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## Scaling laws and spatial effects of Brazilian health regions: a research protocol

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### Abstract

The literature has already consolidated the importance of health regions for Brazilian public health. Complexity properties strongly mark such regions. In this context, there are abundant indications that health regions should be analyzed with approaches linked to the sciences of complexity. One of these approaches, the estimation of scaling laws, can describe important properties of socio-spatial elements. However, no studies estimate the scaling laws of Brazilian health regions. This research protocol can remedy this limitation, proposing the estimation of scaling laws of the previously mentioned regions, mainly considering variables relevant to Brazilian public health. Still, this paper can substantially mitigate other relevant limitations of usual research that estimate scaling laws of socio-spatial elements. These mitigations, which provide advances in the literature on estimating scaling laws, are given by the proposal of modeling (if necessary) spatial effects and estimating scaling laws for the entire population of the socio-spatial elements. According to the theory, the expected results are non-linear scaling laws, which will likely vary with space and time and coexist with relevant spatial effects. From such laws and effects, it will be possible to accurately characterize the performance of each health region through Spatial and Scale Adjusted Metropolitan Indicators and unravel spatio-temporal properties, stabilities, and instabilities of sets composed of health regions. The expected findings of this paper can help rearrange health regions and improve the quality of information used in Brazilian public health planning.

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## 1. Introduction

The regionalization of the Brazilian public health system, the Unified Health System (Sistema Único de Saúde [SUS]), is the construction of health regions as territorial cuts in continuous geographic spaces [1,2]. Health regions have a broad planning perspective [3], configuring themselves as a space for negotiation, coordination, technical regulation, and health policy [3,4]. In addition to these utilities, health regions act to mitigate the problem of fragmentation of health actions and services in Brazil [5] and are associated with greater rationality in the use of resources [6].

Health regions form a complex network of interactions of their stakeholders (patients, managers, and health professionals, among others). In this context, several researchers adopt paradigms, theories, laws, and methods related to complexity to study such regions [7-10]. One of these laws stands out in the context of Brazilian public health [11,12]. Given the relevance of socio-spatial elements for Brazilian public health, with emphasis on health regions [4,6], this law, called scaling law [11], can accurately describe properties and dynamics of socio-spatial elements [10]. Scaling law refers to how a system behaves as its size changes, which is measured (in the case of socio-spatial elements) by the Number of Inhabitants (NI) [13].

Usual researches that estimate scaling laws present relevant limitations. In this paper, we focus on three of these, which are briefly described in this section and further elaborated in section 2.3. The first two of these limitations allude to the methods often used to carry out the estimations of such laws. The first is the adoption of criteria to select valid observations (commonly cities) that exclude the vast majority of the latter [10,11,14]. This engenders limitations to the use of identified results for all the socio-spatial elements to which the estimated scaling law refers. The second limitation is that the methods assume that the observations are independent and show no spatial effects\*. There are theoretical [15] and empirical indications [10,16] that these assumptions are false, which could lead to bias [16,17] and inconsistency in the estimates [17]. Finally, the third limitation is that the estimation of scaling laws valid for regions (instead of cities) is very scarce, which is even more severe when referring to health regions.

Given this scenario, the main objective of this protocol is to describe and analyze the properties of scaling laws, possible spatial effects, and the performance of all the Brazilian health regions. This considers especially variables relevant to Brazilian public health and refers separately to distinct periods. The performance of health regions will be measured by Spatial and Scale Adjusted Metropolitan Indicators (SSAMIs). The achievement of the described main objective guarantees coherence between the studied element (health regions) and the method proposed (estimation of scaling laws). Such achievement, combined with the proposed methodology in this paper, enables relevant mitigation of the three limitations mentioned earlier of usual research that estimates scaling laws, engendering progress in this literature.

In order to understand more deeply the context in which this article is embedded, its background is presented in sequence (section 2). After that, the proposed materials and methods to achieve the main objective of this paper (section 3) are detailed. In sequence, expected results, contributions, possible limitations (and mitigations thereof) are highlighted (section 4). Finally, conclusions and future work are expressed (section 5).

## 2. Background

### 2.1. Definition and basic properties of urban scaling

Considering NI as a measure of the size of cities, the latter follows scaling laws, called urban scaling [11]. Much high-impact research estimates and analyzes scaling laws of socio-spatial elements [10,13,14]. Given cities located in the same country, scaling laws can be estimated. These laws, captured in different countries, show substantial

regularities compared to each other [11,13,14,18]. The mathematical representation can best describe the scaling laws and their regularities. This representation is expressed by equation 1.

$$Y(t) = Y_0 N(t)^\beta \quad (1)$$

The  $Y(t)$  is a variable of interest (such as the Gross Domestic Product, GDP; numbers of AIDS cases; number of homicides),  $N(t)$  is NI at time  $t$ ,  $Y_0$  is a normalization constant, and  $\beta$  is the scaling exponent [2,13,14,18].

