The Use of Brain Activity Estimated Graphs on the Inference of Behavioral Measures: A Goalkeeper Game Study

(Preliminary Data)

Paulo Roberto Cabral-Passos[1], Italo Ivo Lima Dias Pinto[1], Claudia Domingues Vargas[2], Florencia Graciela Leonardi[1]

[1]Institute of Mathematics, Statistics and Computer Sciences, University of São Paulo, Brazil
[2]Carlos Chagas Institute of Biophysics, Federal University of Rio de Janeiro, Brazil

Introduction

- We analyze narrowband EEG signals, focusing on the mu-alpha rhythm (approximately 8-12 Hz)
- We apply a discretization to the EEG recorded from participants playing a prediction game.
- We use Markov Random Fields (MRF) to model relationships between EEG activity and subject's choices.

Materials and Methods

- **Reactive Index:** Measures changes in EEG power between resting and active states
- **Reactive Region:** Identifies the reactivity region of a frequency band for each subject/electrode.
- **Discretization:** Converts continuous EEG activity into binary states

Algorithm 1 Reactive Region Search

Require: α, β, ϵ
Ensure: a, b

$a \leftarrow \alpha$
$b \leftarrow \beta$
repeat
  $\rho_{\text{old}} \leftarrow \rho^{w}_{a,b}$
  $\rho_{\text{new}} \leftarrow \rho^{w}_{a-1,b}$
  $d \leftarrow \rho_{\text{new}} - \rho_{\text{old}}$
  if $\rho_{\text{old}} > 0$ and $d > 0$ and $\left(\frac{d}{\rho_{\text{old}}} > \epsilon\right)$ **then**
    $a \leftarrow a - 1$
  end if
until not $(\rho_{\text{old}} > 0$ and $d > 0$ and $\left(\frac{d}{\rho_{\text{old}}} > \epsilon\right))$
repeat
  $\rho_{\text{old}} \leftarrow \rho^{w}_{a,b}$
  $\rho_{\text{new}} \leftarrow \rho^{w}_{a,b+1}$
  $d \leftarrow \rho_{\text{new}} - \rho_{\text{old}}$
  if $\rho_{\text{old}} > 0$ and $d > 0$ and $\left(\frac{d}{\rho_{\text{old}}} > \epsilon\right)$ **then**
    $b \leftarrow b + 1$
  end if
until not $(\rho_{\text{old}} > 0$ and $d > 0$ and $\left(\frac{d}{\rho_{\text{old}}} > \epsilon\right))$
return a, b

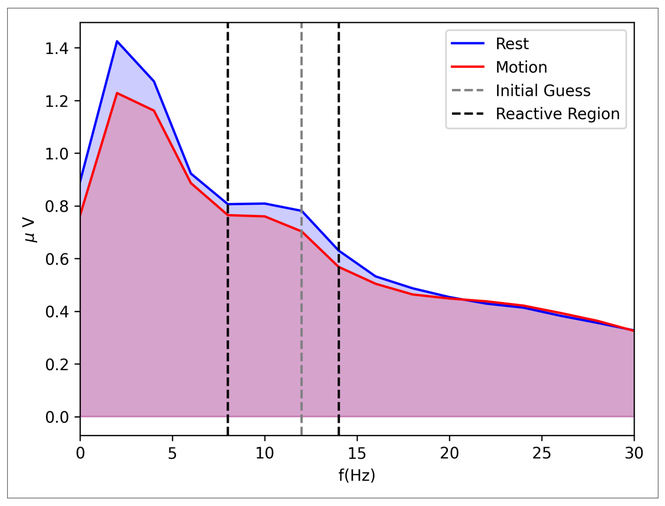

Figure: Example of the reactive region for a single subject and channel.

Acknowledgements

This work is part of the activities of the São Paulo Research Foundation (FAPESP)'s Research, Innovation and Dissemination Center for Neuromathematics (NeuroMat, grant 2013/07699-0). This study was supported by FAPESP (2022/00699-3; 2025/07274-6; 2023/13453-5; 2025/09647-4), FAPERJ (CNE 202.785/2018; E-26/010.002418/2019; E-26/204.076/2024; E-26/200.350/2025; CNE E-26/204.076/2024), CNPq (3310397/2021-9; 310397/2021-9; 407092/2023-4; 441884/2023-7; 305688/2025-1; 408389/2024-9), FINEP (18.569-8), CAPES (88887.671450/2022-00), and CAPES MathAmsud (88881.179479/2025-01)

Markov Random Field modeling approach

- We model EEG activity and responses as a graphical model $G = (V, E)$
- V contains 32 EEG nodes + 3 response nodes (total 35 nodes)
- Each EEG node X_i takes binary values $\{0, 1\}$ based on discretized activity
- Each response node Y_j (j=0,1,2) takes binary values:

$$Y_j = \begin{cases} 1 & \text{if response } j \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

- Edges represent statistical dependencies between nodes

Neighborhood Estimation

For a target node v, we estimate its Markov neighborhood $ne(v)$ as:

$$\widehat{ne}(v) = \underset{W \subset (V \setminus \{v\})}{\arg\max} \left\{ \log\left(\mathbb{P}(X_v^{(1:n)} \mid \mathbf{X}_W^{(1:n)})\right) - c|A|^{|W|}\log(n) \right\}$$

where $c = 0.1$ is the penalty constant and $|A| = 2$ is the alphabet size.

