X^2(t)\},$$

with $N = S R$.

Under certain mixing assumptions, the estimate is asymptotically unbiased, consistent and normal.

3.3. General Modulated Sequence.

In what follows we shall consider three cases of the general sequence $\{a(n)\}$: Deterministic, random (independent and correlated) and the case which includes both of these.

3.3.1. *Deterministic case.*

Parzen (1963) assumes that $\{X(n)\}$ is a stationary normal process, with mean zero and covariance function $\gamma_X(v)$ satisfying

$$\lim_{N\to\infty} \frac{1}{N} \sum_{v=1}^{N} \gamma_X^2(v) = 0,$$

$\{a(n)\}$ a deterministic bounded sequence, such that $C_a(\ell)$ given by (2.3) exists. Then, it is shown that $\hat{\gamma}_X(\ell)$ is consistent in mean square for $\gamma_X(\ell)$, and from this consistent estimates of $f_X(\lambda)$ are derived, using $\hat{\gamma}_X(\ell)$ in (3.1). A formula is obtained for the asymptotic variance of the spectral estimator.

As we already mentioned in section 3.1.3., Jones (1971) used a deterministic sequence $\{a(n)\}$, not necessarily formed of 0's and 1's. The use of a general sequence does not change the form of the estimator, but changes its variance.

Toloi (1988) assumes that $\{X(n)\}$ is a strictly stationary process, with $E\{|X(n)|^k\} < \infty$, $k > 0$, and $\{a(n)\}$ is real sequence, deterministic, of bounded variation and asymptotically stationary. The proposed estimator is of the type of (3.2), with $I_N(\lambda)$ replaced by

$$I_N'(\lambda) = [2\pi \sum_{n=1}^{N} a^2(n)]^{-1} \left| \sum_{j=1}^{N} Y(j)e^{-i\lambda j} \right|^2, \qquad (3.7)$$

which is also a modified periodogram.

If $\sum_{n=0}^{N-1} a^2(n) = O(N)$, $\mu_X = 0$ and $\{X(n)\}$ satisfies certain mixing regularity conditions, it is shown that the estimator is asymptotically unbiased, consistent and asymptotically normal.

3.2.2. *The Random Case.*

The case of $\{a(n)\}$ forming a sequence of independent, identically distributed random variables, is considered by Toloi (1988). Here $X(n)$ is assumed to be strictly stationary, with $E\{|X(n)|^k\} < \infty$, $k > 0$. The proposed estimator is given by (3.2) with $I_N(\lambda)$ substituted by

$$I'_N(\lambda) = \frac{1}{2\Pi N\mu_a^2} \left| \sum_{n=0}^{N-1} Y(n)e^{-i\lambda n} \right|^2 - (2\Pi\mu_a^2)^{-1}\ \sigma_a^2(\sigma_X^2+\mu_X^2), \qquad (3.8)$$

where $E\{a(n)\} = \mu_a \neq 0$ and $Var\{a(n)\} = \sigma_a^2 > 0$. The estimator (3.8) is a modified periodogram, corrected for bias.

Under certain conditions on $\{X(n)\}$, the estimator $\hat{f}_X(\lambda)$ is shown to be asymptotically unbiased consistent and asymptotically normal. Moreover, if we replace the unknown parameters of (3.8) by consistent estimators, then the new spectral estimator will be asymptotically equivalent to the former one (in distribution sense).

The situation where both $\{a(n)\}$ and $\{X(n)\}$ are strictly stationary processes, independent is also considered by Toloi (1988). The estimator of $f_X(\lambda)$ is of type (3.1), with $\gamma_X^*(\ell)$ given by

$$\gamma_X^*(\ell) = \frac{\gamma_Y^*(\ell)}{E\{a(n)a(n+\ell)\}},$$

with $\gamma_Y^*(\ell) = (N-\ell)/N\ \hat{\gamma}_Y(\ell)$. This is a generalization of the estimator proposed by Bloomfield (1970), for the case of correlated sequence (see section 3.2.2). To derive properties of the estimator, several additional assumptions on $\{a(n)\}$ and $\{X(n)\}$ are necessary. It is then shown that the estimator is consistent in mean square and if $E\{a(n)a(n+\ell)\}$ is replaced by $NC_a(\ell)/(N-\ell)$, the modified estimator is asymptotically equivalent to the former.

The case of a general modulated sequence (deterministic or stochastic) is considered by Dahlhaus (1980). The estimator is of the form (3.2), with the periodogram replaced by (3.7). To prove consistency and asymptotic normality a large number of assumptions on {a(n)} has to be made.

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