The regularities of  $\beta$  hold for the two components of cities: physical structure and socioeconomic dynamics. Such regularities also refer to individual human needs. The valid scaling law for physical structure (such structure manifests, for example, in streets and buildings) is described by  $\beta \approx 0.85$  (sublinear, i.e.  $\beta < 1$ ). Thus, there are economies of scale in cities [2,10,13]. Regarding socioeconomic dynamics, the metrics of its manifestations are socioeconomic quantities (such as the number of AIDS cases and GDP). About these quantities, the scaling law is described by  $\beta \approx 1.15$  (superlinear, that is,  $\beta > 1$ ) [10,13,14], expressing increasing returns to scale [10]. When dealing with individual needs (e.g. number of houses or water consumption), the scaling law is linear ( $\beta \approx 1$ ) [2,13,14]. Deviations from the scaling laws are called Scale-Adjusted Metropolitan Indicators (SAMIs) [10] and serve as more accurate per capita measures.

In a comprehensive view, the non-linear scaling laws are manifestations of non-linear interactions in social dynamics [10], which are anchored in infrastructure networks [13,18]. That is, the amalgamation of interactions between the two components of cities (physical structure and socioeconomic dynamics) explains the non-linear scaling laws presented [18].

Some researchers aim to analyze urban scaling in Brazil, referring to health-related variables. The scaling laws estimated in these researches demonstrate substantial consistency with the theoretical expectation. Thus, Brazilian cities scale, when dealing with variables related to health, in a non-linear way in several aspects [11,12]. Examples are infectious diseases (superlinear), heart attacks (superlinear), and deaths from diabetes (sublinear) [12].

## 2.2 Methods usually adopted to estimate scaling laws

Cities must satisfy specific criteria to be considered valid observations in estimating scaling laws. Criteria frequently used are a minimum NI (usually 50,000) [2], usually combined with criteria that focus on social interactions to define cities [10,14,19]. Cities that do not meet the criteria adopted are completely excluded from the estimation of scaling laws [10,11,14]. In sequence, logarithmic transformation of  $(t)$  and  $(t)$  (referring to equation 1) is performed. The transformed  $(t)$  is considered the dependent variable and  $N(t)$  transformed as the independent variable, that of simple linear regression. This regression is usually estimated by ordinary least squares (OLS). In this regression, the slope of the line represents the value of  $\beta$ , the scaling exponent [10,11,14].

Only one study [16] deals with spatial autocorrelation while estimating scaling laws. Its authors used eigenvector spatial filtering and dealt with the GDP of cities located in the US or China. The residuals identified in this research are Spatial and Scale Adjusted Metropolitan Indicators (SSAMIS). They are SAMIs after eliminating the spatial dependency between observations. Compared with non-spatial models of scaling laws, this research indicates that treating the spatial autocorrelation could generate better-fitted results.

There are also (few) papers investigating scaling laws referring to socio-spatial elements similar to regions. In this investigation, the methods adopted differ from those previously described by not excluding observations, all other procedures are very similar [20,21]. No research was identified that estimates the scaling laws of health regions.

## 2.3 Limitations of usual research that estimate scaling laws of socio-spatial elements

Three relevant limitations mark usual research that estimate scaling laws. The first, referring to the method usually adopted to estimate scaling laws, is caused by excluding observations according to the aforementioned criteria [10,11,14]. Such excluded cities tend to be the vast majority of the total number of cities in a locality. This exclusion greatly limits the amount of socio-spatial elements that can directly use the estimated scaling laws. Also,

as cities are selected non-randomly, but using well-defined criteria, the pre-selection of cities generates problems in extrapolation (to the excluded cities) of the regression findings.

The second limitation also alludes to the method usually adopted for estimating scaling laws. This limitation is initially described by the mainly theoretical perspective that cities (especially those that are geographically close) often interact [15] and are correlated with each other [16]. Consistent with this perspective, empirically, a correlation was found in spatial dimension [10] and spatial autocorrelation in procedures that aim to capture scaling laws [16,22]. In most cases, spatial proximity between cities also explains observations in ( $\square$ ) and affects the scaling exponent [23]. Despite this scenario, to estimate scaling laws, the usual estimation method (OLS) and specification (equation 1) assume that the observations are independent and have no significant spatial effects. This can lead to bias [16,17] and inconsistency in the estimates [17]. The only research that dealt with spatial autocorrelation did not analyze any health-related variables. The same study considered only cities, not regions, and did not address the possibility of spatial heterogeneity [16].