Response Prediction

For a given configuration $\mathbf{x}_N$ of the neighborhood N of a response node Y_j:

$$P(\text{response} = j) = \frac{\hat{p}(Y_j = 1 \mid \mathbf{X}_N = \mathbf{x}_N)}{\sum_{k=1}^{3} \hat{p}(Y_k = 1 \mid \mathbf{X}_N = \mathbf{x}_N)}$$

where $\hat{p}(Y_j = 1 \mid \mathbf{X}_N = \mathbf{x}_N)$ is estimated from the data.

Parameter Optimization

Table: AUC for different penalty constants and neighborhood sizes.

Max Size	c=0.1	c=0.2	c=0.3	c=0.4
1	0.704	0.886	0.75	0.868
2	0.842	0.776	0.98	0.948
3	0.84	0.976	0.96	0.702
4	0.874	0.98	0.84	0.684
5	0.91	0.79	0.528	0.506
6	0.868	0.598	0.5	0.5
7	0.778	0.5	0.5	0.5
8	0.862	0.5	0.5	0.5

Selected parameters: $c = 0.1$, maximum size $= 8$.

Summary graphical model

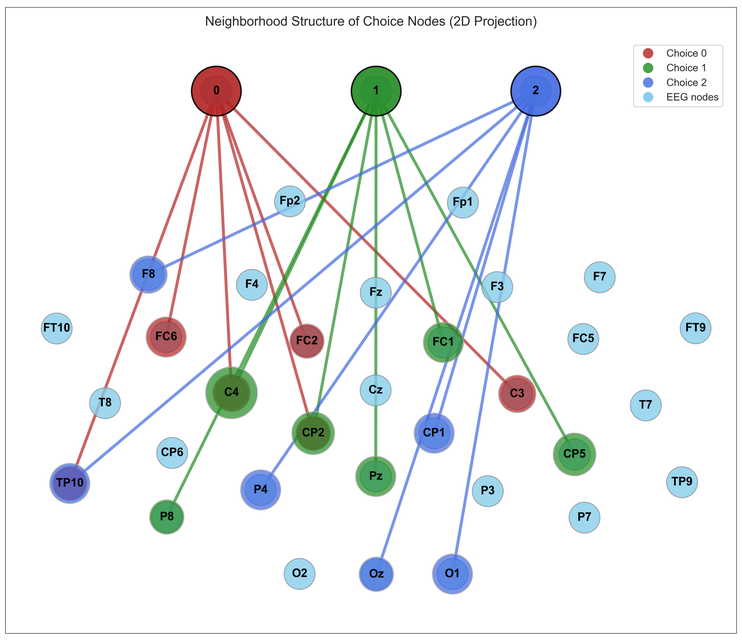

Figure: **Summary graphical model of the neighborhood of choice nodes**

Summary graphical model

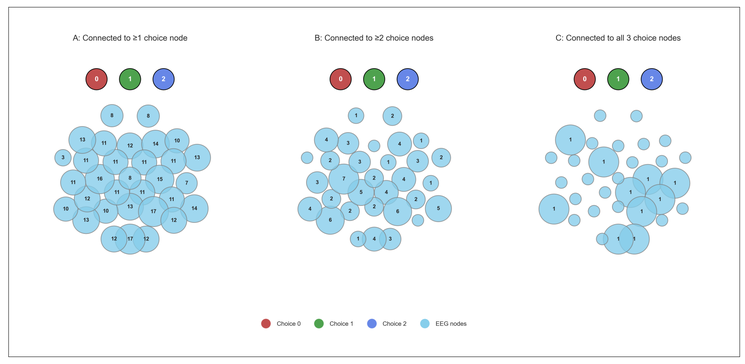

Figure: **Summary graphical models of EEG-choice node connectivity across subjects.** Each panel shows EEG nodes (blue circles) with sizes proportional to the number of subjects exhibiting connections to choice nodes (red=0, green=1, blue=2). **(A)** Nodes connected to at least one choice node, **(B)** nodes connected to at least two choice nodes, **(C)** nodes connected to all three choice nodes. The numbers on EEG nodes indicate how many subjects showed each connectivity pattern, revealing consistent patterns of functional connectivity between EEG channels and decision choices in the Goalkeeper Game task.

Results

Table: Classification accuracy comparison: MRF algorithm vs. random chance. Positive differences indicate better MRF performance.

Subj	MRF	Random	Diff
1	0.392	0.377	0.015
2	0.422	0.389	0.034
3	0.381	0.380	0.000
4	0.331	0.367	-0.036
5	0.394	0.394	0.000
6	0.397	0.375	0.022
7	0.378	0.356	0.021
8	0.389	0.373	0.015
9	0.406	0.363	0.042
10	0.358	0.347	0.011
11	0.414	0.363	0.051
12	0.353	0.355	-0.002
13	0.408	0.366	0.043
14	0.372	0.368	0.004
15	0.428	0.382	0.046
16	0.461	0.387	0.074
17	0.342	0.372	-0.030
18	0.408	0.356	0.052
19	0.364	0.391	-0.028
20	0.389	0.372	0.017
21	0.417	0.365	0.052
22	0.347	0.371	-0.024
23	0.361	0.340	0.021
24	0.403	0.383	0.020
25	0.350	0.350	0.000
26	0.358	0.386	-0.027

Statistical Significance: $t(29) = 2.65$, $p < 0.01$. Accuracy differences where significantly greater than zero.

Conclusion

- The MRF approach identifies EEG patterns that predict participants' choices, even with a trait that typically does not discriminate between different motor actions (alpha-mu band activity).
- Preliminary data shows connectivity patterns yet to be explored.
- Future work will refine the analysis technique, possibly leading to better prediction accuracies. Combining characteristics from different bands and time intervals has the potential to improve accuracy.

References

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