Finally, the third and last highlighted limitation refers to the type of socio-spatial elements analyzed in scaling laws. Discussing it, it is stated that social dynamics [10], anchored in infrastructure networks [13,18], mark socio-spatial elements at the spatial scale of cities [10,13] and beyond [24], such as regions. As these dynamics (and their infrastructure bases) explain and predict scaling laws, including nonlinear ones [18], there is theoretical support for regions to present nonlinear scaling laws. In this way, empirical findings indicate the existence of such laws valid for regions [20,21]. In contrast with the possibility of exploring scaling laws of regions, there is very little research on such laws. The relevance of this shortage is intensified in the context of Brazilian public health. Such aggravation is due to the even greater scarcity of research investigating scaling laws valid for Brazilian health regions, adding to the relevance of such regions for Brazilian public health [3-6].

The limitations discussed in this section are mitigated by achieving the main objective of this research protocol. The proposed methods for achieving this objective are detailed below.

### 3. Materials and methods proposed

The methods proposed to achieve the main objective of this research protocol can be categorized into three stages: (1) selection and data collection; (2) pre-analysis, treatment and data modeling and, finally; (3) description and analysis of the results obtained from data treatment and modeling.

#### 3.1. Selection and data collection

The first stage will be performed by a pre-selection of variables followed by a more definitive selection which will determine the choice of variables used in this research. The pre-selected variables are those described as relevant to evaluating Brazilian health [25]. In addition, a variable that measures NI of Brazilian health regions should also be selected.

Subsequently, the (more definitive) selection of variables is carried out. Such selection considers the pre-selected variables. Six variables very similar to the six of the pre-selected ones had their scaling laws, referring to Brazilian cities, identified by [11] or [12]. This makes it possible to compare scaling laws valid for cities with those valid for health regions for the same country (Brazil). Given this possibility, added to the relevance of these variables for Brazilian health [25], they were selected. They are total beds, suicide mortality, proper garbage collection, AIDS incidence, incidence of dengue, and infant mortality. Two more variables were selected. The first contributes to the indirect reach of information relevant to the research priorities of the Ministry of Health [26], which is the incidence of tuberculosis. Finally, the last selected variable measures the total length of land transport infrastructure in health regions. It was selected because of the substantial importance that physical infrastructure has for scaling laws [13,18] and because it makes it possible to compare the scaling exponent (referring to infrastructure) valid for health regions with that valid for cities [11,13,14].

Data regarding the length of land transport infrastructure are made available for download by the OSMnx package of the Python software [27] and by Open Street Maps (OSM). All other mentioned variables can be collected on the Health System Performance Assessment Project (PROADESS) and Department of Informatics of the SUS (DATASUS) online pages. For all selected variables, there is a wide availability of data; that is, the selected

variables present a very mild existence of missing data. Most of the pre-selected variables (as well as the eight selected ones) present data annually. Such periodicity is also used to execute research that estimates scaling laws [10,13,14]. In this context, the annual frequency will be selected to collect variables. Data from three periods (2006, 2012, and 2018) will be collected as they have minimal missing data and, given the relevant time interval (6 years) between them, facilitate the analysis of the temporal variability of properties referring to the scaling laws. Considering studies similar to this proposal [2,11,28], great methodological feasibility is observed for carrying out the research described in this article. In addition to these data, a file must also be obtained containing spatial information on the territorial arrangement of health regions in Brazil. This file can be obtained in conjunction with the TabWin software. As performed by Albuquerque et al. (2017) [29], the most recent arrangement of health regions can be adopted.

The execution of the procedures of the first stage allows carrying out the procedures that compose the second stage. The latter is described below.

### 3.2 *Pre-analysis, treatment and data modeling*

The second stage of the methods consists of data cleaning, exploratory spatial data analysis, data treatment and data modeling. As the usual methods used to capture scaling laws use regressions [11,14] and the latter, in the field of spatial econometrics, prove to be adequate to deal with spatial effects [17,30], this stage is based especially on methods located in the field of spatial econometrics. This stage is initiated by data cleaning. This procedure may involve replacing variables (especially considering the pre-selected variables) with many missing data or interpolation to estimate missing data. It is unlikely that this step will significantly impact the selected variables since they have a tiny amount of missing data.

In sequence, ensuring congruence with [10,14], data will be organized in cross-sectional format. Also, data that do not express absolute values vis-à-vis the population will be transformed (based on the total population) to express absolute values. In these terms, as recommended, an Exploratory Spatial Data Analysis (ESDA) will be performed prior to performing econometric-spatial modeling [17]. The ESDA will analyze relationships between NI and another variable of interest, both logarithmically transformed and referring to the same year. Other functional forms can also compose the ESDA. Subsequently, it should be noted that substantially different locations (in different aspects, such as socioeconomic) should not be considered together in a scaling law estimation [13]. Thus, given the significant heterogeneity of Brazil [5], added especially to the results obtained in the ESDA and the regionalizations of Brazil [31,32], this country can be divided into sub-regions, this for each period and variable. If this is done, each sub-region (composed of a set of health regions) will have a scaling law identified separately from all other sub-regions.

Next, treating health regions as observations and using the methods usually adopted to estimate scaling laws, such estimations are carried out separately for each variable, period, and sub-region [10,11,13,14]. Subsequently, spatial weighting matrices can be identified using the criterion of maximum centroid distance [17,33] from health regions. Spatial weight matrices will be identified considering different threshold distances. The selected matrix will be the one that maximizes the spatial dependence, measured by Moran's I [17,34]. Again, this will be done separately for each variable, period, and sub-region.

Subsequently, procedures will be carried out to identify the specification (referring to possible spatial lags) of the most appropriate model for the phenomenon addressed. As “most of the spatial econometrics literature on comparing model specification relies on likelihood ratio or Lagrange Multiplier Test” [30, p.16], mainly based on these tests, more adequate models will be selected. Furthermore, the specific theory for each variable can cooperate in choosing an appropriate model. Procedures included in the ESDA and Brazil's division into sub-regions help to address spatial heterogeneity. This treatment can be improved by carrying out heteroscedasticity tests (in the last-mentioned selected models) and, according to the result, calculating robust standard errors or adopting suitable estimators [17]. After carrying out these procedures and selecting the most appropriate model for the phenomenon under consideration, the resulting regressions (which express scaling laws) will be used to collect SSAMIs of health regions. SSAMIs are the simple residues of the selected regressions [10,16].

At this point, the second stage of the methods is finished. The results obtained by the execution of this stage make it possible to carry out the third and final stage of the methods.

### 3.3 Description and analysis of results obtained from data treatment and modeling

Finally, the third stage of the methods is described. At its beginning, this stage consists of collecting the properties of the selected regressions. These properties refer to each period and sub-region individually. According to these guidelines, such properties are the statistical significance, direction, and magnitude of the estimated scaling exponents and the coefficients that describe possible spatial lags. Information obtained by the ESDA (such as the sub-regions potentially captured) and the SSAMIs of the health regions will also be collected. In these terms, as the main objective involves the analysis of results, the collected results will be analyzed according to the literature, especially on scaling laws [10,11,13,14], public health, and the regionalization of Brazilian health [3,35,36].

At this point, the description of all the stages of the methods is finished. The sequential execution of these stages consolidates the achievement of the main objective of this protocol. A simplified visual representation of the proposed methods is presented in Fig. 1.

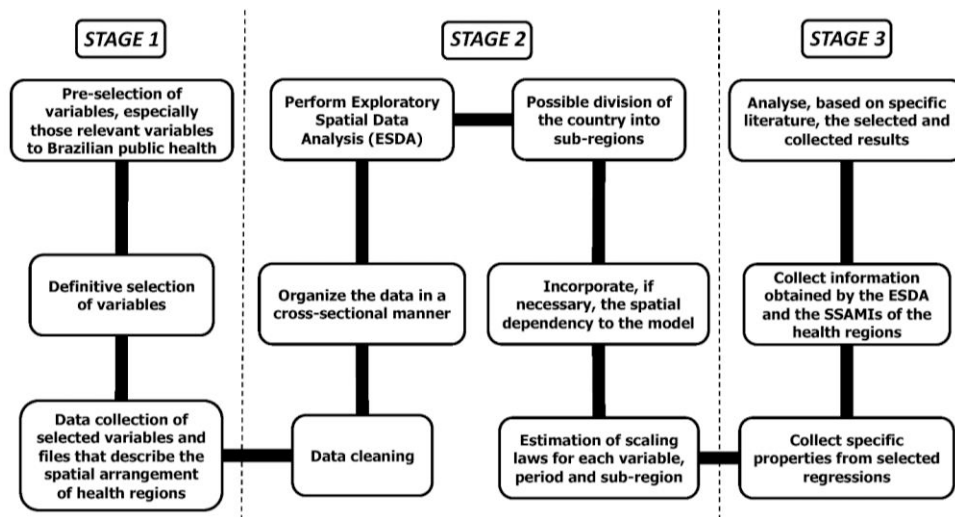


Fig. 1. Simplified Representation of the Proposed Methods.

Combining the materials and methods proposed with theories related to the topic of this paper, it is possible to find out the likely results that this protocol will identify. The expected results are presented in the next section.

## 4. Expected results and discussion

Discussing the scaling exponent initially, it is emphasized that there is theoretical support [18] and empirical findings [20,21] that regions have non-linear scaling laws. Thus, health regions should present non-linear scaling laws. Estimating scaling laws enables the identification of SSAMIs of health regions [10]. These SSAMIs, that precisely characterize the performance of each health region, makes it possible to improve the health status of the population [37] and refine information used in strategic planning [38] of public health. Furthermore, the analysis of the scaling exponent can unravel spatio-temporal properties, stabilities, and instabilities of sub-regions.

Regarding spatial dependence, it is stated that socio-spatial elements have a tendency to interact and be dependent on each other [15,17]. Also, empirical findings of spatial dependence in estimating scaling laws have already been reported [10,16]. Thus, the results are expected to show significant spatial dependence and spatial lags. The investigation of this result helps reveal spatial relationships between health regions. This relationship can be considered in planning [38], especially at the regional level [17]. Also, as a certain internal coherence (in health-related aspects) is sought within a health region [3,24] and an intense spatial dependence may indicate a lack of such coherence [17], the results can be useful to rearrange health regions, i.e., identify a spatial ordering that is more adequate to the objectives and phenomena dealt with.

Regarding spatial heterogeneity, it is highlighted that geographically distant locations may present different scaling laws [13]. Furthermore, the regionalization of Brazil considering factors that strongly correlate with public health (such as human development) results in the identification of five sub-regions [29,31,32]. Thus, we hope the results will show spatial heterogeneity and a number close to five sub-regions. Such results can cooperate to better identify structural differences [17] in health regions scaling laws [14]. Also, it is emphasized that, for each variable, the  $\beta$  and the composition of sub-regions will be identified for different periods and locations. As these results refer to sub-regions, the analysis of the former can unravel spatio-temporal stabilities and instabilities seen in the latter. Still, it is noteworthy that the main objective of this paper, combined with the methods proposed to achieve it, is one of its most important innovations. Such a combination can mitigate three relevant limitations of usual research that estimate scaling laws and enable the advancement of this literature.

As a potential limitation, it is possible that the selected models do not completely eliminate spatial autocorrelation. This problem is mild, since there are procedures for specifying spatial lag that, although not optimal, are valid and consider only the mitigation of such autocorrelation as sufficient to model it [17]. To mitigate this problem, several variables relevant to health were pre-selected, based on [25]. Thereby, at the same time that results of variables that the modeling does not completely eliminate spatial autocorrelation can be reported, others variables (in which such total elimination is possible) can be sought in the large "reservoir" based on [25]. Thus, it is unlikely that a relevant amount of scaling laws that rely on the complete elimination of spatial autocorrelation will not be estimated.

## 5. Conclusion and future work

This research protocol presents the main objective of describing and analyzing the properties of scaling laws, possible spatial effects, and the SSAMIs of Brazilian health regions, dealing especially with variables relevant to Brazilian public health and referring separately to distinct periods. The achievement of this objective, combined with the methodological approach described in this article, engenders the mitigation of relevant limitations of typical research that estimate scaling laws. In this way, it cooperates for the advancement of the literature on estimation of scaling laws.

The theory expressed in this protocol indicates that the expected results are non-linear scaling laws. Such laws should vary according to space and time and coexist with spatial effects. The results can be used to improve information used in Brazilian public health planning, refine the spatial arrangement of health regions and identify properties and dynamics of sub-regions. It is expected that these properties and dynamics can be better understood especially under the lens of the theory of scaling laws, public health, and the regionalization of Brazilian health.

As future work, it is noteworthy that health regionalization was adopted in several countries [39]. Thus, the procedures described in this article can be adopted to estimate scaling laws of health regions located in manifold countries. Therefore, the benefits of this paper can be expanded beyond Brazil, involving Italy, Portugal, Canada, among other countries.

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